MODEL PREDICTIVE CONTROL

CONCLUSIONS

Alberto Bemporad

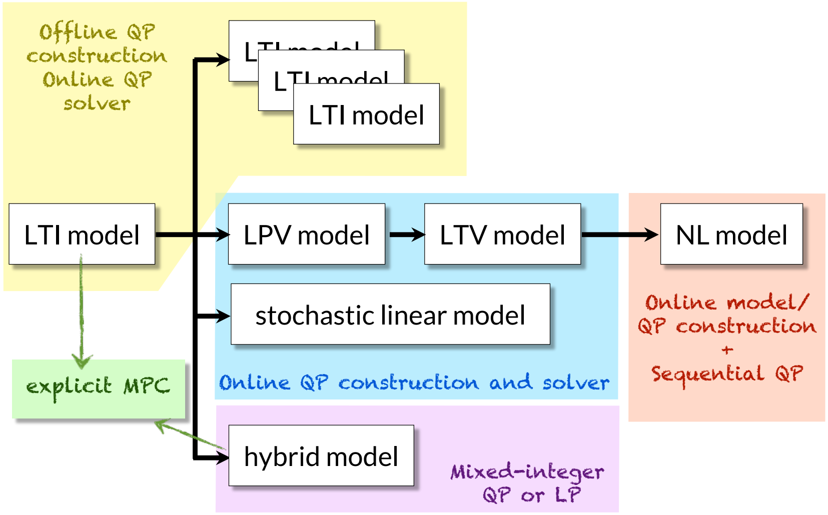
`http://cse.lab.imtlucca.it/~bemporad/mpc_course.html`

IMT SCHOOL FOR ADVANCED STUDIES LUCCA

COURSE STRUCTURE

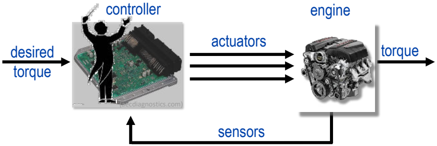
- **Basic concepts** of model predictive control (MPC) and **linear MPC**
- **Linear time-varying** and **nonlinear** MPC
- **Quadratic programming** (QP) and **explicit MPC**
- **Hybrid** MPC
- **Stochastic** MPC
- **Learning-based** MPC

CONCLUSIONS


Offline QP construction Online QP solver
LTI model
LTI model
LTI model
LTI model
LPV model
LTV model
NL model
stochastic linear model
explicit MPC
Online QP construction and solver
Online model/ QP construction + Sequential QP
hybrid model
Mixed-integer QP or LP

Perspective of the automotive industry:

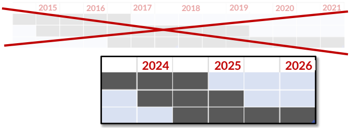
- Increasingly demanding **requirements** (emissions/consumption, passenger safety and comfort, ...)



- Better control performance only achieved by better **coordination** of actuators:
  - **increasing number** of actuators (e.g., due to electrification)

  - take into account **limited range** of actuators

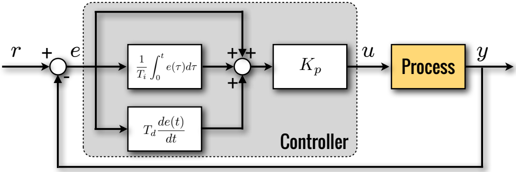
  - resilience in case of some **actuator failure**



- **Shorter development time** for control solution (market competition, changing legislation)

PROPORTIONAL INTEGRATIVE DERIVATIVE (PID) CONTROLLER

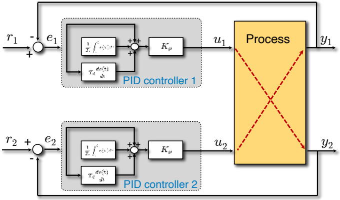
- PIDs are the most used controllers in industrial automation since the '30s



Pros:

- ✔ **Single-loops** are very easy to tune, just **3 parameters to calibrate**
- ✔ **Few lines of C code**, minimal memory and throughput requirements
- ✔ **No process model** required, just output measurements
- ✔ **Offset-free set-point tracking** thanks to integral action

PROPORTIONAL INTEGRATIVE DERIVATIVE (PID) CONTROLLER

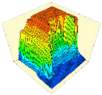


Cons: (1/2)

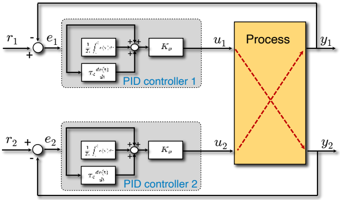
✖ **Multi-input/multi-output** systems: dynamical coupling requires tuning multiple PID loops together

- ☹ Surgically changing a PID loop tuning may have bad consequences on other loops, due to dynamical **interactions**
- ☹ Lookup-table complexity increases **exponentially** (e.g.: 5 inputs, 10 values each $\rightarrow 10^5$ entries)
- ☹ Hard to coordinate multiple actuators **optimally**
- ☹ The calibration might need to be completely redone for a new model

	A	B	C	D	E
1	Input 1	Input 2	Input 3	Input 4	Input 5
2	0.0119	0.0046	0.0287	0.0155	0.0012
3	0.0318	0.0154	0.0292	0.0225	0.0067
4	0.0344	0.043	0.0305	0.0326	0.0336
5	0.0357	0.0497	0.0377	0.0424	0.0358
6	0.0462	0.0598	0.0855	0.0527	0.068
7	0.054	0.076	0.0987	0.0596	0.0688
8	0.0759	0.0782	0.1068	0.0605	0.0908
9	0.0971	0.0811	0.1111	0.0714	0.0911
10	0.0975	0.0838	0.1174	0.0835	0.0942
11	0.119	0.0844	0.1366	0.0967	0.1056



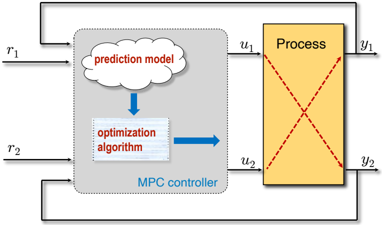
PROPORTIONAL INTEGRATIVE DERIVATIVE (PID) CONTROLLER



Cons: (2/2)

- ✖ Handling **input constraints** require additional **anti-windup** design
- ✖ **Output constraints** are much harder to handle
- ✖ Limited **preview** (derivative term =1st order extrapolation of future output)
- ✖ No explicit performance index optimized at runtime
- ✖ Resilience to **actuator faults** requires further design effort

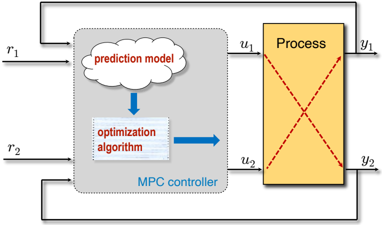
Multivariable PID control design & calibration might be time consuming



$$
\begin{aligned}
\min \quad & \sum_{k=0}^{N-1} \|y_k - r_{t+k}\|_2^2 + \rho \|u_k - u_{r,t+k}\|_2^2 \\
\text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k \\
& y_k = Cx_k \\
& u_{\min} \leq u_k \leq u_{\max} \\
& y_{\min} \leq y_k \leq y_{\max}
\end{aligned}
$$

Pros:

- ✔ Naturally coordinates **multiple inputs and outputs**
- ✔ Naturally handles **input and output constraints**
- ✔ Very easily includes **preview** on references/measured disturbances
- ✔ **Performance index** optimized at runtime

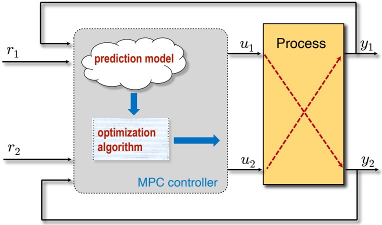


$$\begin{aligned} \min \quad & \sum_{k=0}^{N-1} \|y_k - r_{t+k}\|_2^2 + \rho\|u_k - u_{r,t+k}\|_2^2 \\ \text{s.t.} \quad & x_{k+1} = Ax_k + Bu_k \\ & y_k = Cx_k \\ & u_{\min} \leq u_k \leq u_{\max} \\ & y_{\min} \leq y_k \leq y_{\max} \end{aligned}$$

Pros:

- ✔ **Offset-free set-point tracking** thanks to **disturbance models** and observers
- ✔ Design easy to **transfer** to new models (**no lookup tables**)
- ✔ Controller easily reconfigurable online to **handle faults** (resilience)

MODEL PREDICTIVE CONTROL (MPC)



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\end{aligned}
$$

Cons:

- ✖ Multiple parameters to calibrate (models, weights, solver tolerances, ...)

  → **Automatic calibration**

- ✖ **Nontrivial C code** (QP solver), need to consider **memory** and **throughput** issues

  → **Certifiable QP code**

- ✖ Requires a **process model** (physical modeling, system identification) as all **model-based control-design** methods

  → **Model learning tools**

CONCLUSIONS

- MPC is a **universal control methodology**, same approach used for different
  - **models** (linear, nonlinear, hybrid, stochastic, ...)
  - **performance indices** (quadratic, convex, nonlinear, stochastic)
  - **constraints** (linear, nonlinear, robust, in probability)
- **MPC research**:
  1. Linear, uncertain, explicit, hybrid, nonlinear MPC: **mature theory**
  2. Stochastic MPC, economic MPC: **still open issues**
  3. Embedded optimization methods for MPC: **still room for many new ideas**
  4. System identification for MPC: there is **a lot to "learn"** from machine learning
  5. Data-driven MPC: still **a lot of open issues**
- **MPC technology**: rather mature, widely spread in many industrial sectors

General references on MPC

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- V. Breschi, D. Piga, and A. Bemporad, "Piecewise affine regression via recursive multiple least squares and multicategory discrimination," Automatica, 73, pp. 155–162, 2016

Explicit MPC

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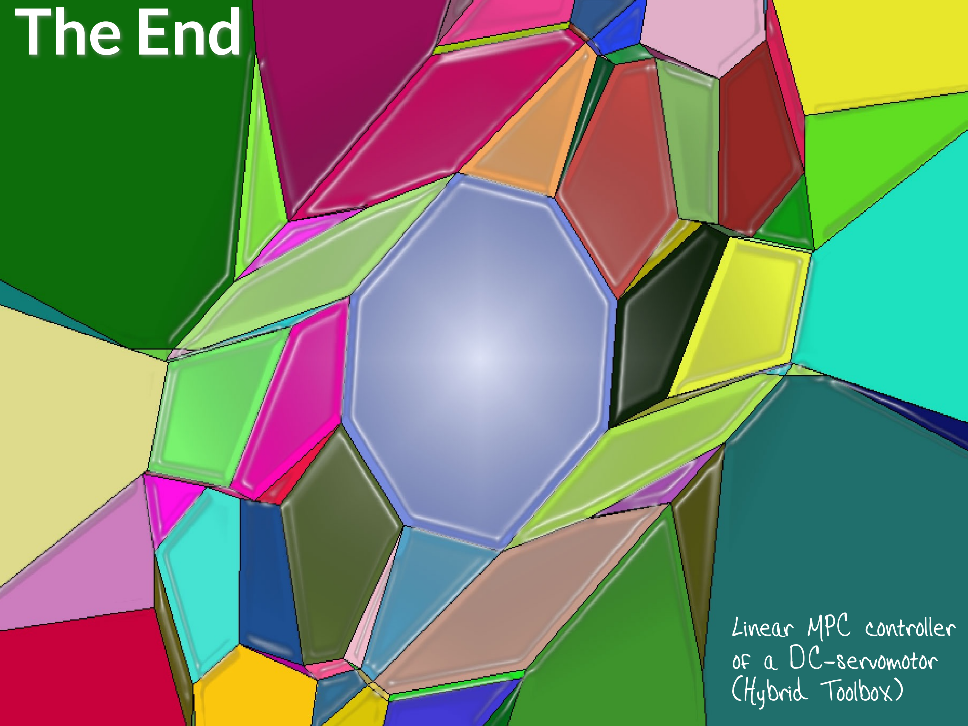

Linear MPC controller of a DC-servomotor (Hybrid Toolbox)