University of Trento

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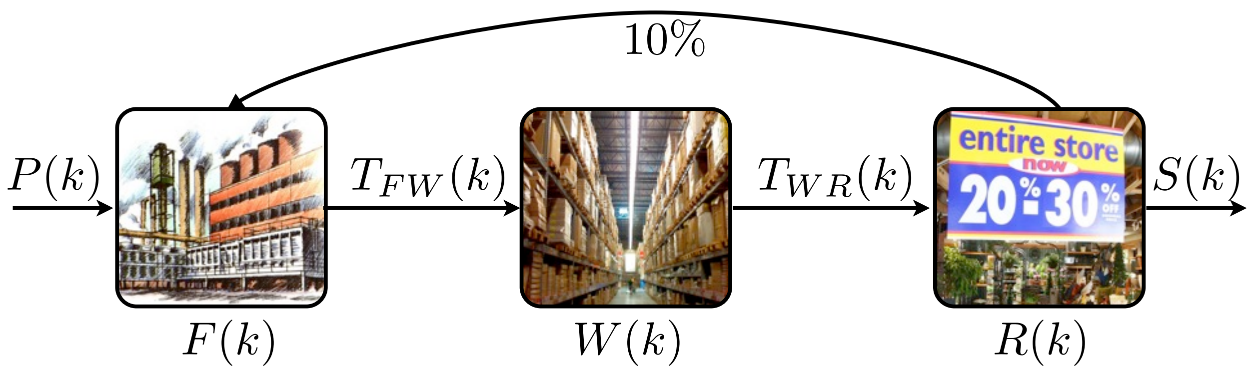
Automatic Control 1

<u>Exercise 1</u> (13 points)



Consider the supply chain depicted below. It consists of a factory F, a warehouse W and a retailer R. At each day k, a quantity $P(k)$ of raw materials is delivered to the factory. At the same time, the factory produces $T_{FW}(k)$ of finished product, that is delivered to the warehouse. A quantity $T_{WR}(k)$ of stored product is delivered to the retailer. Undelivered products are stored for the next days, denote by $F(k)$, $W(k)$ and $R(k)$ the quantities stored in the factory, warehouse, and retailer, respectively. The retailer sells a quantity $S(k)$ of product, and returns 10% of the entire product stored in R back to the factory for further refinements. Each transaction from one stage to the other occurs in one time step (one day).

1. Obtain a discrete-time state-space model of the system, considering that the quantities of product in the three stages (F, W, P) are measurable, the quantities T_{FW} and T_{WR} are decided by the owner of the supply chain, while the injected raw materials P and the sold products S are modeled as additive disturbances.

2. Assume now that the factory does not acquire any raw material and the retailer does not sell any product. Assume that only the quantity $R(k)$ of products stored by the retailer is measurable. Study the observability properties of the system (observability, reconstructability, detectability).

3. Study the stability of the system.

Exercise 2 (10 points)

Consider the continuous-time system

$$\dot{x}_1(t) = -x_1(t) + x_3(t)$$
$$\dot{x}_2(t) = -x_2(t)$$
$$\dot{x}_3(t) = x_1(t) - x_3(t) + u(t)$$

1. Is the system reachable, controllable, stabilizable ?
2. Design (if possible) a state feedback controller, using pole-placement techniques, placing all the poles of the closed-loop system in -1.

Exercise 3 (7 points)

1. Given a SISO (single-input, single-output) continuous-time dynamical system in state-space form

$$\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{cases}$$

   define what are the "poles" and the "zeros" of its transfer function $G(s)$. Discuss the relation between the poles of $G(s)$ and the eigenvalues of A.

2. Given the continuous-time system

$$\dot{x}(t) = \sqrt{2}u(t)$$

   with output $y(t) = x(t)$, find the poles and the zeros of its transfer function. Which are the stability properties of the system ?