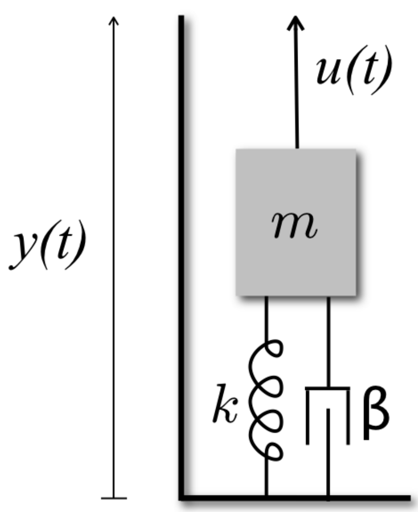
Prof. Alberto Bemporad January 27, 2011

Automatic Control 1

<u>Exercise 1</u> (13 points)

Consider the mechanical system depicted below, consisting of a mass m which moves on the vertical axis, connected to the ground by a spring of elastic contant k and a damper with viscous friction coefficient β. Assume that is is possible to act on the vertical position $y(t)$ of the mass mass with an external force $u(t)$, and take into account that the mass is also subject to gravity.

- Considering the gravity force $-mg$ as a constant and unmeasurable disturbance d (i.e. $d = -mg$), obtain a state-space representation (A, B, C, D) of the system, with the signal $(u+d)$ as input variable, and the measured position $y(t)$ as output.
- Determine for which values of the parameters $m > 0$, $k \geq 0$, $\beta \geq 0$ the system is reachable, controllable, stabilizable.
- Let $m = 2$ Kg, $k = 2$ Kg/s, $\beta = 1$ Kg/s^2. By using pole-placement techniques design a state-feedback control law that is able to regulate the mass position $y(t)$ on a given constant reference r with zero steady-state error, placing all the poles of the closed-loop system in -1. Before designing the controller, verify the reachability of the extended system for the given numerical values of the parameters.



Exercise 2 (11 points)

Consider the continuous-time system

$$\begin{aligned}\dot{x}_1(t) &= e^{-x_1(t)} - x_2(t) + u(t)\\ \dot{x}_2(t) &= x_1(t)\\ y(t) &= x_1(t) + x_2(t)\end{aligned}$$

- Find the equilibrium state $(\bar{x}_1, \bar{x}_2)$ obtained with a constant input $\bar{u} = 0$.
- Determine the linearized system (A, B, C, D) around the equilibrium state $(\bar{x}_1, \bar{x}_2)$, and study its stability.
- Obtain the transfer function $G(s)$ of the linearized system, and indicate its DC gain, zeros and poles.

Exercise 3 (7 points)

- Recalling that, given a linear discrete-time system, the pair of states $x_1 \neq x_2 \in \mathbb{R}^n$ is called *indistinguishable from the output* $y(\cdot)$ if for any input sequence $u(\cdot)$ one has $y(k, x_1, u(\cdot)) = y(k, x_2, u(\cdot))$, $\forall\, k \geq 0$, give the definition of observability. Also, define the *observability matrix* Θ and describe how it is possible to check the observability of the system through the analysis of Θ.
- Is it possible to determine the current state $x(k)$ even if the system is not completely observable? In which case?