University of Trento

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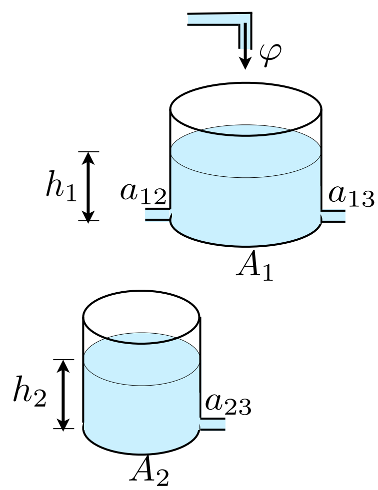
Automatic Control 1

Exercise 1 (15 points)

Consider the tank system depicted below. The control variable (input) is the volume flow φ, while the measured variable (output) is the fluid level h_2 in the second tank. The areas of the two tanks are A_1 and A_2, while the areas of the orifices indicated in the figure are a_{12}, a_{13} and a_{23}. The fluid is perfect (no shear stresses, no viscosity, no heat conduction), and subject only to gravity. The tanks are filled with water (incompressible fluid), and the external pressure is constant (atmospheric pressure).



- Obtain a continuous-time state-space model of the system.
- Given $a_{12} = a_{23} = a_{13} = 1\ m^2$, $A_1 = 200\ m^2$, $A_2 = 100\ m^2$, find the equilibrium state $(\bar{x}_1, \bar{x}_2)$ obtained with a constant input $\bar{u} = 10\ m^3/s$, approximating gravity acceleration as $g \simeq 10\ m/s^2$.
- Determine the linearized system (A, B, C, D) around the equilibrium state $(\bar{x}_1, \bar{x}_2)$, and study its stability.
- Is the linearized system reachable? Controllable? Stabilizable?

Exercise 2 (13 points)

Consider the continuous-time system

$$\begin{cases} \dot{x}(t) &= \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ \\ y(t) &= \begin{bmatrix} 1 & 1 \end{bmatrix} x(t) \end{cases}$$

- Determine (if possible) a control law, using pole-placement techniques, placing the poles of the closed-loop system in $(-2, -2)$.
- Determine (if possible) a state observer, using pole-placement techniques, with poles in $(-1, -2)$. Is it possible to place the poles of the observer in $(0.5, 0.5)$ instead? What would be the (qualitative) behavior of the estimation error $x(t) - \hat{x}(t)$ in such a case?

Exercise 3 (5 points)

Enunciate the initial and final value theorems for the Laplace transform and for the Z-transform.