University of Trento

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Automatic Control 2: Solutions

Exercise 1 (10 points)

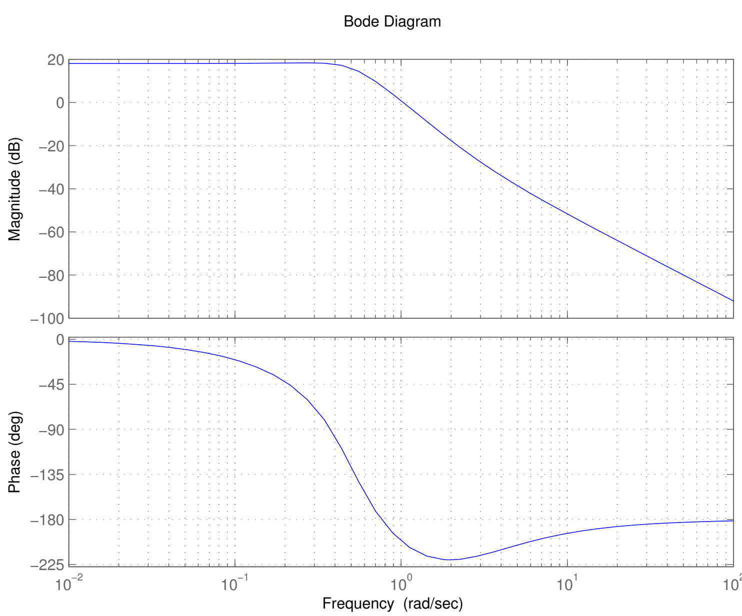
The Bode form of P(s) is the following

$$P(s) = \frac{8(s/2+1)^3}{(s+1)^3(4s^2+2s+1)}$$

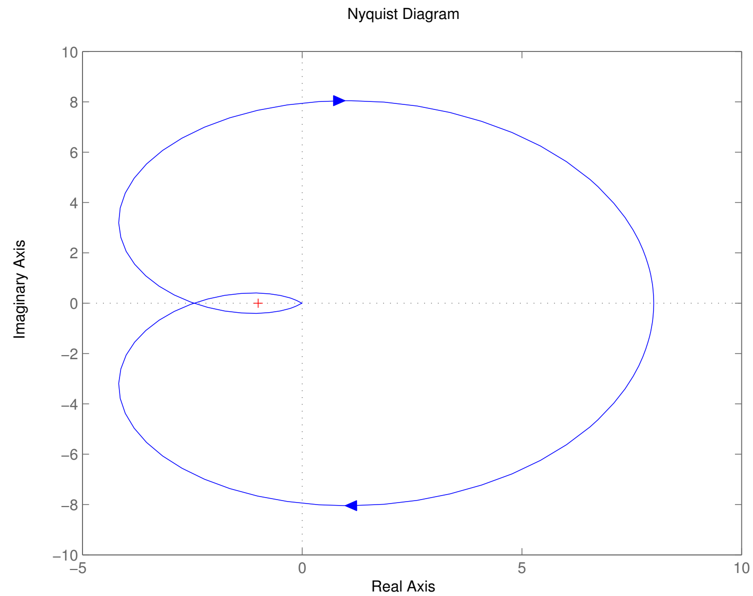
The second order polynomial is a complex and conjugate pair of poles, then, for the polynomial identity principle, $\omega_n = 0.5$ and $\zeta = 0.5$.
For the sake of simplicity, a table of the magnitude and phase contribution of the basic components is depicted in the following

	Magnitude dB	Phase deg
K_B	$20\log_{10}(8) = 18.1$	0
$(s+2)^3$	$+60$ dB/dec, $\omega_z = 2$	$+\frac{3}{2}\pi$
$(s+1)^3$	-60 dB/dec, $\omega_p = 1$	$-\frac{3}{2}\pi$
$(4s^2+2s+1)$	-40 dB/dec, $\omega_n = 0.5$	$-\pi$

The real Bode diagram is depicted in the following Figure



The Nyquist diagram is shown in the following Figure



As the nominal plant is asymptotically stable, exploiting the Nyquist criterion, the stability of the closed loop is ensured iff there are no (clockwise) encirclements of the point -1, while moving the point $-1/K$. The closed loop is stable iff $-\frac{1}{K} < -2.46$, or, conveniently, $K < \frac{1}{2.46}$.

Exercise 2 (10 points)

The design specifications must be translated as follows.

- $M_r \simeq 1.65$ dB
  By recovering the approximate formulae $M_p = \frac{2.3-M_r}{1.25}$, where M_r is not in dB, the desired phase margin is $M_p \simeq 50$ deg
- $t_r \leq 2.5$ s
  Since $t_r B_3 \simeq 3$ and $\omega_c = [0.5 \dots 0.8]\, B_3$, assuming $t_r = 2.5$ s, the desired bandwidth is $B_3 = 1.2$ dB and $\omega_c = [0.6 \dots 0.96]$. A good choice is $\omega_c = 0.6$ rad/s

In order to satisfy the steady state specification, since the $P(s)$ is of *type* 0, there is no need to add integrator. Hence K_c can be chosen as $K_c = 1/(e_d K_B)$, where K_B is the dc-gain of $P(s)$. In this case $K_c = 0.3846$. By looking at the phase and the magnitude of $P(j\omega_c)$, the gap to compensate can be calculated as follows

$$\angle(K_c P(j6)) = -152 \text{ deg}, \ |K_c P(j6)|_{\text{dB}} = 4.7 \text{ dB}$$

$$\Delta M = 0 \text{ dB} - 4.7 \text{ dB}, \ \Delta\phi = M_p - (180 - 152) \simeq 22 \text{ deg}$$

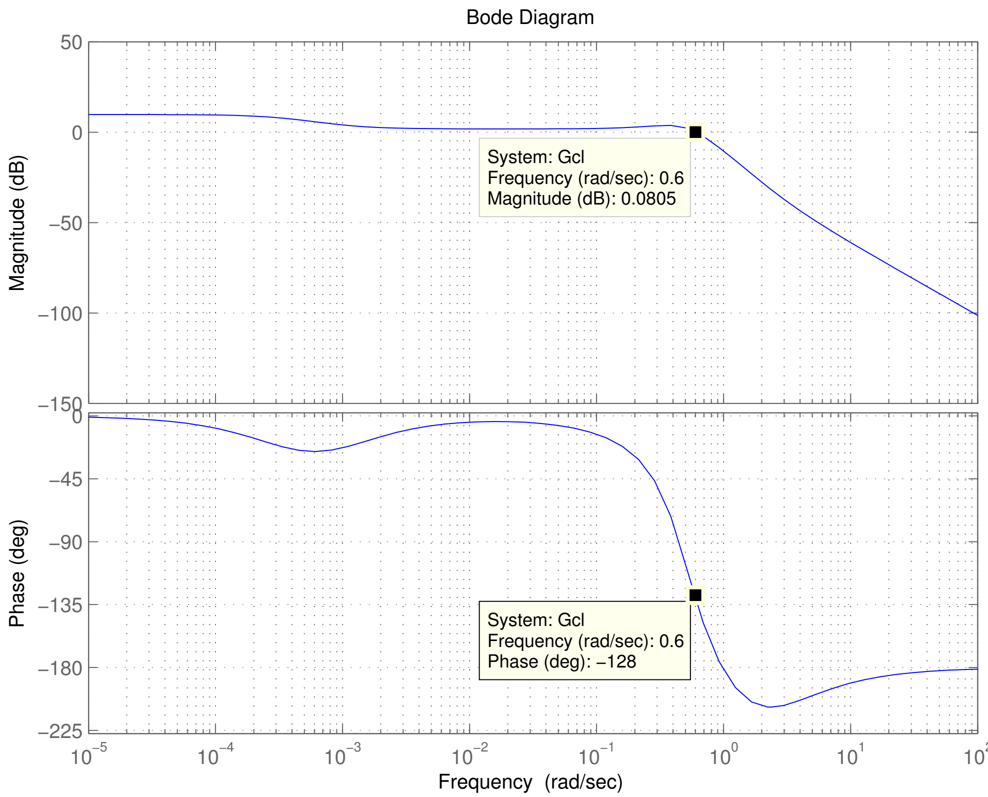
In order to satisfy the phase specification, a *lead* network with $\alpha_1 = 0.45$, centered in w_c for which $\tau_1 = \frac{1}{w_c\sqrt{\alpha_1}} = 2.4845$. This network will increase the phase for about 23 deg and the magnitude for about 3.5 dB. The phase specification is satisfied, while the new magnitude specification is $\Delta M' = \Delta M - 3.5 = -8.2$ dB. In order to shift the magnitude diagram while preserving the phase diagram at $\omega_c = 0.6$ rad/s, a *lag* network is posed at $\omega_{low} = \omega_c/1000$ rad/s, with $\alpha_2 = 0.4$ and $\tau_2 = \frac{1}{w_{low}\sqrt{\alpha_2}} = 2635$.
The corresponding network functions have the form

$$C_{lead}(s) = \frac{1+\tau_1 s}{1+\alpha_1\tau_1 s} \quad , \quad C_{lag}(s) = \frac{1+\alpha_2\tau_2 s}{1+\tau_2 s}$$

At this point, the design specifications are verified for the closed loop transfer function

$$F(s) = \frac{L(s)}{1 + L(s)}, \text{ where } L(s) = K_c C_{lead}(s) C_{lag}(s) G(s)$$

In the following Figure is shown the step response of the closed loop $F(s)$



<u>Exercise</u> 3 (6 points)

Setting $V(x) = x'Px$, with $P = P' > 0$, and imposing the decreasing of $V(x)$, one obtains that $\Delta V(x) = V(x(k+1)) - V(x(k))$ is negative definite if and only if for some $Q = Q' > 0$

$$A'PA - P = -Q$$

Now, we take $Q = I$, and defining $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$ we obtain

$$\begin{bmatrix} p_{11}/4 + p_{12} + p_{22} & p_{12}/4 + p_{22}/2 \\ p_{12}/4 + p_{22}/2 & p_{22}/4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

leading to

$$P = \begin{bmatrix} -20 & 8 \\ 8 & -4 \end{bmatrix}$$

which is positive definite. Therefore, the origin is a globally asymptotically stable equilibrium point for the system.

Exercise 4 (7 points)

- Given the discrete-time system of order n

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

  a deadbeat controller is a state feedback controller $u(k) = Kx(k)$ which places all the closed-loop poles at the origin, i.e. $\det(zI - A - BK) = z^n$. Since by Cayley-Hamilton theorem $(A + BK)^n$, the state vanishes after n steps: $x(n) = (A + BK)^n x(0) = 0$, and then remains at the origin. The procedure to obtain the vector K is exactly the same as in an eigenvalue assignment problem.

- System identification is a procedure to build a mathematical model of the dynamics of a system from measured data. Such procedure is often used in practice because the dynamical model is difficult to obtain due to the complexity of the system and/or to lack of knowledge on it. Also, a first principle model is sometime too complex for the design of a controller.

  Three kinds of models: white box (model structure based on first principles, model parameters estimated from measured data); grey box (model structure partially known from first principles, the rest reconstructed from data), black box (model structure and parameters unknown, all reconstructed from measured I/O data).