University of Trento

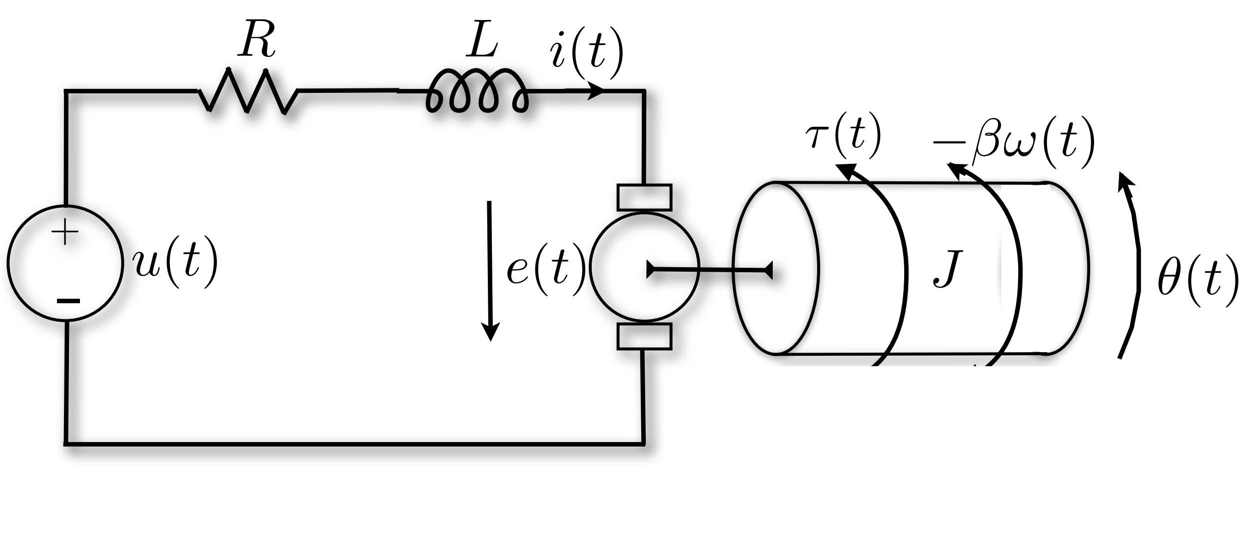
Prof. Alberto Bemporad July 19, 2010

Automatic Control 1

Exercise 1 (13 points)

Consider the DC motor depicted below. We recall that the back-EMF is related to rotor angular velocity as $e(t) = -K\omega(t)$, while the torque $\tau(t)$ is related to the armature current as $\tau(t) = Ki(t)$. The mechanical part is also subject to a viscous friction torque $-\beta\omega(t)$. The positive constants R, L, J, represent the armature resistance and inductance, and the rotor moment of inertia, respectively. The input is the voltage $u(t)$, and the output is the angular position $\theta(t)$.

- Obtain a continuous-time state-space model of the system.
- Given $J = 0.01\ Kg \cdot m^2$, $R = 1\ \Omega$, $L = 0.5\ H$, $\beta = 0.1\ Nms$, study the observability of the system (observabiliy, reconstructability, detectability) for $K = [0, +\infty)$.
- Study the stability of the system with the same parameters at the previous point, and $K = 0$.



Exercise 2 (13 points)

Consider the discrete-time system

$$\begin{aligned} x_1(k+1) &= x_1(k) + x_3(k) \\ x_2(k+1) &= -x_2(k) + (1+\alpha)u(k) \\ x_3(k+1) &= x_1(k) + x_3(k) + u(k) \end{aligned}$$

- Study the system reachability for $\alpha \in \mathbb{R}$.
- For $\alpha = 0$, design (if possible) a state feedback controller, using pole-placement techniques, placing all the poles of the closed-loop system in zero.

Exercise 3 (7 points)

- Given a linear continuous-time dynamical system (A, B, C, D), with input $u(t)$ and output $y(t)$, define the transfer function of the system.
- Given the problem of regulating the output $y(k)$ of a discrete-time dynamical system $(A, B, C, 0)$ to zero under the action of a constant input disturbance $d(k)$, describe a possible state-feedback design strategy to obtain zero steady-state error.