DC-Servomechanism

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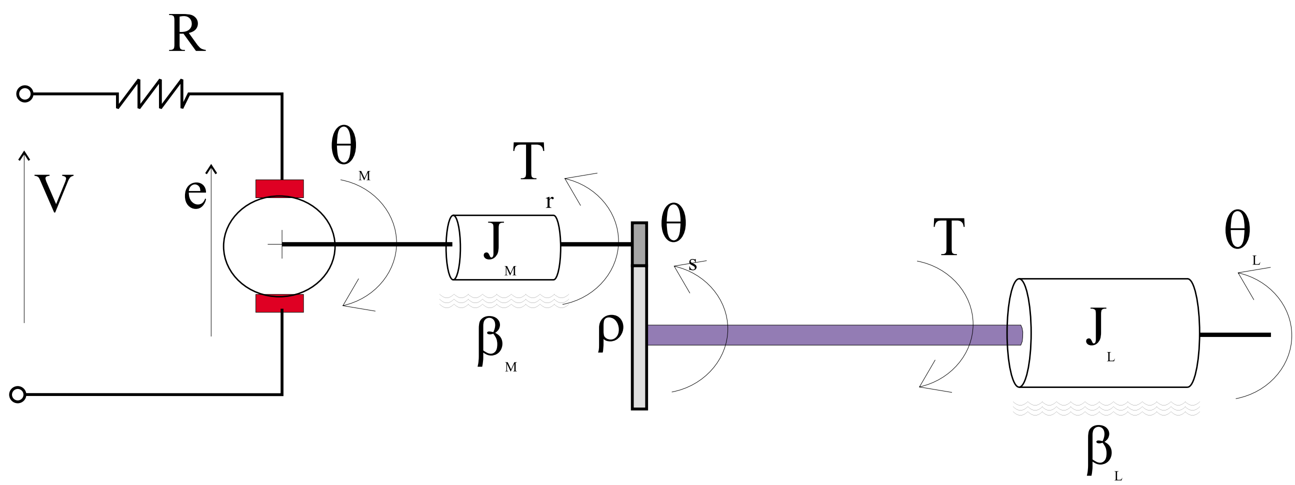

Figure 1: Servomechanism model (*modello del servomeccanismo*).

1 Servomechanism Model

The position servomechanism is schematically described in Fig. 1. It consists of a DC-motor (*motore in corrente continua*), gear-box (*scatola di riduzione*), elastic shaft (*albero di trasmissione elastico*) and load (*carico meccanico*). We derive the equations of the system, in order to obtain a model suitable for control tasks. To this aim, let us consider each physical component of the system.

1.1 DC Motor - Armature Control (*Controllo in Tensione di Armatura*)

1.

$$V = Ri + e$$

where $V =$ applied armature voltage (*tensione di armatura*), $i =$armature current (*corrente di armatura*), $R =$resistance, $e =$back e.m.f. (*forza contro-elettromotrice*)

2.

$$e = k_b \omega_M$$

where $\omega_M = \dot{\theta}_M$ =angular velocity of motor-shaft, k_b =constant

3.

$$\phi = k_f i_f$$

where ϕ =air gap flux, i_f =field current (*corrente di campo*), k_f =constant

4.

$$T_M = k_1 \phi i$$

where T_M =torque (*coppia*) developed by the motor, k_1 =constant

5. By steady-state power balance (*equilibrio della potenza in condizioni di regime*)

$$T_M \omega_M = ei$$

and therefore $k_1 k_f i_f = k_b = k_T$, k_T =motor constant (*costante del motore*)

6.

$$J_M \dot{\omega}_M = T_M - \beta_M \omega_M - T_r$$

where J_M =equivalent moment of inertia of motor, β_M =equivalent viscous friction coefficient (*coefficiente di attrito viscoso*) of motor, T_r =other torques.

In summary:

$$V = Ri + k_T \omega \quad (1)$$

$$J_M \dot{\omega}_M = k_T i - \beta_M \omega_M - T_r \quad (2)$$

1.2 Gear Box (*Scatola di Riduzione*)

Consider the gear box depicted in Fig. 2.

1. Same distance traveled by the two wheels

$$\theta_1 r_1 = \theta_2 r_2$$

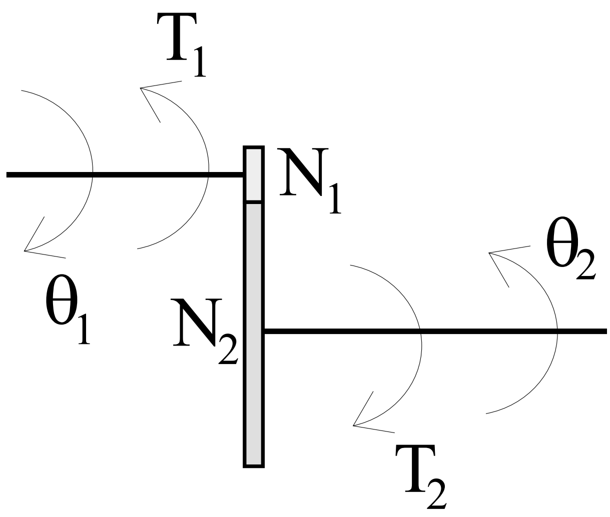

Figure 2: Gear box.

where r_1, r_2 = wheel radii (*raggi delle ruote dentate*)

2. and therefore
$$\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1} = \rho$$
where N_1, N_2 =number of teeth (*numero di denti*), ρ =gear ratio (*rapporto di riduzione*).

3. By differentiation,
$$\omega_1 = \rho\omega_2$$

4. Power transmission (no loss) (*potenza trasmessa, trascu-*

rando le perdite)

$$T_1\omega_1 = T_2\omega_2$$

and therefore

$$\frac{T_1}{T_2} = \frac{1}{\rho}$$

By referring again to Fig. 1, we have for $\theta_1 = \theta_M$, $\theta_2 = \theta_s$, $T_1 = -T_r$, $T_2 = T$

$$\theta_s = \frac{1}{\rho}\theta_M \quad (3)$$

$$T_r = -\frac{1}{\rho}T \quad (4)$$

where θ_M =angular displacement of motor shaft (*posizione angolare dell'angolo motore*), θ_s =angular displacement of load-side gear (*posizione angolare a valle della scatola di riduzione*), T =torque acting on the load (*coppia agente sul carico*).

1.3 Elastic Shaft (*Albero di trasmissione elastico*)

The shaft has finite torsional rigidity (*rigidità torsionale*) k_θ:

$$T = k_\theta(\theta_L - \theta_s) \tag{5}$$

where θ_L =angular displacement of load (*posizione angolare del carico*).

1.4 Load Dynamics (*Dinamica del Carico Meccanico*)

$$J_L \dot{\omega}_L = -\beta_L \omega_L - T \tag{6}$$

where $\omega_L = \dot{\theta}_L$ =angular load velocity (*velocità angolare del carico*), J_L =equivalent moment of inertia of load, β_L =equivalent viscous friction coefficient of load.

1.5 Differential Equations of the System

By collecting the previous relation, we can write the

$$\dot{\omega}_L = -\frac{k_\theta}{J_L}\left(\theta_L - \frac{\theta_M}{\rho}\right) - \beta_L\omega_L \tag{7}$$

$$\dot{\omega}_M = \frac{k_T}{J_M}\left(\frac{V - k_T\omega_M}{R}\right) - \frac{\beta_M\omega_M}{J_M} + \frac{k_\theta}{\rho J_M}\left(\theta_L - \frac{\theta_M}{\rho}\right) \tag{8}$$

1.6 State-Space Model (*Modello in Forma Spazio di Stato*)

By setting $x_p \triangleq [\theta_L\ \dot{\theta}_L\ \theta_M\ \dot{\theta}_M]'$, the model can be described by the following state-space form

$$\dot{x}_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_\theta}{J_L} & -\frac{\beta_L}{J_L} & \frac{k_\theta}{\rho J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_\theta}{\rho J_M} & 0 & -\frac{k_\theta}{\rho^2 J_M} & -\frac{\beta_M + k_T^2/R}{J_M} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_T}{R J_M} \end{bmatrix} V$$

$$\theta_L = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x_p$$

$$\dot{\theta}_L = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} x_p$$

$$T = \begin{bmatrix} k_\theta & 0 & -\frac{k_\theta}{\rho} & 0 \end{bmatrix} x_p$$

Since the steel shaft (*albero in acciaio*) has finite shear strength (*resistenza al taglio*), determined by a maximum admissible $\tau_{adm} = 50N/mm^2$, the torsional torque T must satisfy the constraint

$$|T| \leq 78.5398 \ Nm \tag{9}$$

Moreover, the input DC voltage V has to be constrained within the range

$$|V| \leq 220 \ V \tag{10}$$

The model is transformed in discrete time by sampling (*campionando*) every $T_s = 0.1s$ and using a zero-order holder on the input voltage.

1.7 Parameters

The parameters of the system are reported in Table 1.

2 Esercizio

1. Descrivere con uno script Matlab il modello del sistema (tempo continuo)
2. Calcolare la risposta di θ_L, $\dot{\theta}_L$ e T dovuta ad un gradino di tensione $V = 120$ V
3. Calcolare la risposta all'impulso
4. Calcolare l' evoluzione libera da condizione iniziale $\theta_L(0) = \theta_M(0) = 0$, $\dot{\theta}_L = 1$ rad/s, $\dot{\theta}_M$=20 rad/s
5. Ottenere un modello a tempo discreto del sistema (tempo di campionamento $T_s = 0.1$ s)

Table 1: Model parameters

Symbol	Value (MKS)	Meaning
L_S	1.0	shaft length
d_S	0.02	shaft diameter
J_S	negligible	shaft inertia
J_M	0.5	motor inertia
β_M	0.1	motor viscous friction coefficient
R	20	resistance of armature
k_T	10	motor constant
ρ	20	gear ratio
k_θ	1280.2	torsional rigidity
J_L	$50J_M$	nominal load inertia
β_L	25	load viscous friction coefficient
T_s	0.1	sampling time