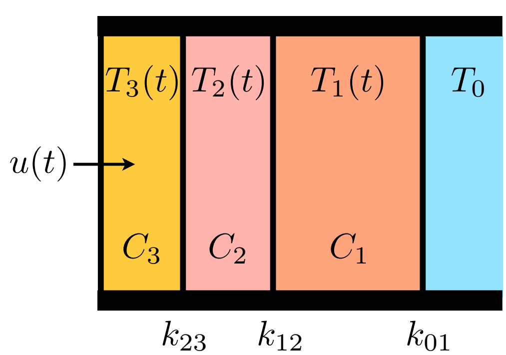
Prof. Alberto Bemporad February 17, 2011

Automatic Control 1

<u>Exercise 1</u> (14 points)

Consider the thermal system depicted below. The system consists of three bodies of thermal capacitance C_1, C_2, and C_3 and temperature $T_1(t)$, $T_2(t)$, and $T_3(t)$, respectively. Body 1 exchanges heat with body 2 and with a body of constant temperature T_0. Body 2 exchanges heat with bodies 1 and 3, while body 3 exchanges heat with body 2 and receives a heat flow $u(t)$ from an external source. The heat exchange coefficients between the different bodies, as indicated in the figure below, are referred to as k_{01}, k_{12}, and k_{23}, respectively.

- Obtain a state-space representation (A, B, C, D) of the system, with input $u(t)$ and output $y(t) = T_1(t) - T_0$.
- Given $C_1 = C_2 = C_3 = 10$ J/K, $k_{01} = k_{12} = 10$ W/K, $k_{23} = 0$ W/K, study the stability of the system.
- Given $C_1 = C_2 = C_3 = 10$ J/K, $k_{01} = k_{12} = 10$ W/K, determine for which values of the parameter $k_{23} \in [0, 10]$ W/K the system is observable, reconstructable, detectable.



<u>Exercise 2</u> (13 points)

Consider the discrete-time system

$$
\begin{aligned}
x_1(k+1) &= -x_1(k) + x_2(k) \\
x_2(k+1) &= x_1(k) + u(k)
\end{aligned}
$$

- Study the reachability properties of the system.
- Determine (if possible) a state-feedback control law using pole-placement techniques by placing both poles of the closed-loop system in 0.5.
- Given the output equation $y(k) = x_1(k) + u(k)$, obtain the transfer function $G(z)$ of the system and indicate its DC gain, zeros, and poles.

<u>Exercise 3</u> (4 points)

Describe the difference between open-loop (feedforward) control and closed-loop (feedback) control.