University of Trento

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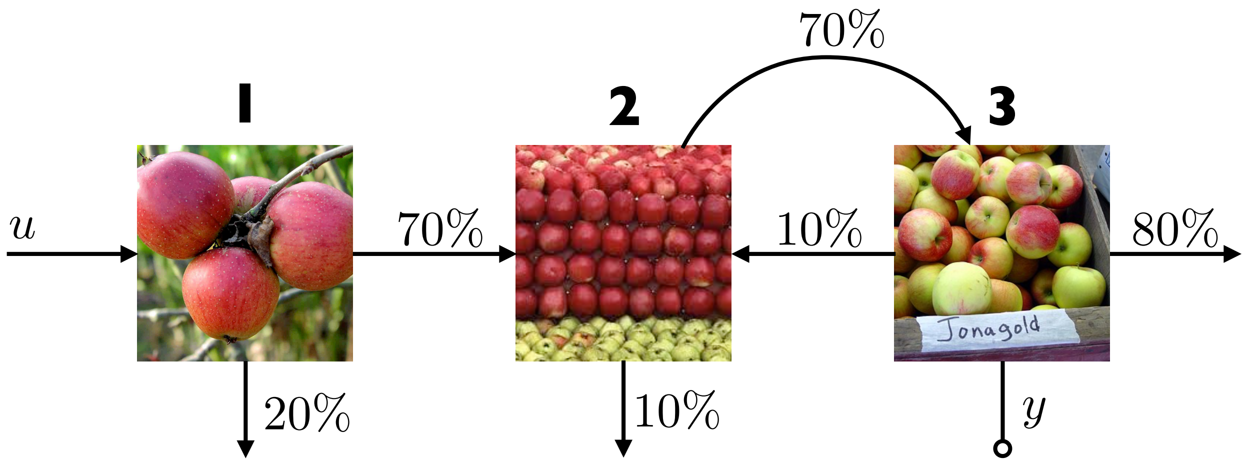
Automatic Control 1

Exercise 1 (9 points)



Consider the supply chain depicted above. Every day stock farm #1 receives a quantity u of apples, gives 70% of its apples to the consortium #2, while 20% are wasted. Consortium #2 discards 10% of its apples, gives 70% to store #3. Store #3 sells 80% of apples, while gives back 10% of unsold apples to consortium #2. Consider the apples stored in store #3 as the output of the system.

- Provide a state space representation (A, B, C, D) of the supply chain.
- Study the stability of the system.

Exercise 2 (13 points)

Consider the continuous-time nonlinear system

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= 3x_1(t)(1-x_2(t)) - x_2(t) + u(t) + d(t)\\ y(t) &= x_1(t)\end{aligned}$$

where $u(t)$ is the control input and $d(t)$ is a disturbance term.

- Considering the term $v(t) = u(t)+d(t)$ as the input of the system, find the equilibrium point $(\bar{x}_1, \bar{x}_2) \in \mathbb{R}^2$ obtained by setting the constant value $v(t) \equiv \bar{v} = 0$, $\forall t \in \mathbb{R}$.

- Determine the system (A, B, C, D) linearized around the equilibrium point $(\bar{x}_1, \bar{x}_2)$ and study its stability.
- Consider the system linearized around $(\bar{x}_1, \bar{x}_2) = (0, 0)$ and assume a constant (and unmeasurable) value of the disturbance $d(t) \equiv \bar{d}$. Using pole-placement techniques design (if possible) a state-feedback control law for $u(t)$ that is able to regulate $y(t)$ on a given constant reference r with zero steady-state error, placing all the poles of the closed-loop system in -1. Before designing the controller, verify the reachability of the extended system.

Exercise 3 (9 points)

Consider the discrete-time linear system

$$\begin{aligned} x_1(k+1) &= ax_1(k) + bx_2(k) + u(k) \\ x_2(k+1) &= x_2(k) + bu(k) \\ y(k) &= 2x_2(k) \end{aligned}$$

where $a, b \in \mathbb{R}$.

- Study the observability properties of the system (observability, reconstructability, detectability).
- Obtain the transfer function $G(z)$ of the system.
- For a generic linear system describe how the observability properties of the system are linked to the possible zero/pole cancellations in the transfer function. Use these properties to briefly analyze the case of the given system.