University of Trento

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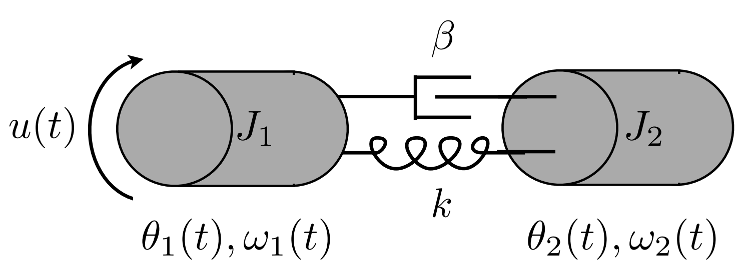
Automatic Control 1

Exercise 1 (8 points)

Consider the mechanical system depicted below, consisting of two bodies with moment of inertia equal to J_1 and J_2, respectively. The angular position and velocity of each body is denoted by θ_i and ω_i, with $i = 1, 2$. The two bodies are connected through a rotational spring of elastic constant k, and a damper with viscous friction coefficient β. Assume that it is possible to act with an external torque $u(t)$ on the first body, and consider $y(t) = \omega_2(t)$ as the system output.

- Obtain a state-space representation (A, B, C, D) of the system by taking $x = \begin{bmatrix} \theta_2 - \theta_1 \\ \omega_1 \\ \omega_2 \end{bmatrix}$ as the state vector.
- Given $J_1 = J_2 = 1$ kg·m^2, $k = 2$ Nm/rad, and $\beta = 1$ Nms/rad, study the stability of the system



Exercise 2 (9 points)

Consider the continuous-time nonlinear system

$$\begin{aligned} \dot{x}_1(t) &= x_1^3(t) - x_2(t) + 1 \\ \dot{x}_2(t) &= e^{x_2(t)} + u(t) \\ y(t) &= -x_1(t) + u(t) \end{aligned}$$

- Find the equilibrium state $(\bar{x}_1, \bar{x}_2) \in \mathbb{R}^2$ obtained with a constant input $\bar{u} = -1$.

- Determine the linearized system (A, B, C, D) around the equilibrium state $(\bar{x}_1, \bar{x}_2)$.
- Obtain the transfer function $G(s)$ of the linearized system, and determine its zeros and poles.
- Discuss the stability properties of the linearized system.

<u>**Exercise**</u> 3 (10 points)

Consider the discrete-time linear system

$$\begin{aligned} x_1(k+1) &= x_1(k) + u(k) \\ x_2(k+1) &= 2x_1(k) + ax_2(k) + u(k) \end{aligned}$$

where a is a constant parameter.

- Study the reachability properties of the system (reachability, controllability, stabilizability) for all $a \in \mathbb{R}$.
- In case $a = 0$, is it possible to steer the state from $x_1(0) = x_2(0) = 1$ to $x_1(2) = x_2(2) = 2$ in two time steps, and why? If so, find the sequence $u(0)$, $u(1)$ that determines such a state transition.
- In the same case $a = 0$, design (if possible) a state-feedback control law using pole-placement techniques by placing both poles of the closed-loop system at $\frac{1}{2}$.

<u>**Exercise**</u> 4 (4 points)

For the discrete-time linear system

$$\begin{aligned} x(k+1) &= \begin{bmatrix} A_{uo} & A_{12} \\ 0 & A_o \end{bmatrix} x(k) + \begin{bmatrix} B_{uo} \\ B_o \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0 & C_o \end{bmatrix} x(k) \end{aligned}$$

with $A_o \in \mathbb{R}^{n_o \times n_o}$, $A_{uo} \in \mathbb{R}^{n-n_o \times n-n_o}$, $1 \leq n_o \leq n$, specify the conditions for which the system is observable, reconstructable, detectable.