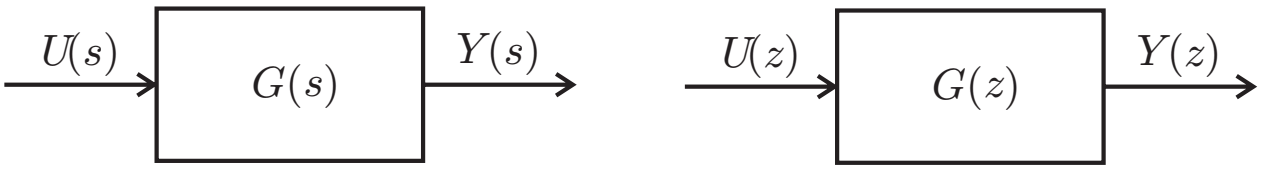
7. Diagrammi a Blocchi

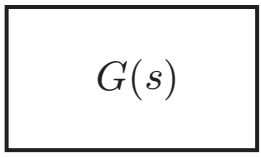
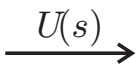
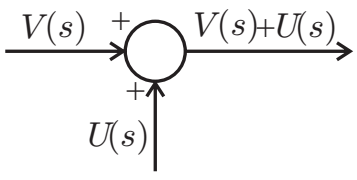
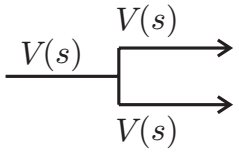
7 Diagrammi a Blocchi



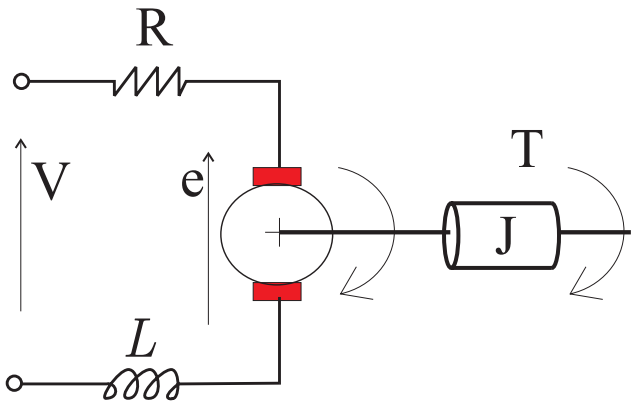
Sistema tempo continuo Sistema tempo discreto

- Esprimono le *interconnessioni* fra sistemi lineari (tempo-continuo o tempo discreto)
- Permettono di determinare in maniera diretta le funzioni di trasferimento da una qualsiasi grandezza ad un'altra
- Esprimono relazioni ingresso-uscita (risposta forzata o regime permanente): gli effetti delle condizioni iniziali non sono considerati.

Elementi fondamentali:

Blocco	
Freccia	
Sommatore	
Punto di diramazione	

Esempio



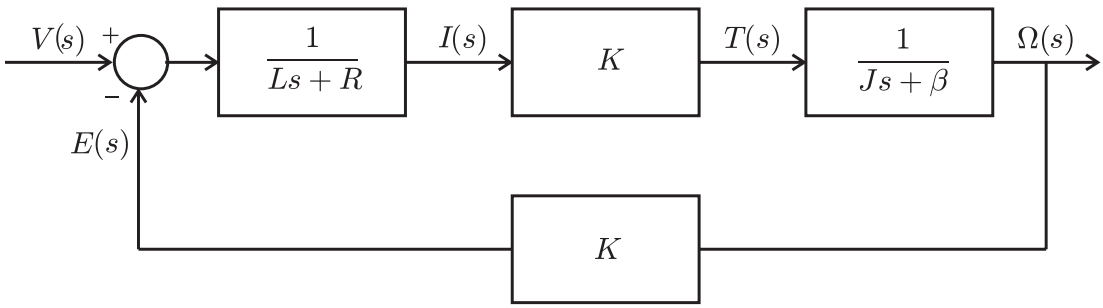
- Equazioni nel dominio del tempo:

$$\begin{cases} V - Ri - e - L\frac{di}{dt} = 0 \\ e = k\omega \\ J\frac{d\omega}{dt} = T - \beta\omega \\ T = ki \end{cases}$$

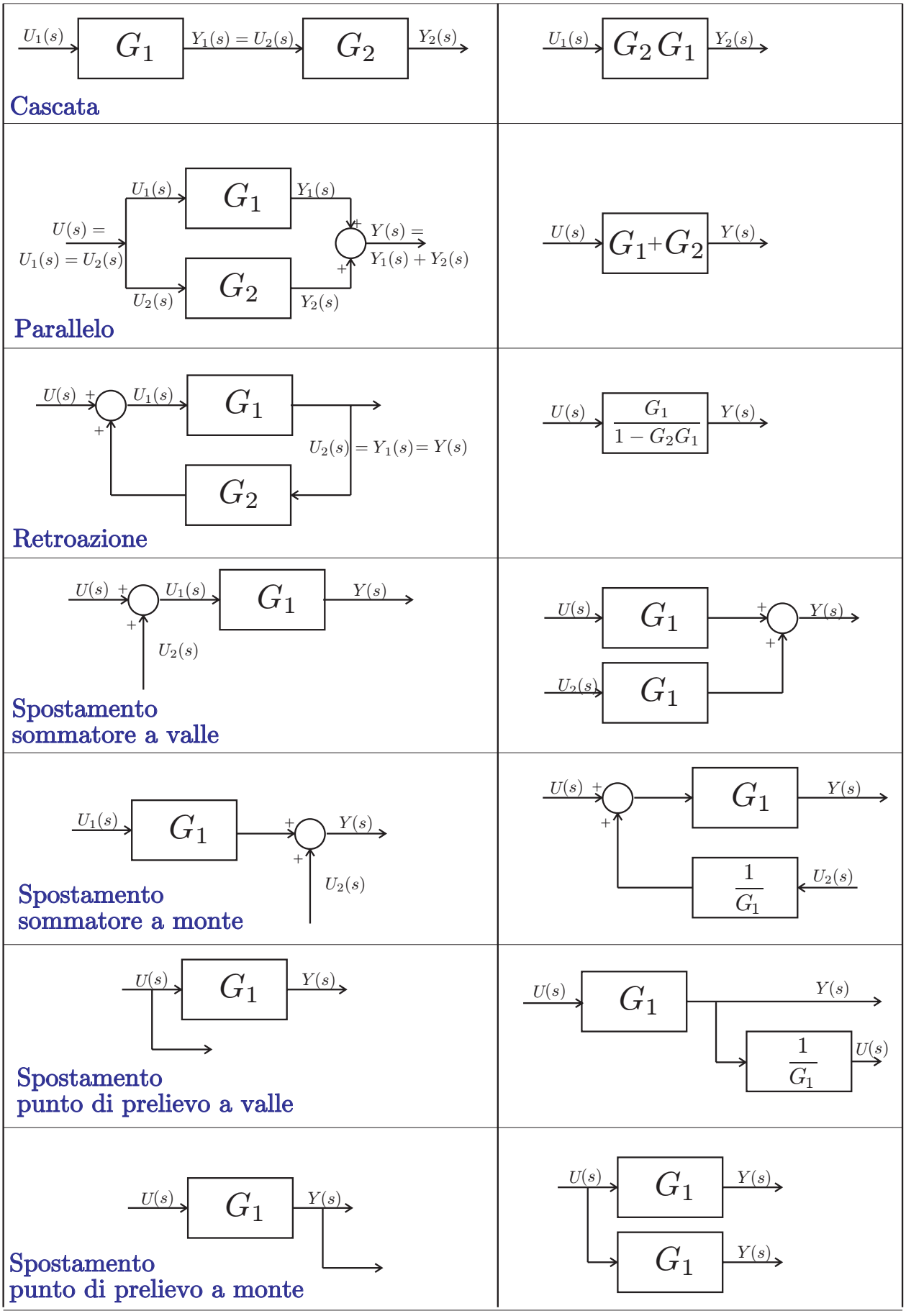
- Equazioni nel dominio di Laplace:

$$\begin{cases} (sL + R)I(s) = V(s) - E(s) \\ E(s) = k\Omega(s) \\ (Js + \beta)\Omega = T \\ T(s) = kI(s) \end{cases}$$

- Diagramma a blocchi:

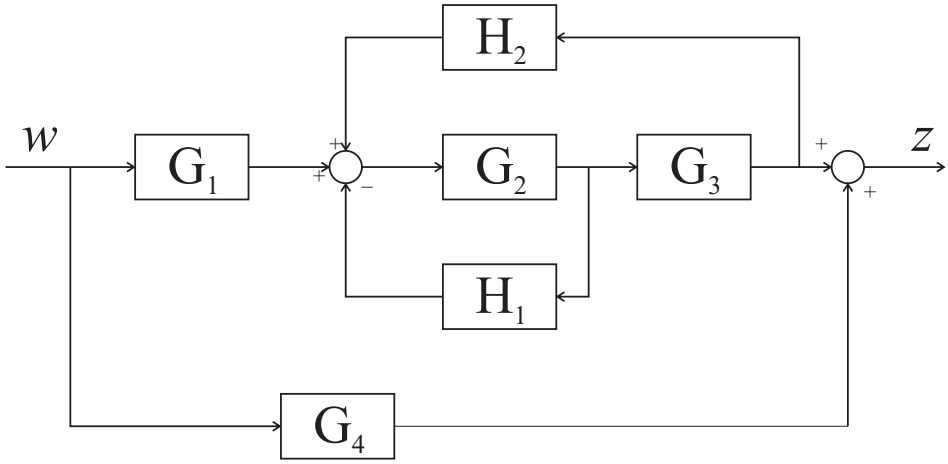


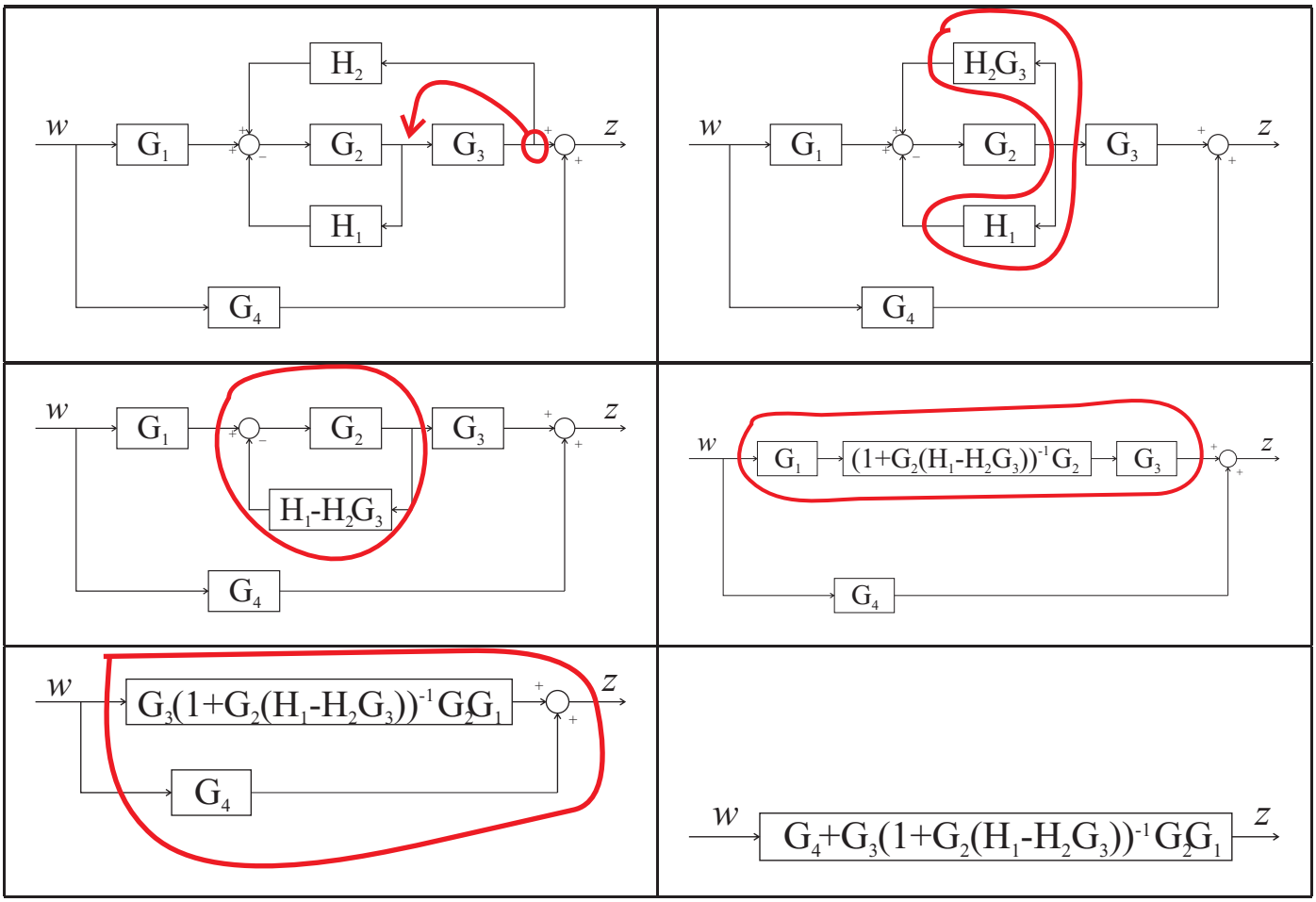
7 Diagrammi a Blocchi



7 Diagrammi a Blocchi

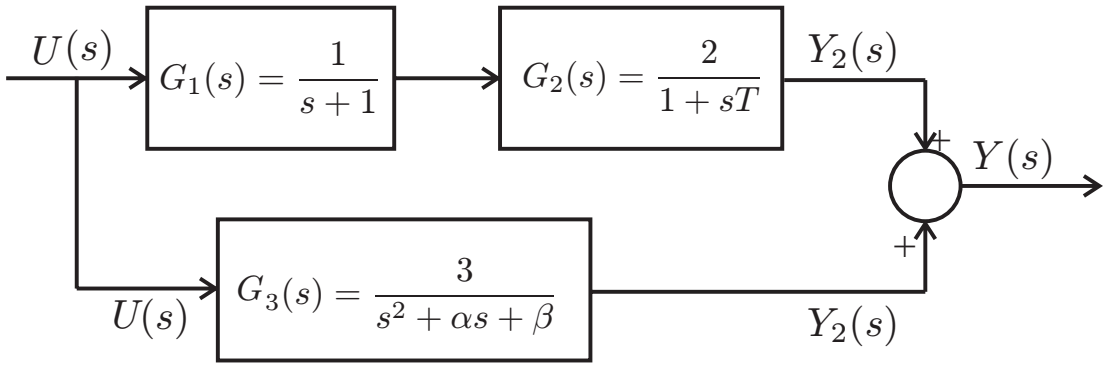
Esempio





Teorema. Un sistema costituito da un numero qualunque di sottosistemi fra loro connessi in cascata e/o parallelo è asintoticamente stabile se e solo se sono asintoticamente stabili tutti i sottosistemi che lo compongono.

Esempio

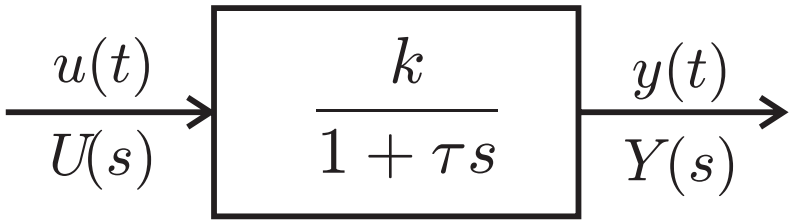


Il sistema è asintoticamente stabile se e solo se $T > 0$, $\alpha > 0, \beta > 0$.

8. Risposte Tipiche nel Tempo

8 Risposte Tipiche nel Tempo

Sistema del primo ordine



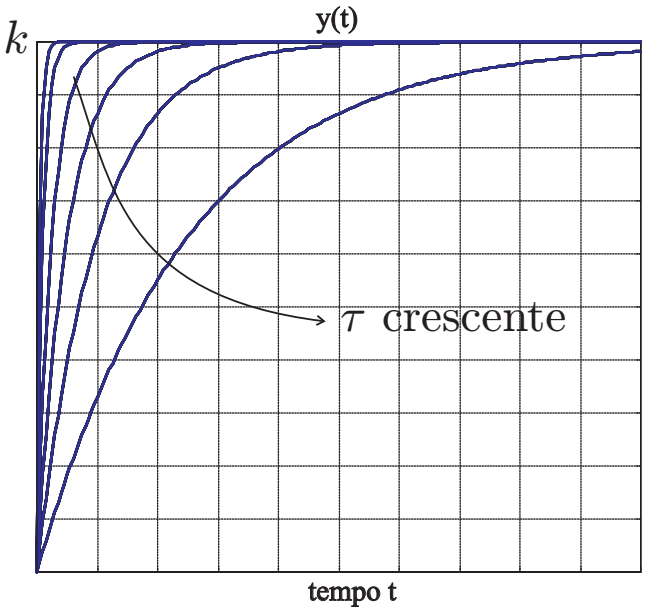
$(k, \tau > 0)$

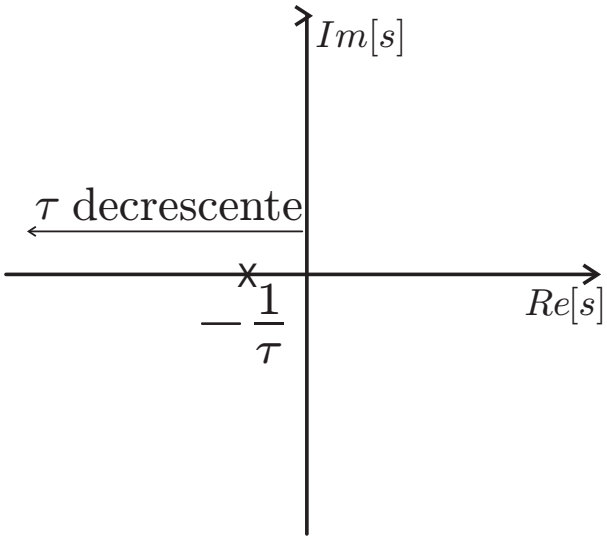
- *Risposta all'impulso* : $u(t) = \delta(t), U(s) = 1,$

$$Y(s) = \frac{k}{1+\tau s},\ y(t) = \frac{k}{\tau} e^{-\frac{t}{\tau}}\ \mathbb{1}(t)$$

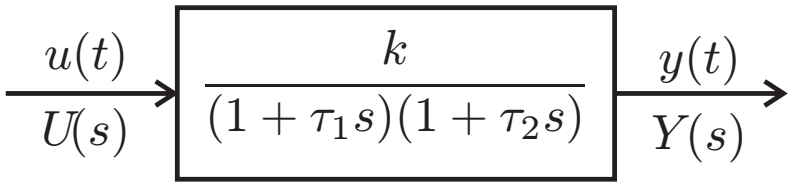
- *Risposta al gradino* : $u(t) = \mathbb{1}(t), U(s) = \frac{1}{s},$

$$Y(s) = \frac{k}{s(1+\tau s)},\ y(t) = k(1 - e^{-\frac{t}{\tau}})\ \mathbb{1}(t)$$





Sistema del secondo ordine (poli reali)

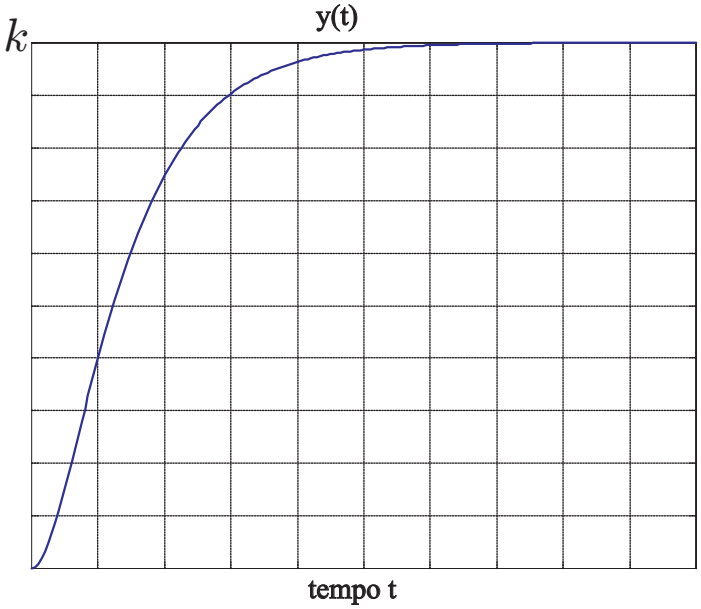


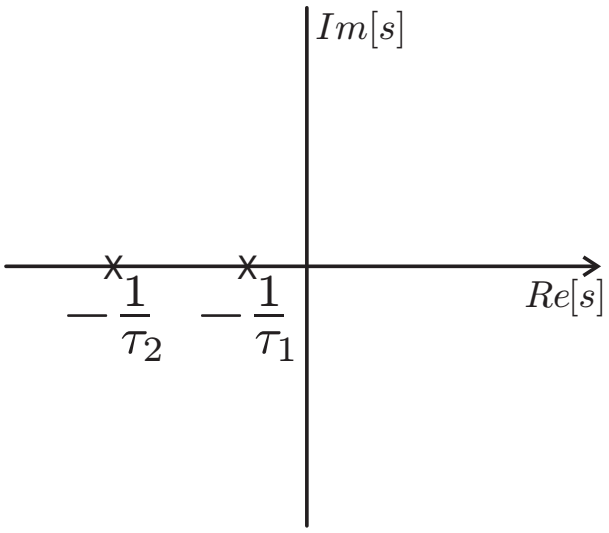
$(k, \tau_1, \tau_2 > 0)$

- *Risposta al gradino* : $u(t) = \mathbb{1}(t), U(s) = \frac{1}{s}$,

$$Y(s) = \frac{k}{s(1+\tau_1 s)(1+\tau_2 s)}$$

$$y(t) = k(1 + \frac{\tau_1}{\tau_2 - \tau_1} e^{-\frac{t}{\tau_1}} - \frac{\tau_2}{\tau_2 - \tau_1} e^{-\frac{t}{\tau_2}}) \, \mathbb{1}(t)$$





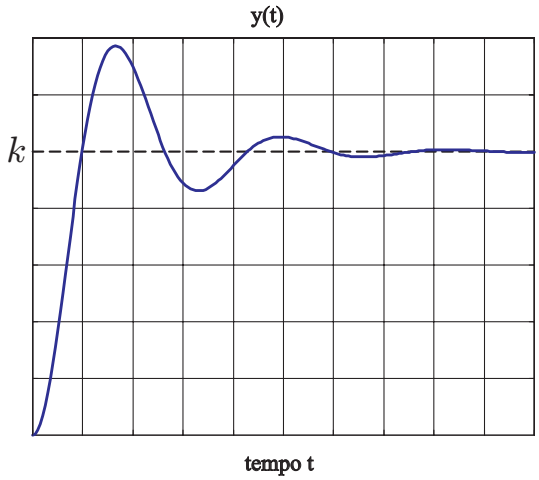
Sistema del secondo ordine (poli complessi)

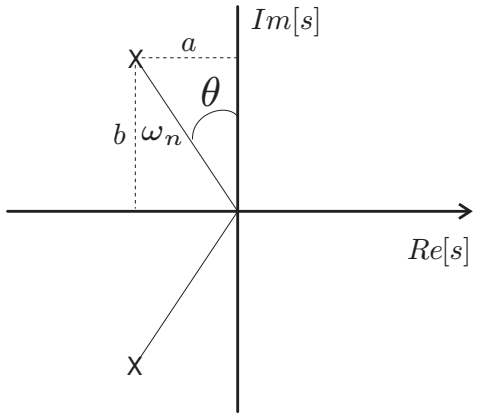
$$\underset{U(s)}{u(t)} \rightarrow \boxed{\frac{k}{1+2\frac{\zeta}{\omega_n}s+\frac{1}{\omega_n^2}s^2}} \rightarrow \underset{Y(s)}{y(t)}$$

$(k, \omega_n > 0, 0 < \zeta < 1)$

- *Risposta al gradino* : $u(t) = \mathbb{1}(t), U(s) = \frac{1}{s}$,

$$\begin{aligned} Y(s) &= \frac{k}{s(1+2\frac{\zeta}{\omega_n}s+\frac{1}{\omega_n^2}s^2)} \\ y(t) &= k\left[1 - e^{-\omega_n\zeta t}\left(\cos\left(\omega_n\sqrt{1-\zeta^2}t\right) + \right.\right. \\ &\quad \left.\left.\frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}t\right)\right)\right]\mathbb{1}(t) \end{aligned}$$

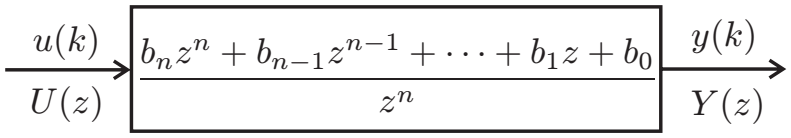




- $s = -a + jb = \omega_n e^{j(\theta+\frac{\pi}{2})}, \theta = \arcsin\zeta$

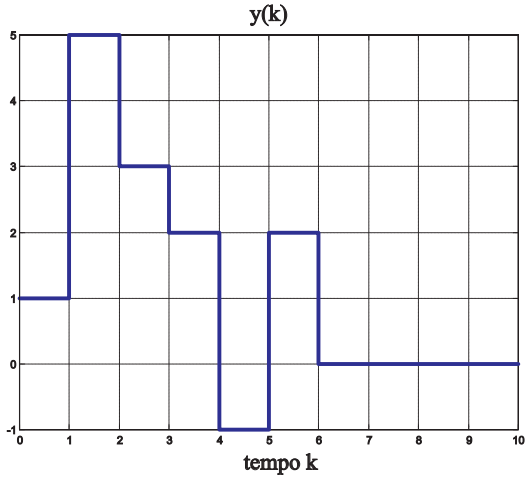
$$\begin{cases} a &= \omega_n\zeta \\ b &= \omega_n\sqrt{1-\zeta^2} \end{cases} \quad \begin{cases} \omega_n &= \sqrt{a^2+b^2} \\ \zeta &= \frac{a}{\sqrt{a^2+b^2}} \end{cases}$$

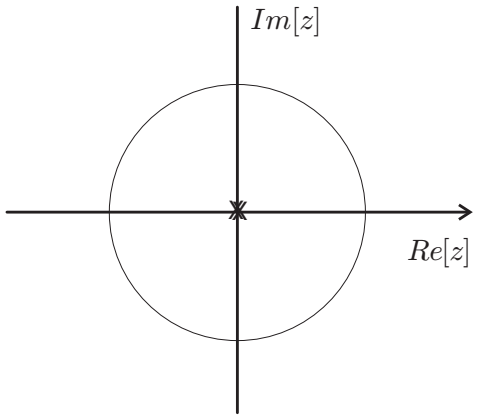
Sistema tempo discreto nilpotente



- *Risposta all'impulso* : $u(k) = \delta(k), U(z) = 1,$

$$
\begin{aligned}
Y(z) &= \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_{n-1} z + b_n}{z^n} \\
y(k) &= \mathcal{Z}^{-1}[Y(z)] = \mathcal{Z}^{-1}\left[b_0 + b_1 z^{-1} + \cdots + b_n z^{-n}\right] \\
&= \begin{cases} b_k & 0 \le k \le n \\ 0 & k > n,\ k < 0 \end{cases}
\end{aligned}
$$





- L'effetto dell'impulso svanisce dopo un numero finito n di passi. Idem per l'effetto delle condizioni iniziali.

8 Risposte Tipiche nel Tempo