MODEL PREDICTIVE CONTROL

HYBRID MPC EXAMPLES

Alberto Bemporad

http://cse.lab.imtlucca.it/~bemporad/mpc_course.html

IMT SCHOOL FOR ADVANCED STUDIES LUCCA

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COURSE STRUCTURE

- ✔ **Basic concepts** of model predictive control (MPC) and **linear MPC**
- ✔ **Linear time-varying** and **nonlinear** MPC
- ✔ **Quadratic programming** (QP) and **explicit MPC**
- **Hybrid** MPC
- **Stochastic** MPC
- **Learning-based** MPC

HYBRID MPC EXAMPLES

Disclaimer: This is an academic example

GOAL:

Command gear ratio, gas pedal, and brakes to **track** a desired speed and minimize fuel consumption

HYBRID MPC FOR CRUISE CONTROL - MODEL

- Vehicle dynamics

$$\boxed{m\ddot{x} = F_e - F_b - \beta\dot{x}}$$

$\dot{x}$ = vehicle speed

F_e = traction force

F_b = brake force

→ discretized with sampling time $T_s = 0.5$ s

- Transmission kinematics

$$\boxed{\omega = \frac{R_g(i)}{k_s}\dot{x}}$$

$$\boxed{F_e = \frac{R_g(i)}{k_s}C}$$

ω = engine speed

C = engine torque

i = gear

power balance:

$$F_e\dot{x} = C\omega$$

HYBRID MPC FOR CRUISE CONTROL - MODEL

- **Gear selection:** for each gear #i, define a binary input $\boxed{g_i \in \{0,1\}}$

  $i = R, 1, 2, 3, 4, 5$

- **Gear selection (traction force):**

$$\boxed{F_e = \frac{R_g(i)}{k_s} C}$$ depends on gear #i

define auxiliary continuous variables:

$$\boxed{\text{IF } g_i = 1 \text{ THEN } F_{ei} = \frac{R_g(i)}{k_s} C \text{ ELSE } 0}$$

$$\Longrightarrow \quad F_e = F_{eR} + F_{e1} + F_{e2} + F_{e3} + F_{e4} + F_{e5}$$

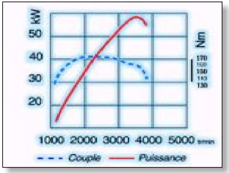
- **Gear selection (engine/vehicle speed):**

$$\boxed{\omega = \frac{R_g(i)}{k_s} \dot{x}}$$ similarly, also requires 6 auxiliary continuous variables

HYBRID MPC FOR CRUISE CONTROL - MODEL

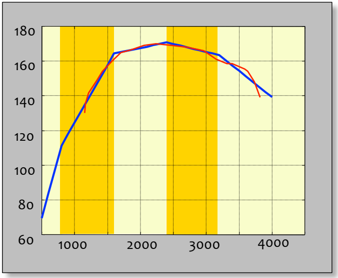
- engine torque $\quad -C_e^-(\omega) \leq C \leq C_e^+(\omega)$

- max engine torque $\quad C_e^+(\omega)$



Piecewise-linearization

(PWL Toolbox, Julián, 2000)



requires: 4 binary aux variables
4 continuous aux variables

- Min engine torque $\quad C_e^-(\omega) = \alpha_1 \omega + \beta_1$

Note: in this case the PWL constraint $C \leq C_e^+(\omega)$ is convex, it could be handled by linear constraints without introducing any binary variable !

HYBRID MPC FOR CRUISE CONTROL - HYSDEL

```
SYSTEM cruisecontrolmodel {

INTERFACE {
    PARAMETER {
        REAL mass = 1020; /* kg */
        REAL beta_friction = 25; /* W/m*s */

        [snip]
    }

    STATE { REAL position [0,10000];
            REAL speed [vmin,vmax]; }

    INPUT { REAL torque [Cmin,Cmax];
            REAL F_brake [0,max_brake_force];
            BOOL gear1, gear2, gear3, gear4, gear5, gearR; }
    }

IMPLEMENTATION {
    AUX {REAL F, Fe1, Fe2, Fe3, Fe4, Fe5, FeR;
         REAL w, w1, w2, w3, w4, w5, wR;
         BOOL dPWL1,dPWL2,dPWL3,dPWL4;
         REAL DCe1,DCe2,DCe3,DCe4;  }

    LINEAR {F = Fe1+Fe2+Fe3+Fe4+Fe5+FeR;
            w = w1+w2+w3+w4+w5+wR; }

    AD { dPWL1 = wPWL1-w<=0;
         dPWL2 = wPWL2-w<=0;
         dPWL3 = wPWL3-w<=0;
         dPWL4 = wPWL4-w<=0;  }

    DA { Fe1 = {IF gear1  THEN torque/speed_factor*Rgear1};
         Fe2 = {IF gear2  THEN torque/speed_factor*Rgear2};
         Fe3 = {IF gear3  THEN torque/speed_factor*Rgear3};
         Fe4 = {IF gear4  THEN torque/speed_factor*Rgear4};
         Fe5 = {IF gear5  THEN torque/speed_factor*Rgear5};
         FeR = {IF gearR  THEN torque/speed_factor*RgearR};
         w1 = {IF gear1  THEN speed/speed_factor*Rgear1};
         w2 = {IF gear2  THEN speed/speed_factor*Rgear2};
         w3 = {IF gear3  THEN speed/speed_factor*Rgear3};
         w4 = {IF gear4  THEN speed/speed_factor*Rgear4};
         w5 = {IF gear5  THEN speed/speed_factor*Rgear5};
         wR = {IF gearR  THEN speed/speed_factor*RgearR};

         DCe1 = {IF dPWL1 THEN (aPWL2-aPWL1)+(bPWL2-bPWL1)*w};
         DCe2 = {IF dPWL2 THEN (aPWL3-aPWL2)+(bPWL3-bPWL2)*w};
         DCe3 = {IF dPWL3 THEN (aPWL4-aPWL3)+(bPWL4-bPWL3)*w};
         DCe4 = {IF dPWL4 THEN (aPWL5-aPWL4)+(bPWL5-bPWL4)*w};
    }

    CONTINUOUS { position = position+Ts*speed;
                 speed = speed+Ts/mass*(F-F_brake-beta_friction*speed);

    MUST {  /* max engine speed */
            /* wemin <= w1+w2+w3+w4+w5+wR <= wemax */

        -w1 <= -wemin; w1 <= wemax;
        -w2 <= -wemin; w2 <= wemax;
        -w3 <= -wemin; w3 <= wemax;
        -w4 <= -wemin; w4 <= wemax;
        -w5 <= -wemin; w5 <= wemax;
        -wR <= -wemin; wR <= wemax;

        -F_brake <=0;
        F_brake <= max_brake_force;

        -torque-(alpha1+beta1*w) <=0;
        torque-(aPWL1+bPWL1*w+DCe1+DCe2+DCe3+DCe4)-1<=0;

        -((REAL gear1)+(REAL gear2)+(REAL gear3)+(REAL gear4)+
                    (REAL gear5)+(REAL gearR))<=-0.9999;
        (REAL gear1)+(REAL gear2)+(REAL gear3)+(REAL gear4)+
                    (REAL gear5)+(REAL gearR)<=1.0001;

        dPWL4 -> dPWL3; dPWL4 -> dPWL2;
        dPWL4 -> dPWL1; dPWL3 -> dPWL2;
        dPWL3 -> dPWL1; dPWL2 -> dPWL1;
        }
    }
}
```

go to demo **/demos/cruise/init.m**

HYBRID MPC FOR CRUISE CONTROL - MLD MODEL

- MLD model

$$\begin{aligned} x(t+1) &= Ax(t) + B_1u(t) + B_2\delta(t) + B_3z(t) + B_5 \\ y(t) &= Cx(t) + D_1u(t) + D_2\delta(t) + D_3z(t) + D_5 \\ E_2\delta(t) + E_3z(t) &\leq E_4x(t) + E_1u(t) + E_5 \end{aligned}$$

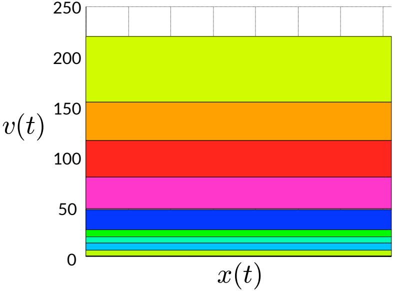
- 2 continuous states: x, v (vehicle position and speed)
- 2 continuous inputs: C, F_b (engine torque, brake force)
- 6 binary inputs: $g_R, g_1, g_2, g_3, g_4, g_5$ (gears)
- 1 continuous output: v (vehicle speed)
- 18 auxiliary continuous vars: (6+1 traction force, 6+1 engine speed, 4 PWL max engine torque)
- 4 auxiliary binary vars: (PWL max engine torque breakpoints)
- 100 mixed-integer inequalities

HYBRID MPC FOR CRUISE CONTROL - CONTROLLER

- **Maximum-speed controller**: the objective is to maximize speed (to reproduce max acceleration plots)

$$\begin{array}{rl} \max_{u(t)} & v(t+1|t) \\ \text{s.t.} & \text{MLD model} \\ & v(t|t)=v(t) \\ & x(t|t)=x(t) \end{array}$$

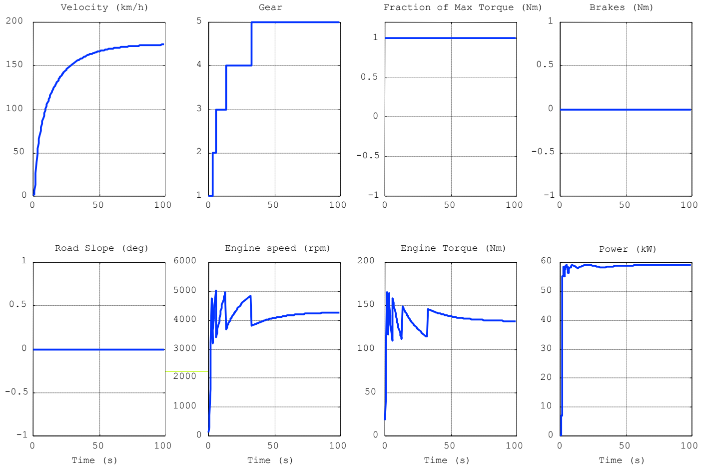
Linear constraints	96
Continuous variables	18
Binary variables	10
Parameters	1
Number of regions	11



$x(t)$ is irrelevant

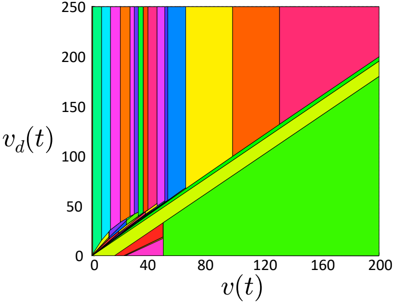
(parameters: Renault Clio 1.9 DTI RXE)

- **Maximum-speed controller**



- **Cruise controller**

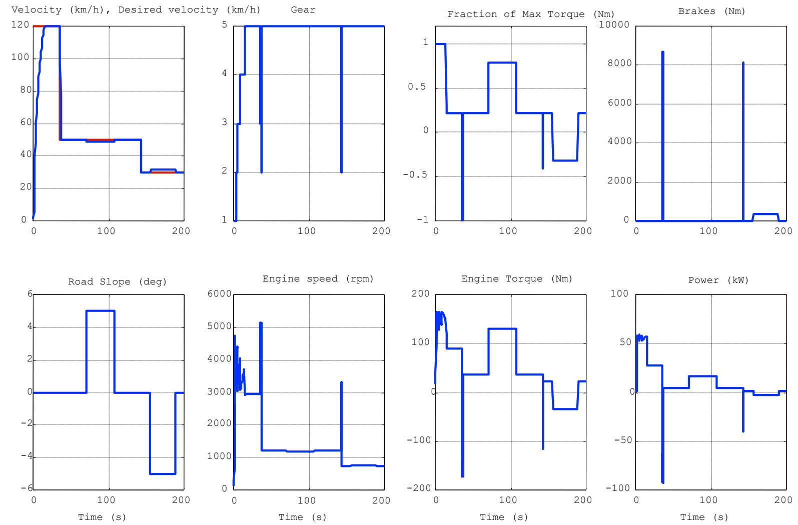
$$\begin{array}{ll} \min_{u(t)} & |v(t+1|t) - v_d(t)| + \rho|\omega| \\ \text{s.t.} & \text{MLD model} \\ & v(t|t) = v(t) \\ & x(t|t) = x(t) \end{array}$$



Linear constraints	98
Continuous variables	19
Binary variables	10
Parameters	2
Number of regions	49

HYBRID MPC FOR CRUISE CONTROL - RESULTS

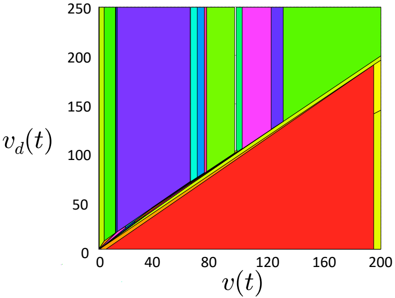
- **Cruise controller** $\min_{u(t)} |v(t+1|t) - v_d(t)| + \rho|\omega|, \rho = 0.001$



HYBRID MPC FOR CRUISE CONTROL - CONTROLLER

- **Smoother cruise controller**

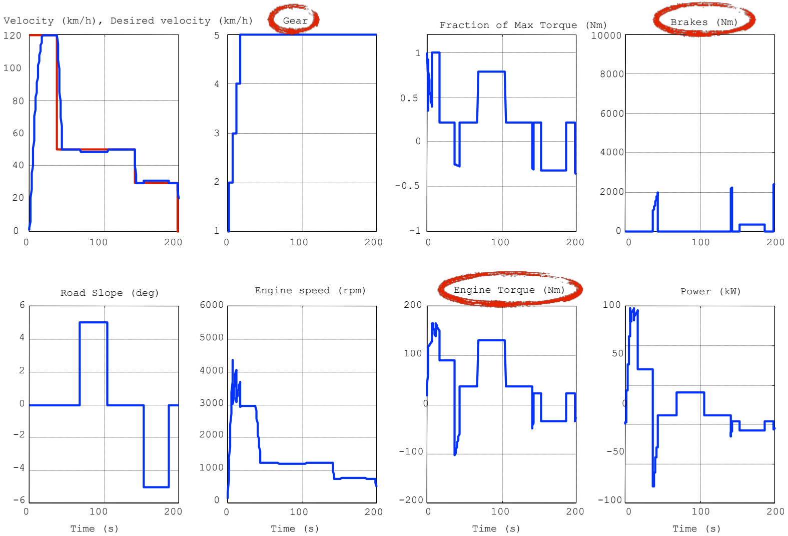
$$
\begin{aligned}
\min_{u(t)} \quad & |v(t+1|t) - v_d(t)| + \rho|\omega| \\
\text{s.t.} \quad & \text{MLD model} \\
& |v(t+1|t) - v(t)| \leq a_{\max} T_s \\
& v(t|t) = v(t) \\
& x(t|t) = x(t)
\end{aligned}
$$



Linear constraints	100
Continuous variables	19
Binary variables	10
Parameters	2
Number of regions	54

HYBRID MPC FOR CRUISE CONTROL - RESULTS

- **Smoother cruise controller**



HYBRID MPC FOR TRACTION CONTROL

VEHICLE TRACTION CONTROL PROBLEM

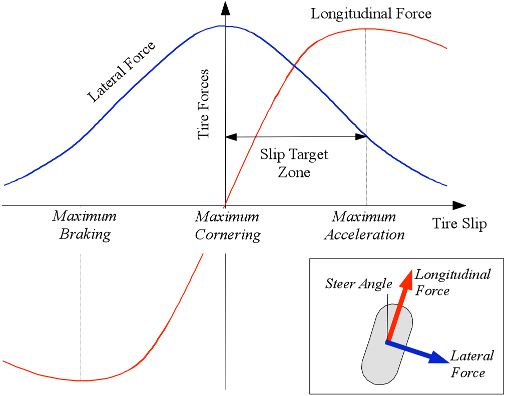
Improve driver's ability to control a vehicle under adverse external conditions (wet or icy roads)

Model: nonlinear, uncertain, constraints

Controller: suitable for real-time implementation

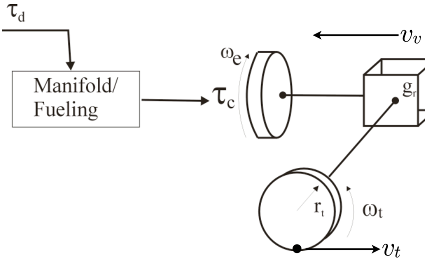
Solution: MLD hybrid framework + explicit hybrid MPC strategy

TIRE FORCE CHARACTERISTICS



SIMPLE TRACTION MODEL

(Borrelli, Bemporad, Fodor, Hrovat, 2006)



- Mechanical system

$$\dot{\omega}_e = \frac{1}{J_e}\left(\tau_c - b_e\omega_e - \frac{\tau_t}{g_r}\right)$$

$$\dot{v}_v = \frac{\tau_t}{m_v r_t}$$

- Manifold/fueling dynamics

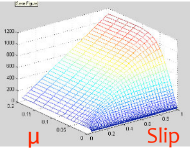
$$\tau_c = b_i \tau_d(t - \tau_f)$$

$$v_t = \omega_t r_t = \frac{\omega_e}{g_r} r_t$$

$$\Delta\omega = \frac{1}{r_t}(v_t - v_v) = \frac{\omega_e}{g_r} - \frac{v_v}{r_t} \qquad \text{wheel slip}$$

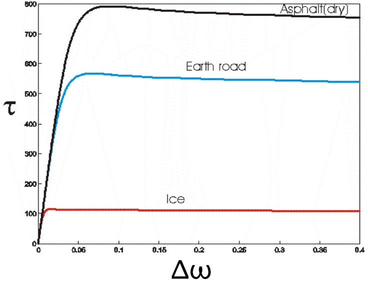
- Tire torque τ_t is a function of slip $\Delta\omega$ and road surface adhesion coefficient μ

HYBRIDIZATION OF TIRE CHARACTERISTICS

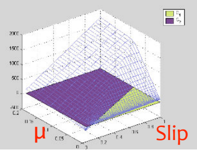


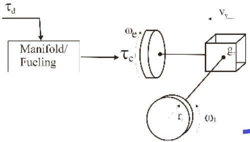
Nonlinear tire torque

$\tau_t = f(\Delta\omega, \mu)$



PWA approximation





HYSDEL

Mixed-Logical

Dynamical (MLD)

Hybrid Model

(discrete time)

MLD TRACTION MODEL

$$
\begin{aligned}
x(t+1) &= Ax(t) + B_1u(t) + B_2\delta(t) + B_3z(t) + B_5 \\
y(t) &= Cx(t) + D_1u(t) + D_2\delta(t) + D_3z(t) + D_5 \\
E_2\delta(t) + E_3z(t) &\leq E_4x(t) + E_1u(t) + E_5
\end{aligned}
$$

state	$x(t) \in \mathbb{R}^4$
input	$u(t) \in \mathbb{R}$
aux. binary	$\delta(t) \in \{0,1\}$
aux. continuous	$z(t) \in \mathbb{R}^3$

number of mixed-integer inequalities = 14

→ The MLD matrices are automatically generated in MATLAB format by HYSDEL

PERFORMANCE AND CONSTRAINTS

- Control objective:

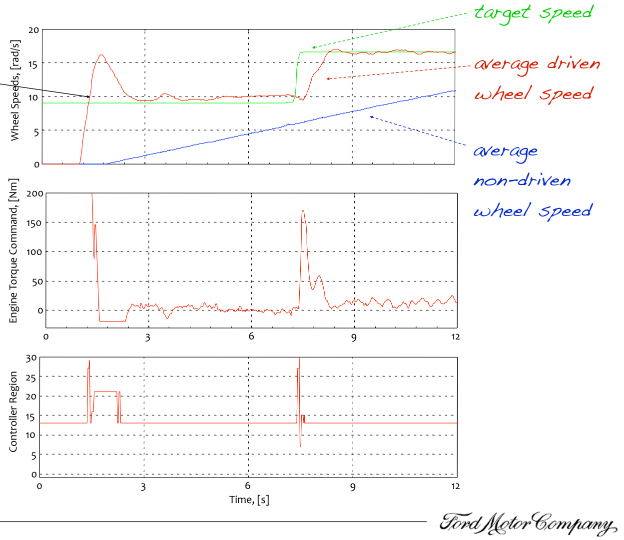
$$\begin{array}{ll} \min & \sum_{k=0}^{N} |\Delta\omega(t+k|t) - \Delta\omega_{\text{des}}| \\ \text{s.t.} & \text{MLD dynamics} \end{array}$$

- Constraints:
  - Limits on the engine torque: $-20 \text{ Nm} \leq \tau_d \leq 176 \text{ Nm}$

EXPERIMENTAL RESULTS

controller is triggered ON

(250 ms delay from commanded to actual engine torque → initial overspin)



EXPERIMENTS

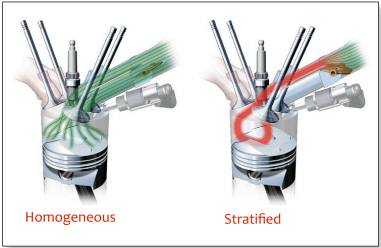
(Borrelli, Bemporad, Fodor, Hrovat, 2006)

HYBRID CONTROL OF A DISC ENGINE

DISC ENGINE CONTROL PROBLEM

Objective: develop a controller for a Direct-Injection Stratified Charge (DISC) engine that:

- automatically chooses operating mode (homogeneous/stratified)
- can cope with nonlinear dynamics
- handles constraints on A/F ratio, air-flow, spark

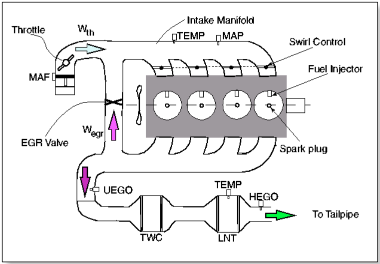


- achieves optimal performance (track desired torque and A/F ratio)

DISC ENGINE

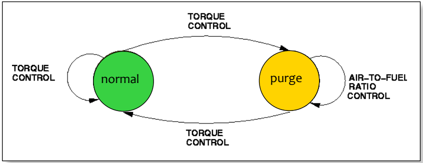
Two distinct regimes:

regime	fuel injection	air-to-fuel ratio
Homogeneous combustion	intake stroke	λ=14.64
Stratified combustion	compression stroke	λ>14.64



- Mode is switched by changing fuel injection timing (late / early)
- Better fuel economy during stratified mode

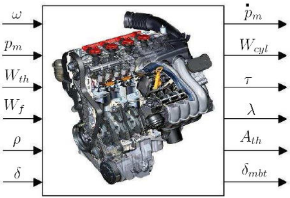
Periodical cleaning of the aftertreatment system needed ($\lambda=14.00$, homogeneous regime)



the stratified operation
can only be sustained in a restricted
part of the engine
operating range

DISC ENGINE

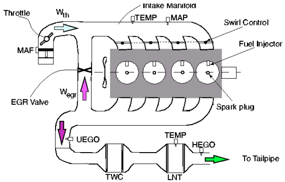
- States: intake manifold pressure (p_m)
- Outputs: Air-to-fuel ratio (λ), torque (τ), max-brake-torque spark timing (δ_{mbt})
- Continuous inputs: spark advance (δ), air flow (W_{th}), fuel flow (W_f)
- Binary input: spark combustion regime (ρ)
- Disturbance: engine speed (ω) [measured]
- Constraints on:
  - Air-to-fuel ratio (due to engine roughness, misfiring, smoke emiss.)
  - Spark timing (to avoid excessive engine roughness)
  - Mass flow rate on intake manifold (constraints on throttle)



Dynamic equations are nonlinear, dynamics and constraints depend on regime ρ

DISC ENGINE DYNAMICS

Nonlinear model of the engine developed and validated at Ford *(Kolmanovsky, Sun, …)*



<u>Assumptions</u>:
- no EGR (exhaust gas recirculation) rate,
- engine speed=2000 rpm.

- Intake manifold pressure: $$\boxed{\dot{p_m} = c_m\left(W_{th} - W_{cyl}\right) = c_m\left(W_{th} - k_{cyl}p_m\right)}$$
- In-cylinder Air-to-Fuel ratio: $$\boxed{\lambda = \frac{W_{cyl}}{W_f} = \frac{k_{cyl}p_m}{W_f}}$$
- Engine torque: $$\boxed{\tau = \tau_{mfr} + \tau_{pump} + \tau_{ind}}$$ with τ_{mfr}, τ_{pump} functions of p_m

$$\tau_{ind} = (\theta_a + \theta_b(\delta - \delta_{mbt})^2)W_f$$ where θ_a, θ_b, δ_{mbt} are functions of λ, δ and ρ

✓ Good for simulation

✖ Not suitable for optimization-based controller synthesis

HYBRIDIZATION OF DISC MODEL

DYNAMICS (intake pressure, air-to-fuel ratio, torque):

- Definition of two operating points;
- Linearization of nonlinear dynamics;
- Time discretization of the linear models.

→ ρ-dependent dynamic equations

CONSTRAINTS on:

- Air-to-Fuel Ratio: $\lambda_{min}(\rho) \leq \lambda(t) \leq \lambda_{max}(\rho)$
- Mass of air through the throttle: $0 \leq W_{th} \leq K$
- Spark timing: $0 \leq \delta(t) \leq \delta_{mbt}(\lambda, \rho)$

→ $\rho-$dependent constraints

↓

Hybrid system with 2 modes (switching affine system)

INTEGRAL ACTION

Integrators on torque error and air-to-fuel ratio error are added to obtain zero offsets in steady-state:

$$
\begin{aligned}
\epsilon_{\tau,k+1} &= \epsilon_{\tau,k} + T_s(\tau_{\text{ref}}(t) - \tau_k) \\
\epsilon_{\lambda,k+1} &= \epsilon_{\lambda,k} + T_s(\lambda_{\text{ref}}(t) - \lambda_k)
\end{aligned}
$$

T_s = sampling time

τ_{ref}, λ_{ref} = references on brake torque and air-to-fuel ratio

Simulation based on nonlinear model confirms zero offsets in steady-state (despite the model mismatch)

MPC OF DISC ENGINE

$$\min_{\xi} J(\xi, x(t)) = \sum_{k=0}^{N-1} u_k' R u_k + y_k' Q y_k + x_{k+1}' S x_{k+1}$$

$$\text{subj. to } \begin{cases} x_0 = x(t), \\ \text{hybrid model} \end{cases}$$

N= control horizon

$x(t)$ = current state

$$\xi = [u_0', \gamma_0', z_0', \ldots, u_{N-1}', \gamma_{N-1}', z_{N-1}']',$$

where:

$$\begin{aligned} u_k &= [W_{th,k} - W_{th,ref},\ W_{f,k} - W_{f,ref}, \delta_k - \delta_{ref},\ \rho_k - \rho_{ref}]' \\ y_k &= [\tau_k - \tau_{ref},\ \lambda_k - \lambda_{ref},\ \delta_{mbt} - \delta_k - \Delta\delta_{ref}]' \\ x_k &= [p_{m,k} - p_{m,ref},\ \epsilon_{\tau,k},\ \epsilon_{\lambda,k}]' \end{aligned}$$

and:

$$R = \begin{pmatrix} r_{W_{th}} & 0 & 0 & 0 \\ 0 & r_{W_f} & 0 & 0 \\ 0 & 0 & r_\delta & 0 \\ 0 & 0 & 0 & r_\rho \end{pmatrix} \quad Q = \begin{pmatrix} q_\tau & 0 & 0 \\ 0 & q_\lambda & 0 \\ 0 & 0 & q_{\Delta\delta} \end{pmatrix} \quad S = \begin{pmatrix} s_{p_m} & 0 & 0 \\ 0 & s_{\epsilon_\tau} & 0 \\ 0 & 0 & s_{\epsilon_\lambda} \end{pmatrix}$$

Reference values are automatically generated from τ_{ref} and λ_{ref} by numerical computations based on the nonlinear model

DISC ENGINE - HYSDEL

```
SYSTEM hysdisc{
   INTERFACE{
   STATE{
     REAL pm       [1, 101.325];
     REAL xtau     [-1e3, 1e3];
     REAL xlam     [-1e3, 1e3];
     REAL taud     [0,     100];
     REAL lamd     [10,     60];
      }
   OUTPUT{
     REAL lambda, tau, ddelta;
          }
   INPUT{
     REAL Wth      [0,38.5218];
     REAL Wf       [0,       2];
     REAL delta    [0,      40];
     BOOL rho;
      }
   PARAMETER{
     REAL Ts, pm1, pm2;
     …
      }
   }

   IMPLEMENTATION{
   AUX{
     REAL lam,taul,dmbtl,lmin,lmax;
      }
   DA{
   lam={IF rho THEN l11*pm+l12*Wth...
               +l13*Wf+l14*delta+l1c
        ELSE   l01*pm+l02*Wth+l03*Wf...
               +l04*delta+l0c      };

     taul={IF rho THEN taul1*pm+...
       tau12*Wth+tau13*Wf+tau14*delta+tau1c
           ELSE   tau01*pm+tau02*Wth...
            +tau03*Wf+tau04*delta+tau0c };

     dmbtl ={IF rho THEN dmbt11*pm+dmbt12*Wth...
             +dmbt13*Wf+dmbt14*delta+dmbt1c+7
             ELSE dmbt01*pm+dmbt02*Wth...
             +dmbt03*Wf+dmbt04*delta+dmbt0c-1};

     lmin ={IF rho THEN 13 ELSE 19};
     lmax ={IF rho THEN 21 ELSE 38};
          }
     CONTINUOUS{
         pm=pm1*pm+pm2*Wth;
         xtau=xtau+Ts*(taud-taul);
         xlam=xlam+Ts*(lamd-lam);
         taud=taud; lamd=lamd;
              }
     OUTPUT{
        lambda=lam-lamd;
        tau=taul-taud;
        ddelta=dmbtl-delta;
        }
     MUST{
        lmin-lam     <=0;
        lam-lmax     <=0;
        delta-dmbtl  <=0;
        }
     }
}
```

MPC – TORQUE CONTROL MODE

$$\min \sum_{k=0}^{N-1} (u_k - u_r)' R (u_k - u_r) + (y_k - y_r)' Q (y_k - y_r) + (x_{k+1} - x_r)' S (x_{k+1} - x_r)$$

$$\text{subj. to} \begin{cases} x_0 = x(t), \\ \text{hybrid model} \end{cases}$$

Solve **MIQP problem** to compute **u(t)**

$$u(t) = [W_{th}(t),\ W_f(t),\ \delta(t),\ \rho(t)]$$

Weights:

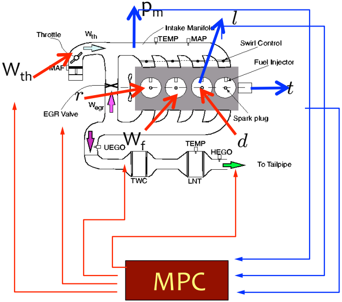
$$R = \begin{pmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

r_ρ (prevents unneeded chattering)

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.01 \end{pmatrix} \qquad S = \begin{pmatrix} 0.04 & 0 & 0 \\ 0 & 1500 & 0 \\ 0 & 0 & 0.01 \end{pmatrix}$$

q_τ q_λ s_{ε_τ} s_{ε_λ}

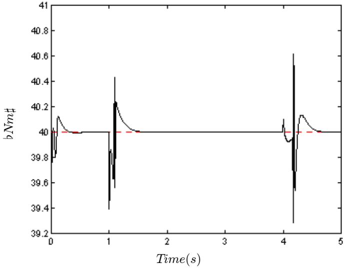
main emphasis on torque



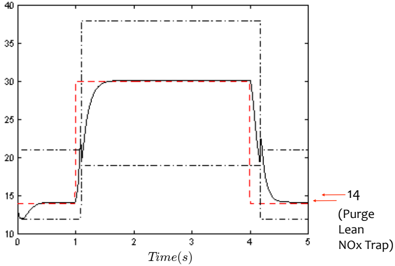
SIMULATION RESULTS (NOMINAL ENGINE SPEED)

ω/2000 rpm

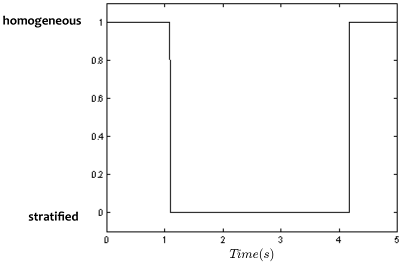
Engine brake torque



Air-to-fuel ratio



Combustion mode

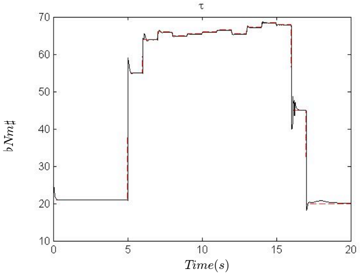


- Control horizon **N=1;**
- Sampling time **Ts=10 ms;**
- PC Xeon 2.8 GHz + Cplex 9.1

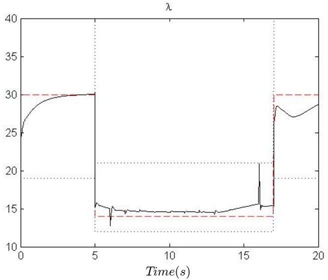
≈ **3 ms** per time step

SIMULATION RESULTS (VARYING ENGINE SPEED)

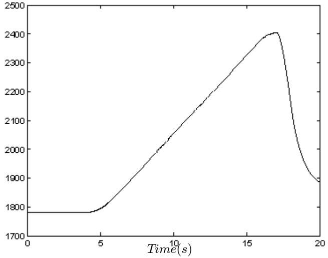
Engine Brake Torque



Air-to-Fuel Ratio



Engine speed

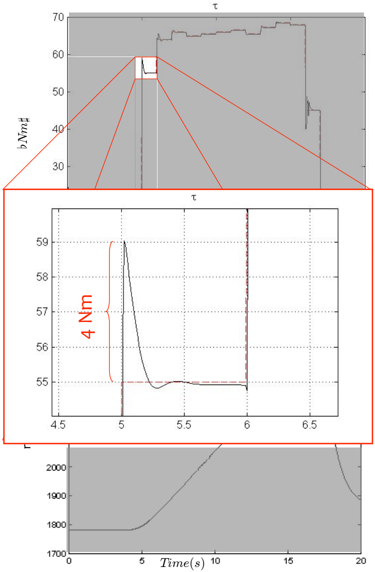


20 s segment of the European drive cycle (NEDC)

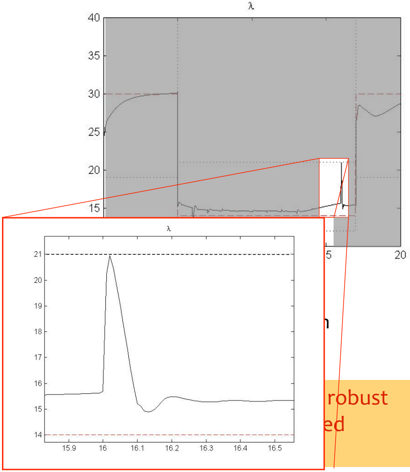
Hybrid MPC design is quite robust with respect to engine speed variations

SIMULATION RESULTS (VARYING ENGINE SPEED)

Engine Brake Torque



Air-to-Fuel Ratio



Control code too complex

(MIQP) → not implementable !

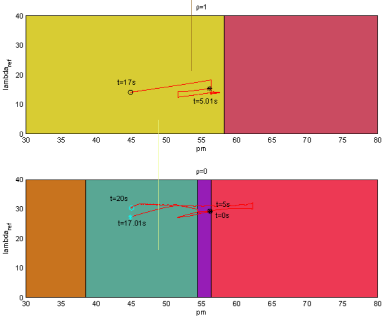
EXPLICIT MPC

Explicit control law: $u(t) = f(\theta(t))$

where: $u = [W_{th}\ W_f\ \delta\ \rho]'$

$$\theta = [p_m\ \epsilon_\tau\ \epsilon_\lambda\ \tau_{\text{ref}}\ \lambda_{\text{ref}}\ p_{m,\text{ref}}\ W_{th,\text{ref}}\ W_{f,\text{ref}}\ \delta_{\text{ref}}]'$$

Cross-section by the τ_{ref}-λ_{ref} plane



ρ/0

ρ/1

N=1 (control horizon)

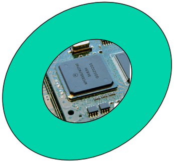
42 partitions

- Time to compute explicit MPC: $\approx$ 3s;
- Sampling time Ts=10 ms;
- PC Xeon 2.8 GHz + Cplex 9.1

$\rightarrow$ **8 μs** per time step

≈3ms on

μ–controller
Motorola
MPC 555
43kb RAM
(custom made for Ford)



EXPLICIT MPC CONTROLLER (N=2)

Explicit control law: $\boxed{u(t) = f(\theta(t))}$

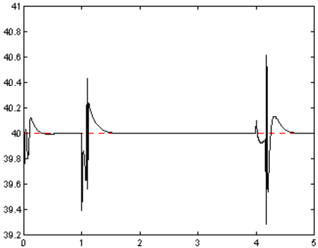
where: $u = [W_{th}\ W_f\ \delta\ \rho]'$

$$\theta = [p_m\ \epsilon_T\ \epsilon_\lambda\ \tau_{\text{ref}}\ \lambda_{\text{ref}}$$
$$p_{m,\text{ref}}\ W_{th,\text{ref}}\ W_{f,\text{ref}}\ \delta_{\text{ref}}]'$$

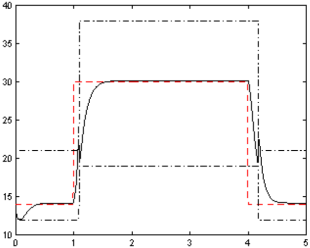
N=2 (control horizon)

747 partitions

Engine Brake Torque



Air-to-Fuel Ratio



Closed-loop N=2 ⟷ Closed-loop N=1 ⟵ adequate !

EXPLICIT HYBRID MPC OF SEMIACTIVE SUSPENSIONS

ACTIVE SUSPENSIONS

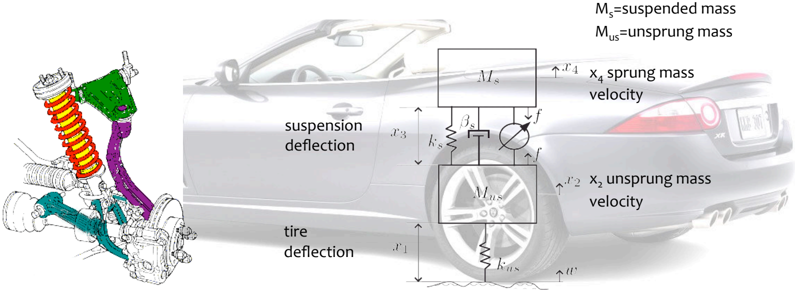
(Giorgetti, Bemporad, Tseng, Hrovat, 2006)

Active Suspension System
Ford Mercur XR 40i

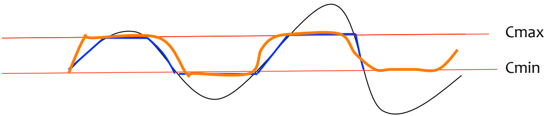
active suspensions

passive suspensions

QUEST OF OPTIMAL SEMI-ACTIVE SUSPENSIONS



For Semi-Active with Variable Damping, $f(x)=C*(x_4-x_2)$



Clipped Optimal

$C=f(x)/(x_4-x_2)$, where $f(x)$ is the optimal active suspension force

$C=\text{sat}[f(x)/(x_4-x_2)]$

Optimal

$\overset{?}{=}$

SUSPENSION MODEL

- State-space model

$$\dot{x} = Ax + B\bar{f} + B_w w$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{us}^2 & -2\rho\zeta\omega_s & \rho\omega_s^2 & 2\rho\zeta\omega_s \\ 0 & -1 & 0 & 1 \\ 0 & 2\zeta\omega_s & -\omega_s^2 & -2\zeta\omega_s \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \rho \\ 0 \\ -1 \end{bmatrix} \qquad B_w = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= \text{tire deflection from equilibrium} \\ x_2 &= \text{unsprung mass velocity} \\ x_3 &= \text{suspension deflection from equilibrium} \\ x_4 &= \text{sprung mass velocity} \\ \bar{f} &= \text{normalized adjustable force} \\ w &= \text{road velocity disturbance} \end{aligned}$$

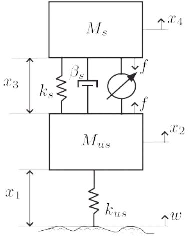
$$\rho = \frac{M_s}{M_{us}},\ \omega_{us} = \sqrt{\frac{k_{us}}{M_{us}}},\ \omega_s = \sqrt{\frac{k_s}{M_s}},\ \zeta = \frac{\beta_s}{2\sqrt{M_s k_s}},\ \bar{f} = \frac{f}{M_s}$$

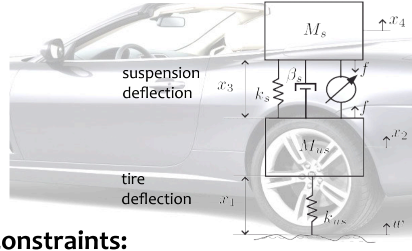
- Output:

$$y = \frac{dx_4}{dt} = \begin{bmatrix} 0 & 2\zeta\omega_s & -\omega_s^2 & -2\zeta\omega_s \end{bmatrix} x - \bar{f}$$

- Cost: $$J = \int (q_{x_1} x_1^2 + q_{x_3} x_3^2 + \dot{x}_4^2)dt = \int (x'Qx + \dot{x}_4^2)dt$$

- Time-discretization: $T_s = 10$ ms



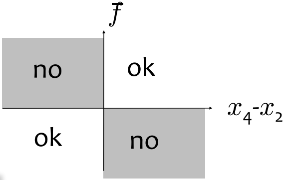


Quarter-car model → linear model

Constraints:

1) Passivity condition:

$$\bar{f}(x_4 - x_2) \geq 0$$



2) Max dissipation power:

$$\bar{f}(x_4 - x_2) \leq (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2)^2$$

3) Saturation:

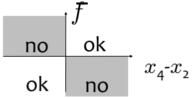
$$|\bar{f}| \leq \sigma$$

(1), (2) are nonlinear & nonconvex physical constraints → **hybrid model**

HYBRID MODEL OF SEMIACTIVE CONSTRAINTS

1) Passivity condition:

$$\boxed{\bar{f}(x_4 - x_2) \geq 0}$$



$$\boxed{\begin{array}{l} [\delta_v = 1] \leftrightarrow [x_4 - x_2 \geq 0] \\ [\delta_{\bar{f}} = 1] \leftrightarrow [\bar{f} \geq 0] \\ [\delta_v = 1] \rightarrow [\delta_{\bar{f}} = 1] \\ [\delta_v = 0] \rightarrow [\delta_{\bar{f}} = 0] \end{array}}$$

2) Max dissipation power:

$$\boxed{\bar{f}(x_4 - x_2) \leq (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2)^2} \qquad \boxed{F \geq 0}$$

where

$$\boxed{F = \begin{cases} \bar{f} - (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2) & \text{if } (x_4 - x_2) \leq 0 \\ -\bar{f} + (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2) & \text{otherwise} \end{cases}}$$

3) Saturation:

$$\boxed{|\bar{f}| \leq \sigma} \qquad \boxed{\begin{array}{ccc} \bar{f} & \leq & \sigma \\ \bar{f} & \geq & -\sigma \end{array}}$$

SEMIACTIVE SUSPENSION MODEL - HYSDEL

```
/* Semiactive suspension system

   (C) 2003-2005 by A.Bemporad, D.Hrovat,
                    E.Tseng, N.Giorgetti
*/

SYSTEM suspension {

 INTERFACE {
    STATE {
        REAL x1 [-0.05,0.05];
        REAL x2 [-5,5];
        REAL x3 [-0.2,0.2];
        REAL x4 [-2,2];
    }
    INPUT{
        REAL u [-10,10];   /* m/s^2 */
    }
    OUTPUT {
        REAL y;
    }
 PARAMETER {
   REAL A1dot,A2dot,A3dot,A4dot,B4dot,ws;
   REAL A11,A12,A13,A14,B1,A21,A22,A23,A24,B2;
   REAL A31,A32,A33,A34,B3,A41,A42,A43,A44,B4;
 }
}
```

```
IMPLEMENTATION {
  AUX {
    BOOL sign;
    BOOL usign;
    REAL F;
  }
  AD {
    sign = x4-x2<=0;
    usign = u<=0;
  }
  DA {
    F={ IF sign THEN u-(2*25.5*ws)*(x4-x2)
              ELSE -u+(2*25.5*ws)*(x4-x2)};
  }
  OUTPUT {    y=A1dot*x1+A2dot*x2+A3dot*x3
                +A4dot*x4+B4dot*u;
  }
  CONTINUOUS {
      x1 = A11*x1+A12*x2+A13*x3+A14*x4+B1*u;
      x2 = A21*x1+A22*x2+A23*x3+A24*x4+B2*u;
      x3 = A31*x1+A32*x2+A33*x3+A34*x4+B3*u;
      x4 = A41*x1+A42*x2+A43*x3+A44*x4+B4*u;
  }
  MUST {
      sign -> usign;
      ~sign -> ~usign;
      F>=0;
  } } }
```

```
>>S=mld('semiact3',Ts)
```

get the MLD model in MATLAB

```
>>[X,T,D,Z,Y]=sim(S,x0,U);
```

simulate the MLD model

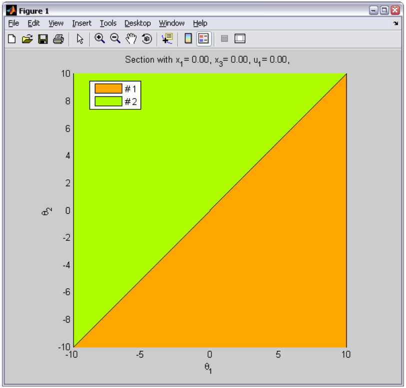
SEMIACTIVE SUSPENSION MODEL – PWA FORM

- PWA model

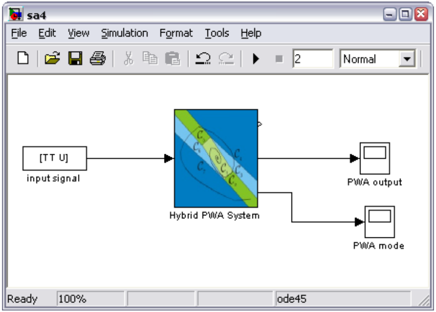
$$
\begin{aligned}
x(k+1) &= A_{i(k)}x(k) + B_{i(k)}u(k) + f_{i(k)} \\
y(k) &= C_{i(k)}x(k) + D_{i(k)}u(k) + g_{i(k)} \\
\\
i(k) \ \ \text{s.t.} & \quad H_{i(k)}x(k) + J_{i(k)}u(k) \leq K_{i(k)}
\end{aligned}
$$

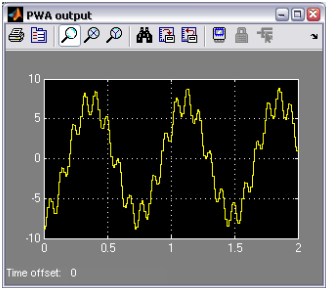
- 4 continuous states
  (x_1, x_2, x_3, x_4)
- 1 continuous input
  (normalized adjustable damping force $\bar{f}$)
- 2 polyhedral regions

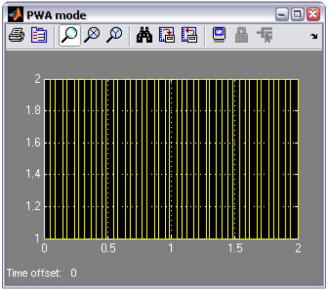
```
>>P=pwa(S);
```



OPEN-LOOP SIMULATION OF PWA SUSPENSION MODEL







MPC PERFORMANCE SPECIFICATIONS

tire deflession

suspension deflession

vertical acceleration

$$\min \left(\sum_{k=1}^{N-1} 1100 x_{1,k}^2 + 100 x_{3,k}^2 + \dot{x}_{4,k}^2 \right) + x_N' P x_N$$

terminal weight (Riccati matrix)

CLOSED-LOOP MPC RESULTS

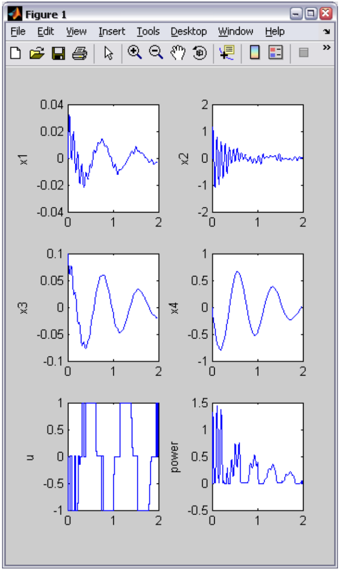
$$J = \int (q_1 x_1^2 + q_3 x_3^2 + \dot{x}_4^2)$$

```
>>refs.y=1;    % weights output #1
>>Q.y=Ts*rx4d;% output weight
…
>>Q.norm=2;    % quadratic costs
>>N=1;         % optimization horizon
>>limits.umin=umin;
>>limits.umax=umax;
```

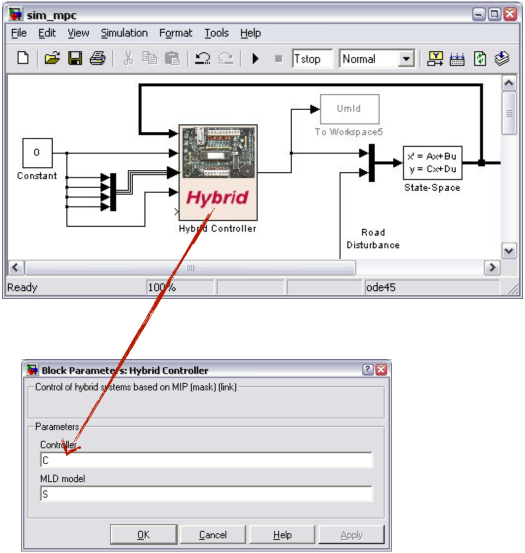
```
>>C=hybcon(S,Q,N,limits,refs);
```

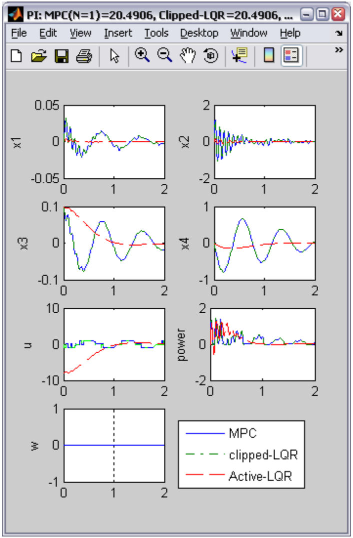
```
>> C

Hybrid controller based on MLD model S <semiact3.hys> [2-norm]

  4 state measurement(s)
  1 output reference(s)
  1 input reference(s)
  4 state reference(s)
  0 reference(s) on auxiliary continuous z-variables

  4 optimization variable(s) (2 continuous, 2 binary)
 13 mixed-integer linear inequalities
sampling time = 0.01, MIQP solver = 'cplex'

Type "struct(C)" for more details.
>>
```

```
>>[XX,UU,DD,ZZ,TT]=sim(C,S,r,x0,Tstop);
```



CLOSED-LOOP MPC RESULTS (SIMULINK)



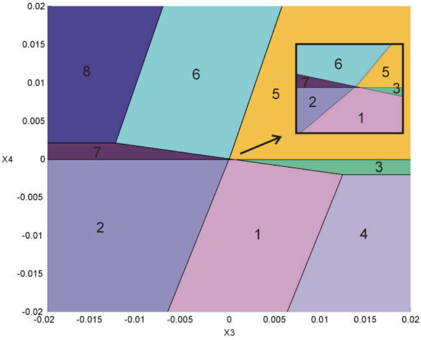


EXPLICIT HYBRID MPC

```
>>E=expcon(C,range,options);
```

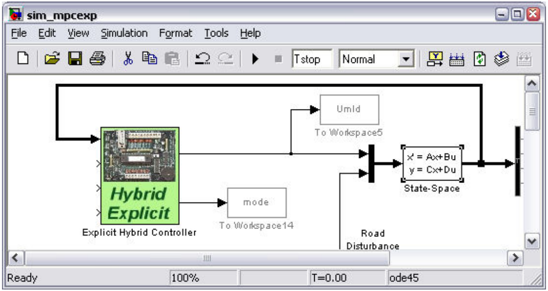
```
>> E

Explicit controller (based on hybrid controller C)
   4 parameter(s)
   1 input(s)
   8 partition(s)
sampling time = 0.01

The controller is for hybrid systems (tracking)
[2-norm]

This is a state-feedback controller.

Type "struct(E)" for more details.
>>
```

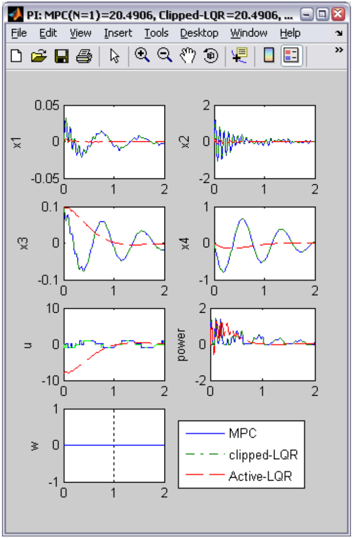
Explicit solution (N=1, x_1=x_2=0):



$$u(x) = \begin{cases} 10.4748x_1 + 0.2446x_2 \ +79.1519x_3 - 3.9235x_4 & \\ \quad (= K_{LQ}) & \text{Regions } \#1, \#6 \\ 0 & \text{Regions } \#2, \#5 \\ (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2) & \text{Regions } \#3, \#7 \\ -1 & \text{Region } \#4 \\ 1 & \text{Region } \#8 \end{cases}$$

EXPLICIT HYBRID MPC

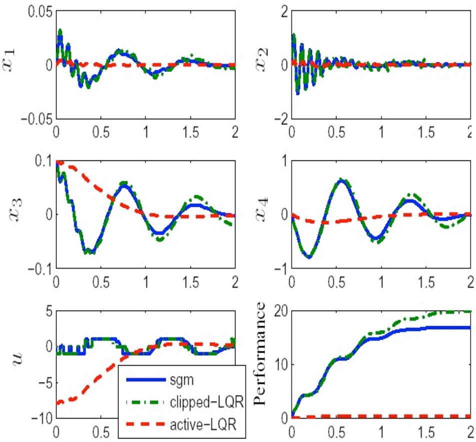




generated
C-code

```
#define EXPCON_NU 1
#define EXPCON_NX 4
#define EXPCON_NY 1
#define EXPCON_TS 0.01000000
#define EXPCON_REG 8
#define EXPCON_NTH 4
#define EXPCON_NYM 4
#define EXPCON_NUC 1
#define EXPCON_NUB 0
#define EXPCON_NGAIN 1
#define EXPCON_NH 21
#define EXPCON_NF 8
static double EXPCON_F[]={
    10.4748,0,0,0,10.4748,0,0,0,-0.244594,0,-
    480.664,0,
    3.92349,0,
    480.664,0
    };

static double EXPCON_G[]={
    0,1e-006,-1e-006,-1,0,0,1e-006,1
```

QUEST OF OPTIMAL SEMIACTIVE SUSPENSIONS



PARAMETER VALUES USED IN SIMULATION

Parameter	Value	Description
T_s	10 ms	Sampling time
ω_s	1.5 Hz	Sprung mass natural frequency
ω_{us}	10 Hz	Wheel-hop natural frequency
ρ	10	Sprung-to-unsprung mass ratio
ζ	0	Damping ratio
σ	1	Maximum force capacity
q_1	1100	Weight on tire deflection
q_3	100	Weight on suspension deflection

TABLE II

SHOCK TEST: MPC COST VALUE FOR DIFFERENT CONTROL HORIZONS SUBJECTED TO I.C.=[0 0 0.1 0]'.

N	MPC	Clipped-LQR	SGM	LQR
1	20.4282	20.4282	17.4944	0.4446
2	20.4054			
3	20.3290			
4	20.1100			
5	19.7380			
10	20.9840			
12	19.3084			
14	18.4842			
15	18.5996			
16	19.3212			
20	18.0764			
30	17.1494			
40	17.1304			

EXPLICIT HYBRID MPC - RESULTS

- Horizon N=1: same as Clipped-LQR !
- Better closed-loop performance for increasing N

Performance Index

N	MPC	Clipped-LQR
1	1.5155	1.5155
5	1.4416	
10	1.5238	
15	1.3083	
20	1.2204	
30	1.1456	
40	1.1462	

N=1, same cost value !

- Simulations with road noise.
- Initial condition x(0)=[0 0 0 0]'
- Simulation time T=20 s, sampling time T_s=10 ms

VEHICLE YAW STABILITY CONTROL

YAW STABILITY CONTROL PROBLEM

- **Problem**: Control vehicle stability while tracking driver's desired trajectory
  - Electronic Stability Control (ESC) (Koibuchi et al., 1996)
  - Active Front Steering (AFS) (Ackermann, 1997)

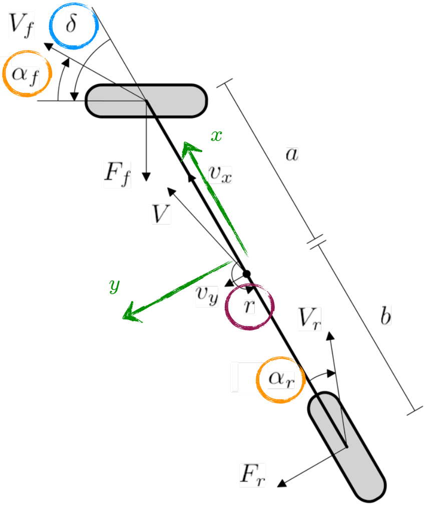
- **Main control objective**: Make the yaw rate of the vehicle track a time-varying reference computed from the driver's steering angle and current velocity

- **Approach**: Consider the steering command as a reference generator and actuate steering and differential braking (=coordinated AFS and ESC action)

  (Bernardini, Di Cairano, Bemporad, Tseng, 2009) (Di Cairano, Tseng, Bernardini, Bemporad, 2012)

VEHICLE MODEL

- **Bicycle model** appropriate in high speed turns (Gillespie, 1992)



- **reference frame** (x,y,z) moving with the vehicle
- **front steering angle** δ [rad]
- **tire slip angles** α_f,α_r [rad]
- **yaw rate** r [rad/s]

$$
\begin{aligned}
\tan(\alpha_f+\delta) &= \frac{v_y+ar}{v_x}\\
\tan\alpha_r &= \frac{v_y-br}{v_x}
\end{aligned}
$$

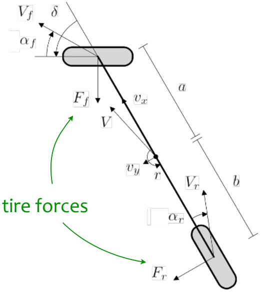
TIRE FORCE MODEL

- **Tire force characteristics:** are nonlinear functions of the **slip angles** and of the **longitudinal slip**

- For a constant longitudinal slip, we use a **piecewise affine model**

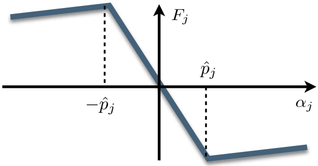
$$F_f(\alpha_f) = \begin{cases} -c_f\alpha_f & \text{if} \quad -\hat{p}_f \leq \alpha_f \leq \hat{p}_f \\ -(d_f\alpha_f + e_f) & \text{if} \quad \alpha_f > \hat{p}_f \end{cases}$$

$$F_r(\alpha_r) = \begin{cases} -c_r\alpha_r & \text{if} \quad -\hat{p}_r \leq \alpha_r \leq \hat{p}_r \\ -(d_r\alpha_r + e_r) & \text{if} \quad \alpha_r > \hat{p}_r \end{cases}$$



- **Critical slip angles** $\hat{p}_f$, $\hat{p}_r$ are threshold values where dynamics switch

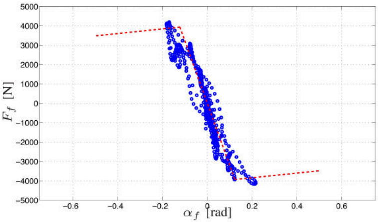
- For symmetry, we can restrict to analyze **clockwise turns** (counter-clockwise turns can be handled by opportunely inverting signs)



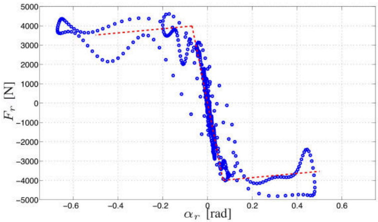
tire forces characteristics

TIRE FORCE MODEL

Sideslip angle-force characteristics



(a) Front tires



(b) Rear tires

Experimental tire data and piecewise linear approximation of the tire

Rear-wheel drive test vehicle equipped with active front steering and differential braking used for experimental validation

VEHICLE DYNAMICAL MODEL

slip angles

$$\begin{aligned}\dot{\alpha}_f &= \frac{\dot{v}_y + a\dot{r}}{v_x} - \dot{\delta}\\ \dot{\alpha}_r &= \frac{\dot{v}_y - b\dot{r}}{v_x}\end{aligned}$$

static yaw rate

$$r = \frac{v_x}{a+b}(\alpha_f - \alpha_r + \delta)$$

overall dynamical model

$$\begin{aligned}\dot{\alpha}_f &= \frac{F_f + F_r}{m v_x} - \frac{v_x}{a+b}(\alpha_f - \alpha_r + \delta) + \frac{a}{v_x I_z}(aF_f - bF_r + Y)\\ \dot{\alpha}_r &= \frac{F_f + F_r}{m v_x} - \frac{v_x}{a+b}(\alpha_f - \alpha_r + \delta) - \frac{b}{v_x I_z}(aF_f - bF_r + Y)\\ r &= \frac{v_x}{a+b}(\alpha_f - \alpha_r + \delta)\end{aligned}$$

$$\dot{v}_y = \frac{F_f \cos\delta + F_r}{m} - r v_x$$

lateral velocity

$$\dot{r} = \frac{aF_f \cos\delta - bF_r + Y}{I_z}$$

yaw rate derivative

VEHICLE DYNAMICAL MODEL - PWA FORM

- **The overall dynamics model** is recast as a **PWA system** by introducing the Boolean variables

$$
\begin{aligned}
\gamma_f = 0 \quad &\leftrightarrow \quad \alpha_f \leq \hat{p}_f \\
\gamma_r = 0 \quad &\leftrightarrow \quad \alpha_r \leq \hat{p}_r
\end{aligned}
$$

- By **discretizing** with sampling period $T_s = 0.1$ s we obtain

$$
\begin{aligned}
x(k+1) \quad &= \quad A_i x(k) + B_i u(k) + f_i \\
y(k) \quad &= \quad C x(k) + D u(k) \\
i \in \{1, \ldots, 4\} \quad &: \quad H_i x(k) \leq K_i
\end{aligned}
$$

where $x = [\alpha_f \ \alpha_r]'$ $u = [Y \ \delta]'$ $y = r$

slip angles

yaw moment

front steering

yaw rate

- **Control goal:**
  stabilize the system at the equilibrium obtained with $\delta(k) = \hat{\delta}(k)$
  while **minimizing** the use of the brake actuator $\left(\hat{Y}(k) = 0\right)$

driver's steering angle

- **Equilibrium** condition in the linear region:

$$
\begin{aligned}
\overset{0}{\cancel{\dot{\alpha}_f}} &= \frac{F_f+F_r}{mv_x} - \frac{v_x}{a+b}(\alpha_f - \alpha_r + \overset{\hat{\delta}}{\cancel{\delta}}) + \frac{a}{v_x I_z}(aF_f - bF_r + \overset{0}{\cancel{Y}}) \\
\cancel{\dot{\alpha}_r} &= \frac{F_f+F_r}{mv_x} - \frac{v_x}{a+b}(\alpha_f - \alpha_r + \cancel{\delta}) - \frac{b}{v_x I_z}(aF_f - bF_r + \cancel{Y})
\end{aligned}
$$

- **Time-varying set-points** are defined using the overall dynamical model

$$
\begin{aligned}
\hat{\alpha}_f &= \frac{m\tilde{v}_x^2 b c_r \hat{\delta}}{m\tilde{v}_x^2(ac_f - bc_r) - c_f c_r (a+b)^2} \\
\hat{\alpha}_r &= \hat{\alpha}_f \frac{ac_f}{bc_r} \\
\hat{r} &= \frac{\tilde{v}_x}{a+b}(\hat{\alpha}_f - \hat{\alpha}_r + \hat{\delta})
\end{aligned}
$$

- Current longitudinal velocity $v_x(k)$ is used to **update set-points**

- **Yaw rate tracking:**
  zero tracking error in steady state is provided by **integral action**

integral of tracking error

yaw rate set-point

$$\begin{array}{rcl} I_r(k+1) & = & I_r(k) + r(k) - r_s(k) \\ r_s(k+1) & = & r_s(k) \end{array}$$

- The **global hybrid dynamical model** of the vehicle is given by

$$\begin{array}{rcl} z(k+1) & = & \tilde{A}_i z(k) + \tilde{B}_i u(k) + \tilde{f}_i \\ y(k) & = & \tilde{C} z(k) + \tilde{D} u(k) \\ i & \in & \{1, \ldots, 4\} : \tilde{H}_i z(k) \leq \tilde{K}_i \end{array}$$

augmented state

$$z = \begin{bmatrix} \alpha_f \\ \alpha_r \\ I_r \\ r_s \end{bmatrix}$$

$$\tilde{A}_i = \begin{bmatrix} \multicolumn{2}{c}{A_i} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \frac{v_x}{a+b} & \frac{-v_x}{a+b} & 1 \quad -1 \\ 0 & 0 & 0 \quad 1 \end{bmatrix}, \quad \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \quad \frac{v_x}{a+b} \\ 0 \quad 0 \end{bmatrix}, \quad \tilde{f}_i = \begin{bmatrix} f_i \\ 0 \\ 0 \end{bmatrix},$$

$$\tilde{C} = \begin{bmatrix} \frac{v_x}{a+b} & \frac{-v_x}{a+b} & 0 & 0 \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} 0 & \frac{v_x}{a+b} \end{bmatrix},$$

$$\tilde{H}_i = \begin{bmatrix} H_i & 0 & 0 \end{bmatrix}, \quad \tilde{K}_i = K_i \ .$$

CONTROL PROBLEM FORMULATION

- The **optimal control problem** solved at every time step k is

$$
\begin{aligned}
\min_{\mathbf{u}_k} \quad & \sum_{j=0}^{N-1} \{(z_{k+j|k} - \hat{z})' Q_z (z_{k+j|k} - \hat{z}) \\
& + (y_{k+j|k} - \hat{y})' Q_y (y_{k+j|k} - \hat{y}) \\
& + (u_{k+j|k} - \hat{u})' Q_u (u_{k+j|k} - \hat{u})\} \\
\text{s.t.} \quad & z_{k|k} = z(k) \\
& \text{hybrid dynamics} \\
& \text{state and input constraints}
\end{aligned}
$$

slip angles tracking & int. action

yaw rate tracking

penalty on actuators action

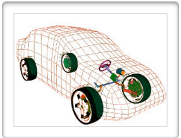
MIQP

- State and input **constraints**:

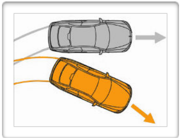
$$
\begin{aligned}
[z]_1(k) &\geq -\hat{p}_f \quad \text{front slip angle} \qquad & |[u]_1(k)| &\leq 1000 \text{ [Nm]} \quad \text{yaw moment} \\
[z]_2(k) &\geq -\hat{p}_r \quad \text{rear slip angle} \qquad & |[u]_2(k)| &\leq 0.35 \text{ [rad]} \quad \text{front steering}
\end{aligned}
$$

SIMULATION SETUP

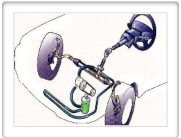
- Simulations run on a **nonlinear vehicle model** including:



longitudinal and lateral vehicle dynamics



yaw rate dynamics



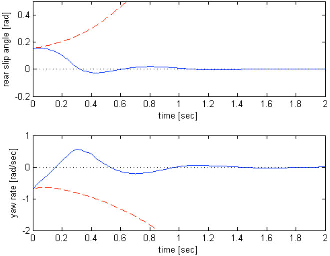
steering actuation dynamics

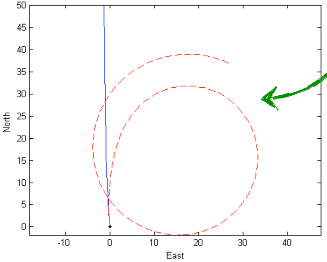
- Driver's steering command **constant** over the simulation interval

- **Controller setup** after calibration:
  - Prediction horizon $N = 3$
  - Weight matrices $Q_z = \begin{bmatrix} .1 & 0 & 0 & 0 \\ 0 & .1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad Q_u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Q_y = 1$

- **Stability analysis** under nominal conditions (with $\hat{\delta} = 0$):

open-loop *vs* closed-loop



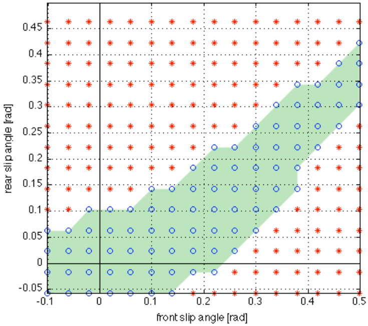


open-loop
spin

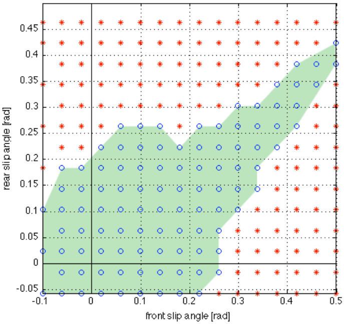
- Controller has to cope with **linearization errors**

- **Stability analysis** under nominal conditions (with $\hat{\delta} = 0$):

open-loop



closed-loop

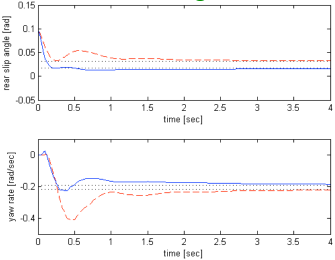


stable initial conditions
unstable initial conditions

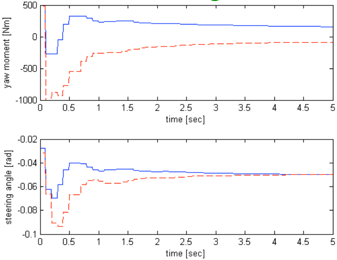
closed-loop provides a **larger** stability region

- **Robustness analysis** w.r.t. model mismatches (with $\hat{\delta} = -0.05$):
  - nominal longitudinal velocity $\hat{v}_x = 20\ \mathrm{m/s}$
  - real longitudinal velocity $v_x = 15\ \mathrm{m/s}$ and $v_x = 25\ \mathrm{m/s}$

state signals



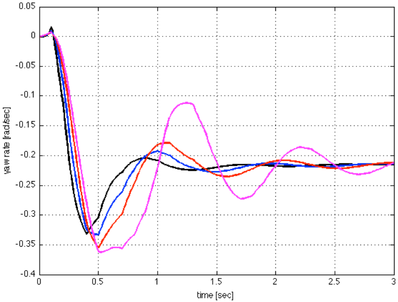
control signals



- Stability and **fast tracking response** are provided

CLOSED-LOOP SIMULATIONS RESULTS

- **Turns on slippery road surface** (with $\hat{\delta} = -0.05$):
  - several values tested: $\hat{s} = 0$, $s = 0.20$, $s = 0.30$, $s = 0.35$

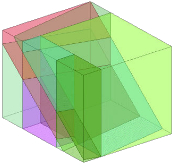


- Good **degree of robustness** with respect to slip mismatches

COMPUTATIONAL ISSUES

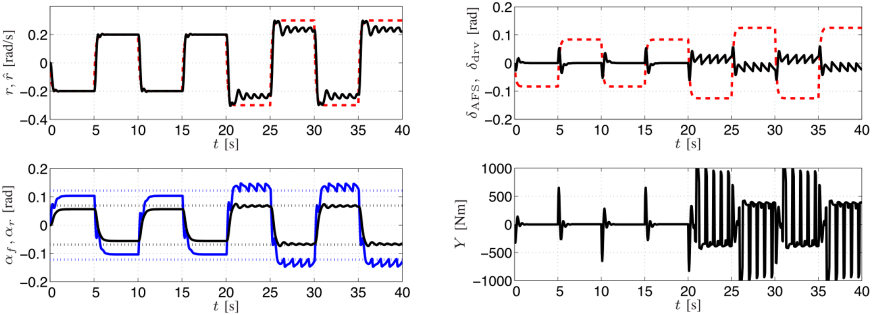
- **Computational issues:**
  - MPC-based approach is **viable for experimental tests** (average CPU time 17ms, worst-case CPU time 63 ms in MATLAB), but requires a MIQP **solver** in the ECU

- **Explicit solution** of hybrid MPC control problem:
  - Exploits **multiparametric programming** techniques to provide a description of the control law as an **explicit** function of the state
  - All the computation is executed **off-line**, only simple set-membership tests and function evaluations are performed on-line to compute the control action
  - However, the explicit solution requires **memory** (around **5000 polytopes** to be stored, in this case!)
  - An **approximation** of the solution is needed for real implementation



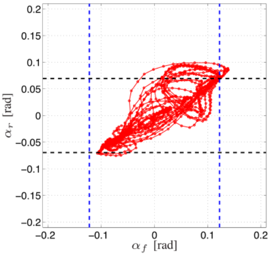
SWITCHED MPC SOLUTION

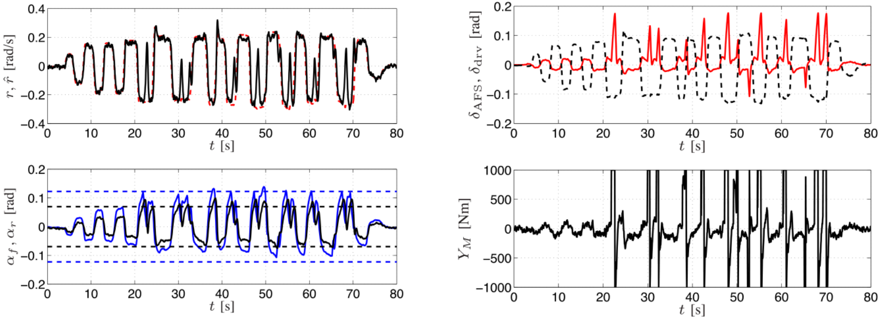
- Simpler solution: assume PWA mode remains constant in prediction
- Design a linear MPC controller for each mode, make it explicit
- 4 linear explicit MPC's are enough (linear/saturation $\times$ front/rear)
- Simulation results:



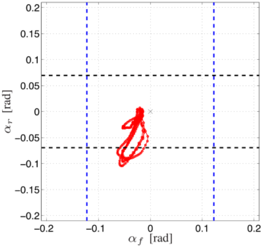
EXPERIMENTAL RESULTS

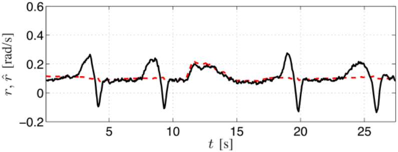
- Experimental results (**slalom test**):

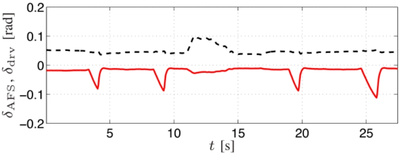


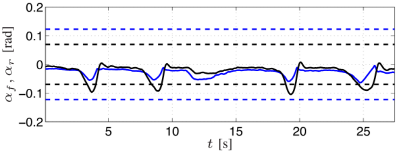


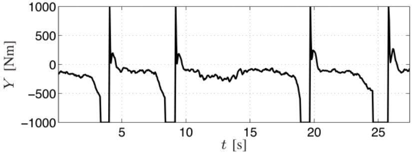
- Experimental results (**stability recovery test**):





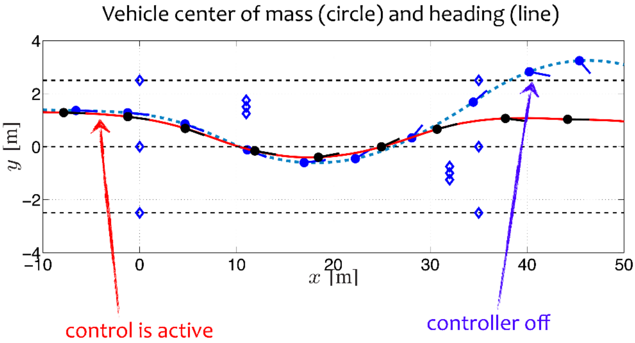






EXPERIMENTAL RESULTS

- Experimental results (**double-lane change**):



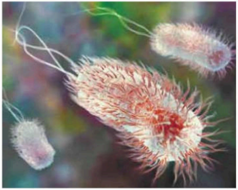
- **Alternative**: use switched linear MPC and mpQP (Di Cairano, Tseng, 2010)

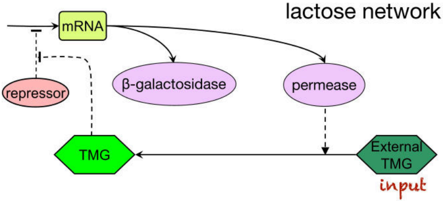
HYBRID MPC OF INDUCTION OF ESCHERICHIA COLI

HYBRID MPC OF INDUCTION OF ESCHERICHIA COLI

(Julius, Sakar, Bemporad, Pappas, 2007)

- **Goal:** control the **lactose regulation system** of a colony of *E. coli*

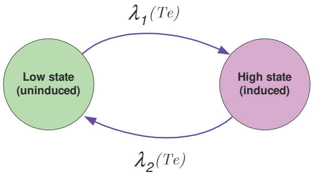




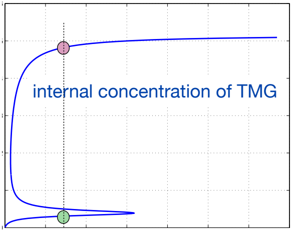
TMG = *thio-methyl galactosidase* concentration

- Model, measurements, and actuation are at the **entire colony** level

- **Bistable** lactose regulation system of E. coli



two-state Markov chain



Te = external concentration of TMG

- The probabilities x_{lo}, x_{hi} to be in low/high state satisfy the dynamics

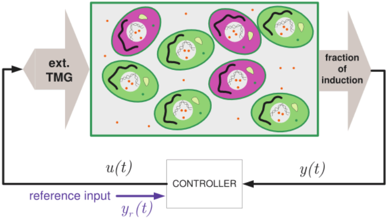
$$\frac{d}{dt}\begin{bmatrix} x_{\text{lo}} \\ x_{\text{hi}} \end{bmatrix} = \begin{bmatrix} -\lambda_1(T_e) & \lambda_2(T_e) \\ \lambda_1(T_e) & -\lambda_2(T_e) \end{bmatrix}\begin{bmatrix} x_{\text{lo}} \\ x_{\text{hi}} \end{bmatrix}$$

- Transition rates λ_1, λ_2 modeled as **piecewise constant** functions of T_e

$T_e\,[10^{-3}\text{mM}]$	$\lambda_1(T_e)\,[\text{min}^{-1}]$	$\lambda_2(T_e)\,[\text{min}^{-1}]$
$[1.4, 1.5)$	$8.68 \cdot 10^{-4}$	$5.91 \cdot 10^{-3}$
$[1.5, 1.6)$	$9.27 \cdot 10^{-4}$	$3.61 \cdot 10^{-3}$
$[1.6, 1.7)$	$1.13 \cdot 10^{-3}$	$2.36 \cdot 10^{-3}$
$[1.7, 1.8)$	$1.39 \cdot 10^{-3}$	$1.54 \cdot 10^{-3}$
$[1.8, 1.9)$	$1.67 \cdot 10^{-3}$	$9.53 \cdot 10^{-4}$
$[1.9, 2.0)$	$1.93 \cdot 10^{-3}$	$5.54 \cdot 10^{-4}$

HYBRID MPC OF INDUCTION OF ESCHERICHIA COLI

- Hybrid MPC problem
  - switched linear system
  - constraints on input T_e and dT_e/dt
  - penalties on tracking error $y-y_r$ and input rate dT_e/dt



- Closed-loop results
  - MPC controller developed with **Hybrid Toolbox** in MATLAB
  - Mixed-Integer Linear Program solver GLPK
  - solution time: 32 ms (worst case=**280 ms**) on 1.2 GHz laptop
  - sampling time = **10 min**