Examples of hybrid MPC

Hybrid MPC for cruise control

GOAL:

command gear ratio, gas pedal, and brakes to **track** a desired speed and minimize consumption

Hybrid Model

- Vehicle dynamics

$$m\ddot{x} = F_e - F_b - \beta\dot{x}$$

$\dot{x}$ = vehicle speed

F_e= traction force

F_b= brake force

⇨ discretized with sampling time $T_s = 0.5$ s

- Transmission kinematics

$$\omega = \frac{R_g(i)}{k_s}\dot{x}$$

$$F_e = \frac{R_g(i)}{k_s}M$$

ω = engine speed

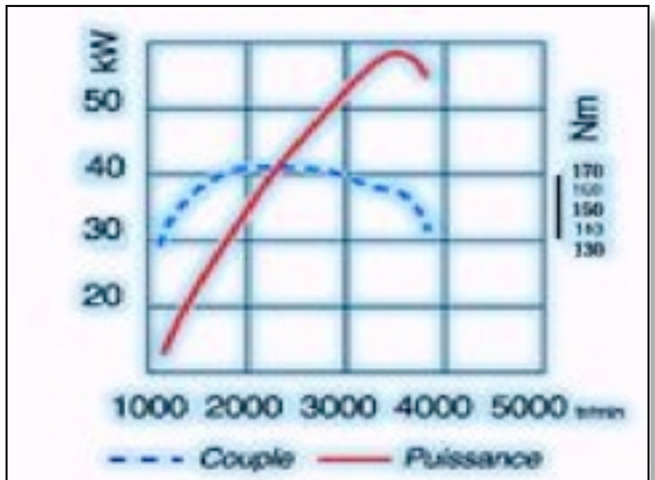
M = engine torque

i = gear

Hybrid Model

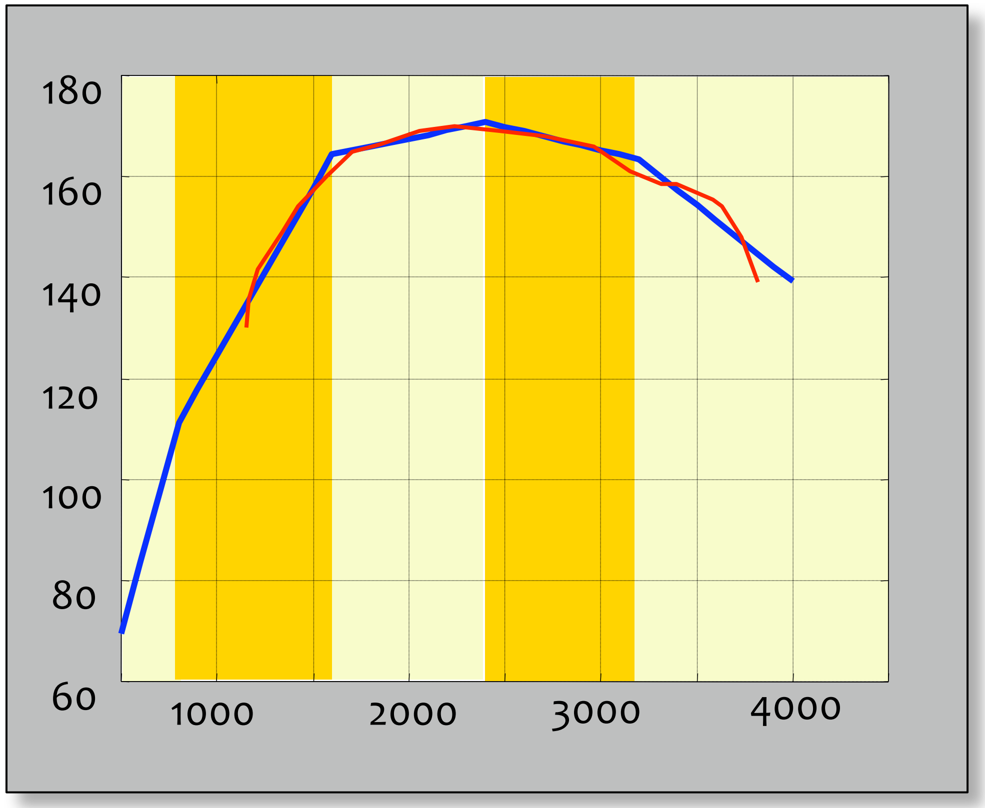
- Engine torque $-C_e^-(\omega) \leq M \leq C_e^+(\omega)$

- Max engine torque $C_e^+(\omega)$



Piecewise-linearization

(PWL Toolbox, Julián, 2000)



requires: 4 binary aux variables
4 continuous aux variables

(Note: in this case PWL function is convex $\Rightarrow$ could be handled by linear constraints without introducing any binary variable !)

- Min engine torque $C_e^-(\omega) = \alpha_1\omega + \beta_1$

Hybrid Model

- Gear selection: for each gear #i,

  define a binary input $g_i \in \{0, 1\}$

- Gear selection (traction force):

$$F_e = \frac{R_g(i)}{k_s} M$$

depends on gear #i

define auxiliary continuous variables:

$$\text{IF } g_i = 1 \text{ THEN } F_{ei} = \frac{R_g(i)}{k_s} M \text{ ELSE } 0$$

$$\Longrightarrow \quad F_e = F_{eR} + F_{e1} + F_{e2} + F_{e3} + F_{e4} + F_{e5}$$

- Gear selection (engine/vehicle speed):

$$\omega = \frac{R_g(i)}{k_s} \dot{x}$$

similarly, also requires 6 auxiliary continuous variables

Hysdel Model

```
SYSTEM car {

INTERFACE {
        STATE { REAL position, speed; }
        INPUT { REAL torque, F_brake;
                BOOL gear1, gear2, gear3, gear4, gear5, gearR;  }

        PARAMETER{
                REAL mass = 1020; /* kg */
                REAL beta_friction = 25; /* W/m*s */
                REAL Rgear1 = 3.7271; REAL Rgear2 = 2.048;
                REAL Rgear3 = 1.321; REAL Rgear4 = 0.971;
                REAL Rgear5 = 0.756; REAL RgearR = -3.545;
                REAL wheel_rim = 14;        /* in */
                ...

                }  }
IMPLEMENTATION {
        AUX {REAL Fe1, Fe2, Fe3, Fe4, Fe5, FeR;
             REAL w1, w2, w3, w4, w5, wR;
             BOOL dPWL1,dPWL2,dPWL3,dPWL4;
             REAL DCe1,DCe2,DCe3,DCe4; }

        AD  { dPWL1  = wPWL1-(w1+w2+w3+w4+w5+wR)<=0;
              dPWL2  = wPWL2-(w1+w2+w3+w4+w5+wR)<=0;
              dPWL3  = wPWL3-(w1+w2+w3+w4+w5+wR)<=0;
              dPWL4  = wPWL4-(w1+w2+w3+w4+w5+wR)<=0; }


        DA  { Fe1 = {IF gear1  THEN torque/speed_factor*Rgear1;
              Fe2 = {IF gear2  THEN torque/speed_factor*Rgear2;
              Fe3 = {IF gear3  THEN torque/speed_factor*Rgear3;
              Fe4 = {IF gear4  THEN torque/speed_factor*Rgear4;
              Fe5 = {IF gear5  THEN torque/speed_factor*Rgear5;
              FeR = {IF gearR  THEN torque/speed_factor*RgearR;

              w1  = {IF gear1  THEN speed/speed_factor*Rgear1;
              w2  = {IF gear2  THEN speed/speed_factor*Rgear2;
              w3  = {IF gear3  THEN speed/speed_factor*Rgear3;
              w4  = {IF gear4  THEN speed/speed_factor*Rgear4;
              w5  = {IF gear5  THEN speed/speed_factor*Rgear5;
              wR  = {IF gearR  THEN speed/speed_factor*RgearR;

              DCe1 = {IF dPWL1 THEN (aPWL2-aPWL1)+(bPWL2-bPWL1)*(w1+w2+w3+w4+w5+wR);
              DCe2 = {IF dPWL2 THEN (aPWL3-aPWL2)+(bPWL3-bPWL2)*(w1+w2+w3+w4+w5+wR);
              DCe3 = {IF dPWL3 THEN (aPWL4-aPWL3)+(bPWL4-bPWL3)*(w1+w2+w3+w4+w5+wR);
              DCe4 = {IF dPWL4 THEN (aPWL5-aPWL4)+(bPWL5-bPWL4)*(w1+w2+w3+w4+w5+wR);
              }

        CONTINUOUS { position = position+Ts*speed;
                     speed = speed+Ts/mass*(Fe1+Fe2+Fe3+Fe4+Fe5+FeR-
                                 F_brake-beta_friction*speed);

        MUST    {  wemin <= w1+w2+w3+w4-w5-wR;
                   w1+w2+w3+w4+w5+wR <= wemax;
                   -F_brake <=0; /* brakes cannot accelerate ! */
                   F_brake <= max_brake_force;

                   -torque-(alpha1+beta1*(w1+w2+w3+w4+w5+wR)) <=0;
                   torque-(aPWL1+bPWL1*(w1-w2-w3+w4+w5+wR)-DCe1+DCe2+DCe3+DCe4)-1<=0;

                   -(gear1-gear2+gear3+gear4+gear5+gearR)<=-1;
                   (gear1+gear2+gear3+gear4+gear5+gearR)<=1;
                   Fe1+Fe2-Fe3+Fe4+Fe5+FeR <= max_force;
                   -Fe1-Fe2-Fe3-Fe4-Fe5-FeR <= -max_force;

                   dPWL4 -> dPWL3; dPWL4 -> dPWL2;
                   dPWL4 -> dPWL1; dPWL3 -> dPWL2;
                   dPWL3 -> dPWL1; dPWL2 -> dPWL1;   }
```

go to demo **/demos/cruise/init.m**

- MLD model

$$\begin{aligned} x(t+1) &= Ax(t) + B_1 u(t) + B_2\delta(t) + B_3 z(t) + B_5 \\ y(t) &= Cx(t) + D_1 u(t) + D_2\delta(t) + D_3 z(t) + D_5 \\ E_2\delta(t) + E_3 z(t) &\leq E_4 x(t) + E_1 u(t) + E_5 \end{aligned}$$

- 2 continuous states: $x,\ v$ (vehicle position and speed)
- 2 continuous inputs: M, F_b (engine torque, brake force)
- 6 binary inputs: g_R, g_1, g_2, g_3, g_4, g_5 (gears)
- 1 continuous output: v (vehicle speed)
- 16 auxiliary continuous vars: (6 traction force, 6 engine speed, 4 PWL max engine torque)
- 4 auxiliary binary vars: (PWL max engine torque breakpoints)
- 96 mixed-integer inequalities

- Max-speed controller

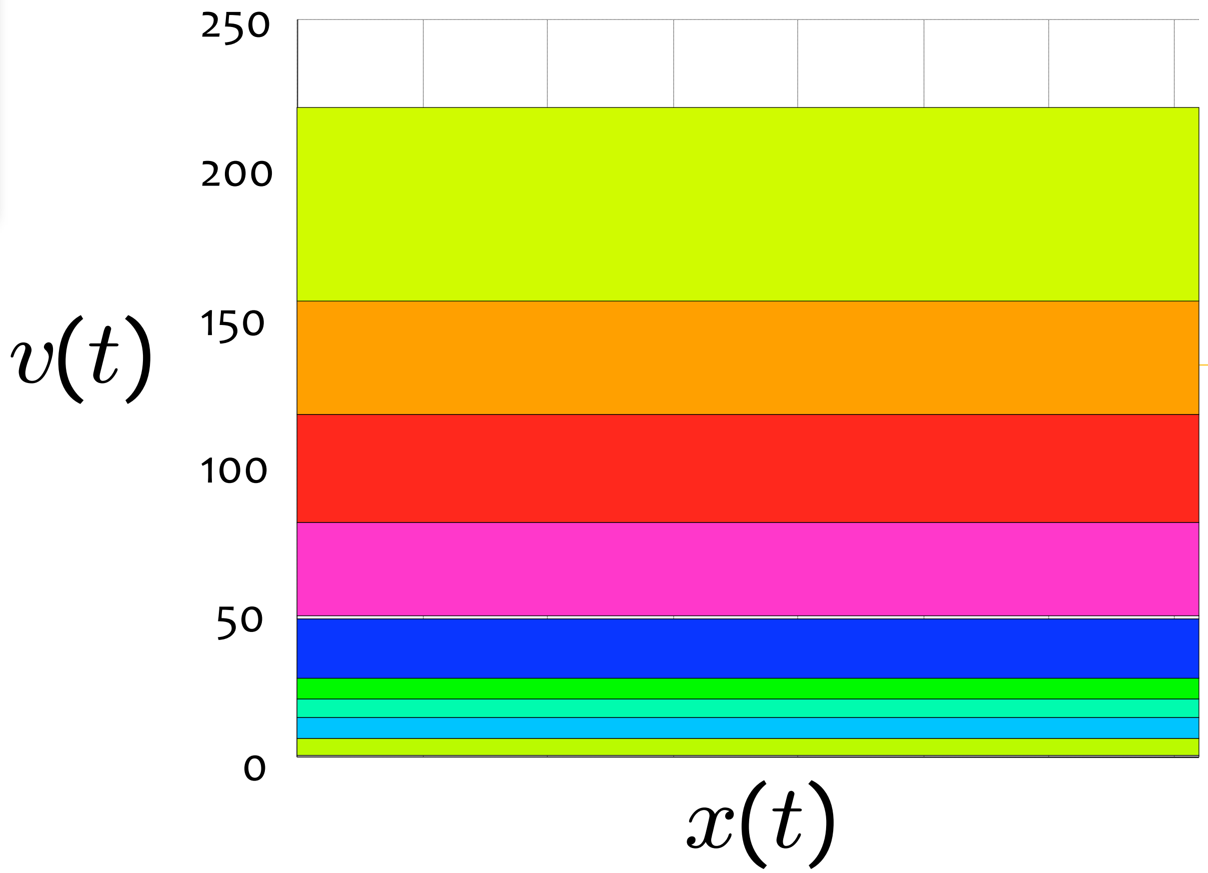
$$\max_{u_t} \quad J(u_t, x(t)) \triangleq v(t+1|t)$$
$$\text{s.t.} \quad \begin{cases} \text{MLD model} \\ x(t|t) = x(t) \end{cases}$$

Objective: maximize speed

(to reproduce max acceleration plots)

MILP optimization problem

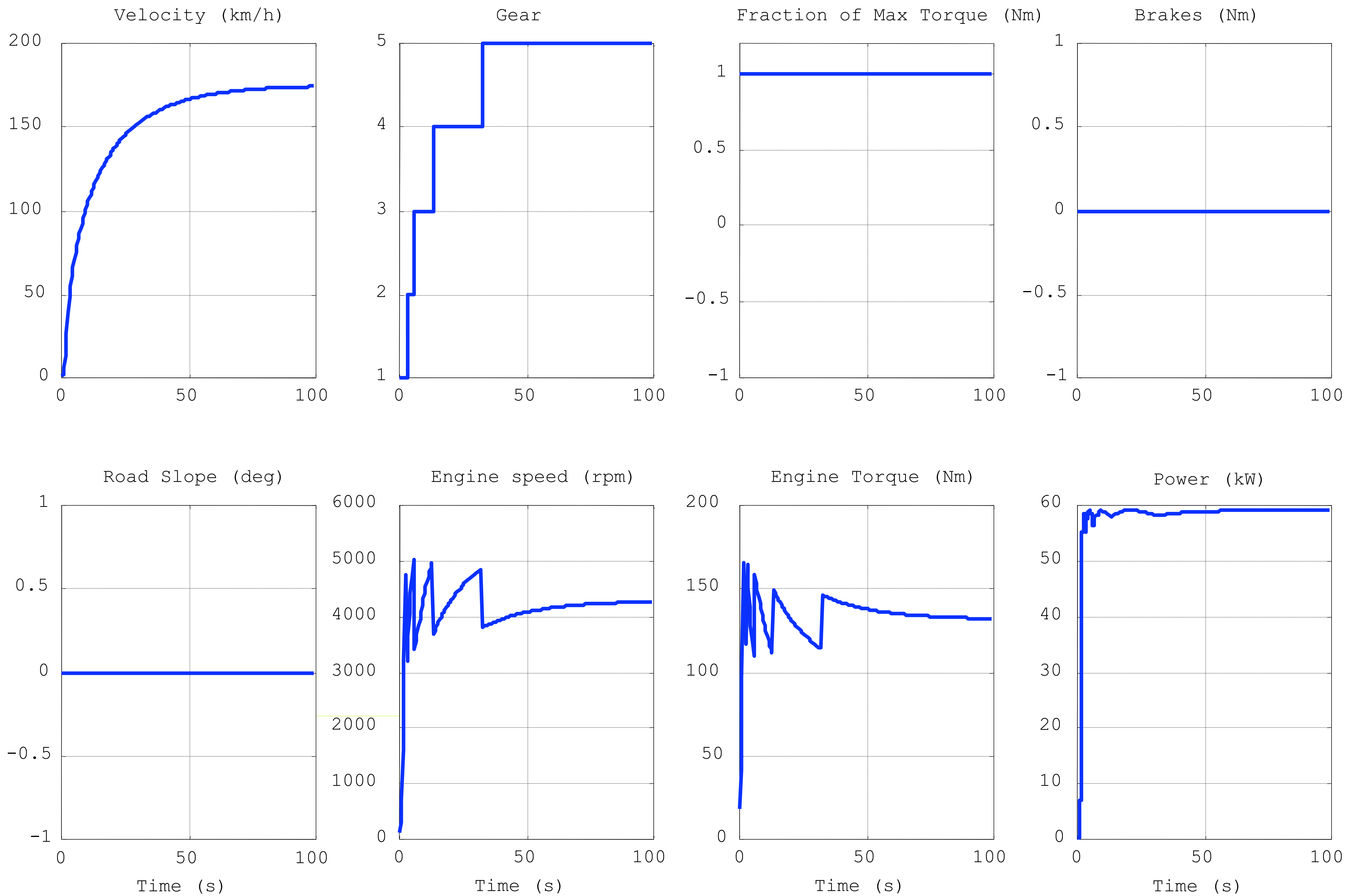
Linear constraints	96
Continuous variables	18
Binary variables	10
Parameters	1
Time to solve mp-MILP (Sun Ultra 10)	45 s
Number of regions	**11**



($x(t)$ is irrelevant)

(Parameters: Renault Clio 1.9 DTI RXE)

- Max-speed controller



Hybrid Controller

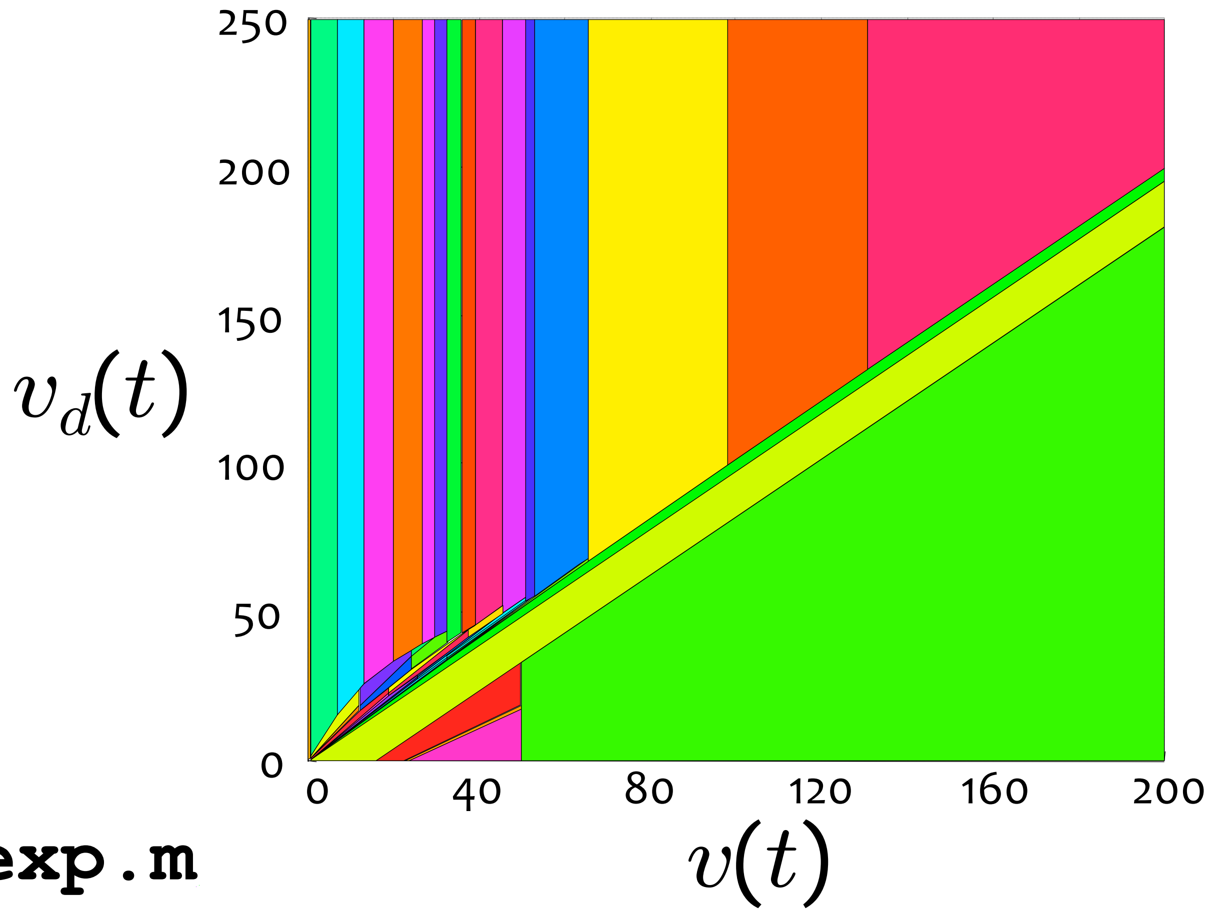
- Tracking controller

$$\min_{u_t} \quad J(u_t, x(t)) \triangleq |v(t+1|t) - v_d(t)| + \rho|\omega|$$

$$\text{s.t.} \quad \begin{cases} \text{MLD model} \\ x(t|t) = x(t) \end{cases}$$

MILP optimization problem

Linear constraints	98
Continuous variables	19
Binary variables	10
Parameters	2
Time to solve mp-MILP (PC 850Mhz)	43 s
Number of regions	**49**

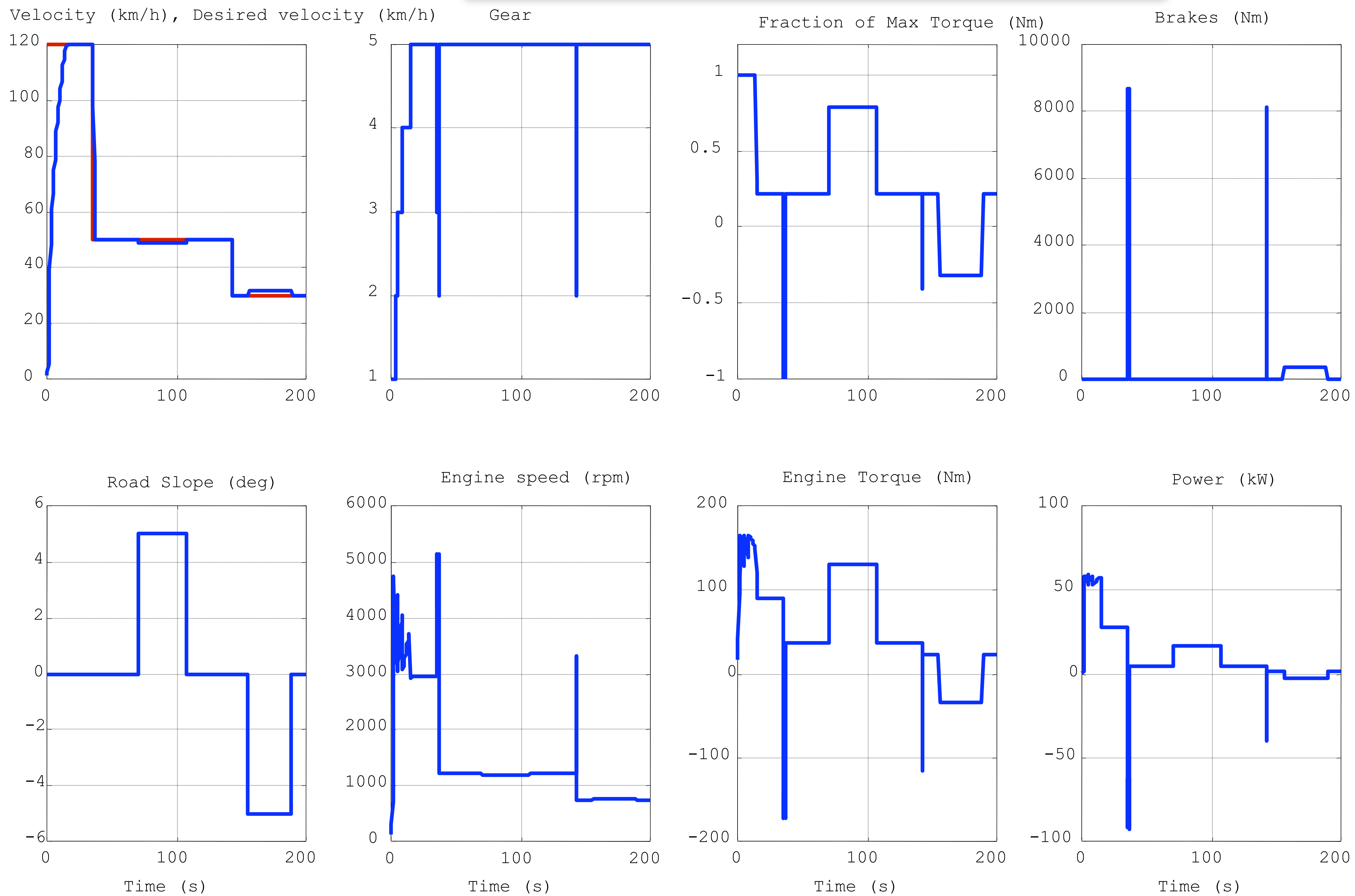


go to demo /**demos/cruise/init_exp.m**

Hybrid Controller

- Tracking controller

$$\min_{u_t} |v(t+1|t) - v_d(t)| + \rho|\omega| \qquad \rho = 0.001$$



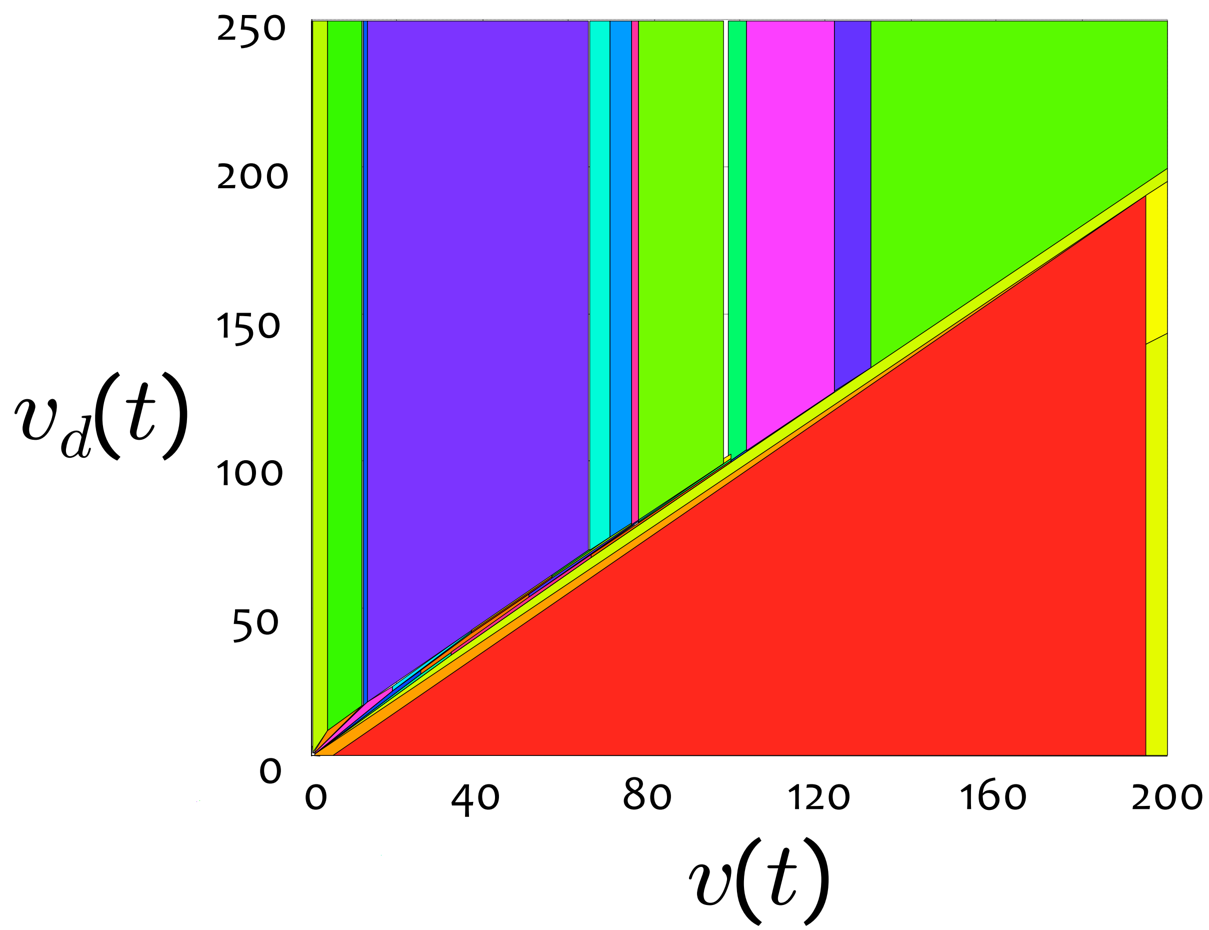
- Smoother tracking controller

$$\min_{u_t} \quad J(u_t, x(t)) \triangleq |v(t+1|t) - v_d(t)| + \rho|\omega|$$

$$\text{s.t.} \quad \begin{cases} \text{MLD model} \\ |v(t+1|t) - v(t)| \leq a_{\max} T_s \\ x(t|t) = x(t) \end{cases}$$

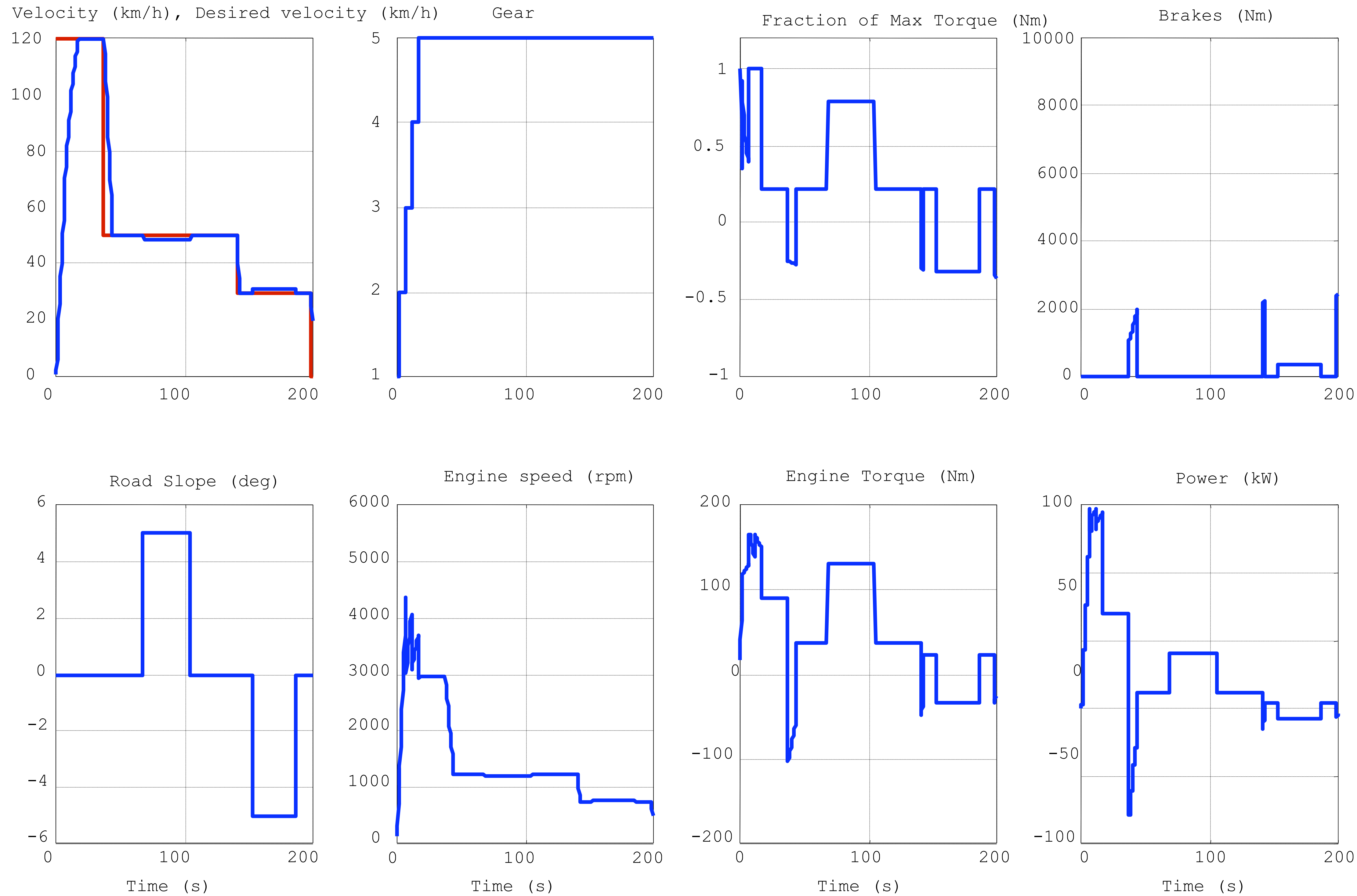
MILP optimization problem

Linear constraints	100
Continuous variables	19
Binary variables	10
Parameters	2
Time to solve mp-MILP (PC 850Mhz)	47 s
Number of regions	**54**



Hybrid Controller

- Smoother tracking controller



Traction Control System

Vehicle Traction Control

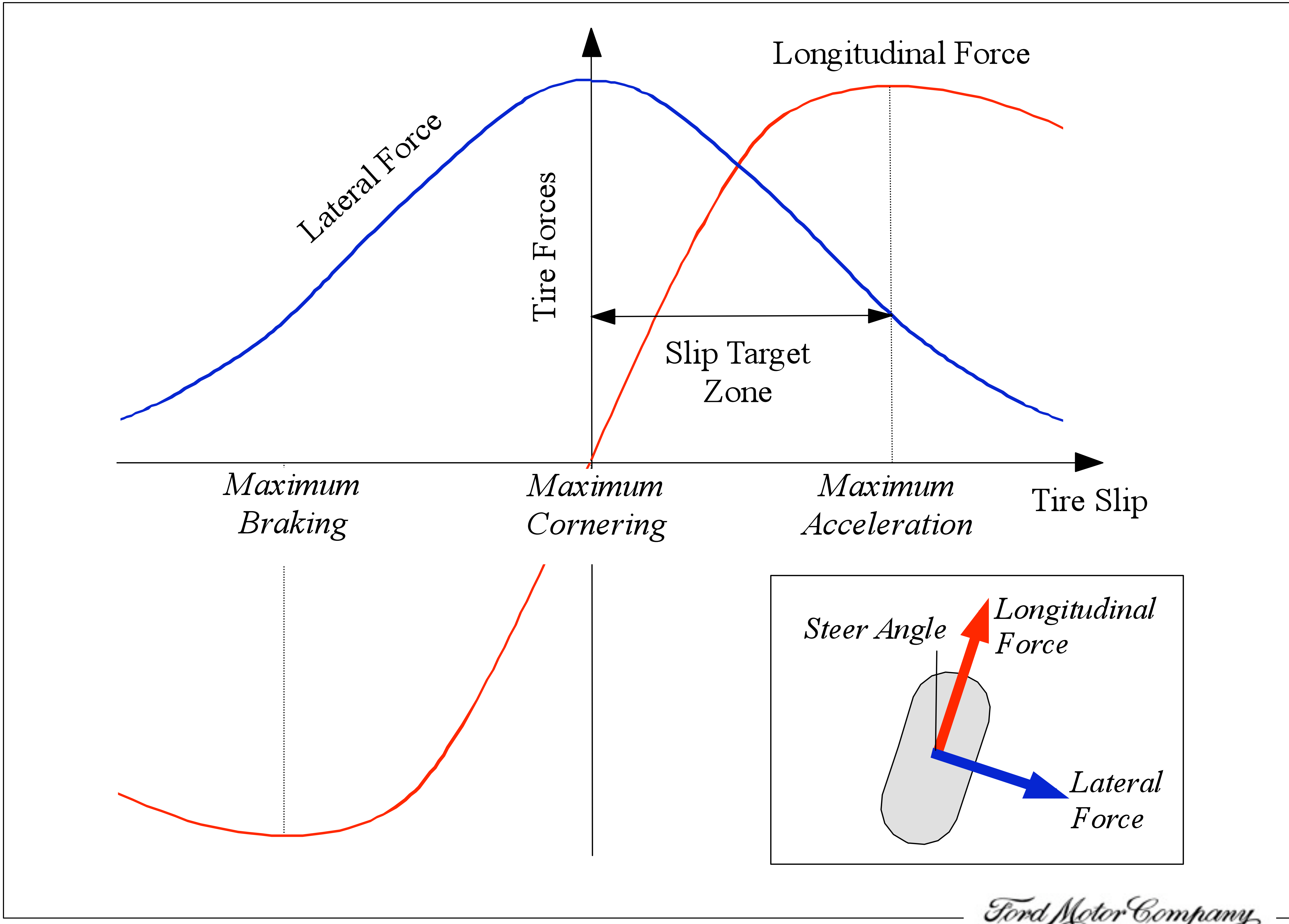
Improve driver's ability to control a vehicle
under adverse external conditions (wet or icy roads)

Model
nonlinear, uncertain,
constraints

Controller
suitable for real-time
implementation

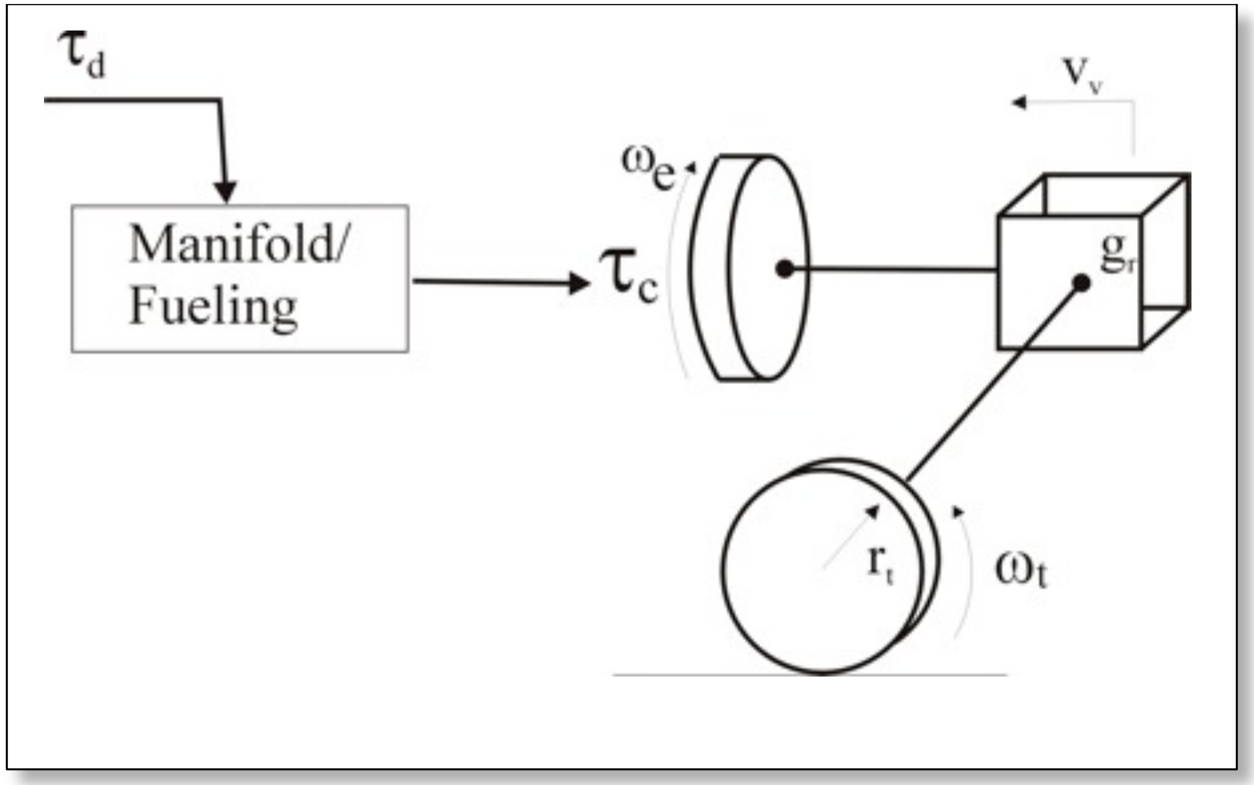
MLD hybrid framework + optimization-based control strategy

Tire Force Characteristics



Simple Traction Model

(Borrelli, Bemporad, Fodor, Hrovat, 2006)

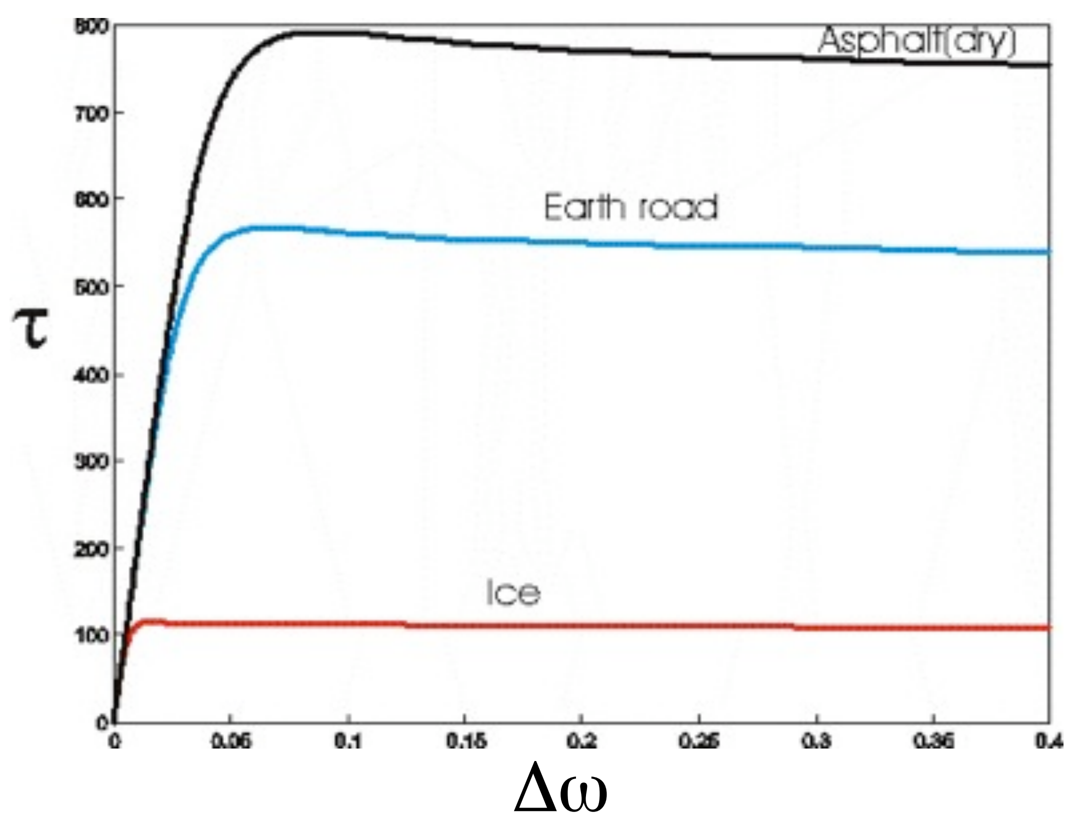


- Mechanical system

$$
\begin{aligned}
\dot{\omega}_e &= \frac{1}{J_e}\left(\tau_c - b_e\omega_e - \frac{\tau_t}{g_r}\right) \\
\dot{v}_v &= \frac{\tau_t}{m_v r_t}
\end{aligned}
$$

- Manifold/fueling dynamics

$$\tau_c = b_i \tau_d(t - \tau_f)$$

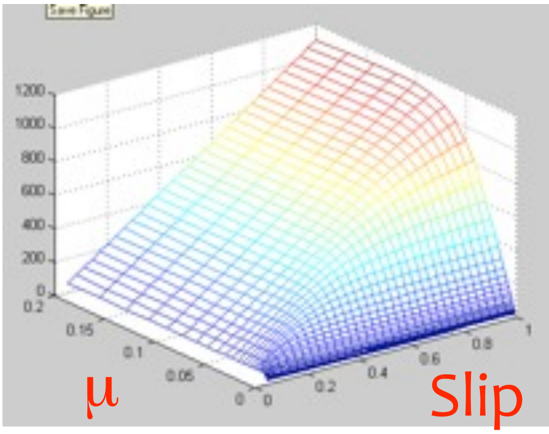


- Tire torque τ_t is a function of slip $\Delta\omega$ and road surface adhesion coefficient μ

$$\Delta\omega = \frac{\omega_e}{g_r} - \frac{v_v}{r_t}$$

wheel slip

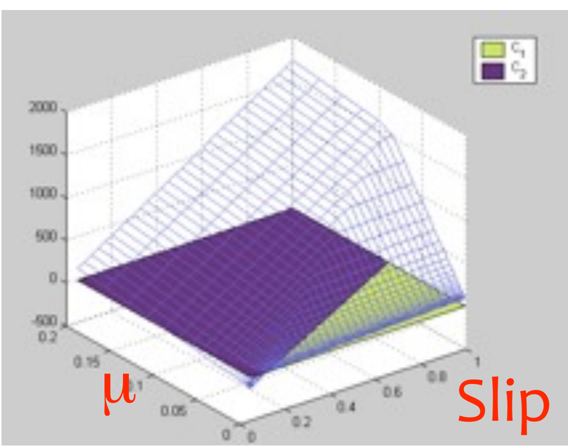
Hybrid model

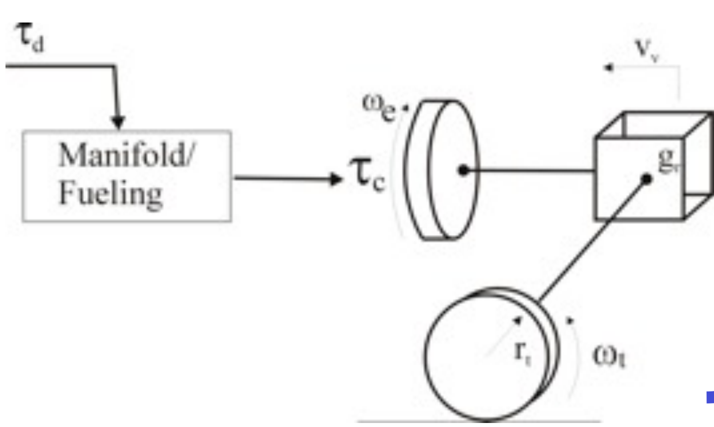


Nonlinear tire torque $\tau_t = f(\Delta\omega\ ,\ \mu)$

PWA Approximation

(PWL Toolbox, Julian, 2000)





HYSDEL

Mixed-Logical
Dynamical (MLD)
Hybrid Model
(discrete time)

MLD model

$$
\begin{aligned}
x(t+1) &= Ax(t) + B_1u(t) + B_2\delta(t) + B_3z(t) + B_5 \\
y(t) &= Cx(t) + D_1u(t) + D_2\delta(t) + D_3z(t) + D_5 \\
E_2\delta(t) + E_3z(t) &\leq E_4x(t) + E_1u(t) + E_5
\end{aligned}
$$

State x(t)	4	variables
Input u(t)	1	variable
Aux. Binary vars δ(t)	1	variable
Aux. Continuous vars z(t)	3	variables
Mixed-integer inequalities	14	

→ The MLD matrices are automatically generated in Matlab format by HYSDEL

go to demo `/demos/traction/init.m`

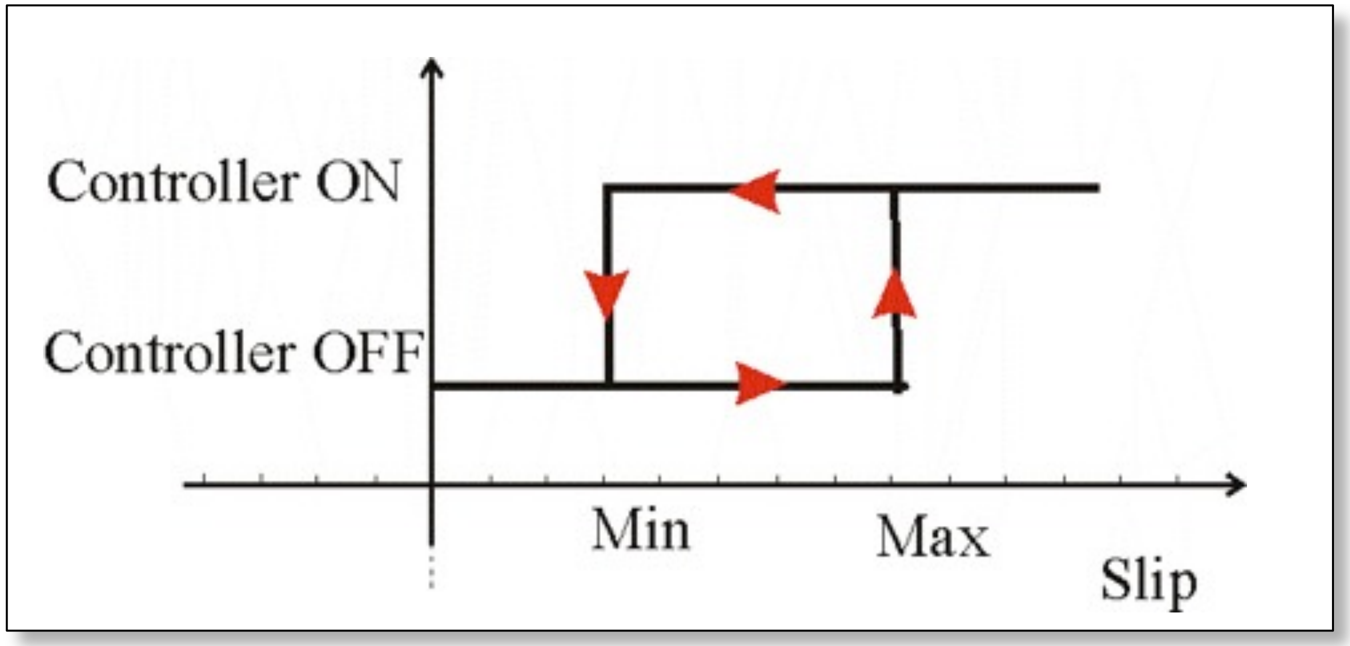
Performance and constraints

- Control objective:

$$\begin{array}{ll} \min & \sum_{k=0}^{N} |\Delta\omega(t+k|t) - \Delta\omega_{\text{des}}| \\ \text{s.t.} & \text{MLD dynamics} \end{array}$$

- Constraints:
  - Limits on the engine torque: $-20 \text{ Nm} \leq \tau_d \leq 176 \text{ Nm}$

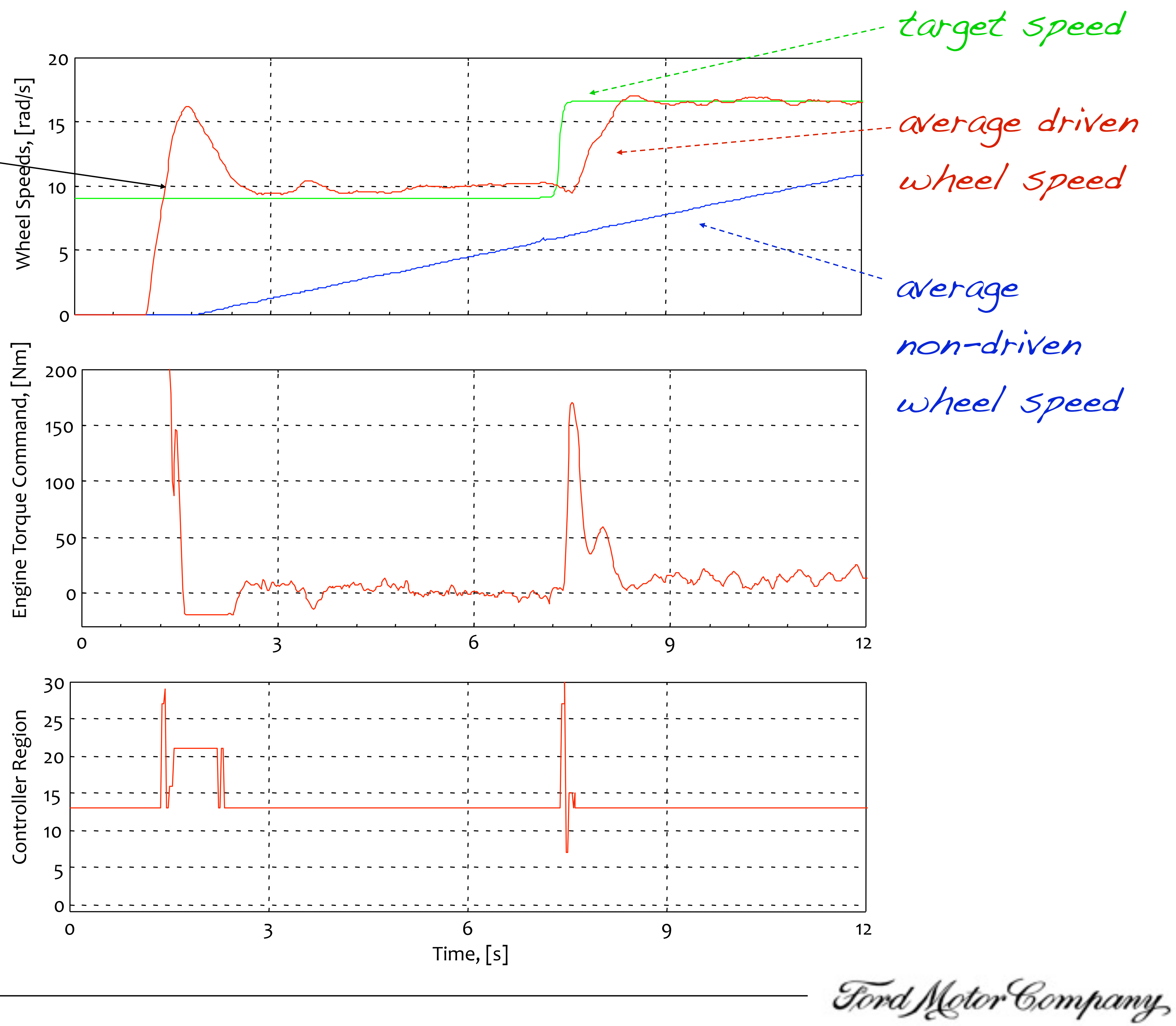
- Note: a logic constraint (hysteresis) may be also taken into account



Experimental results

controller is triggered ON

(250 ms delay from commanded to actual engine torque → initial overspin)



Experiments

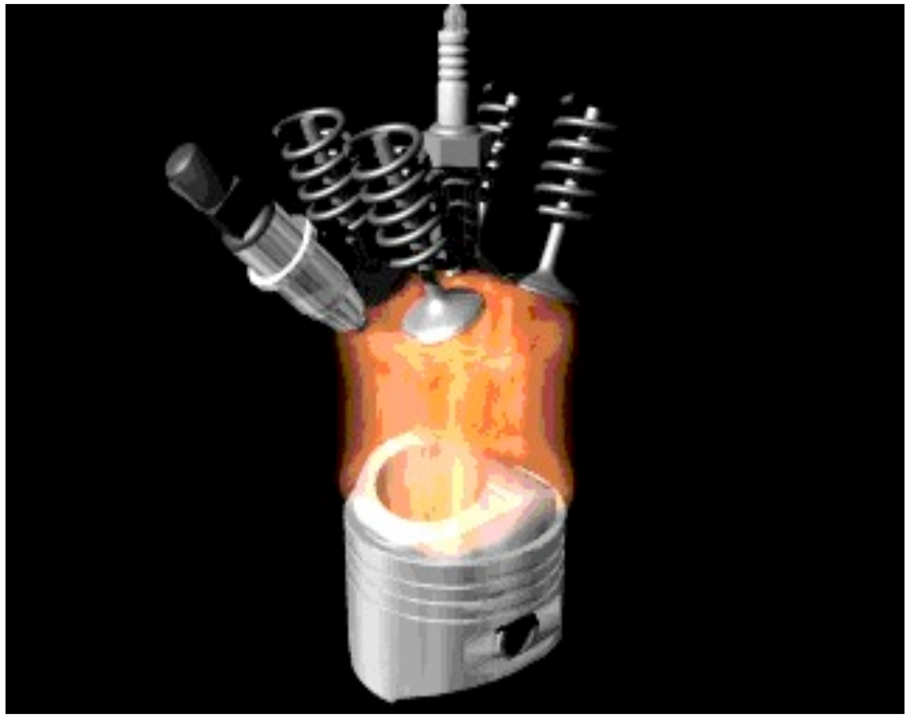
indoor ice arena
($\mu \approx 0.2$)

2000 Ford Focus
2.0l 4-cyl engine
5-speed manual
transmission

- 504 regions
- 20ms sampling time
- Pentium 266Mhz + Labview

Ford Motor Company

Hybrid Control of a DISC Engine



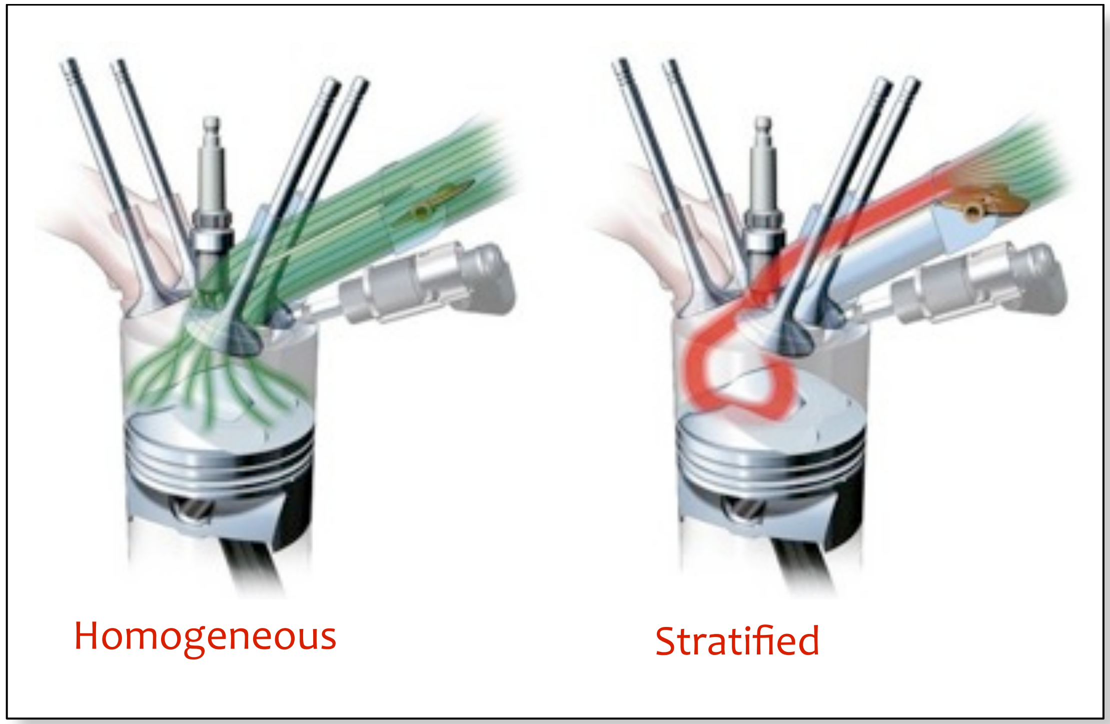
(Photo: Courtesy Mitsubishi)

(N. Giorgetti, G. Ripaccioli, Bemporad, I. Kolmanovsky and D. Hrovat)

DISC engine control problem

Objective: Develop a controller for a Direct-Injection Stratified Charge (DISC) engine that:

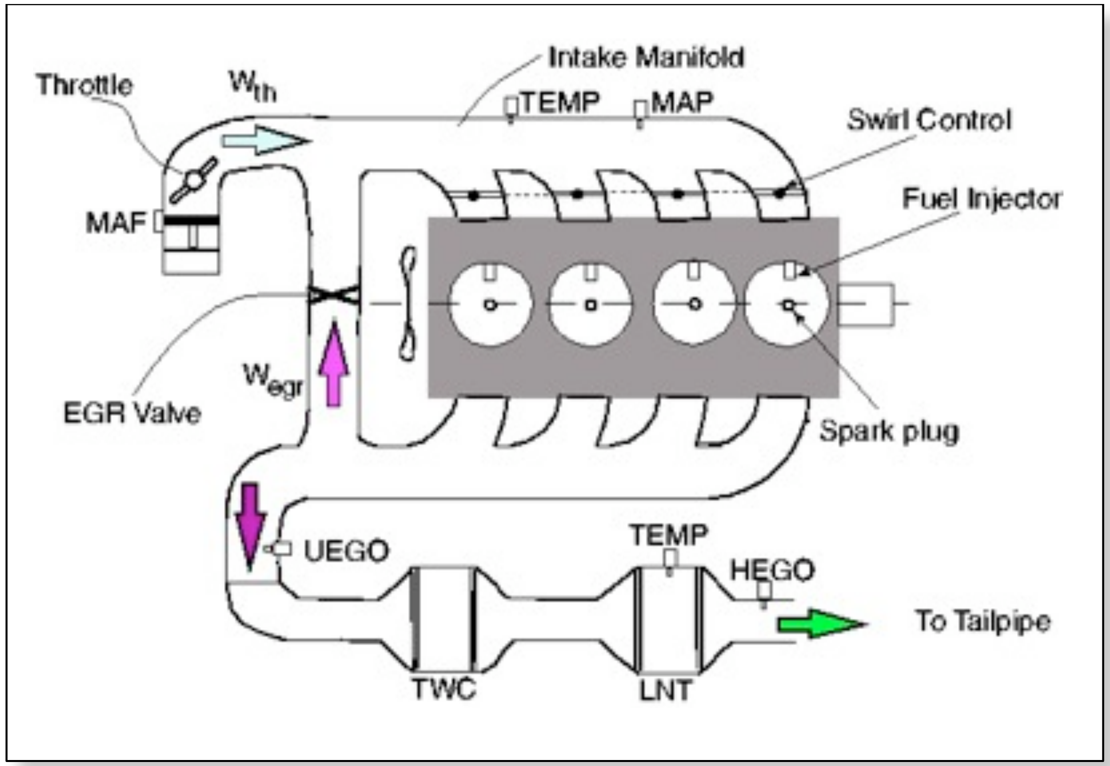
- Automatically chooses operating mode (homogeneous/stratified)
- Can cope with nonlinear dynamics
- Handles constraints (on A/F ratio, air-flow, spark)
- Achieves optimal performance (tracking of desired torque and A/F ratio)



DISC engine

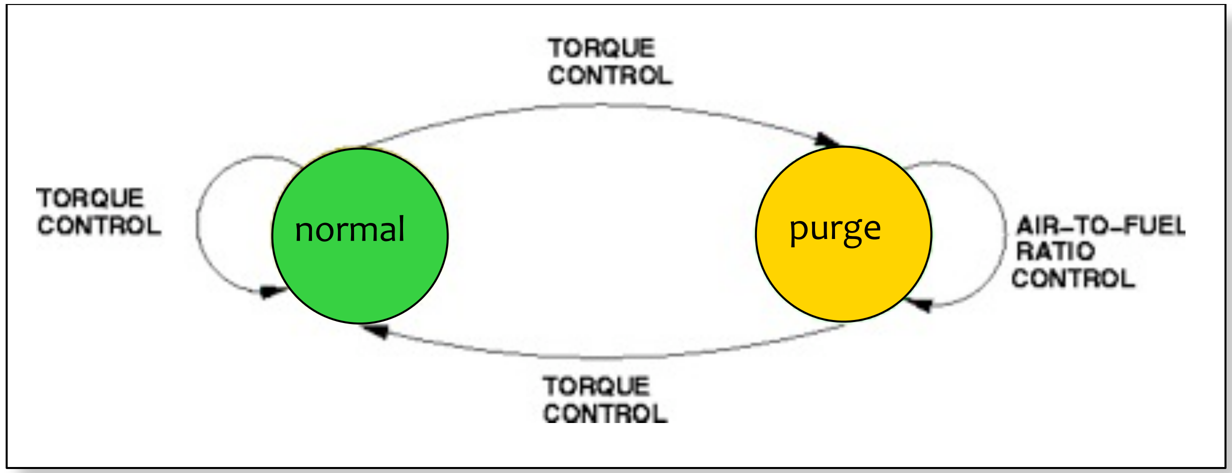
Two distinct regimes:

Regime	fuel injection	air-to-fuel ratio
Homogeneous combustion	intake stroke	λ=14.64
Stratified combustion	compression stroke	λ>14.64



- Mode is switched by changing fuel injection timing (late / early)
- Better fuel economy during stratified mode

Periodical cleaning of the aftertreatment system needed (λ=14.00, homogeneous regime)



the stratified operation
can only be sustained in a restricted
part of the engine
operating range

DISC engine

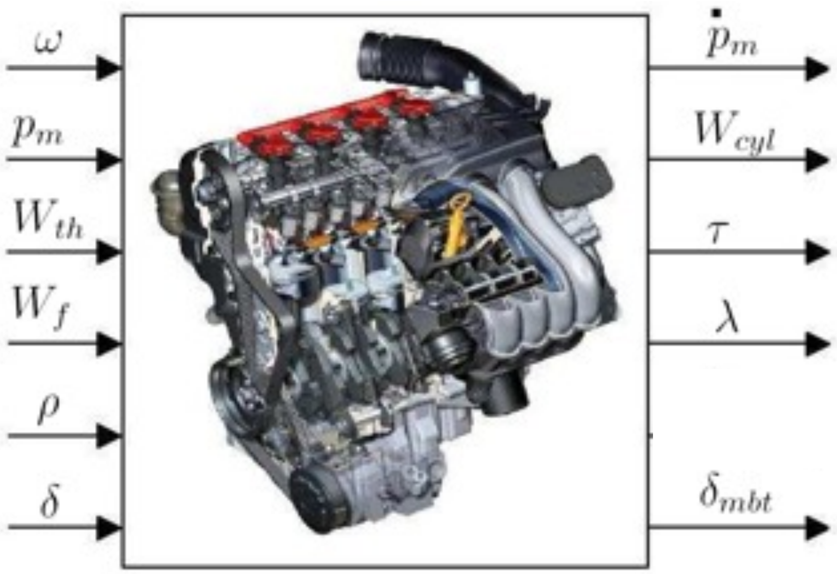
- States: intake manifold pressure (p_m)

- Outputs: Air-to-fuel ratio (λ), torque (τ), max-brake-torque spark timing (δ_{mbt})

- Continuous inputs: spark advance (δ), air flow (W_{th}), fuel flow (W_f)

- Binary input: spark combustion regime (ρ)

- Disturbance: engine speed (ω) [measured]

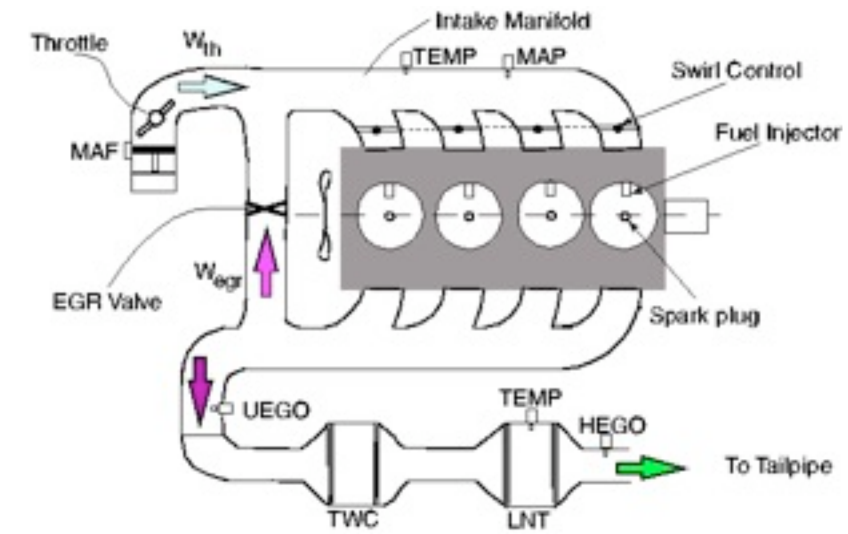


Constraints on:

- Air-to-fuel ratio (due to engine roughness, misfiring, smoke emiss.)
- Spark timing (to avoid excessive engine roughness)
- Mass flow rate on intake manifold (constraints on throttle)

- Dynamic equations are nonlinear
- Dynamics and constraints depend on regime ρ

DISC dynamics



Nonlinear model of the engine developed and validated at Ford *(Kolmanovsky, Sun, …)*

Assumptions:
- no EGR (exhaust gas recirculation) rate,
- engine speed=2000 rpm.

- Intake manifold pressure: $$\dot{p}_m = c_m\left(W_{th} - W_{cyl}\right) = c_m\left(W_{th} - k_{cyl}p_m\right)$$
- In-cylinder Air-to-Fuel ratio: $$\lambda = \frac{W_{cyl}}{W_f} = \frac{k_{cyl}p_m}{W_f}$$
- Engine torque: $$\tau = \tau_{mfr} + \tau_{pump} + \tau_{ind}$$ with τ_{mfr}, τ_{pump} functions of p_m

$$\tau_{ind} = (\theta_a + \theta_b(\delta - \delta_{mbt})^2)W_f$$ where θ_a, θ_b, δ_{mbt} are functions of λ, δ and ρ

✓ Good for simulation

✖ Not suitable for optimization-based controller synthesis

Hybridization of DISC model

DYNAMICS (intake pressure, air-to-fuel ratio, torque):

- Definition of two operating points;
- Linearization of nonlinear dynamics;
- Time discretization of the linear models.

→ ρ-dependent dynamic equations

CONSTRAINTS on:

- Air-to-Fuel Ratio: $\lambda_{min}(\rho) \leq \lambda(t) \leq \lambda_{max}(\rho)$;
- Mass of air through the throttle: $0 \leq W_{th} \leq K$;
- Spark timing: $0 \leq \delta(t) \leq \delta_{mbt}(\lambda, \rho)$

→ ρ–dependent constraints

↓

Hybrid system with 2 modes (switching affine system)

Integral Action

Integrators on torque error and air-to-fuel ratio error are added to obtain zero offsets in steady-state:

$$\epsilon_\tau(t+1) = \epsilon_\tau(t) + T \cdot (\tau_{ref} - \tau)$$
$$\epsilon_\lambda(t+1) = \epsilon_\lambda(t) + T \cdot (\lambda_{ref} - \lambda)$$

T = sampling time

τ_{ref}, λ_{ref} brake torque and air-to-fuel references

Simulation based on nonlinear model confirms zero offsets in steady-state

(despite the model mismatch)

MPC of DISC engine

$$\min_{\xi} J(\xi, x(t)) = \sum_{k=0}^{N-1} u_k' R u_k + y_k' Q y_k + x_{k+1}' S x_{k+1}$$

$$\text{subj. to } \begin{cases} x_0 = x(t), \\ \text{hybrid model} \end{cases}$$

N = control horizon

$x(t)$ = current state

$$\xi = [u_0', \gamma_0', z_0', \ldots, u_{N-1}', \gamma_{N-1}', z_{N-1}']',$$

where:

$$\begin{aligned} u_k &= [W_{th,k} - W_{th,ref},\ W_{f,k} - W_{f,ref}, \delta_k - \delta_{ref},\ \rho_k - \rho_{ref}]' \\ y_k &= [\tau_k - \tau_{ref},\ \lambda_k - \lambda_{ref},\ \delta_{mbt} - \delta_k - \Delta\delta_{ref}]' \\ x_k &= [p_{m,k} - p_{m,ref},\ \epsilon_{\tau,k},\ \epsilon_{\lambda,k}]' \end{aligned}$$

and:

$$R = \begin{pmatrix} r_{W_{th}} & 0 & 0 & 0 \\ 0 & r_{W_f} & 0 & 0 \\ 0 & 0 & r_\delta & 0 \\ 0 & 0 & 0 & r_\rho \end{pmatrix} \quad Q = \begin{pmatrix} q_\tau & 0 & 0 \\ 0 & q_\lambda & 0 \\ 0 & 0 & q_{\Delta\delta} \end{pmatrix} \quad S = \begin{pmatrix} s_{p_m} & 0 & 0 \\ 0 & s_{\epsilon_\tau} & 0 \\ 0 & 0 & s_{\epsilon_\lambda} \end{pmatrix}$$

Reference values are automatically generated from τ_{ref} and λ_{ref} by numerical computations based on the nonlinear model

DISC Engine - HYSDEL List

```
SYSTEM hysdisc{
   INTERFACE{
   STATE{
     REAL pm       [1, 101.325];
     REAL xtau     [-1e3, 1e3];
     REAL xlam     [-1e3, 1e3];
     REAL taud     [0,     100];
     REAL lamd     [10,     60];
      }
   OUTPUT{
     REAL lambda, tau, ddelta;
          }
   INPUT{
     REAL Wth      [0,38.5218];
     REAL Wf       [0,       2];
     REAL delta    [0,      40];
     BOOL rho;
      }
   PARAMETER{
     REAL Ts, pm1, pm2;
     …
      }
   }

   IMPLEMENTATION{
   AUX{
     REAL lam,taul,dmbtl,lmin,lmax;
      }
   DA{
   lam={IF rho THEN l11*pm+l12*Wth...
               +l13*Wf+l14*delta+l1c
        ELSE   l01*pm+l02*Wth+l03*Wf...
               +l04*delta+l0c      };

          taul={IF rho THEN taul1*pm+...
            taul2*Wth+taul3*Wf+taul4*delta+taulc
                ELSE   tau01*pm+tau02*Wth...
                 +tau03*Wf+tau04*delta+tau0c };

          dmbtl ={IF rho THEN dmbtl1*pm+dmbtl2*Wth...
                  +dmbtl3*Wf+dmbtl4*delta+dmbtl1c+7
                  ELSE dmbt01*pm+dmbt02*Wth...
                  +dmbt03*Wf+dmbt04*delta+dmbt0c-1};

          lmin ={IF rho THEN 13 ELSE 19};
          lmax ={IF rho THEN 21 ELSE 38};
               }
          CONTINUOUS{
              pm=pm1*pm+pm2*Wth;
              xtau=xtau+Ts*(taud-taul);
              xlam=xlam+Ts*(lamd-lam);
              taud=taud; lamd=lamd;
                  }
          OUTPUT{
             lambda=lam-lamd;
             tau=taul-taud;
             ddelta=dmbtl-delta;
             }
          MUST{
             lmin-lam     <=0;
             lam-lmax     <=0;
             delta-dmbtl  <=0;
             }
          }
       }
```

MPC - Torque control mode

$$\min \sum_{k=0}^{N-1} (u_k - u_r)' R (u_k - u_r) + (y_k - y_r)' Q (y_k - y_r) + (x_{k+1} - x_r)' S (x_{k+1} - x_r)$$

$$\text{subj. to } \begin{cases} x_0 = x(t), \\ \text{hybrid model} \end{cases}$$

$$u(t) = [W_{th}(t),\ W_f(t),\ \delta(t),\ \rho(t)]$$

Solve **MIQP problem** (mixed-integer quadratic program) to compute **u(t)**

Weights:

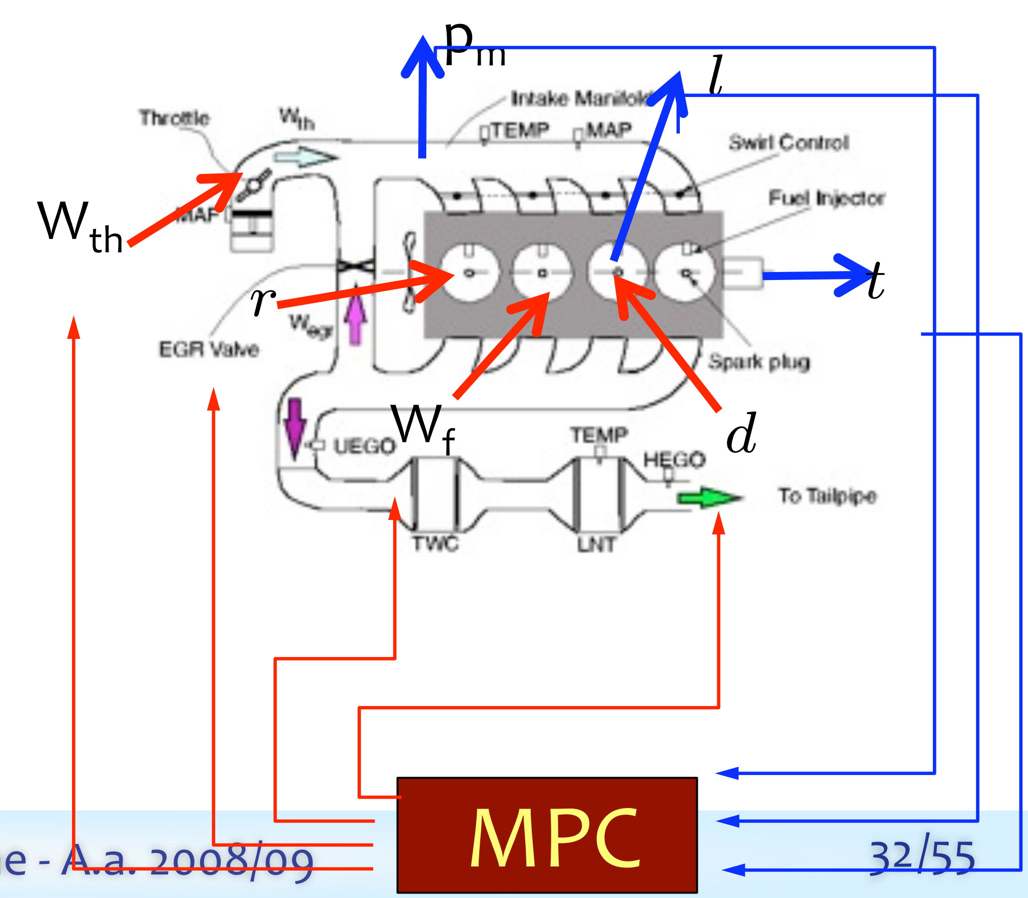
$$R = \begin{pmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

r_ρ (prevents unneeded chattering)

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.01 \end{pmatrix} \quad S = \begin{pmatrix} 0.04 & 0 & 0 \\ 0 & 1500 & 0 \\ 0 & 0 & 0.01 \end{pmatrix}$$

q_τ q_λ s_{ε_τ} s_{ε_λ}

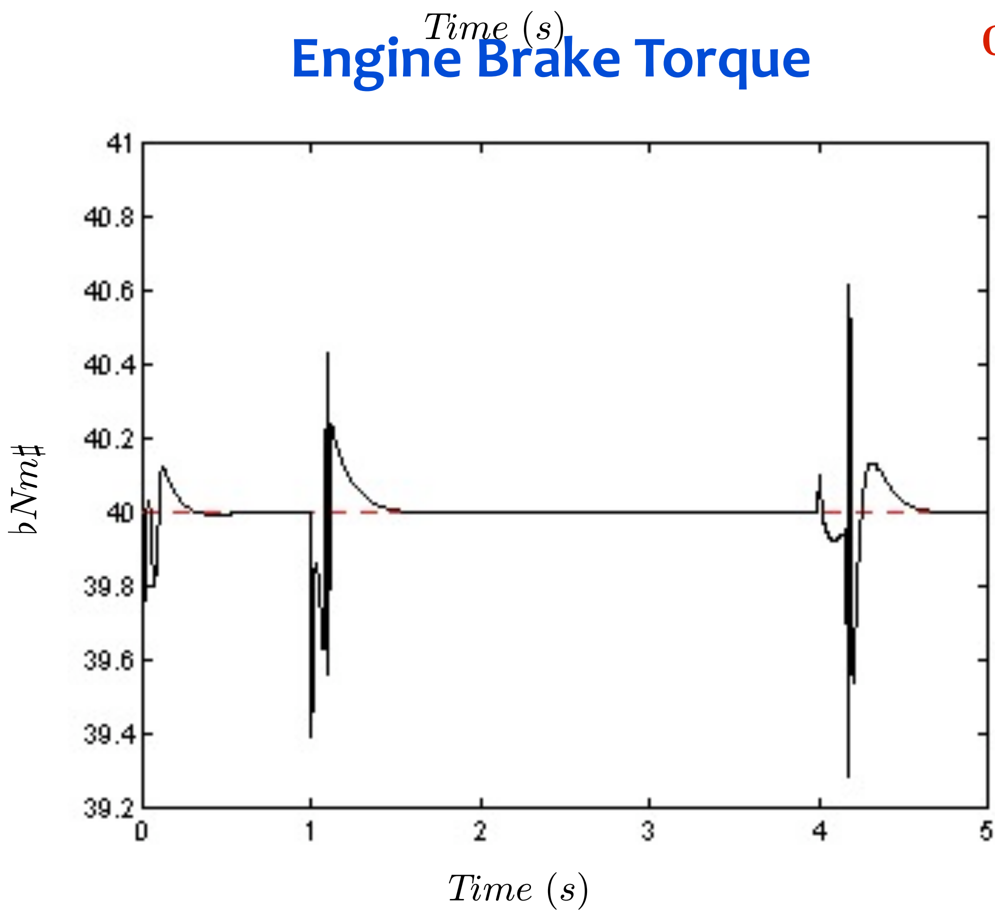
main emphasis on torque



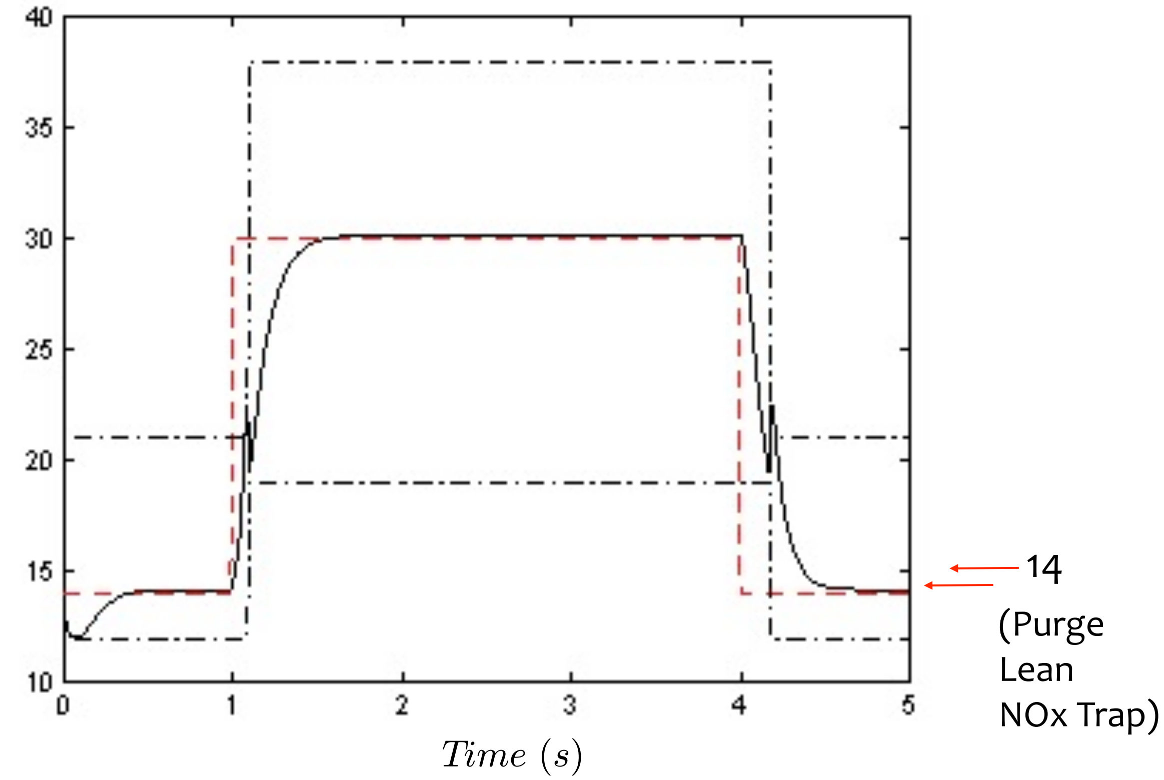
Simulation Results (nominal engine speed)

ω = 2000 rpm

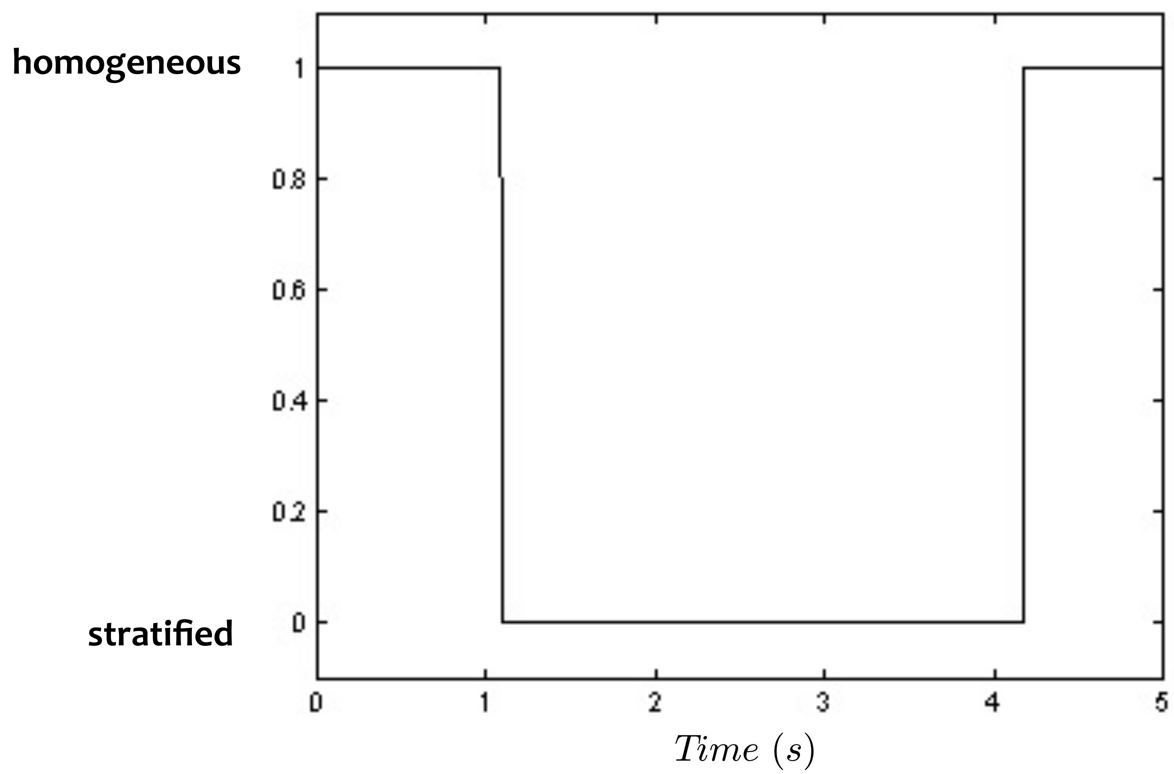
Engine Brake Torque



Air-to-Fuel Ratio



Combustion mode

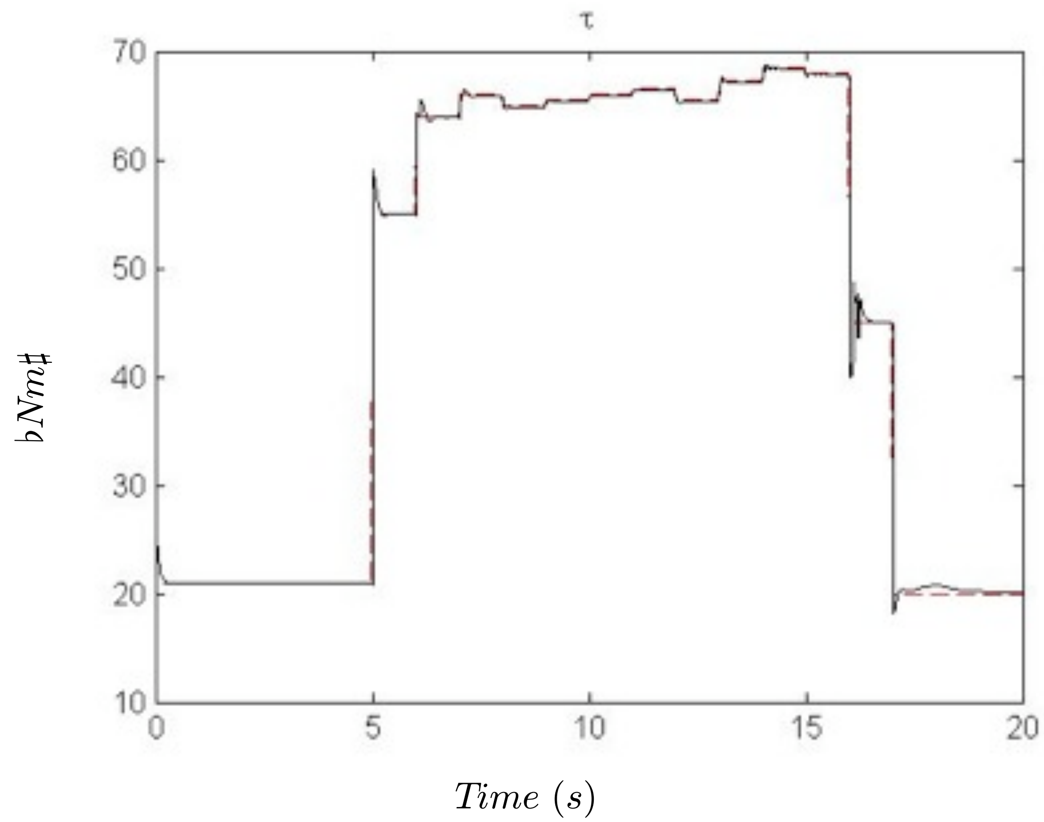


- Control horizon **N=1;**
- Sampling time **Ts=10 ms;**
- PC Xeon 2.8 GHz + Cplex 9.1

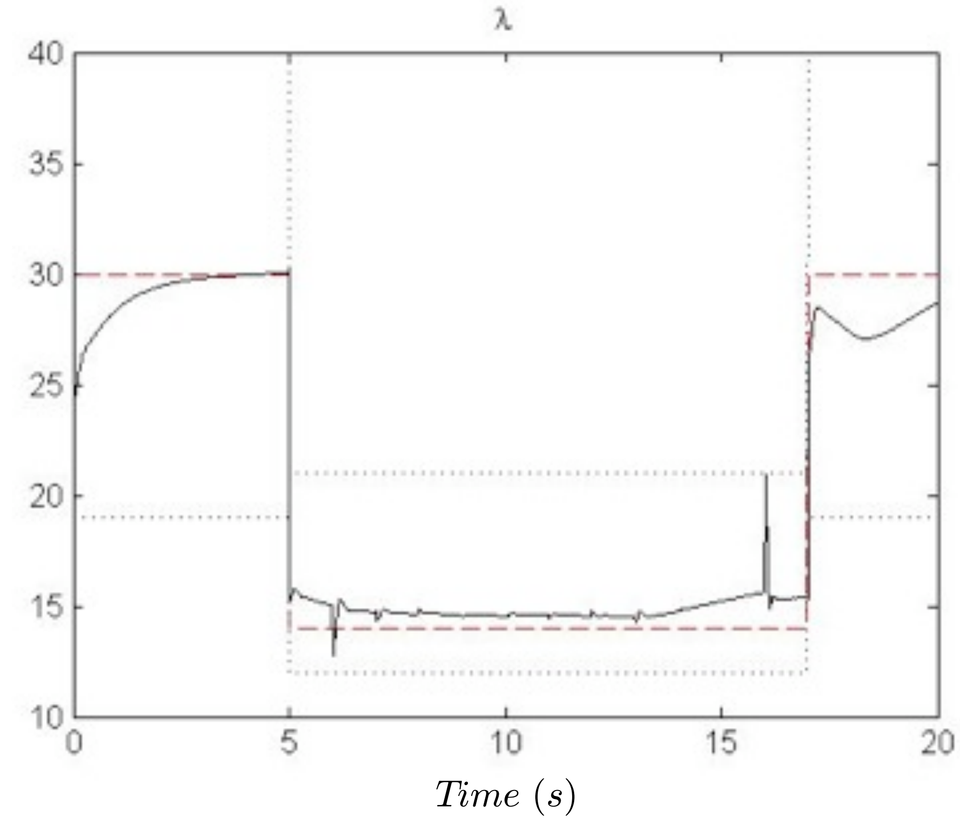
≈ 3 ms per time step

Simulation Results (varying engine speed)

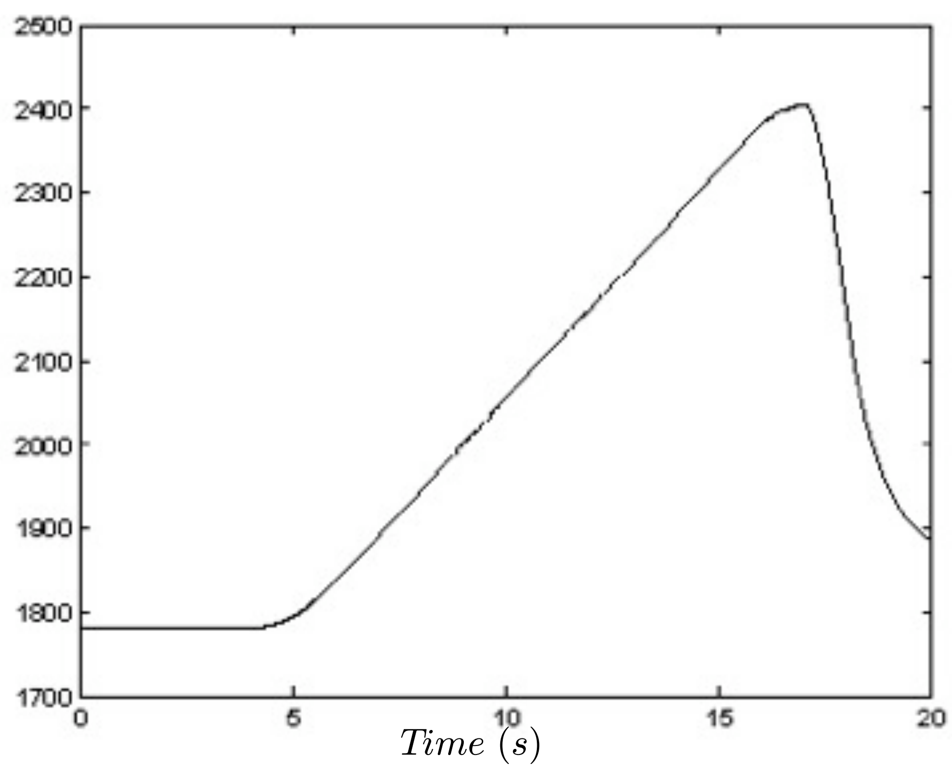
Engine Brake Torque



Air-to-Fuel Ratio



Engine speed

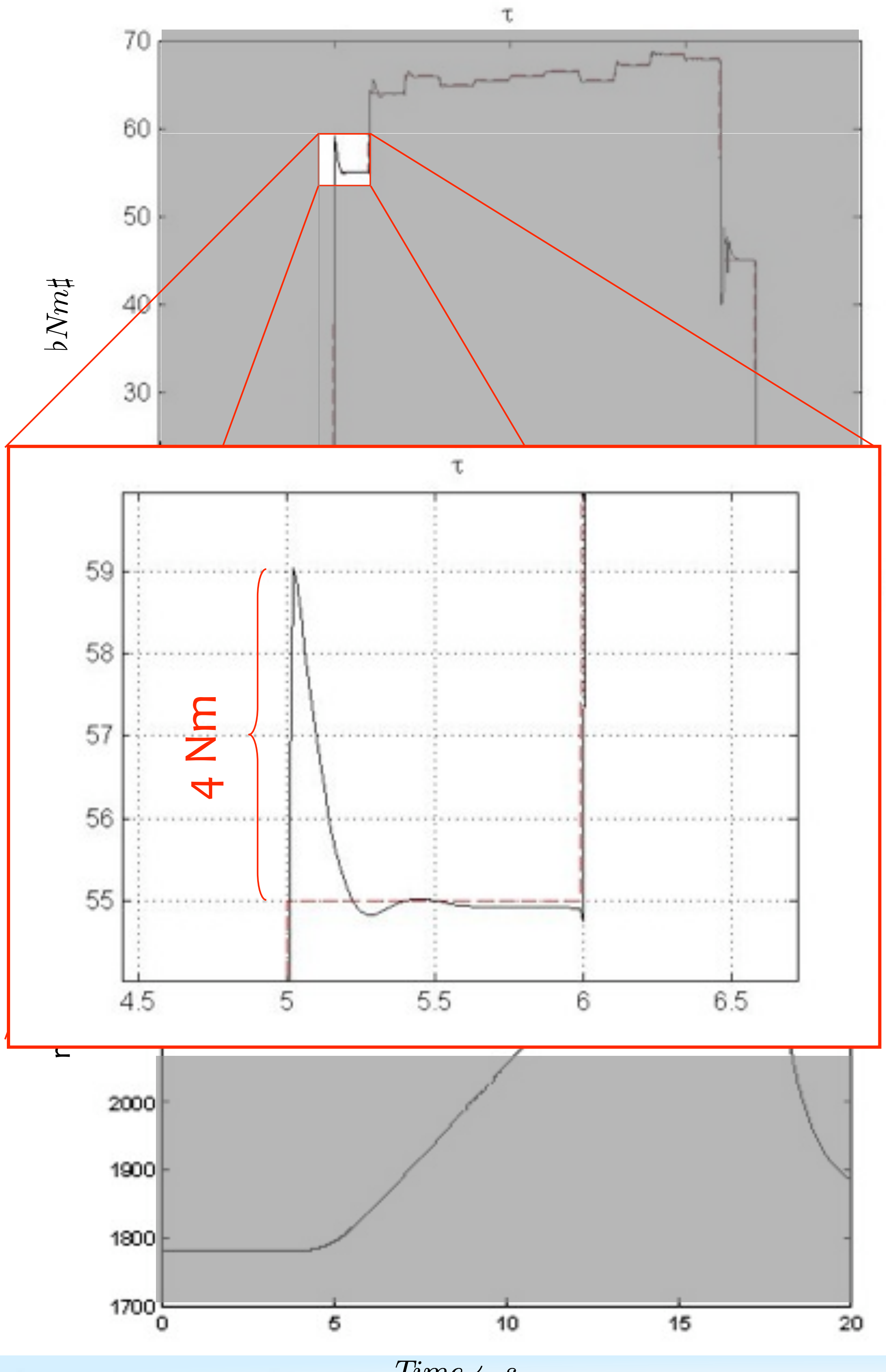


20 s segment of the European drive cycle (NEDC)

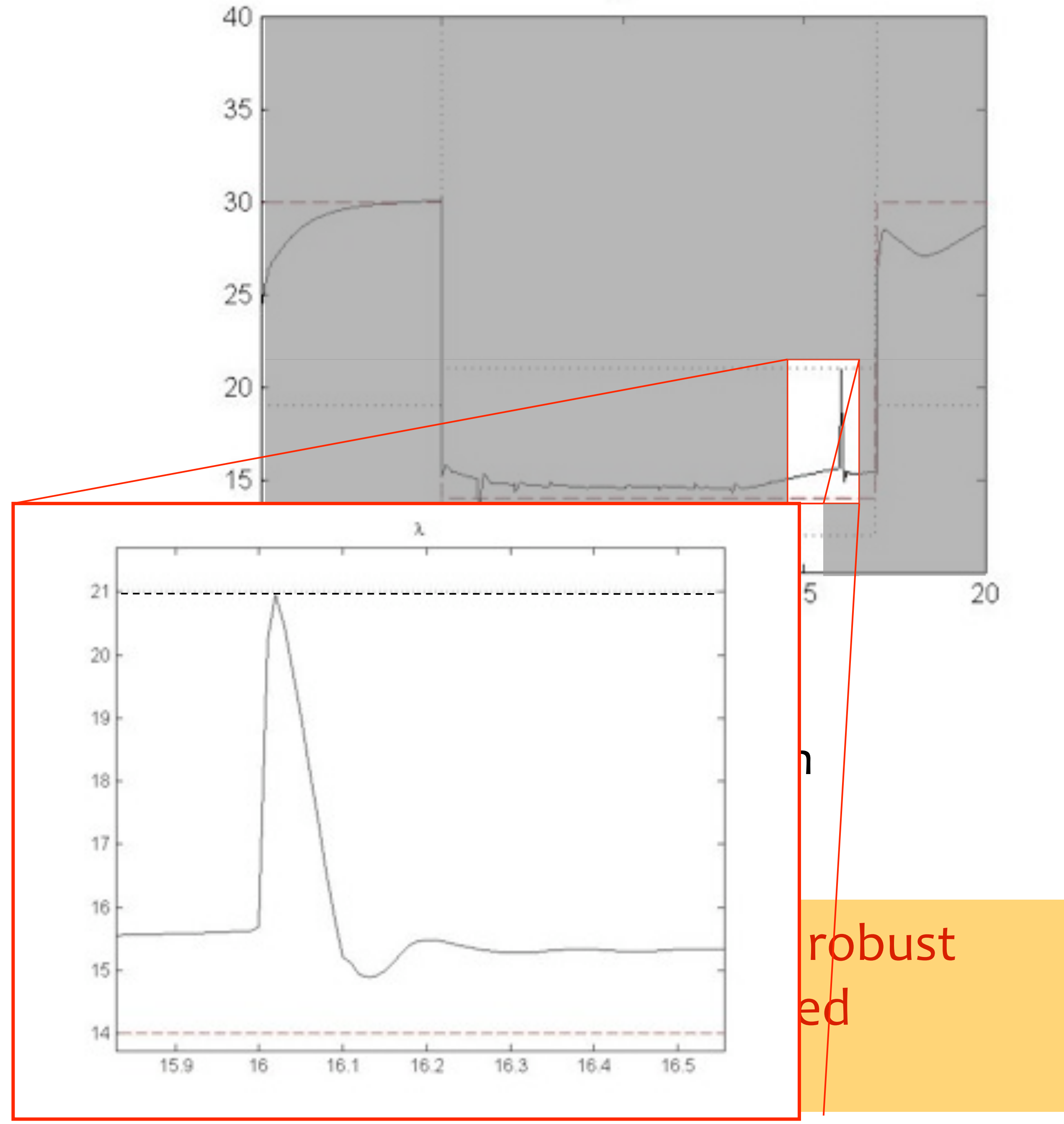
Hybrid MPC design is quite robust with respect to engine speed variations

Simulation Results (varying engine speed)

Engine Brake Torque



Air-to-Fuel Ratio



Control code too complex

(MILP) $\rightarrow$ not implementable !

Explicit MPC Controller

Explicit control law: $u(t) = f(\theta(t))$

where: $u = [W_{th}\ W_f\ \delta\ \rho]'$

$$\theta = [p_m\ \epsilon_\tau\ \epsilon_\lambda\ \tau_{\text{ref}}\ \lambda_{\text{ref}}\ p_{m,\text{ref}}\ W_{th,\text{ref}}\ W_{f,\text{ref}}\ \delta_{\text{ref}}]'$$

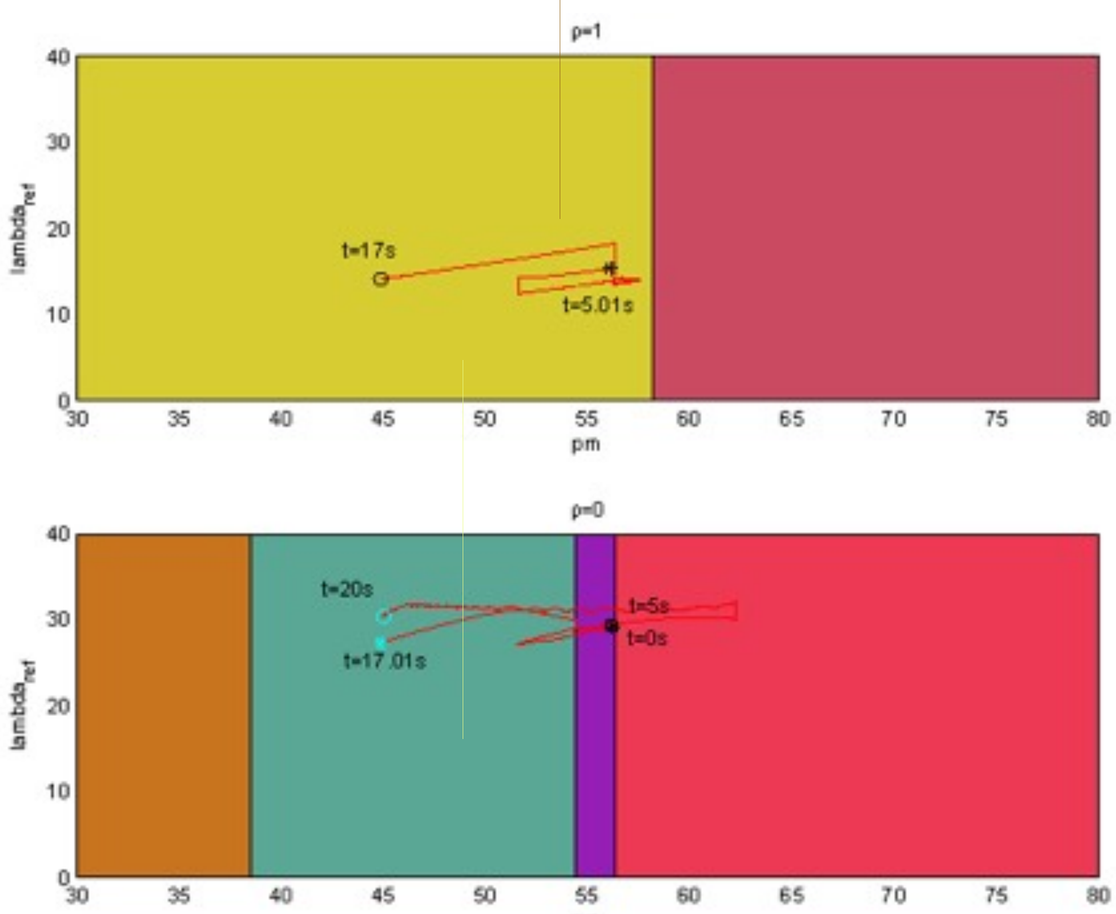
N=1 (control horizon)

42 partitions

- Time to compute explicit MPC: $\approx$ 3s;
- Sampling time Ts=10 ms;
- PC Xeon 2.8 GHz + Cplex 9.1

$\rightarrow$ **8 μs** per time step

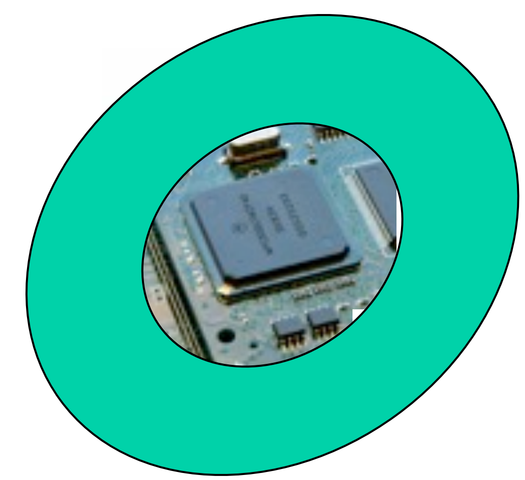
Cross-section by the τ_{ref}-λ_{ref} plane



ρ=0

ρ=1

$\approx$ 3ms on
μ–controller
Motorola
MPC 555
43kb RAM
(custom made for Ford)



Explicit MPC Controller (N=2)

Explicit control law: $u(t) = f(\theta(t))$

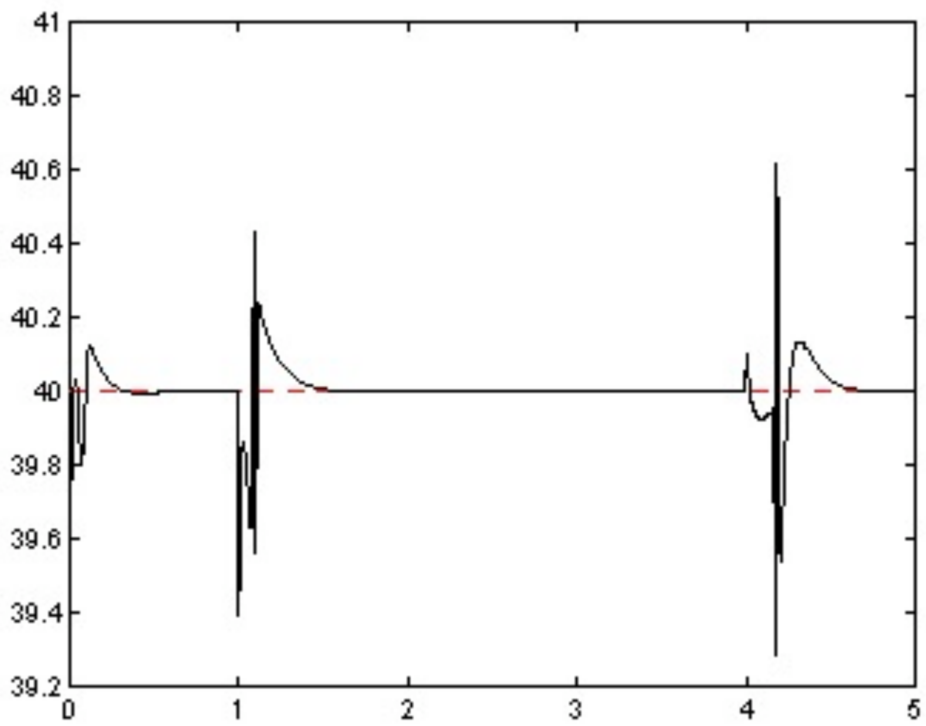
where: $u = [W_{th}\ W_f\ \delta\ \rho]'$

$\theta = [p_m\ \epsilon_\tau\ \epsilon_\lambda\ \tau_{\text{ref}}\ \lambda_{\text{ref}}$
$p_{m,\text{ref}}\ W_{th,\text{ref}}\ W_{f,\text{ref}}\ \delta_{\text{ref}}]'$

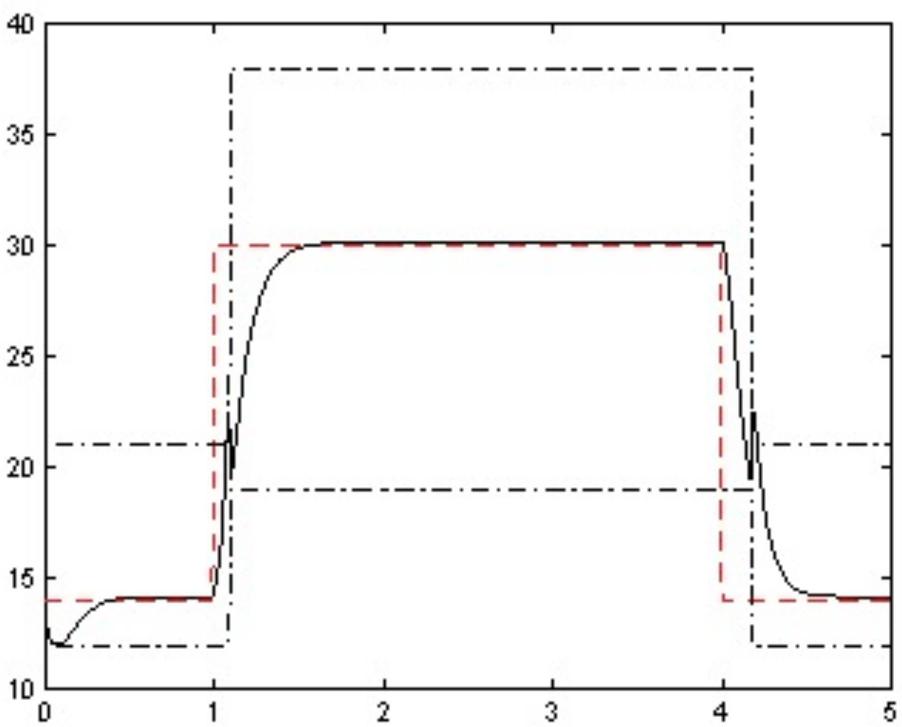
N=2 (control horizon)

747 partitions

Engine Brake Torque



Air-to-Fuel Ratio

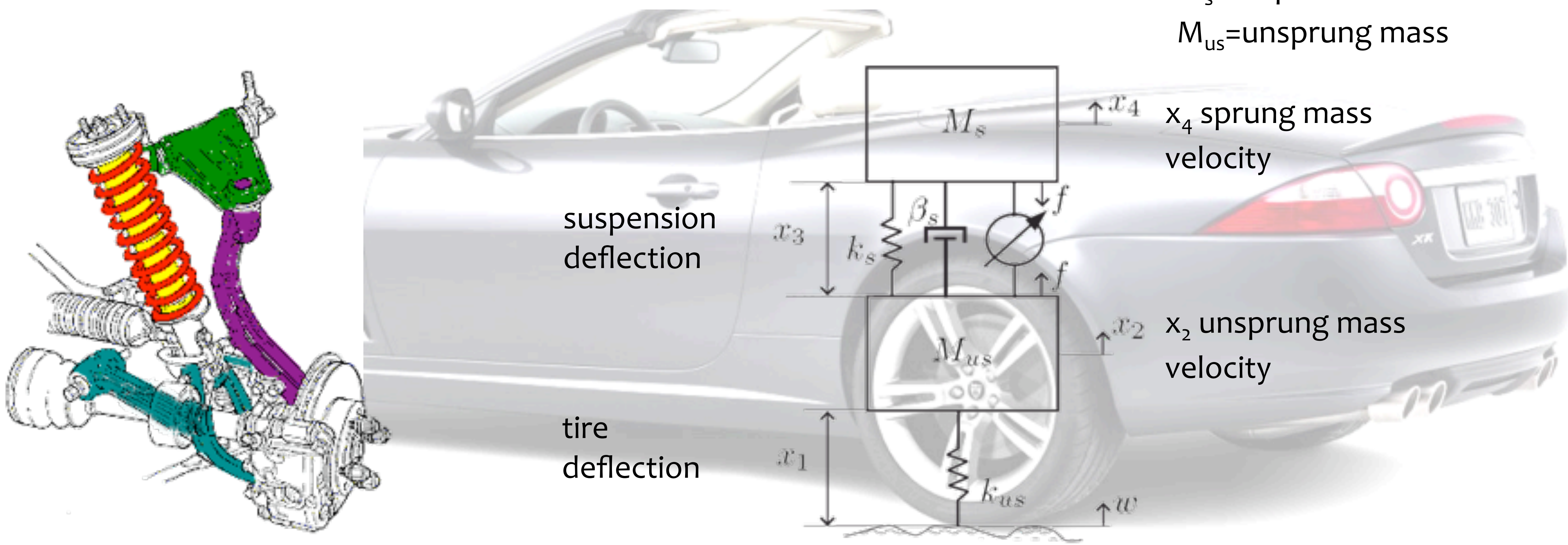


Closed-loop N=2 ⟷ Closed-loop N=1 ⟵ Adequate !

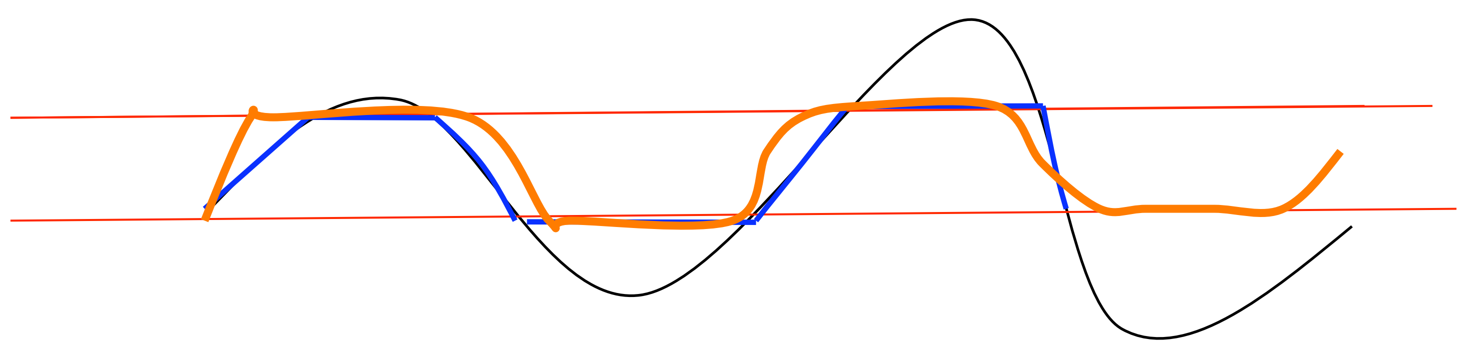
Explicit Hybrid MPC of Semiactive Suspensions

(joint work with N. Giorgetti, H.E. Tseng, D. Hrovat)

M_s=suspended mass
M_{us}=unsprung mass



For Semi-Active with Variable Damping, $f(x)=C^*(x_4-x_2)$



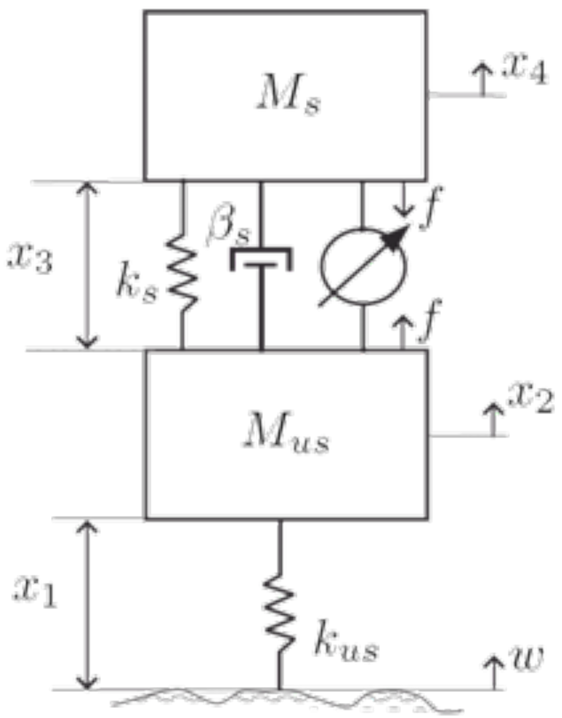
$C=f(x)/(x_4-x_2)$, where $f(x)$ is the optimal active suspension force

$C=\text{sat}[f(x)/(x_4-x_2)]$

Optimal

$\overset{?}{=}$

Quest of Optimal Semi-Active Suspensions



Showed existence by posing as two point boundary problem, Hrovat, Margolis, and Hubbard, 1988.

Showed the optimal solution can be solved from three Riccati Equations (state dependent switching), Butsuen and Hedrick, 1989.

Showed the optimal solution (of unsaturated component) maintains a 'linear' (varying gain) feedback form, Tseng and Hedrick, 1994.

Showed Clipped Optimal cannot be the optimal through a counter example, Tseng and Hedrick, 1994.

Does Closed Loop Form Optimal Solution Exist?

N. Giorgetti, A. Bemporad, H. E. Tseng, and D. Hrovat, "*Hybrid model predictive control application towards optimal semi-active suspension*," International Journal of Control, vol. 79, no. 5, pp. 521–533, 2006.

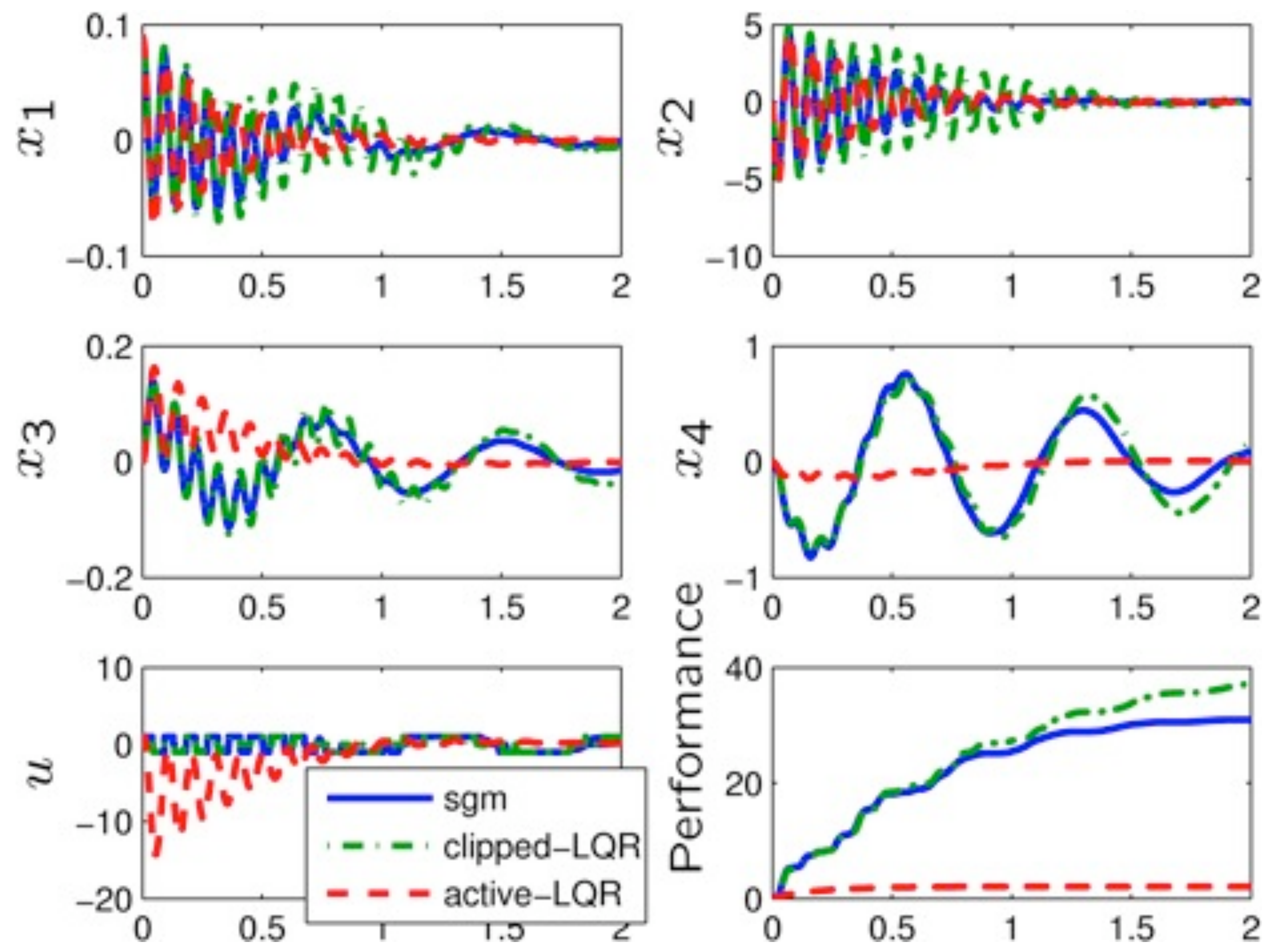
Steepest Gradient (SGM):

$$\bar{f}_{\text{SGM}} = sat[K_{\text{SGM}} x]$$

"Improve the action of a passive suspension"

Shock test of initial condition

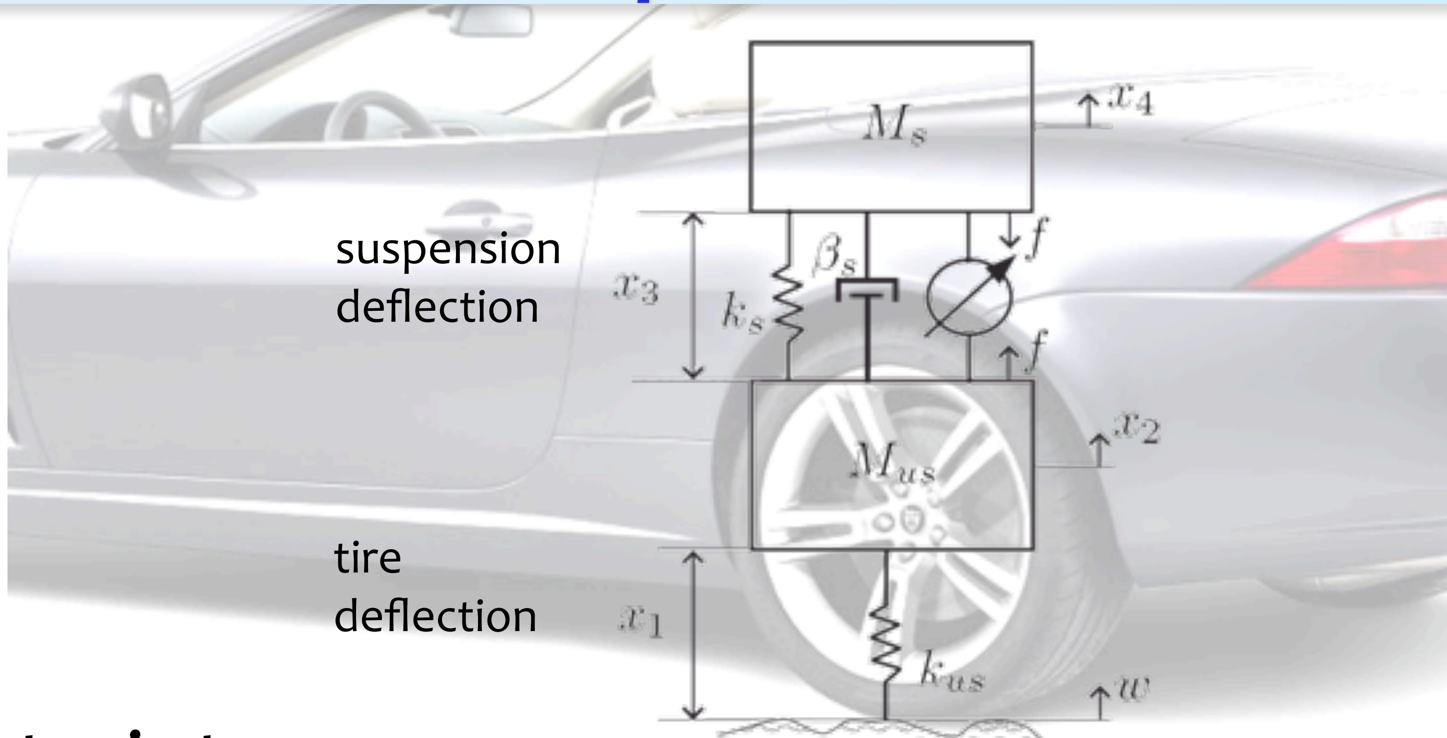
$$x_0 = [0.09\ \ 0\ \ 0\ \ 0]'$$



SGM **16%** better than clipped-LQR

Clipped-LQR is at least 16% from the true optimal

Semiactive Suspensions

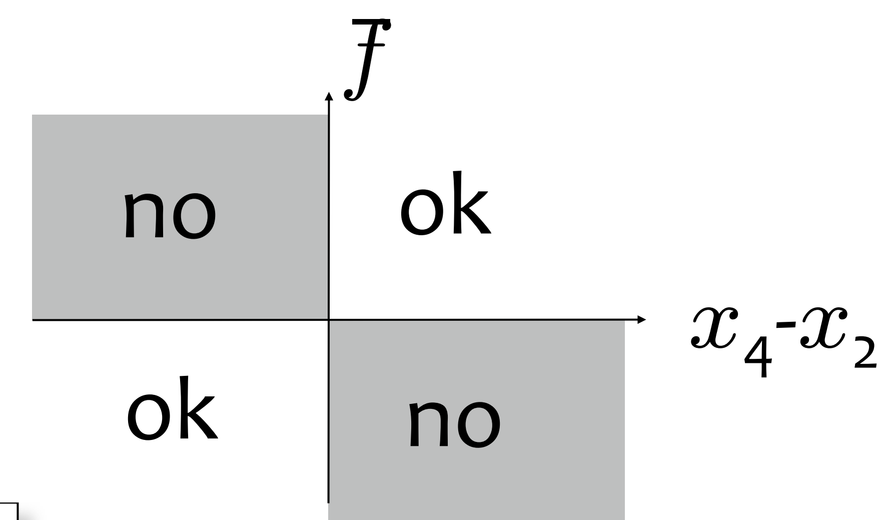


Quarter-car model → linear model

Constraints:

1) Passivity condition:

$$\bar{f}(x_4 - x_2) \geq 0$$



2) Max dissipation power:

$$\bar{f}(x_4 - x_2) \leq (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2)^2$$

3) Saturation:

$$|\bar{f}| \leq \sigma$$

(1), (2) are nonlinear & nonconvex physical constraints → **Hybrid Model**

Model

- State-space model

$$\dot{x} = Ax + B\bar{f} + B_w w$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{us}^2 & -2\rho\zeta\omega_s & \rho\omega_s^2 & 2\rho\zeta\omega_s \\ 0 & -1 & 0 & 1 \\ 0 & 2\zeta\omega_s & -\omega_s^2 & -2\zeta\omega_s \end{bmatrix}$$

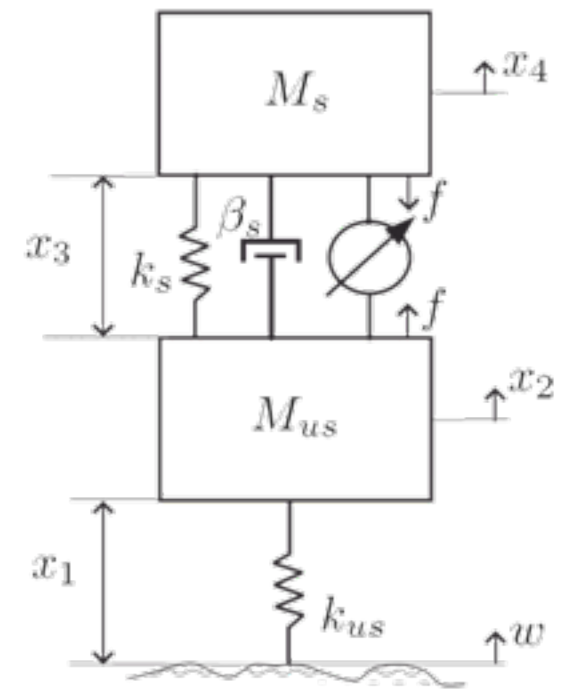
$$B = \begin{bmatrix} 0 \\ \rho \\ 0 \\ -1 \end{bmatrix} \quad B_w = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

x_1 = tire deflection from equilibrium
x_2 = unsprung mass velocity
x_3 = suspension deflection from equilibrium
x_4 = sprung mass velocity
$\bar{f}$ = normalized adjustable force
w = road velocity disturbance

$$\rho = \frac{M_s}{M_{us}},\ \omega_{us} = \sqrt{\frac{k_{us}}{M_{us}}},\ \omega_s = \sqrt{\frac{k_s}{M_s}},\ \zeta = \frac{\beta_s}{2\sqrt{M_s k_s}},\ \bar{f} = \frac{f}{M_s}$$

- Output:

$$y = \frac{dx_4}{dt} = \begin{bmatrix} 0 & 2\zeta\omega_s & -\omega_s^2 & -2\zeta\omega_s \end{bmatrix} x - \bar{f}$$



- Cost:

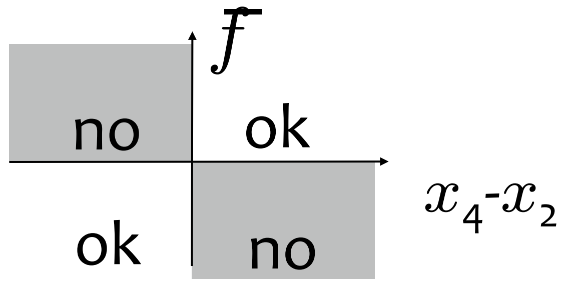
$$J = \int (q_{x_1} x_1^2 + q_{x_3} x_3^2 + \dot{x}_4^2)dt$$
$$= \int (x'Qx + \dot{x}_4^2)dt$$

- Time-discretization: $T_s = 10$ ms

Constraints

1) Passivity condition:

$$\bar{f}(x_4 - x_2) \geq 0$$



$\Longleftrightarrow$

$$\begin{array}{l} [\delta_v = 1] \leftrightarrow [x_4 - x_2 \geq 0] \\ [\delta_{\bar{f}} = 1] \leftrightarrow [\bar{f} \geq 0] \\ [\delta_v = 1] \rightarrow [\delta_{\bar{f}} = 1] \\ [\delta_v = 0] \rightarrow [\delta_{\bar{f}} = 0] \end{array}$$

2) Max dissipation power:

$$\bar{f}(x_4 - x_2) \leq (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2)^2$$

$\Longleftrightarrow$

$$F \geq 0$$

where

$$F = \begin{cases} \bar{f} - (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2) & \text{if } (x_4 - x_2) \leq 0 \\ -\bar{f} + (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2) & \text{otherwise} \end{cases}$$

3) Saturation:

$$|\bar{f}| \leq \sigma$$

$\Longleftrightarrow$

$$\begin{array}{lcl} \bar{f} & \leq & \sigma \\ \bar{f} & \geq & -\sigma \end{array}$$

HYSDEL Model

```
/* Semiactive suspension system

   (C) 2003-2005 by A.Bemporad, D.Hrovat,
                    E.Tseng, N.Giorgetti
*/

SYSTEM suspension {

 INTERFACE {
    STATE {
        REAL x1 [-0.05,0.05];
        REAL x2 [-5,5];
        REAL x3 [-0.2,0.2];
        REAL x4 [-2,2];
    }
    INPUT{
        REAL u [-10,10];  /* m/s^2 */
    }
    OUTPUT {
        REAL y;
    }
 PARAMETER {
   REAL A1dot,A2dot,A3dot,A4dot,B4dot,ws;
   REAL A11,A12,A13,A14,B1,A21,A22,A23,A24,B2;
   REAL A31,A32,A33,A34,B3,A41,A42,A43,A44,B4;
 }
}
```

```
IMPLEMENTATION {
  AUX {
    BOOL sign;
    BOOL usign;
    REAL F;
  }
  AD {
    sign = x4-x2<=0;
    usign = u<=0;
  }
  DA {
    F={ IF sign THEN u-(2*25.5*ws)*(x4-x2)
               ELSE -u+(2*25.5*ws)*(x4-x2)};
  }
  OUTPUT {    y=A1dot*x1+A2dot*x2+A3dot*x3
                +A4dot*x4+B4dot*u;
  }
  CONTINUOUS {
      x1 = A11*x1+A12*x2+A13*x3+A14*x4+B1*u;
      x2 = A21*x1+A22*x2+A23*x3+A24*x4+B2*u;
      x3 = A31*x1+A32*x2+A33*x3+A34*x4+B3*u;
      x4 = A41*x1+A42*x2+A43*x3+A44*x4+B4*u;
  }
  MUST {
      sign -> usign;
      ~sign -> ~usign;
      F>=0;
  } } }
```

```
>>S=mld('semiact3',Ts)
```

get the MLD model in Matlab

```
>>[X,T,D,Z,Y]=sim(S,x0,U);
```

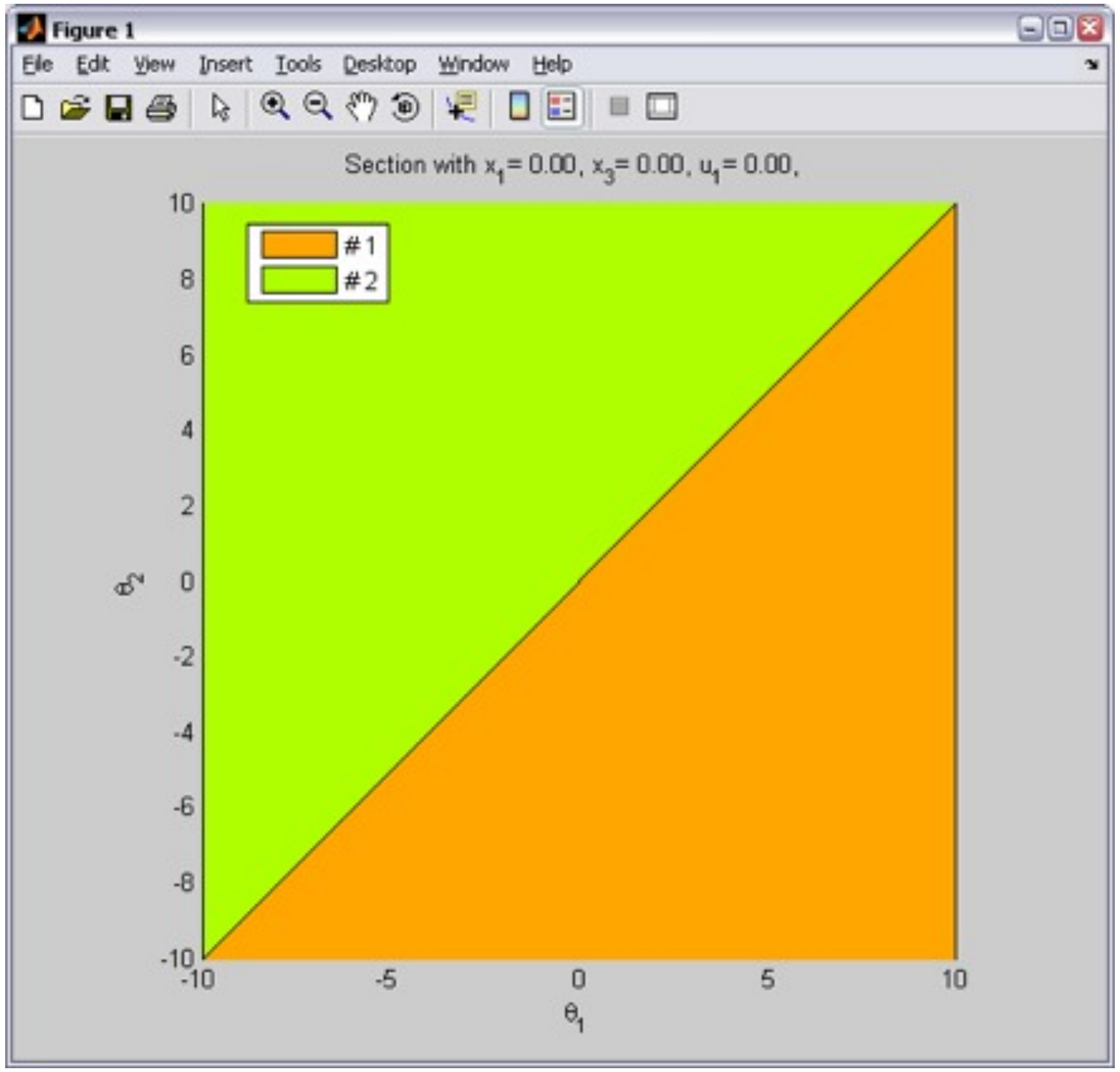
simulate the MLD model

Hybrid PWA Model

- PWA model

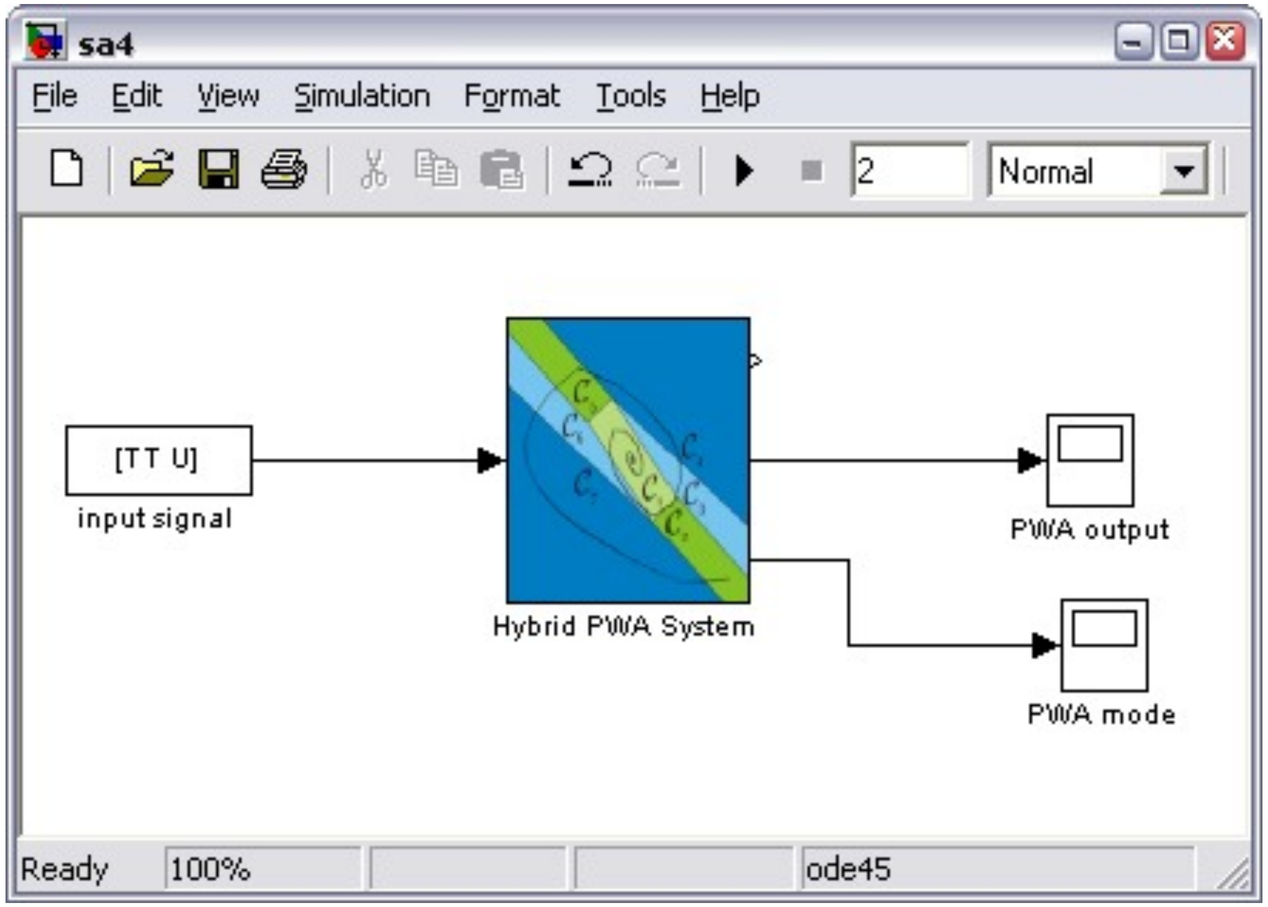
$$
\begin{aligned}
x(k+1) &= A_{i(k)}x(k) + B_{i(k)}u(k) + f_{i(k)} \\
y(k) &= C_{i(k)}x(k) + D_{i(k)}u(k) + g_{i(k)} \\
\\
i(k) \ \text{s.t.} &\ H_{i(k)}x(k) + J_{i(k)}u(k) \leq K_{i(k)}
\end{aligned}
$$

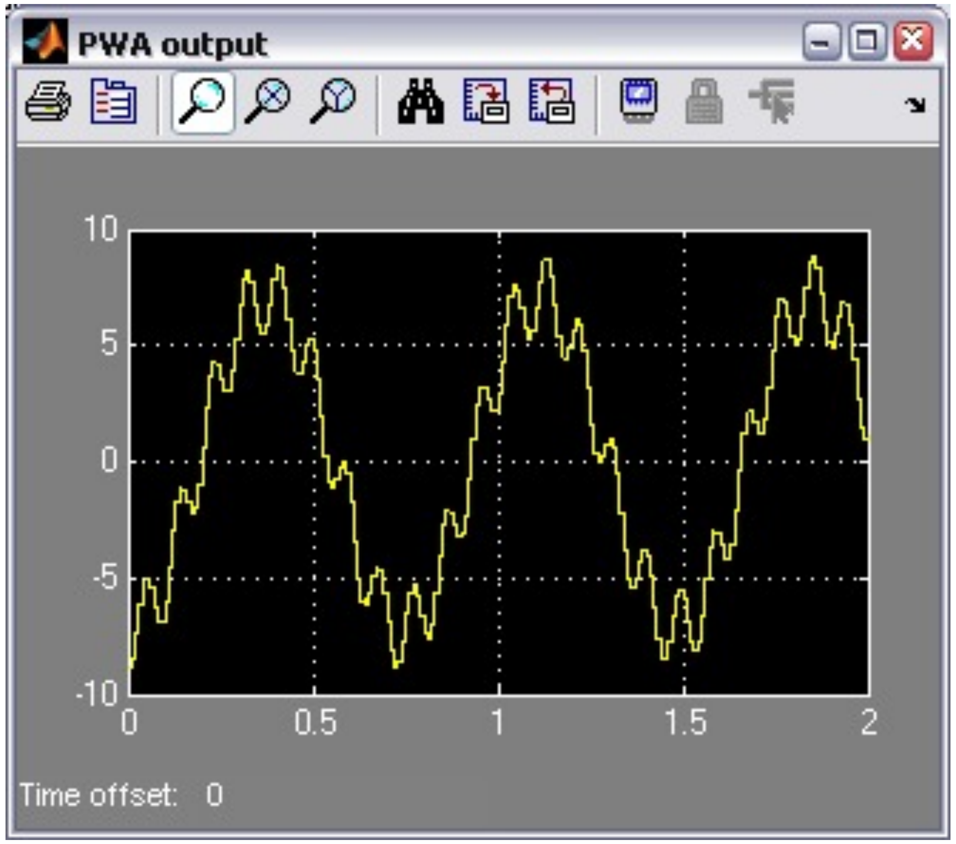
- 4 continuous states (x_1, x_2, x_3, x_4)
- 1 continuous input (normalized adjustable damping force $\bar{f}$)
- 2 polyhedral regions

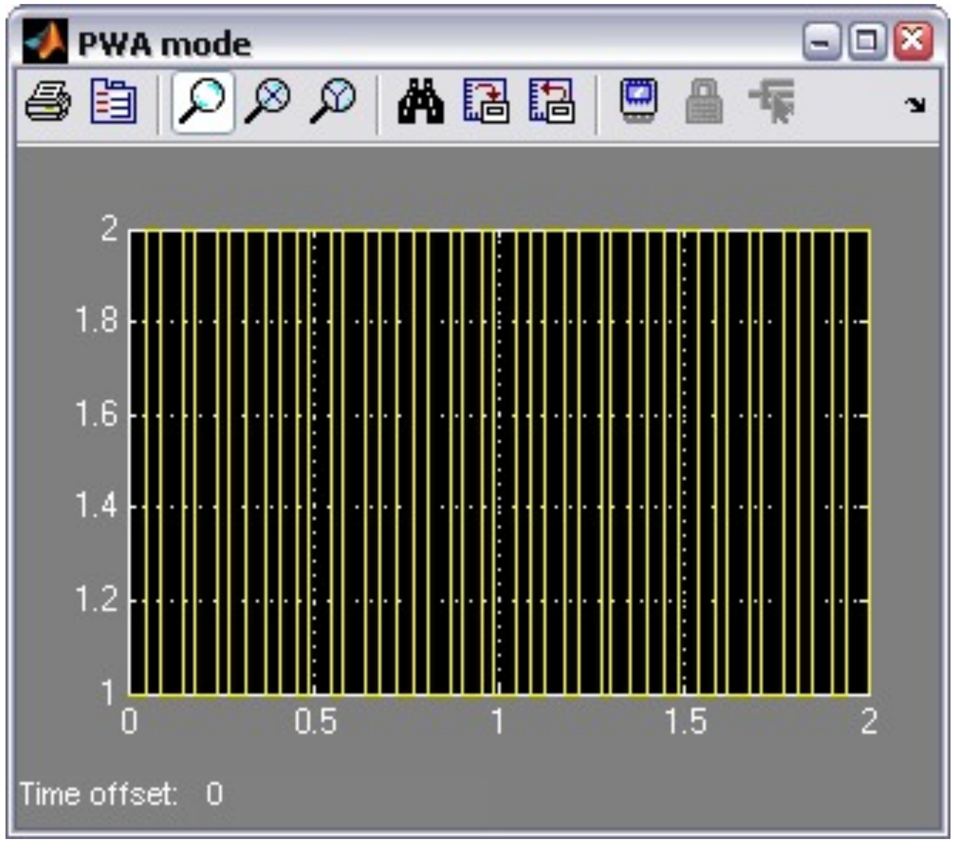



```
>>P=pwa(S);
```

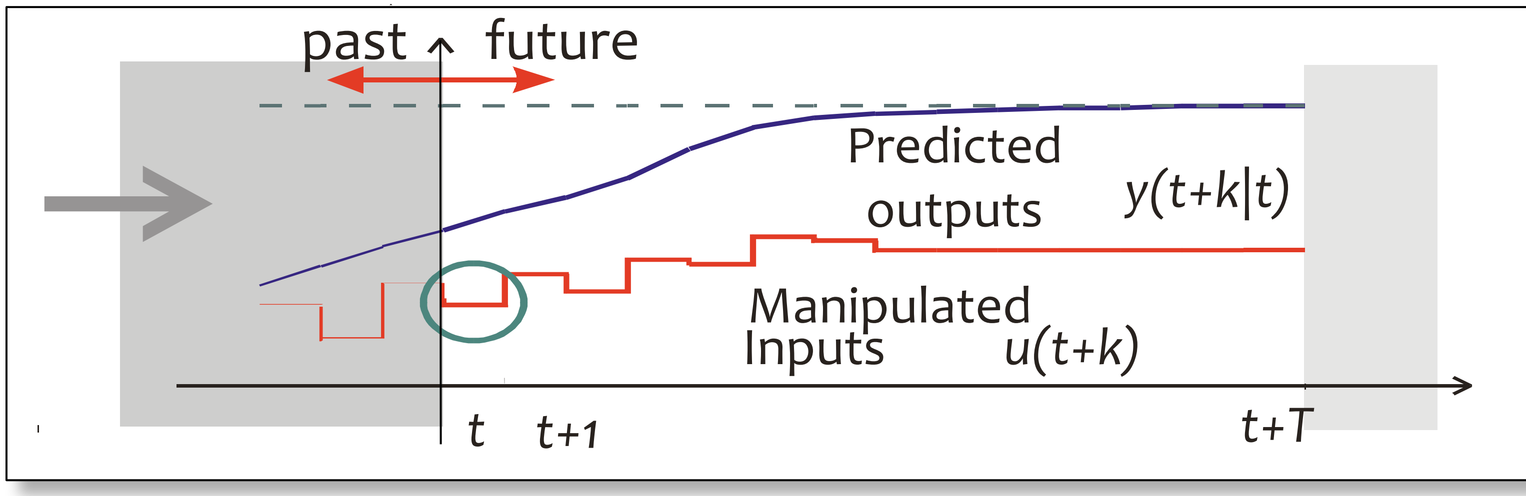
Simulation in Simulink







Control Strategy: MPC



Model
Predictive (MPC)
Control

- At time t solve with respect to th $u_0,\ldots,u_{N-1}$ on open-loop, optimal control problem:

$$\min_{u_0,\ldots,u_{N-1}} \quad x'(N)Q_N x(N) + \sum_{k=0}^{N-1}\left(q_1 x_1^2(k) + q_3 x_3^2(k) + y^2(k)\right)$$

subject to hybrid (MLD or PWA) model

Q_N = terminal Riccati weight (for infinite horizon cost)

- Apply $u(t)=u_0^*$ ptimal control move) and discard the remaining optimal inputs);

- Repeat the whole optimization at time t+1

Performance Specs

$$\min \left(\sum_{k=0}^{N-1} 1100 x_1^2(k) + 100 x_3^2(k) + \dot{x}_4^2(k) \right) + x'(N) P x(N)$$

tire deflession

suspension deflession

vertical acceleration

terminal weight (Riccati matrix)

Hybrid MPC - Example

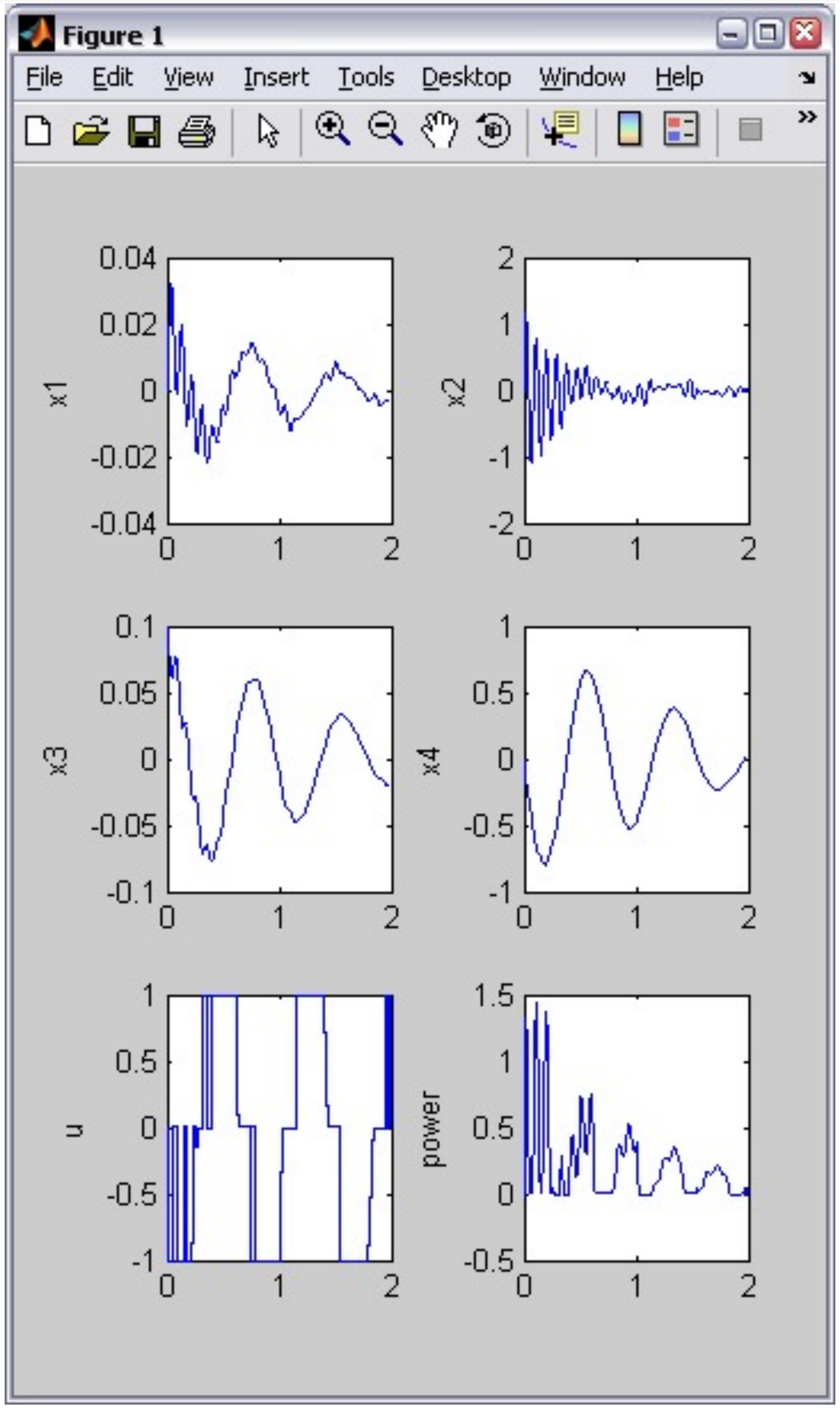
$$J = \int (q_1 x_1^2 + q_3 x_3^2 + \dot{x}_4^2)$$

```
>>refs.y=1;     % weights output #1
>>Q.y=Ts*rx4d;% output weight
…
>>Q.norm=2;     % quadratic costs
>>N=1;          % optimization horizon
>>limits.umin=umin;
>>limits.umax=umax;
```

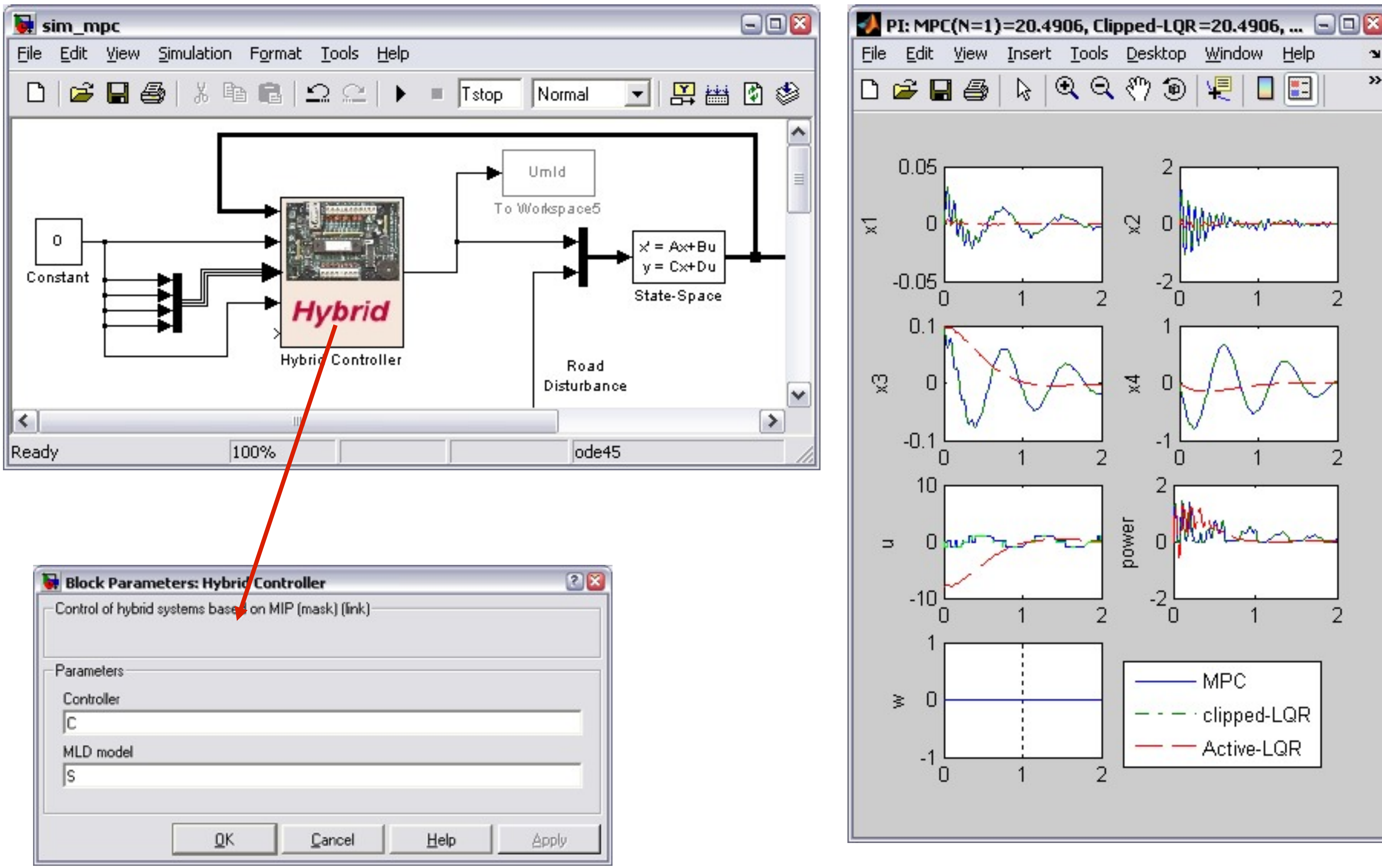
```
>>C=hybcon(S,Q,N,limits,refs);
```

```
>> C

Hybrid controller based on MLD model S <semiact3.hys> [2-norm]

   4 state measurement(s)
   1 output reference(s)
   1 input reference(s)
   4 state reference(s)
   0 reference(s) on auxiliary continuous z-variables

   4 optimization variable(s)  (2 continuous, 2 binary)
  13 mixed-integer linear inequalities
sampling time = 0.01, MIQP solver = 'cplex'

Type "struct(C)" for more details.
>>
```

```
>>[XX,UU,DD,ZZ,TT]=sim(C,S,r,x0,Tstop);
```

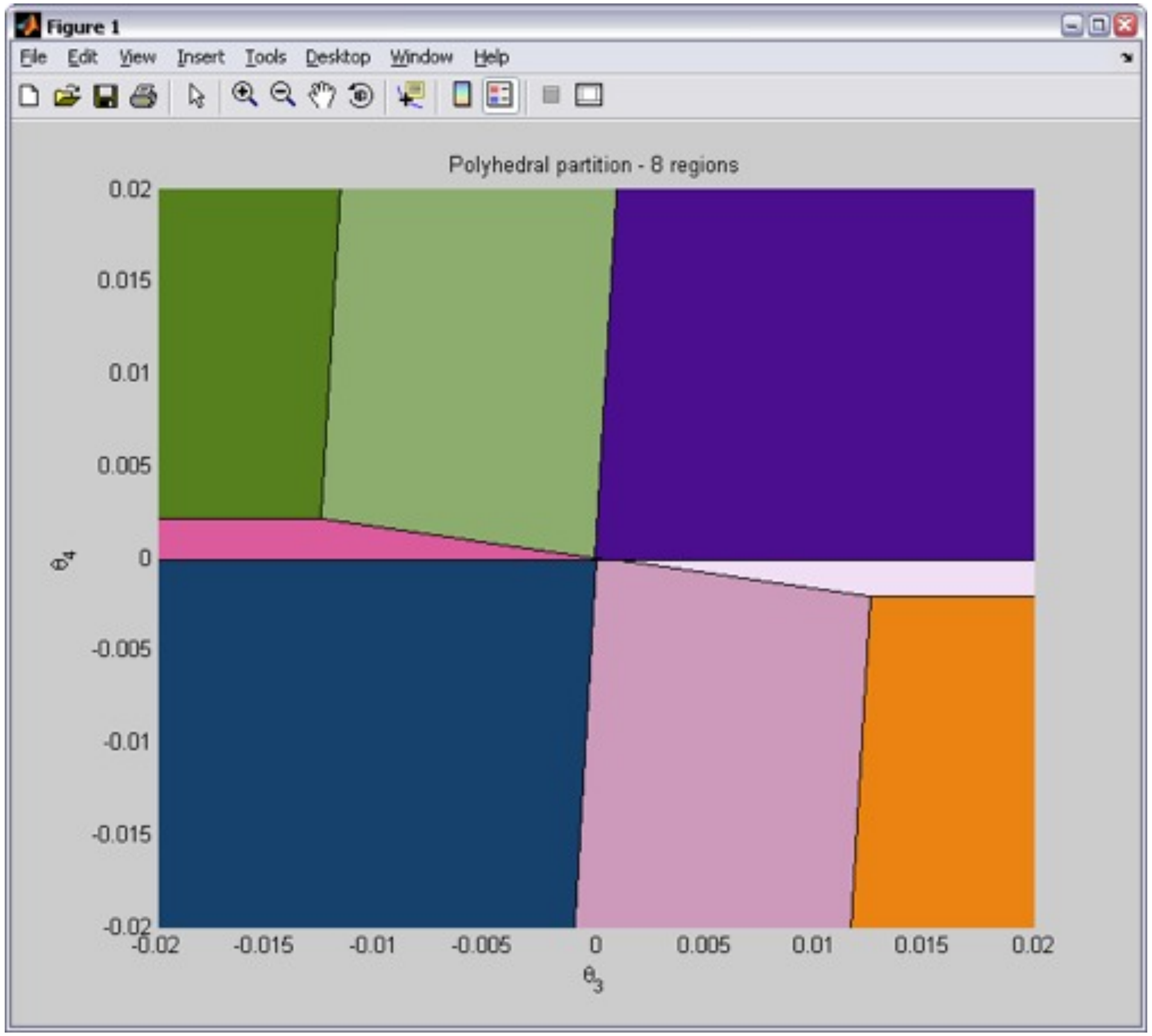


Closed-loop MPC in Simulink



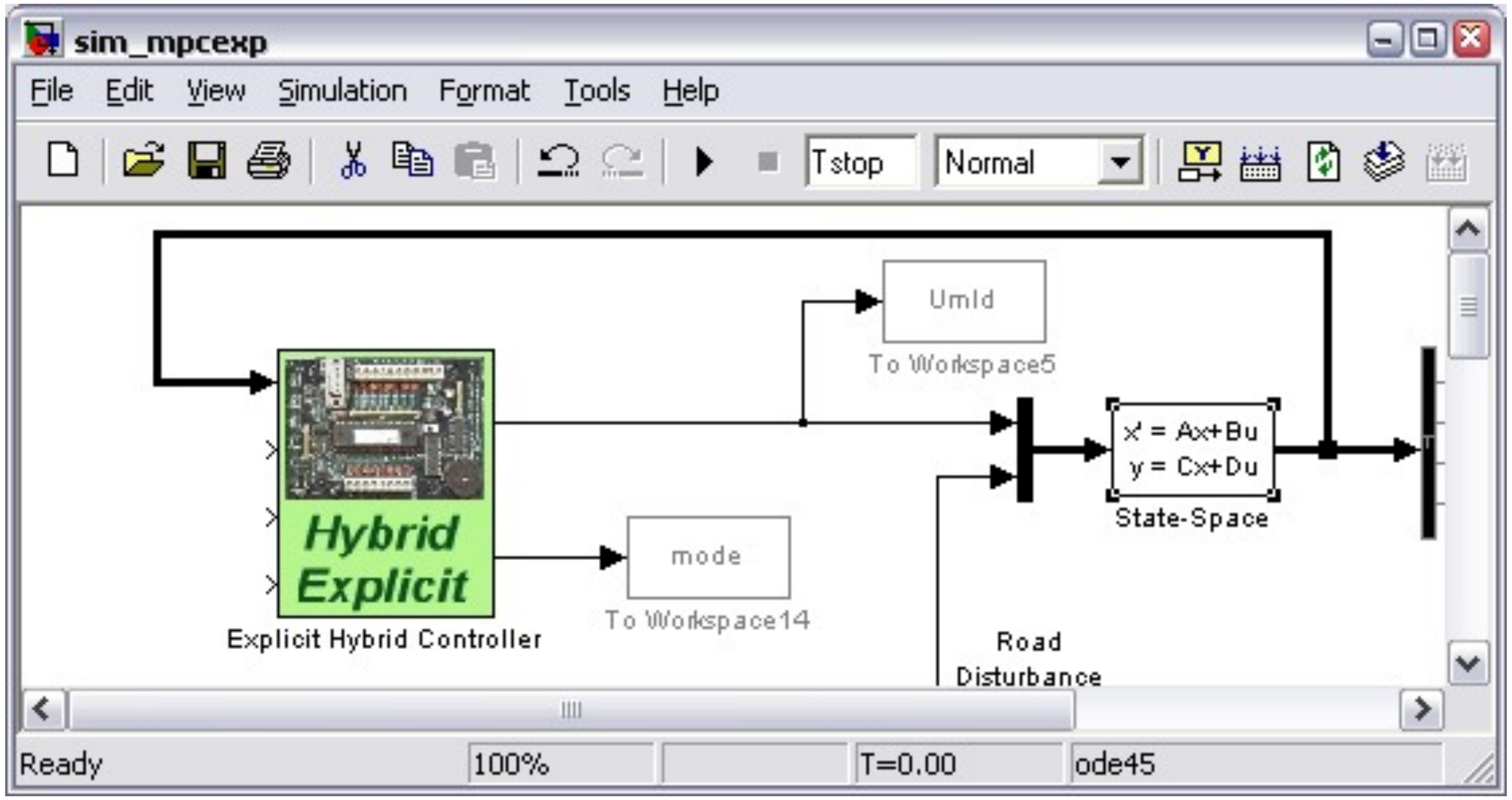
```
>>E=expcon(C,range,options);
```

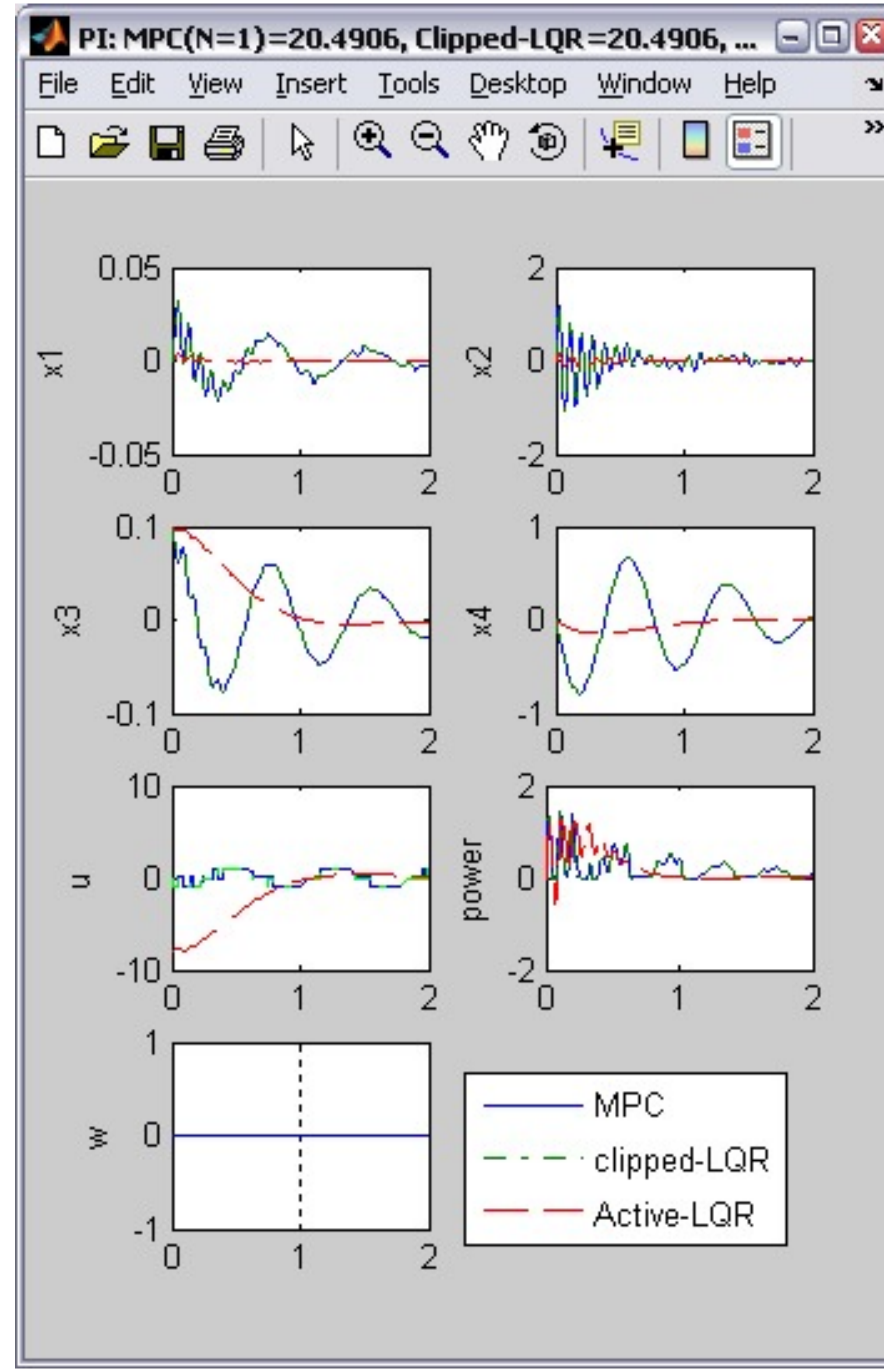
```
>> E

Explicit controller (based on hybrid controller C)
   4 parameter(s)
   1 input(s)
   8 partition(s)
sampling time = 0.01

The controller is for hybrid systems (tracking)
[2-norm]

This is a state-feedback controller.

Type "struct(E)" for more details.
>>
```



Section in the (x_3,x_4)-space for $x_1=x_2=0$

Explicit Hybrid MPC

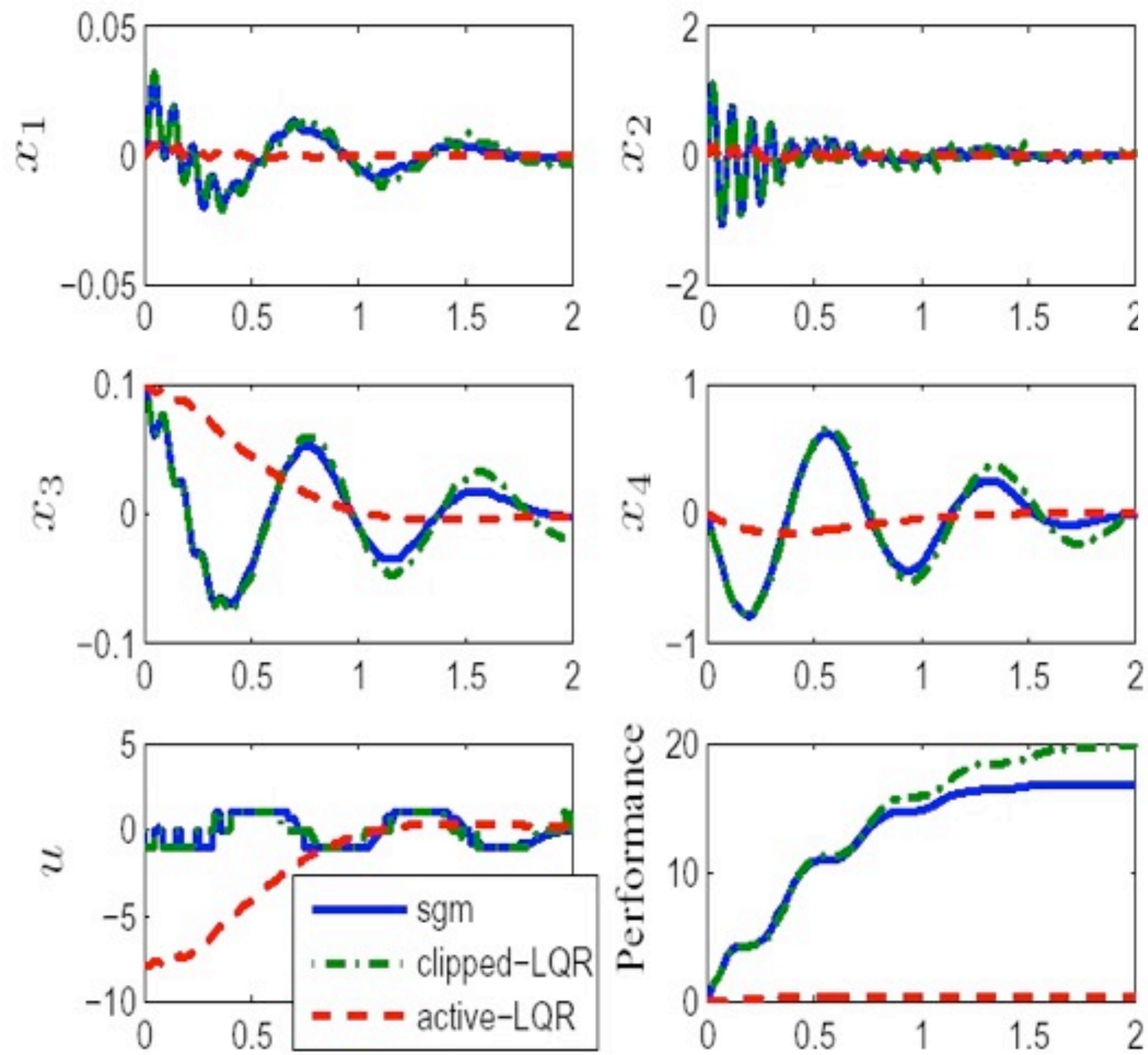




Generated C-code

```
#define EXPCON_NU 1
#define EXPCON_NX 4
#define EXPCON_NY 1
#define EXPCON_TS 0.01000000
#define EXPCON_REG 8
#define EXPCON_NTH 4
#define EXPCON_NYM 4
#define EXPCON_NUC 1
#define EXPCON_NUB 0
#define EXPCON_NGAIN 1
#define EXPCON_NH 21
#define EXPCON_NF 8
static double EXPCON_F[]={
    10.4748,0,0,0,10.4748,0,0,0,-0.244594,0,-
    480.664,0,
    3.92349,0,
    480.664,0
    };

static double EXPCON_G[]={
    0,1e-006,-1e-006,-1,0,0,1e-006,1
```

Quest of Optimal Semi-Active Suspensions



PARAMETER VALUES USED IN SIMULATION

Parameter	Value	Description
T_s	10 ms	Sampling time
ω_s	1.5 Hz	Sprung mass natural frequency
ω_{us}	10 Hz	Wheel-hop natural frequency
ρ	10	Sprung-to-unsprung mass ratio
ζ	0	Damping ratio
σ	1	Maximum force capacity
q_1	1100	Weight on tire deflection
q_3	100	Weight on suspension deflection

TABLE II

SHOCK TEST: MPC COST VALUE FOR DIFFERENT CONTROL HORIZONS SUBJECTED TO I.C.=[0 0 0.1 0]'.

N	MPC	Clipped-LQR	SGM	LQR
1	20.4282	20.4282	17.4944	0.4446
2	20.4054			
3	20.3290			
4	20.1100			
5	19.7380			
10	20.9840			
12	19.3084			
14	18.4842			
15	18.5996			
16	19.3212			
20	18.0764			
30	17.1494			
40	17.1304			

Simulation Results

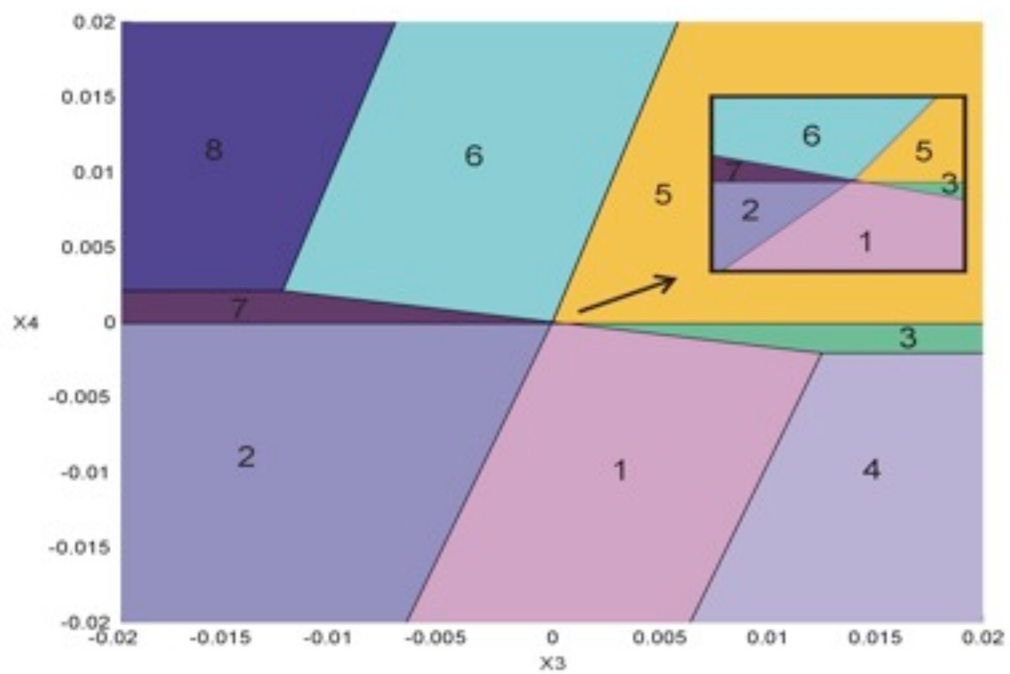
- Horizon N=1: same as Clipped-LQR !
- For increasing N: better closed-loop performance

Performance Index

N	MPC	Clipped-LQR
1	1.5155	1.5155
5	1.4416	
10	1.5238	
15	1.3083	
20	1.2204	
30	1.1456	
40	1.1462	

N=1, Same Cost Value !

Explicit solution (N=1, x_1=x_2=0):



$$u(x) = \begin{cases} 10.4748x_1 + 0.2446x_2 + 79.1519x_3 - 3.9235x_4 \; (= K_{LQ}) & \text{Regions } \#1, \#6 \\ 0 & \text{Regions } \#2, \#5 \\ (2 \cdot 25.5 \cdot \omega_s)(x_4 - x_2) & \text{Regions } \#3, \#7 \\ -1 & \text{Region } \#4 \\ 1 & \text{Region } \#8 \end{cases}$$

- Simulations with road noise.
- Initial condition x(0)=[0 0 0 0]'
- Simulation time T=20 s, sampling time T_s=10 ms