University of Trento

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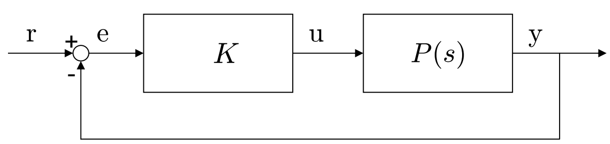
Automatic Control 2

Exercise 1 (10 points)

Consider the system described by the transfer function

$$P(s) = \frac{(s+5)(s^2+2s+1)}{(s+1)(s+4)^2(s+30)(s+100)}$$

- Sketch the asymptotic and real Bode diagrams (magnitude and phase) of $P(s)$.
- After sketching the Nyquist diagram of $P(s)$, consider the feedback system below



where K is a static feedback controller, $K > 0$. By using Nyquist criterion, compute the number of unstable poles of the closed-loop system as a function of K, and determine the values of $K > 0$ for which the closed-loop system is internally stable.

Exercise 2 (10 points)

Consider the system described by the transfer function

$$P(s) = \frac{12/50}{0.2s^2 + 2s + 10}$$

For the same feedback system depicted above design a dynamic feedback controller $K(s)$ satisfying the following specifications

- Rise time $t_r \leq 0.01$ s
- Resonant peak $M_r \simeq 1.5$ dB

<u>**Exercise**</u> <u>3</u> (8 points)

Given the continuous-time scalar system

$$\dot{x}(t) = -x(t) - x^3(t) + u(t)\sin(x(t)) + 2u(t)$$

- Prove the asymptotic stability of the origin of the autonomous system obtained imposing $u(t) = 0$ by finding a suitable Lyapunov function. Determine also a domain of attraction.
- Design (if possible) a feedback linearization law such that the resulting linear closed-loop dynamics has the only pole in -1.

<u>**Exercise**</u> <u>4</u> (3 points)

Given a nonlinear system $\dot{x} = f(x)$ with $f(0) = 0$, describe shortly what Lyapunov's direct method is about in order to determine if the origin is an asymptotically stable equilibrium point and estimate a subset B_ϵ of the domain of attraction. Is the existence of a Lyapunov function a sufficient condition or a necessary condition for the asymptotic stability of an equilibrium point ?