Dual MPC for quasi-Linear Parameter Varying systems

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Abstract— **We present a dual Model Predictive Control (MPC) framework for the simultaneous identification and control of quasi-Linear Parameter Varying (qLPV) systems. The framework is composed of an online estimator for the states and parameters of the qLPV system, and a controller that leverages the estimated model to compute inputs with a dual purpose: tracking a reference output while actively exciting the system to enhance parameter estimation. The core of this approach is a robust tube-based MPC scheme that exploits recent developments in polytopic geometry to guarantee recursive feasibility and stability in spite of model uncertainty. The effectiveness of the framework in achieving improved tracking performance while identifying a model of the system is demonstrated through a numerical example.**

I. Introduction

The closed-loop performance of a Model Predictive Control (MPC) scheme relies heavily on the quality of the model used. While robust MPC can be used to deal with minor prediction inaccuracies [1], its conservativeness grows when the dynamical models are highly nonlinear and identified directly from data [2]. This can be ameliorated by adapting the model online $-$ such MPC schemes [3] combine parameter estimation and MPC schemes whose stability holds with model changes. Passive adaptation can be enhanced by actively generating control inputs that also facilitate parameter adaptation $-$ This principle forms the core of dual control [4]–[6]. For linear systems with input-state measurements, this is generally achieved using persistently exciting inputs [7]–[9], or minimizing the prediction uncertainty [10]. For nonlinear systems with input-state measurements, bounded-uncertainty estimators along with active exploration strategies are used to shrink parameter uncertainty sets [11]–[15].

When restricted to input-output measurements, the dual-control problem becomes markedly more challenging [16]. A prevalent strategy involves using an Extended Kalman Filter (EKF) to perform joint state-parameter mean and variance estimation along, which are propagated within the MPC framework to minimize uncertainty alongside the control objective, and ensure probabilistic constraint satisfaction [6], [17]–[19]. Propagating this uncertainty however yields a nonlinear nonconvex problem to solve online. Furthermore, these schemes typically lack stability guarantees in the presence of online parameter variations.

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Contribution. We present an approach to synthesize dual MPC schemes for quasi-Linear Parameter Varying (qLPV) systems, in which the scheduling function is restricted to the simplex using a $\mathrm{softmax}$ operator. This parameterization effectively models complex nonlinear dynamics [20], while being suitable for the synthesis of robust MPC schemes since the nonlinearity is bounded by construction. We use a constrained estimator to estimate the state and parameters, with the constraints designed to ensure that the tube-based robust MPC (TMPC) scheme remains feasible. We solve the dual MPC problem in two stages, with the first stage computing a *tracking* control input to drive the model to a neighborhood of an output reference, and the second stage a *perturbing* input to promote active exploration. The bounds on this perturbation are user-specified, which is a standard exploration-exploitation tradeoff in dual MPC. To develop the TMPC scheme, we extend the approach of [21], [22] to compute configuration-constrained polytopic tubes [23]. We demonstrate that the closed-loop is input-to-state (ISS) Lyapunov stable [24] against parameter variations, and validate its effectiveness using a simple numerical example.

Notation. We denote by $\mathbb{N}$ the set of natural numbers. For $n \in \mathbb{N}$, we define $\Delta_n := \{x \in \mathbb{R}^n | x \in [0,1], \sum_{i=1}^n x_i = 1\}$ as the simplex set. We denote by $\mathbb{I}_n^m$ the set of integers between n and m, and $\mathbf{I}_n$ as the identity matrix in $\mathbb{R}^{n\times n}$. We define $\mathrm{softmax}(x) : \mathbb{R}^n \to \mathbb{R}^n$ as the function with components $e^{x_i} / \sum_{j=1}^n e^{x_j}$ for $i \in \mathbb{I}_1^n$. Given a vector $a \in \mathbb{R}^n$, $|a| \in \mathbb{R}^n$ denotes the element-wise absolute value vector. Given compact convex sets $M_1, \cdots, M_N \subseteq \mathbb{R}^n$, we define $\mathrm{CH}\{M_i, i \in \mathbb{I}_1^N\} := \left\{\sum_{i=1}^N \lambda_i x_i \in \mathbb{R}^n \middle| x_i \in M_i, \lambda \in \Delta_N\right\}$ as the convex hull of the individual sets, and $M_i \oplus M_j \subseteq \mathbb{R}^n$ as the Minkowski sum of the sets M_i and M_j. If $M_i = \{x\}$, i.e., its a singleton, then we denote the sum as $x \oplus M_j$. Boldface vectors represent optimization variables unless specified otherwise.

II. Problem Setup

We want to control the nonlinear discrete-time system

$$\mathbf{z}^+ = \mathbf{f}(\mathbf{z}, u), \qquad y = \mathbf{g}(\mathbf{z}) \tag{1}$$

where $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$ and $\mathbf{z} \in \mathbb{R}^{n_\mathbf{z}}$ are the input, output, and state vectors at time $t \in \mathbb{N}$, respectively, and $\mathbf{z}^+$ denotes the state at the next time step, under the following setting: (a) The functions $\mathbf{f}$ and $\mathbf{g}$, the state dimension $n_\mathbf{z}$ are unknown; (b) $\mathbf{f}$ and $\mathbf{g}$ are continuously differentiable with $0 = \mathbf{f}(0,0)$ and $0 = \mathbf{g}(0)$; (c) (1) is subject to constraints $u \in \mathbb{U} := \{u : |u| \leq \epsilon^u\}$ and $y \in \mathbb{Y}$, ; (d) Only the output y is measurable.

Under these assumptions, we design an output-reference tracking controller within a *dual-control* framework to simultaneously identify and control the plant (1). Our scheme comprises: (a) a dynamical model with state $x \in \mathbb{R}^{n_x}$ and parameters $\theta \in \mathbb{R}^{n_\theta}$; (b) a dual-objective MPC computing an input $u_t \in \mathbb{U}$ that balances tracking with active exploration; and (c) a state and parameter estimator utilizing the plant output y_{t+1}. We endow this closed-loop scheme with recursive feasibility and asymptotic stability guarantees using modern computational tools from polytopic geometry.

III. Modelling

We model the dynamics of (1) using the qLPV system

$$x^+ = A(p(x,u))x + B(p(x,u))u, \qquad \hat{y} = Cx, \quad (2)$$

where $x \in \mathbb{R}^{n_x}$, $C \in \mathbb{R}^{n_y \times n_x}$, and the matrix-valued functions $(A(p), B(p))$ are parameterized as

$$A(p) = \sum_{i=1}^{n_p} p_i A_i, \qquad B(p) = \sum_{i=1}^{n_p} p_i B_i, \quad (3)$$

with $A_i \in \mathbb{R}^{n_x \times n_x}$ and $B_i \in \mathbb{R}^{n_x \times n_u}$. We parameterize the scheduling function $p : \mathbb{R}^{n_x+n_u} \to \mathbb{R}^{n_p}$ as

$$p(x,u) = \mathrm{softmax}(\mathcal{N}(x,u)), \quad (4)$$

where $\mathcal{N} : \mathbb{R}^{n_x+n_u} \to \mathbb{R}^{n_p}$ is a feedforward neural network (FNN) with continuously differentiable activation units. The parameterization in (4) results in $p(x,u) \in \Delta_{n_p}$, and is hence bounded by construction. We denote by $\theta \in \mathbb{R}^{n_\theta}$ the parameters of (2), which include the system matrices $\{A_i, B_i\}$ and FNN parameters, and assume instead that C is fixed and is a full row-rank matrix, such as a collection of n_y rows of the identity matrix of order n_x, with $n_y \leq n_x$.

A. Combined state and parameter estimation

We recall the general approach to estimate (x,θ) of (2) using input-output measurements from the plant. Denoting $f(x,u,\theta) := A(p(x,u))x + B(p(x,u))u$, we introduce

$$\zeta^+ = F(\zeta,u) + w, \qquad y = \tilde{C}\zeta + v, \quad (5)$$

where $\zeta = (x,\theta)$ is the augmented state with dynamics $F(\zeta,u) = (f(x,u,\theta),\theta)$ and process noise $w \in \mathbb{R}^{n_x+n_\theta}$, $\tilde{C} = [C \ 0]$, and output noise $v \in \mathbb{R}^{n_y}$. Using measurements $u_{0:t} := (u_0,\cdots,u_t)$, $y_{0:t+1} := (y_0,\cdots,y_{t+1})$, an observer estimates the state and parameter of (2) at time $t+1$ as

$$\hat{\zeta}_{t+1} = \hat{F}(u_{0:t}, y_{0:t+1}). \quad (6)$$

For an EKF, the noises are assumed to satisfy $w \sim \mathcal{N}(0, Q_\mathrm{e})$ and $v \sim \mathcal{N}(0, R_\mathrm{e})$. Then, if $\zeta_t \sim \mathcal{N}(\hat{\zeta}_t, \hat{P}_t)$, the mean and covariance of the state of (5) are updated as

$$\tilde{\zeta}_{t+1} = F(\hat{\zeta}_t, u_t), \quad (7a)$$

$$\tilde{P}_{t+1} = \nabla_\zeta F(\hat{\zeta}_t,u_t)\hat{P}_t \nabla_\zeta F(\hat{\zeta}_t,u_t)^\top + Q_\mathrm{e}, \quad (7b)$$

$$K_{t+1} = \tilde{P}_{t+1}\tilde{C}^\top(\tilde{C}\tilde{P}_{t+1}\tilde{C}^\top + R_\mathrm{e})^{-1}, \quad (7c)$$

$$\hat{\zeta}_{t+1} = \tilde{\zeta}_{t+1} + K_{t+1}(y_{t+1} - \tilde{C}\tilde{\zeta}_{t+1}), \quad (7d)$$

$$\hat{P}_{t+1} = (\mathbf{I}_{n_x+n_\theta} - K_{t+1}\tilde{C})\tilde{P}_{t+1}. \quad (7e)$$

Alternative approaches, e.g., the Moving Horizon Estimator (MHE) [25], [26] can be used to model (6). We assume that given a constraint set $\Theta \subseteq \mathbb{R}^{n_x+n_\theta}$, estimator (6) satisfies

$$\hat{\zeta}_{t+1} \in \Theta. \quad (8)$$

While the MHE can explicitly handle such constraints, the EKF can be modified [27], [28] to handle them by modifying the correction step in (7d) as

$$\hat{\zeta}_{t+1} = \arg\min_{\zeta \in \Theta} \|\zeta - \tilde{\zeta}_{t+1}\|^2_{\tilde{P}^{-1}_{t+1}} + \|y_{t+1} - \tilde{C}\zeta\|^2_{R_\mathrm{e}^{-1}}. \quad (9)$$

In the next section, we formulate the constraint set Θ to guarantee stability of the proposed dual-control scheme.

IV. Dual MPC

Towards developing a dual-control framework, we first formulate the MPC problem:

$$\begin{aligned}
\min_{\mathbf{v}, v^\mathrm{s}} \ & \sum_{k=0}^{N-1} \left\| \begin{bmatrix} z_k - z^\mathrm{s} \\ v_k - v^\mathrm{s} \end{bmatrix} \right\|^2_Q + \left\| \begin{bmatrix} z_N - z^\mathrm{s} \\ v_N - v^\mathrm{s} \end{bmatrix} \right\|^2_P + \ell(v^\mathrm{s}, y^\mathrm{r}) + \alpha a(\mathbf{v}) \\
\text{s.t. } & z_{k+1} = f(z_k, v_k, \hat{\theta}), \ z^\mathrm{s} = f(z^\mathrm{s}, v^\mathrm{s}, \hat{\theta}), \\
& Cz_k \in \mathbb{Y}, \ v_k \in \mathbb{U}, \ k \in \mathbb{I}_0^{N-1}, \\
& Cz^\mathrm{s} \in \mathbb{Y}, \ v^\mathrm{s} \in \mathbb{U}, \ z_0 = \hat{x}, \ z_N \in \mathcal{O}(v^\mathrm{s}, \hat{\theta}),
\end{aligned} \quad (10)$$

which depends on the state $\hat{x}$ and parameters $\hat{\theta}$ of (2) estimated using (6), and the output reference $y^\mathrm{r} \in \mathbb{R}^{n_y}$. The optimization vector is the input sequence $\mathbf{v} = (v_0, \cdots, v_N)$. Problem (10) is formulated using an *artificial reference* approach [29], in which the steady-state z^s corresponding to an input v^s is optimized online based on y^r through $\ell(v^\mathrm{s}, y^\mathrm{r})$, such as $\ell(v^\mathrm{s}, y^\mathrm{r}) = \|y^\mathrm{r} - Cz^\mathrm{s}\|_2^2$. The function $a(\mathbf{v})$ is the *active exploration* objective, e.g., as in [30], weighted with $\alpha \geq 0$ against tracking. The cost matrices $Q, P \succeq 0$, along with the terminal set $\mathcal{O}(v^\mathrm{s}, \hat{\theta})$ with terminal control input v_N ensure recursive feasibility and stability of the scheme with $u_t = v_0^\star(\hat{x}_t, \hat{\theta}_t, y_t^\mathrm{r})$, the first input of the parametric optimizer. Unfortunately, solving Problem (10) entails: (a) Using NLP solvers; (b) Loss in stability guarantees under parameter adaptation. We now present a TMPC approximation of (10) that addresses these challenges.

Remark 1: Problem (10) is formulated using a *certainty-equivalence* approach, e.g. , [10]. While this formulation does not account for the effect of unmodeled dynamics on output constraint satisfaction, it serves as a theoretical baseline. In practice, uncertainties are typically addressed via constraint tightening [19], [31]–[33]. Rather than removing output constraints to trivially guarantee recursive feasibility, we retain them. This establishes the stability properties of the dual-control scheme, and provides the framework necessary for integration with constraint-tightening techniques.

A. Tube-based MPC

We construct our TMPC scheme based on the following observations and modifications:

Perturbed input: If the initial model $\hat{\theta}_0$ is *good enough*, a practical implementation of (10) would require a small $\alpha \geq 0$. Under sufficient smoothness of the dynamics and cost functions, the optimizer of (10) is continuous in α. Consequently, for small α, it lies in a neighborhood of the pure-tracking solution, i.e., with $\alpha = 0$. Motivated by this intuition, we propose a two-stage strategy. Given a tuning parameter $\beta \in [0, 1)$, we split the input as $u = u^{\mathrm{c}} + u^{\mathrm{p}}$ with $u^{\mathrm{c}} \in (1-\beta)\mathbb{U}$ and $u^{\mathrm{p}} \in \beta\mathbb{U}$. First, we design a robust controller to compute u^{c} using the uncertain model

$$x^+ \in A(p(x,u))x + B(p(x,u))u^{\mathrm{c}} \oplus W, \tag{11}$$

where the additive disturbance W is defined as

$$w \in W := \mathrm{CH}\left\{\beta B_i \mathbb{U}, i \in \mathbb{I}_1^{n_p}\right\}. \tag{12}$$

Then, we utilize active exploration to compute perturbation $u^{\mathrm{p}} \in \beta\mathbb{U}$, and apply the input $u = u^{\mathrm{c}} + u^{\mathrm{p}}$ to the plant. The effect introducing this two-stage approach on optimality is left for future study.

Model encapsulation: From (3) and (4), observe that from any $(x,u) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_u}$, propagated state x^+ satisfies

$$x^+ \in \mathrm{CH}\left\{A_i x + B_i u^{\mathrm{c}} \oplus W, i \in \mathbb{I}_1^{n_p}\right\} \tag{13}$$

as per (11) since $p(x,u) \in \Delta_{n_p}$. Then, a *tube* constructed for (13) will encapsulate the set-valued trajectories of (11). A tube for (13) is a sequence of sets $\{X_0, \cdots, X_M\} \subseteq \mathbb{R}^{n_x}$ satisfying $A_i x + B_i u^{\mathrm{c}} \oplus W \subseteq X_{t+1}$ for all $x \in X_t$ and $i \in \mathbb{I}_1^{n_p}$ with some $u^{\mathrm{c}} \in (1-\beta)\mathbb{U}$. This implies that if $x_0 \in X_0$, then there exist inputs $\{u_0^{\mathrm{c}}, \cdots, u_{M-1}^{\mathrm{c}}\} \in (1-\beta)\mathbb{U}$ such that the resulting state trajectory of (11) satisfies $x_t \in X_t$ for $t \in \{1, \cdots, M\}$ for any $\{w_0, \cdots, w_{M-1}\} \in W$. TMPC schemes construct feasible tubes that converge to a *robust control invariant* (RCI) set $X^{\mathrm{s}} \subseteq \mathbb{R}^{n_x}$ that satisfies

$$\begin{aligned} &\forall x \in X^{\mathrm{s}},\ \exists u^{\mathrm{c}} \in (1-\beta)\mathbb{U}: \\ &\qquad CX^{\mathrm{s}} \subseteq \mathbb{Y},\ A_i x + B_i u^{\mathrm{c}} \oplus W \subseteq X^{\mathrm{s}}, \quad \forall\, i \in \mathbb{I}_1^{n_p}. \end{aligned} \tag{14}$$

B. Configuration-constrained TMPC

We develop our TMPC scheme using polytopes

$$X \leftarrow X(z,s) := z \oplus \{x \in \mathbb{R}^{n_x} | Fx \leq s\}, \tag{15}$$

where $F \in \mathbb{R}^{\mathsf{f} \times n_x}$ is fixed, and the center-offset vector $(z,s) \in \mathbb{R}^{n_x + \mathsf{f}}$ is optimized online. We enforce configuration constraints [23] on s, $\mathcal{E} := \{s \geq 0 | Es \leq 0\}$ to preserve the combinatorial structure of $X(z,s)$. Then, there exist matrices $V := \{V_j \in \mathbb{R}^{n_x \times \mathsf{f}}, j \in \mathbb{I}_1^{\mathsf{v}}\}$ such that for any $z \in \mathbb{R}^{n_x}$,

$$s \in \mathcal{E} \Rightarrow X(z,s) = \mathrm{CH}\{z + V_j s, j \in \mathbb{I}_1^{\mathsf{v}}\}. \tag{16}$$

Thus, the set $\mathcal{E}$ thus enables a linear parameterization of the halfspace and vertex representations of $X(z,s)$.

Proposition 1: The polytope $X^{\mathrm{s}} = X(z^{\mathrm{s}}, s)$ satisfies the RCI condition in (14) if there exist vectors $v^{\mathrm{s}} \in \mathbb{R}^{n_u}$, $c \in \mathbb{R}^{\mathsf{v} n_u}$ and $q \in \mathbb{R}^{\mathsf{f}}$ verifying the inequalities

$$F(A_i(z^{\mathrm{s}} + V_j s) + B_i(v^{\mathrm{s}} + U_j c)) + d + q \leq s + Fz^{\mathrm{s}}, \tag{17a}$$

$$C(z^{\mathrm{s}} + V_j s) \in \mathbb{Y},\ v^{\mathrm{s}} + U_j c \in (1-\beta)\mathbb{U},\ q \geq 0, \tag{17b}$$

$$s \in \mathcal{E},\ \forall (i,j) \in \mathbb{I}_1^{n_p} \times \mathbb{I}_1^{\mathsf{v}}, \tag{17c}$$

where the disturbance vector $d := \max\{\beta|FB_i|\epsilon^u, i \in \mathbb{I}_1^{n_p}\}$, and matrices $U_j := e_j \otimes \mathbb{I}_{\mathsf{v}} \in \mathbb{R}^{n_u \times \mathsf{v} n_u}$.

Proof: The result follows from [23, Corollary 4], where $c = (c_1, \cdots, c_{\mathsf{v}})$ stacks the vertex control inputs, and (17) contains an additional disturbance vector $q \geq 0$. ■

Given parameters $\hat{\theta}$ estimated by (6) and reference $y^{\mathrm{r}} \in \mathbb{R}^{n_y}$, the optimal RCI set is defined as $X(z_{\mathrm{o}}^{\mathrm{s}}(\hat{\theta}, y^{\mathrm{r}}), s_{\mathrm{o}}(\hat{\theta}, y^{\mathrm{r}}))$, where $(z_{\mathrm{o}}^{\mathrm{s}}, s_{\mathrm{o}})$ are part of the optimizers of the problem

$$r(\hat{\theta}, y^{\mathrm{r}}) := \min_{\mathbf{x}^{\mathrm{r}} \in (17)} \ell(\mathbf{x}^{\mathrm{r}}, y^{\mathrm{r}}) = \|y^{\mathrm{r}} - Cz^{\mathrm{s}}\|_{Q_1}^2 + \|\mathbf{x}^{\mathrm{r}}\|_{Q_2}^2 \tag{18}$$

defined over $\mathbf{x}^{\mathrm{r}} = (z^{\mathrm{s}}, v^{\mathrm{s}}, s, c, q) \in \mathbb{R}^{n_x + n_u + 2m + \mathsf{v} n_u}$. Our goal is to formulate a TMPC scheme which computes a tube $\{X(z_t, s), t \in \mathbb{N}\}$ that converges to $X(z_{\mathrm{o}}^{\mathrm{s}}(\hat{\theta}, y^{\mathrm{r}}), s_{\mathrm{o}}(\hat{\theta}, y^{\mathrm{r}}))$.

Proposition 2: Suppose $\mathbf{x}^{\mathrm{r}}$ satisfies (17). Then, a tube is formed by $\{X(z_0, s), \cdots, X(z_M, s)\}$ if there exist vectors $\{v_0, \cdots, v_{M-1}\} \in \mathbb{R}^{n_u}$ satisfying

$$\begin{aligned} &F(A_i(z_t - z^{\mathrm{s}}) + B_i(v_t - v^{\mathrm{s}})) \leq q + F(z_{t+1} - z^{\mathrm{s}}), \\ &C(z_t + V_j s) \in \mathbb{Y},\ v_t + U_j c \in (1-\beta)\mathbb{U}, \\ &\forall (i,j,t) \in \mathbb{I}_1^{n_p} \times \mathbb{I}_1^{\mathsf{v}} \times \mathbb{I}_0^{M-1}. \end{aligned} \tag{19}$$

Proof: Found in [34, Proposition 2]. ■

Following Propositions 1 and 2, we formulate our TMPC controller based on the QP

$$\begin{aligned} \min_{\mathbf{x}}\ & \sum_{k=0}^{N-1} \left\| \begin{bmatrix} z_k - z^{\mathrm{s}} \\ v_k - v^{\mathrm{s}} \end{bmatrix} \right\|_Q^2 + \left\| \begin{bmatrix} z_N - z^{\mathrm{s}} \\ v_N - v^{\mathrm{s}} \end{bmatrix} \right\|_P^2 + \ell(\mathbf{x}^{\mathrm{r}}, y^{\mathrm{r}}) \qquad (20) \\ \text{s.t. }\ & F(A_i(z_k - z^{\mathrm{s}}) + B_i(v_k - v^{\mathrm{s}})) \leq q + F(z_{k+1} - z^{\mathrm{s}}), \\ & C(z_k + V_j s) \in \mathbb{Y},\ v_k + U_j c \in (1-\beta)\mathbb{U}, \\ & F(A_i(z_N - z^{\mathrm{s}}) + B_i(v_N - v^{\mathrm{s}})) \leq q + \gamma F(z_N - z^{\mathrm{s}}), \\ & C(z_N + V_j s) \in \mathbb{Y},\ v_N + U_j c \in (1-\beta)\mathbb{U}, \\ & F\hat{x} \leq q + Fz_0,\ (17),\ (i,j,k) \in \mathbb{I}_1^{n_p} \times \mathbb{I}_1^{\mathsf{v}} \times \mathbb{I}_0^{N-1}, \end{aligned}$$

where $\gamma \in (0,1)$ is some user-specified constant. The QP is parametric in state-parameter $(\hat{x}, \hat{\theta})$ of (2) estimated using (6), along with reference y^{r}. The optimization vector is $\mathbf{x} = (z_0, v_0, \cdots, z_N, v_N, \mathbf{x}^{\mathrm{r}})$. Denoting $\mathbf{x}^\star(\hat{x}, \hat{\theta}, y^{\mathrm{r}})$ as the optimizer of (20), the closed-loop scheme is defined as

$$u_t = v_0^\star(\hat{x}_t, \hat{\theta}_t, y_t^{\mathrm{r}}) + \sum_{j=1}^{\mathsf{v}} \lambda_j c_j^\star(\hat{x}_t, \hat{\theta}_t, y_t^{\mathrm{r}}) + u_t^{\mathrm{p}}, \tag{21a}$$

$$(\hat{x}_{t+1}, \hat{\theta}_{t+1}) \text{ updated using (6) subject to (8),} \tag{21b}$$

for some $u_t^{\mathrm{p}} \in \beta\mathbb{U}$ computed using an active exploration criterion, where $\lambda \in \mathbb{R}^{\mathsf{v}}$ is computed as

$$\min_{\lambda \in \Delta_{\mathsf{v}}} \|\lambda\|_2^2 \ \text{ s.t. }\ \hat{x} = z_0^\star + \sum_{j=1}^{\mathsf{v}} \lambda_j V_j s^\star. \tag{22}$$

C. Recursive feasibility and stability

To guarantee recursive feasibility when $(\hat{x}, \hat{\theta})$ are updated via (6), we restrict the update step in (8). With the optimizer $\mathbf{x}$ of (20) we define a polytope $\Theta(\mathbf{x}, d)$ as

$$\Theta(\mathbf{x}, d) := \left\{ \begin{pmatrix} x \\ \theta \end{pmatrix} \middle| \begin{array}{l} Fx \leq q + Fz_1,\ \beta|FB_i|\epsilon^u \leq d,\ (17\text{a}) \\ F(A_i(z_k - z^{\mathrm{s}}) + B_i(v_k - v^{\mathrm{s}})) \leq q + F(z_{k+1} - z^{\mathrm{s}}), \\ F(A_i(z_N - z^{\mathrm{s}}) + B_i(v_N - v^{\mathrm{s}}) \leq q + F(z^+ - z^{\mathrm{s}}), \\ F(A_i(z^+ - z^{\mathrm{s}}) + B_i(v^+ - v^{\mathrm{s}}) \leq q + \gamma F(z^+ - z^{\mathrm{s}}), \\ \forall\ (i,j,k) \in \mathbb{I}_1^{n_p} \times \mathbb{I}_1^{\mathsf{v}} \times \mathbb{I}_1^{N-1} \end{array} \right\}$$

where z^+, v^+ are the shifted variables

$$z^+ = z^{\rm s} + \gamma(z_N - z^{\rm s}),\ v^+ = v^{\rm s} + \gamma(v_N - v^{\rm s}). \tag{23}$$

Lemma 1: Suppose Problem (20) is feasible with initial state-parameter estimate $(\hat{x}_0, \hat{\theta}_0)$. Then, (20) is feasible for all $t \in \mathbb{N}$ with $(\hat{x}_t, \hat{\theta}_t)$ generated by the closed-loop scheme in (21) for any reference sequence $\{y_t^{\rm r}, t \in \mathbb{N}\}$ and perturbation sequence $\{u_t^{\rm p} \in \beta\mathbb{U}, t \in \mathbb{N}\}$ when (8) is defined as

$$(\hat{x}_{t+1}, \hat{\theta}_{t+1}) \in \Theta(\mathbf{x}^\star(\hat{x}_t, \hat{\theta}_t, y_t^{\rm r}), \max\{\beta|FB_i|\epsilon^u, i \in \mathbb{I}_1^{n_p}\}). \tag{24}$$

Proof: The proof follows from the vertex control law, and noticing that the estimator constraints are the same as the MPC problem. Full proof found in [34, Lemma 1]. ■

In the following result on stability, we assume a constant $y_t^{\rm r} \equiv y^{\rm r}$, $\forall t \in \mathbb{N}$, and hence drop its time dependence.

Theorem 2: Suppose Problem (20) is feasible with initial state and parameter estimate $(\hat{x}_0, \hat{\theta}_0)$, and (8) is such that (24) holds. Suppose further that matrices $Q, P \succ 0$ are chosen such that $P \succeq (1-\gamma^2)^{-1}Q$, and denote the optimal value of (20) as $\mathcal{C}(\hat{x}, \hat{\theta})$. Then, $\mathcal{L}(\hat{x}, \hat{\theta}) := \mathcal{C}(\hat{x}, \hat{\theta}) - r(\hat{\theta})$ serves as an ISS-Lyapunov function for (21) with respect to the optimal RCI set against $\|\hat{\theta}_{t+1} - \hat{\theta}_t\|$ irrespective of the perturbation sequence $\{u_t^{\rm p} \in \beta\mathbb{U}, t \in \mathbb{N}\}$ in (21a), where $r(\hat{\theta})$ is the optimal RCI cost from (18).

Proof: Denoting $m_k = (z_k^\star - z^{\rm s*}, v_k^\star - v^{\rm s*})$, from Lemma 1 and hypothesis on (Q, P) it follows that

$$\mathcal{L}(\hat{x}_{t+1}, \hat{\theta}_{t+1}) - \mathcal{L}(\hat{x}_t, \hat{\theta}_t) \leq -\|m_0\|_Q^2 + r(\hat{\theta}_{t+1}) - r(\hat{\theta}_t). \tag{25}$$

The proof then concludes using Lipschitz continuity of $r(\cdot)$. Full proof found in [34, Theorem 2]. ■

If (6) gives bounded updates $\|\hat{\theta}_{t+1} - \hat{\theta}_t\| < \infty$, then the cost increase is bounded. For the EKF (7), this occurs if the covariance matrix $\hat{P}_t$ remains bounded, which is expected when the linearized system is uniformly observable [35]. Estimator convergence for the qLPV model class in the presence of constraints (24) is left for future research.

D. Active exploration

From Lemma 1, we know that for any perturbation input $\{u_t^{\rm p} \in \beta\mathbb{U}, t \in \mathbb{N}\}$ defining the control law in (21a), (20) remains feasible. Accordingly, we define the problem

$$\max_{\mathbf{u}}\ a(\mathbf{u}) \text{ s.t.} \begin{cases} x_{k+1} = f(x_k, u_k, \hat{\theta}),\ x_0 = \hat{x} \\ x_{k+1} \in X(z_{k+1}^\star, s^\star),\ u_k \in \mathbb{U}, k \in \mathbb{I}_0^{N^{\rm p}-1} \end{cases} \tag{26}$$

parametric in $(\hat{x}, \hat{\theta})$ from (6), along with the optimizer $\hat{\mathbf{x}}^\star = \hat{\mathbf{x}}^\star(\hat{x}, \hat{\theta}, y^{\rm r})$ of (20). For some $N^{\rm p} \in \mathbb{N}$, the tube sequence in the constraints is defined with $s^\star = s^\star(\hat{x}, \hat{\theta}, y^{\rm r})$, $z_k^\star = z_k^\star(\hat{x}, \hat{\theta}, y^{\rm r})$ if $k \in \mathbb{I}_1^N$ and $z_{k+1}^\star = \gamma z_k^\star + (1-\gamma)z^{\rm s*}(\hat{x}, \hat{\theta}, y^{\rm r})$ otherwise. The active exploration objective $a(\mathbf{u})$ is defined over $\mathbf{u} = (u_0, \cdots, u_{N^{\rm p}-1})$. Some examples include the persistence of excitation criterion, e.g., [7], [36]; Fisher information, e.g., [17]; Predicted covariance minimization, e.g., [10], [18], [37]; General nonlinear regression-type criteria, e.g., [38]. Defining $\mathbf{u}^\star(\hat{x}, \hat{\theta}, y^{\rm r})$ as the optimizer of Problem (26), the closed-loop scheme is defined as

$$u_t = \mathbf{u}_0^\star(\hat{x}_t, \hat{\theta}_t, y_t^{\rm r}), \tag{27a}$$

$$(\hat{x}_{t+1}, \hat{\theta}_{t+1}) \text{ updated using (6) subject to (24).} \tag{27b}$$

Corollary 1: Suppose that Problem (20) with the initial state and parameter estimate $(\hat{x}_0, \hat{\theta}_0)$ is feasible. Then, (20), (26) and the constrained estimator (6) subject to (24) remain recursively feasible irrespective of the reference $\{y_t^{\rm r}, t \in \mathbb{N}\}$, and ISS-Lyapunov stable if the reference is held constant.

Proof: Follows from Lemma 1 and Theorem 2. ■

Optimizing the input sequence online in (26) freely might be computationally expensive. To overcome this difficulty, we approximately solve (26) using a sample-based approach. We first sample the set of perturbation trajectories

$$\mathrm{U^p} := \{\mathrm{u}_j^{\rm p} = (u_0^{\rm p}, \cdots, u_{N^{\rm p}-1}^{\rm p}) \in \beta\mathbb{U}, j \in \mathbb{I}_1^{M^{\rm p}}\}, \tag{28}$$

and, for each $\mathrm{u^p} \in \mathrm{U^p}$, we compute the corresponding nominal input sequence as $u_k^{\rm c} = v_k^\star + \sum_{j=1}^{\mathsf{v}} \lambda_j c_j^\star$, with $v_k^\star$ defined similarly as $z_k^\star$ for $k > N$ and λ computed as in (22) with $(\hat{x}, z_0^\star) \leftarrow (x_k, z_k^\star)$, with the state x_k propagated using the input $u_k = u_k^{\rm c} + u_k^{\rm p}$. Evaluating the maximizing sequence, we apply the corresponding input to the plant.

V. Numerical example

For the purpose of numerical illustration, we consider the nonlinear mass-spring-damper system with dynamics[1]

$$\mathrm{m}_{[1]}\ddot{\mathrm{x}}_{[1]} = 10u - \mathrm{k^s}(\mathrm{x}_{[1]}) - \mathrm{k^d}(\dot{\mathrm{x}}_{[1]}) - \mathrm{k^s}(\delta\mathrm{x}) - \mathrm{k^d}(\dot{\delta\mathrm{x}}),$$
$$\mathrm{m}_{[2]}\ddot{\mathrm{x}}_{[2]} = -\mathrm{k^s}(\mathrm{x}_{[2]}) - \mathrm{k^d}(\mathrm{x}_{[2]}) + \mathrm{k^s}(\delta\mathrm{x}) + \mathrm{k^d}(\dot{\delta\mathrm{x}}),$$

where $(\mathrm{x}_{[1]}, \mathrm{x}_{[2]})$ are the positions of the masses. The input u is the force applied on the first mass, and the output is the position of the second mass. Denoting $\delta\mathrm{x} = \mathrm{x}_{[1]} - \mathrm{x}_{[2]}$, the reaction forces are $\mathrm{k^s}(x) = \mathrm{a}x + \mathrm{b}x^3$ and $\mathrm{k^d}(v) = \mathrm{d}v + \mathrm{e}\cdot\tanh(v/\mathrm{v}_0)$ respectively. The plant parameters are $(m_{[1]}, m_{[2]}, \mathrm{a}, \mathrm{b}, \mathrm{d}, \mathrm{e}, \mathrm{v}_0) = (0.1, 0.01, 1, 1, 0.5, 0.5, 0.01)$ in appropriate units. We obtain (1) by discretizing this system, with integration performed using the Runge-Kutta integrator (Tsit5) from the Diffrax library [39], with a time-step of 0.02s using inputs $u \in \mathbb{U} = [-1, 1]$. While this system is chosen for illustrative purposes with a nonlinear system, applying the methodology to more complex systems and comparing with other dual MPC schemes is a subject of future study. We parameterize (2) with $n_x = 2$, $n_p = 3$, and the FNN in (4) with a single hidden layer containing 3 swish activation units, such that $n_\theta = 42$, and set $C = [1\ 0]$. For the TMPC controller, we set $F = [\mathbf{I}_{n_x}\ \ -\mathbf{I}_{n_x}]^\top$, resulting in $\mathsf{f} = \mathsf{v} = 4$, and compute the matrix $E \in \mathbb{R}^{8\times\mathsf{f}}$ using [23]. We tune the tracking costs as $Q = \mathbb{I}_{n_x+n_u}$ and $P = Q/(1-\gamma^2)$, and the optimal RCI set costs as $Q_1 = 10$ and $Q_2 = \mathrm{blkdiag}(10^{-6}\mathbb{I}_{n_x+n_u}, 10\mathbb{I}_{\mathsf{f}}, \mathbb{I}_{\mathsf{v}n_u}, \mathbb{I}_{\mathsf{f}})$.

For our simulations, we initialize the model and plant at the origin, and compute $\hat{\theta}_0$ following [20] using an input-output dataset $\mathcal{D}_{\rm train}$ of length $T = 100$ points such that

[1]Code to reproduce the results is found on `https://github.com/samku/DMPC_qLPV`

(20) is feasible at $(\hat{x}_0, \hat{\theta}_0)$ with $\beta = 0.3, \gamma = 0.95$, $N = 2$. Evaluating the model quality using the mean-squared error (MSE) loss $\ell_2 = \sum_{t=0}^{T-1} \|y_t - \hat{y}_t\|_2^2/T$, $\hat{\theta}_0$ achieves MSE = 0.003 over the training dataset. We also measure another dataset $\mathcal{D}_{\text{test}}$ of length 10000 points, over which $\hat{\theta}_0$ achieves MSE = 0.198. We scale the plant output by 2.5 to aid in learning. In the test set, the output tightly belongs in $[-1.487, 1.329]$. We define $\mathbb{Y} = [-0.8, 0.8]$, such that $\mathbb{Y}$ is smaller than the reachable set. We use the EKF in (7)-(9) as the estimator in (6) with $Q_{\text{e}} = \text{blkdiag}(10^{-6}\mathbf{I}_{n_x}, 0\mathbf{I}_{n_\theta})$, $R_{\text{e}} = 0.1$ and $\hat{P}_0 = \text{blkdiag}(\mathbf{I}_{n_x}, 10^{-4}\mathbf{I}_{n_\theta})$. A small value of $\hat{P}_{0,\theta\theta}$ avoids large variations $\|\hat{\theta}_{t+1} - \hat{\theta}_t\|$ during simulation. As the active exploration criterion in Problem (26), we adopt the persistence-of-excitation condition for linear systems, given using the minimal eigenvalue of the input gram matrix

$$a(\mathbf{u}) = \min \text{eigval} \left(\sum_{j=t-T}^{t} u_{j:j+N} u_{j:j+N}^T \right), \tag{29}$$

from [7], where $T \in \mathbb{N}$ is the horizon of past inputs, and $u_{j:j+N} = (u_j, \cdots, u_{j+N})$ is the vector formed using the input sequence. We select $T = 10$ in our simulations. Alternative criteria designed for nonlinear systems might result in improved performance. We solve (26) with $N^{\text{p}} = N$ through enumeration using $M^{\text{p}} = 20$ samples defining (28).

In Figure 1 (top), we demonstrate the performance of the closed-loop scheme (27), in which y_t^{r} is piecewise constant varying between 0.8 and -0.8. The dull lines indicate model output $\hat{y}$, which is typically different from the plant output y shown in bold colors. By "No adapt" we indicate the case when EKF is not used to update $(\hat{x}_t, \hat{\theta}_t)$, such that $\hat{x}_{t+1} = f(\hat{x}_t, u_t, \hat{\theta}_0)$, $\forall t \in \mathbb{N}$. No benefit was observed if only state estimation is performed with fixed parameters. Instead, when (27b) is executed to perform joint estimation, the performance improves over time: both $\beta = 0$ (no additional perturbation) and $\beta = 0.3$ result in improved tracking performance, with $\beta = 0.3$ giving faster convergence indicated by $y \to \hat{y}$ earlier. As noted in Remark 1, we do not account for output constraint violation by the plant in the certainty-equivalent framework. However, the violation reduces as the model quality improves. In Figure 1 (middle), we see that the Lyapunov cost reduces when the reference is constant, validating Theorem 2. Exponential decrease is observed since the tracking costs dominate, because $\|\hat{\theta}_{t+1} - \hat{\theta}_t\|$ is small (Figure 1 (bottom)). In Figure 1 (middle), we plot optimality loss due to projection (24), where $\hat{\zeta}_t^{\text{u}}$ denotes the unconstrained update following (7), and the constrained update is computed following [27] as $\hat{\zeta}_t = \arg\min_{\zeta \in \Theta_t} \|\zeta - \hat{\zeta}_t^{\text{u}}\|^2_{\hat{P}_t^{-1}}$. The average computation time per step is 8 ms using the qpax QP-solver [40] implemented in jax [41] on a Windows 11 laptop with Intel Core i9-14900HX processor and 32GB of RAM.

We analyze the performance of (27) over 10 randomly generated piecewise constant reference trajectory realizations of 5000 time steps. Using the plant output, we evaluate the tracking cost as $\text{t} = \sum_{t=0}^{5000} \|y_t - y_t^{\text{r}}\|_2^2$, and constraint violation as $\text{c} = \sum_{t=0}^{5000} \left\|\max_{i=1,2}\{H_i y_t - h_i, 0\}^2\right\|_1$, where

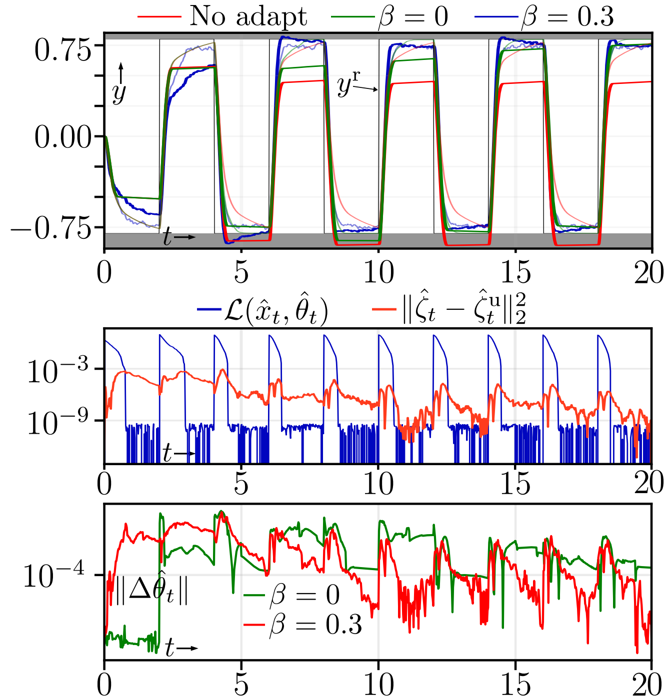

Fig. 1: *(Top) Tracking performance of* (27). *The dull red, green and blue lines indicate the output trajectory $\hat{y}_t$ of* (2), *and the gray region indicates constraints; (Middle) Lyapunov cost defined in Theorem 2, and optimality loss due to projection* (24); *(Bottom) Variation in estimated parameter $\Delta\hat{\theta}_t = \hat{\theta}_{t+1} - \hat{\theta}_t$.*

$\mathbb{Y} = \{y | Hy \leq h\}$. Setting $\hat{\theta}^\star = \hat{\theta}_{5000}$ as the identified model, we compute the MSE over the test dataset. In Table I, we present the mean and standard deviation statistics of these metrics over these runs. First, we run the closed-loop without model adaptation resulting in a high tracking cost since $\hat{\theta}_0$ is of poor quality. Adapting the model with (6), as the value of the active exploration constant β increases, the tracking performance, constraint satisfaction, and final model quality improve. For small values of β, the constraint violation is worse since the system is excited beyond the output bounds before the model converges. For $\beta = 0.3$, the fast model convergence results in the best overall tracking performance along with reduced constraint violation. We report that for $\beta > 0.3$, the model output fails to track the reference because of excessive noise in TMPC, resulting in worse performance.

VI. Conclusions

We have presented a dual MPC framework for nonlinear systems based on qLPV models, which is composed of a constrained estimator for state and parameters of the model, a TMPC controller, and a mechanism to perturb the inputs for the dual-control effect. The TMPC scheme is designed using configuration-constrained polytopes, and is guaranteed to be recursively feasible (Lemma 1) and ISS-stable (Theorem 2) against parameter updates. Through a simple numerical example, the effectiveness of the framework is demonstrated using an EKF for parameter estimation, and active exploration criterion chosen to be the persistency-of-excitation condition. Future research will focus on the incorporation

Strategy	t	c	$\ell_2^{\text{test}}(\hat{\theta}^\star)$
No adapt	389.840 ± 59.262	0.146 ± 0.208	0.198 ± 0.000
$\beta = 0$	350.037 ± 76.337	1.164 ± 1.378	0.045 ± 0.037
$\beta = 0.1$	307.916 ± 60.129	3.668 ± 3.617	0.009 ± 0.003
$\beta = 0.2$	243.600 ± 37.850	0.929 ± 1.003	0.007 ± 0.002
$\beta = 0.3$	203.995 ± 27.431	0.180 ± 0.230	0.006 ± 0.001

TABLE I: *Performance statistics (Mean and standard deviation) of* (27) *over different reference realization.*

of constraint tightening techniques to provide output constraint satisfaction guarantees, the analysis of suboptimality induced by the two-stage approach, and convergence of the constrained estimator scheme for qLPV systems.

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