Is MPC a Mature Technology for Guidance and Navigation ?

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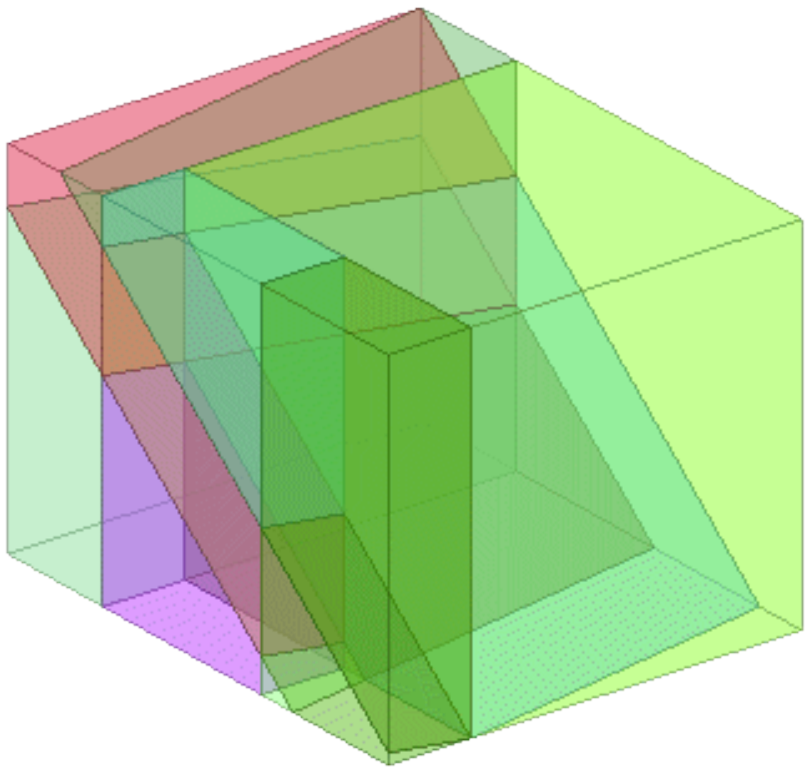
IMT Institute for Advanced Studies Lucca

Outline

- What is Model Predictive Control (MPC) (in a nutshell)
- Computation and memory requirements of MPC
- MATLAB tools for MPC design and code-generation
- Applications of MPC
- Conclusions



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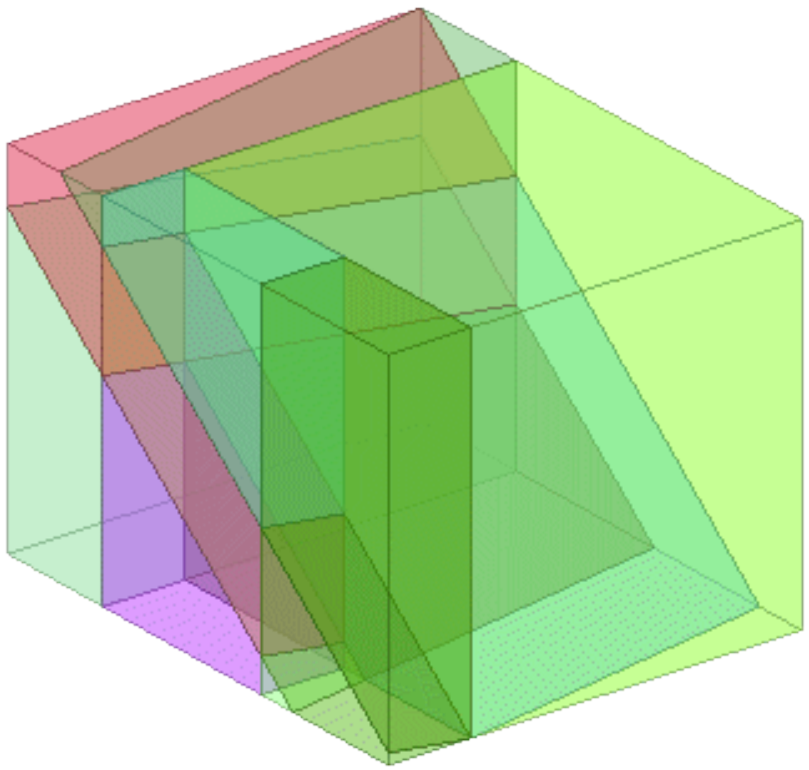
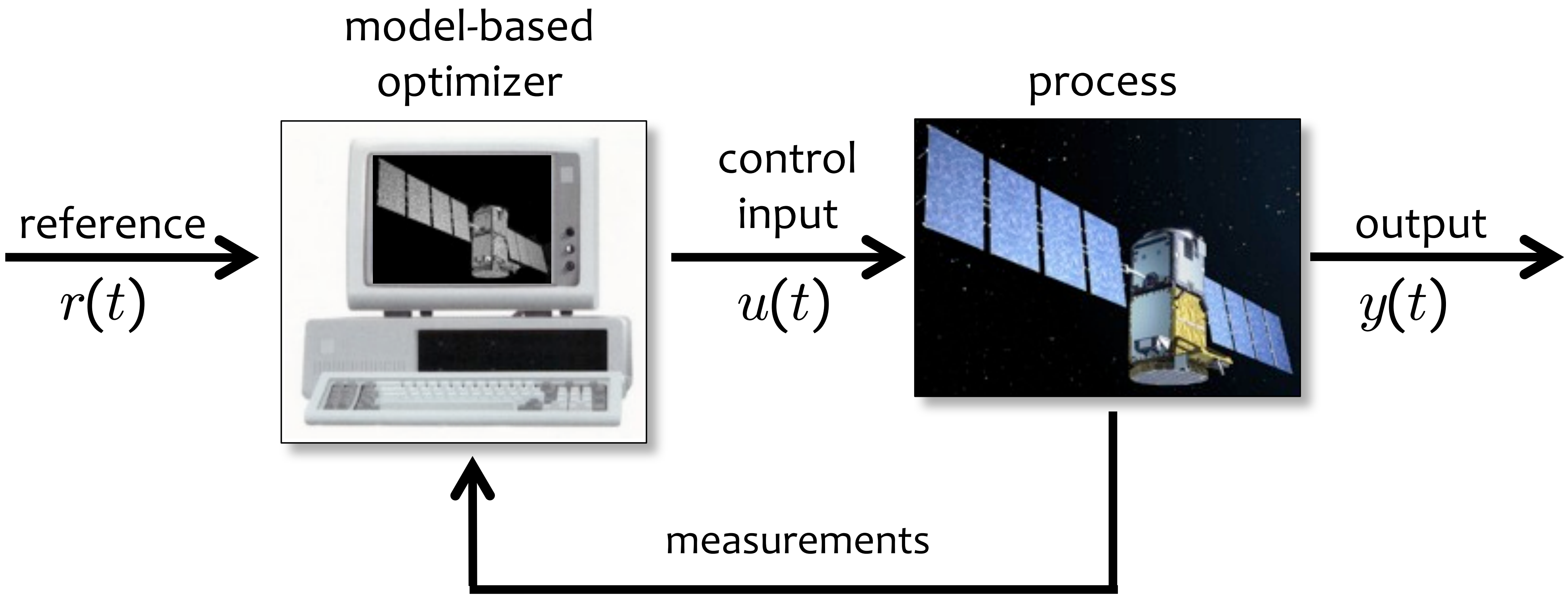


Model Predictive Control (MPC)

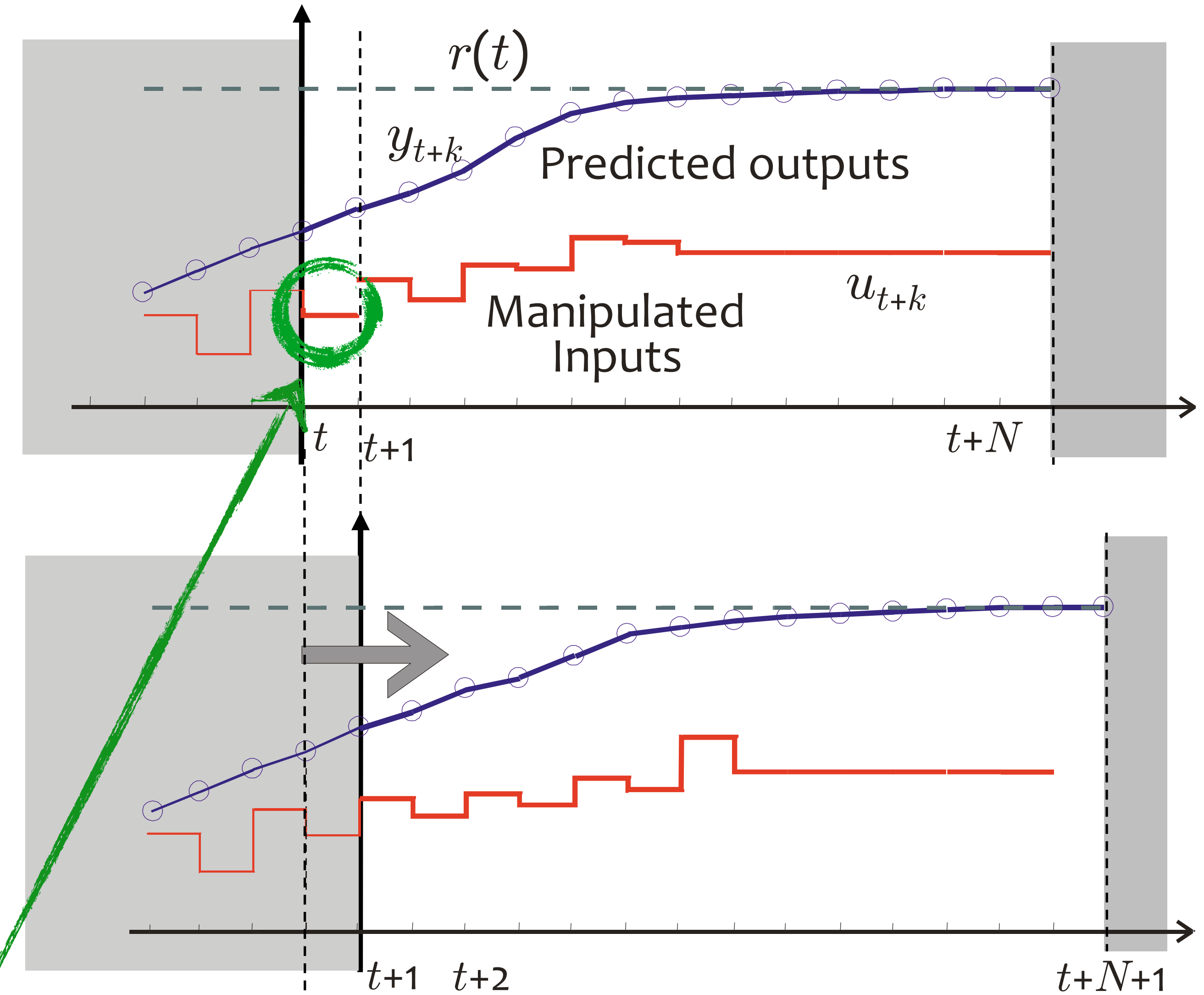


By using a dynamical ***model*** *of the process* ***predict*** *its future evolution to choose the "best"* ***control*** *action*

MPC algorithm

- **At time t:** solve an **optimal control** problem over a future horizon of N steps

$$\begin{array}{ll} \min & \sum_{k=0}^{N-1} \ell(y_{t+k} - r_{t+k}, u_{t+k}) \\ \text{s.t.} & x_{t+k+1} = f(x_{t+k}, u_{t+k}) \\ & y_{t+k} = g(x_{t+k}, u_{t+k}) \\ & \text{constraints on } u_{t+k}, x_{t+k}, y_{t+k} \\ & x_t = x(t) \end{array}$$



- Apply only the first optimal move $u^*(t)$, throw the rest of the sequence away
- **At time t+1:** Get new measurements, repeat the optimization. And so on …

MPC transforms open-loop optimal control into a **feedback** control law

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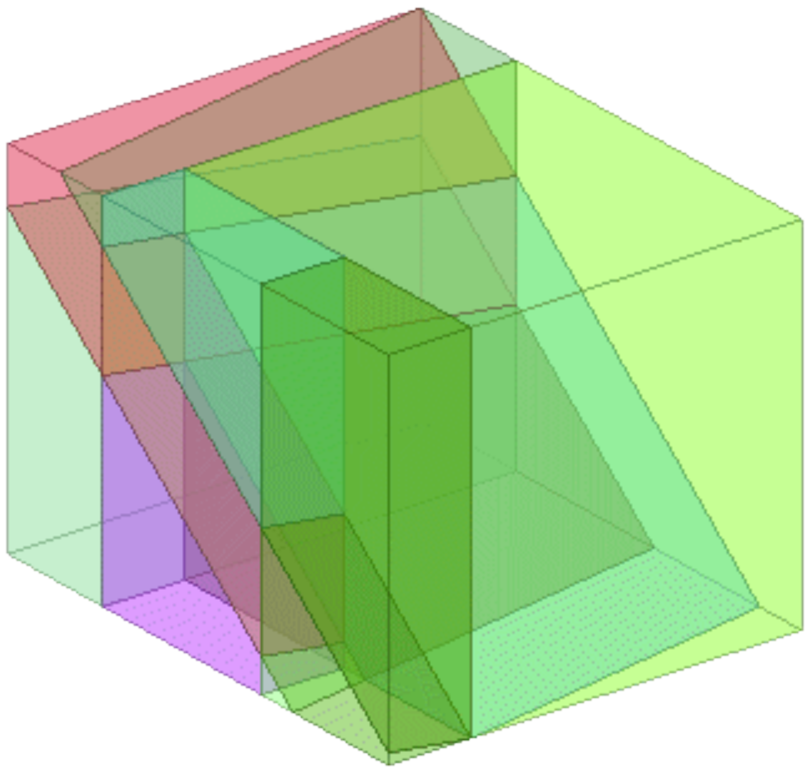


MPC optimization problem

Problem nature depends on prediction model, cost function, constraints:

$$\min \sum_{k=0}^{N-1} \ell(y_{t+k} - r_{t+k}, u_{t+k})$$
$$\text{s.t.} \quad x_{t+k+1} = f(x_{t+k}, u_{t+k})$$
$$y_{t+k} = g(x_{t+k}, u_{t+k})$$
$$\text{constraints on } u_{t+k}, x_{t+k}, y_{t+k}$$
$$x_t = x(t)$$

- **Linear** model and constraints, **quadratic** costs

  → **(convex) Quadratic Program (QP)**

- Linear model and constraints, **"linear"** costs (e.g.: infinity norms)

  → **Linear Program (LP)**

- **Nonlinear** models, costs, constraints

  → **Nonlinear Program (NLP)**

- **Hybrid** dynamical models

  → **Mixed-Integer Program (MIP)**

Pros and cons of on-line optimization

✓ Continuously update the best decision, reacting to unexpected events (disturbances, faults, obstacles,...)

✓ Excellent LP/QP/MIP/NLP solvers exist today
("LP is a technology" – S. Boyd)

✖ Computation time may be too long: ok for large sampling times (>10 ms) but not for fast-sampling applications (< 1 ms).

✖ Requires relatively expensive hardware (microprocessor)

✖ Software complexity: solver code must be embedded in the application

✖ Real-time: Worst-case CPU time often hard to estimate

Any way to use MPC **without** on-line solvers ?

$$\begin{array}{ll} \min\limits_{U} & \frac{1}{2}U'HU + \mathbf{x}'(t)F'U + \cancel{\frac{1}{2}x'(t)Yx(t)} \\ \text{subj. to} & GU \leq W + S\mathbf{x}(t) \end{array}$$

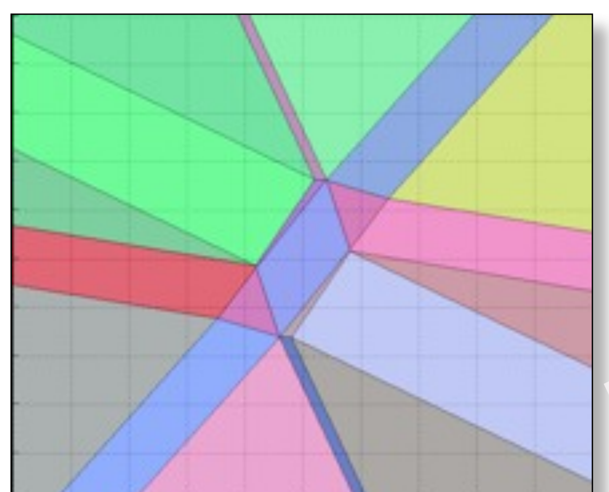
Idea: solve the QP **for all** $x(t)$ within a given range of $\mathbb{R}^n$ **off-line**

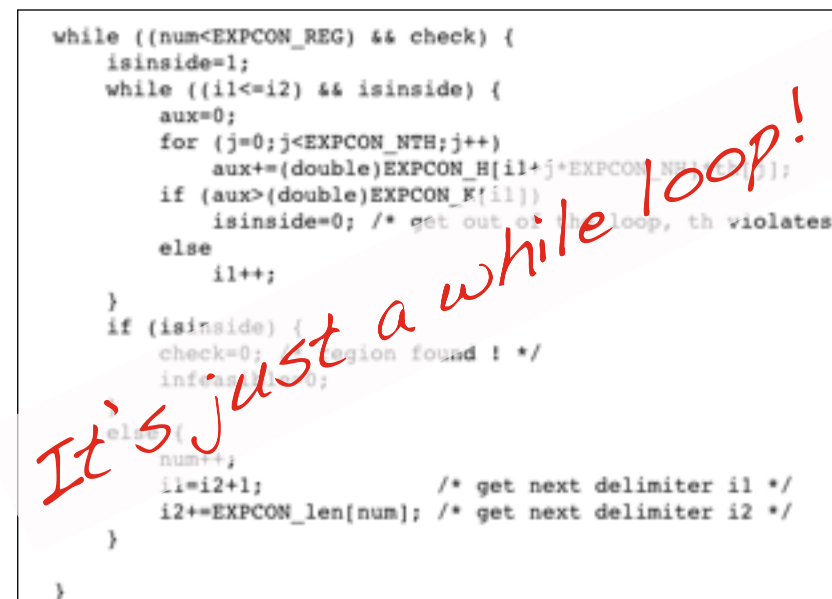
→ multi-parametric programming problem

The linear MPC controller is a **continuous piecewise affine** function of the state vector

$$u(x) = \begin{cases} F_1 x + g_1 & \text{if } H_1 x \leq K_1 \\ \vdots & \vdots \\ F_M x + g_M & \text{if } H_M x \leq K_M \end{cases}$$

(Bemporad, Morari, *et al.*, 2002)





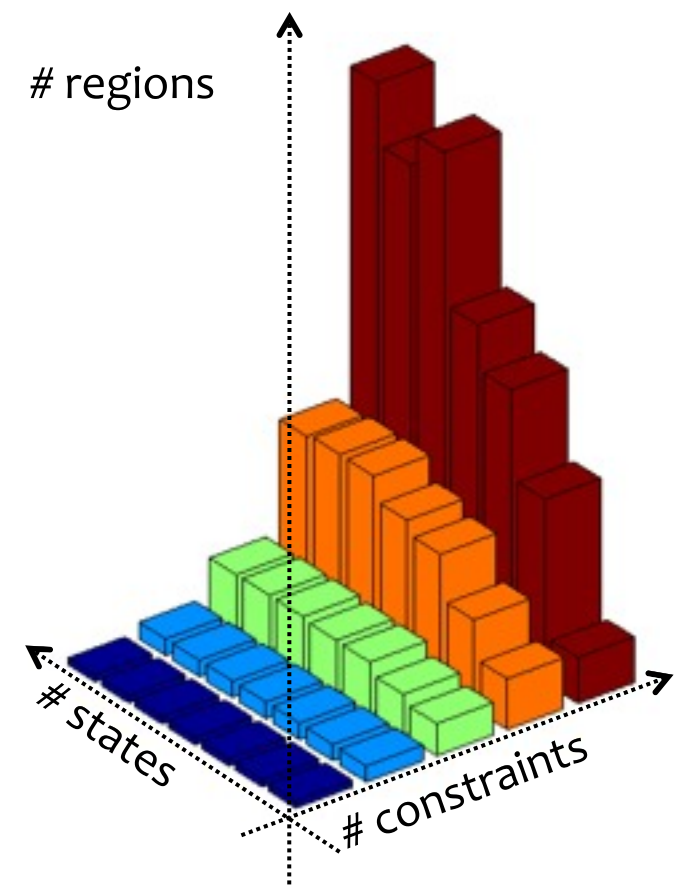
Applies to linear and hybrid MPC formulations !

Complexity of explicit MPC

- Number of regions depends on # possible combinations of active constraints
- Weak dependence on #states
- QP-based vs. Explicit MPC:

$2N$	QP (ms) average	worst	explicit (ms) average	worst	regions	[storage kb]
4	1.1	1.5	0.005	0.1	25	16
8	1.3	1.9	0.023	1.1	175	78
20	2.5	2.6	0.038	3.3	1767	811
30	5.3	7.2	0.069	4.4	5162	2465
40	10.9	13.0	0.239	15.6	11519	5598

(Intel Centrino 1.4 GHz)



Explicit MPC typically limited to 6-8 free control moves and 8-12 states+references

Linear MPC: summary of computation effort

MPC type	off-line computations	on-line computations
LTI model, **explicit** MPC	build QP/LP problem, solve multiparametric problem	evaluate PWA function
LTI model, **implicit**	build QP/LP problem	solve QP/LP problem
LTV model	none	build QP/LP problem solve QP/LP problem

LTI = Linear Time-Invariant

LTV = Linear Time-Varying

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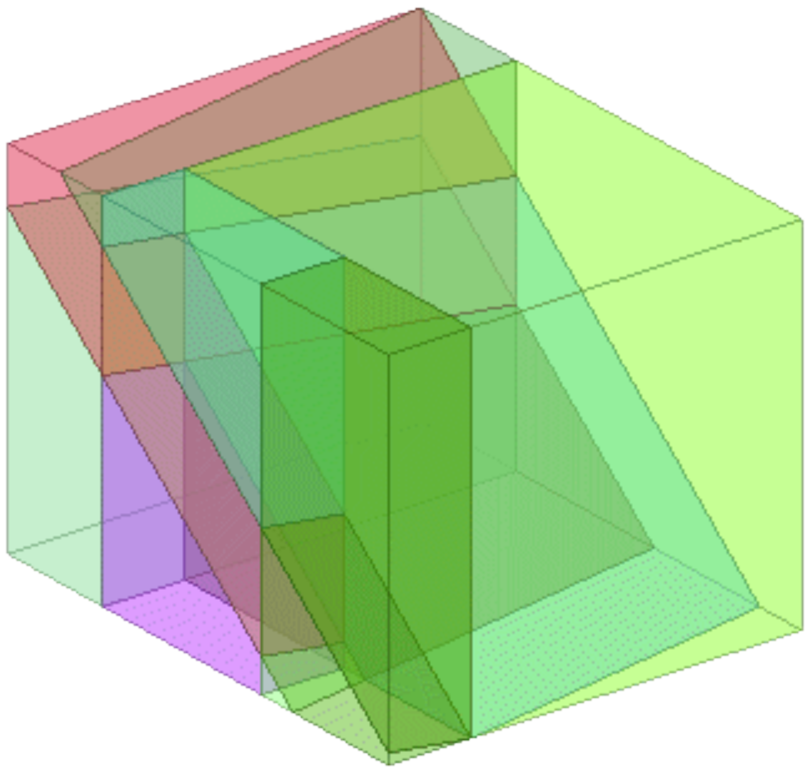
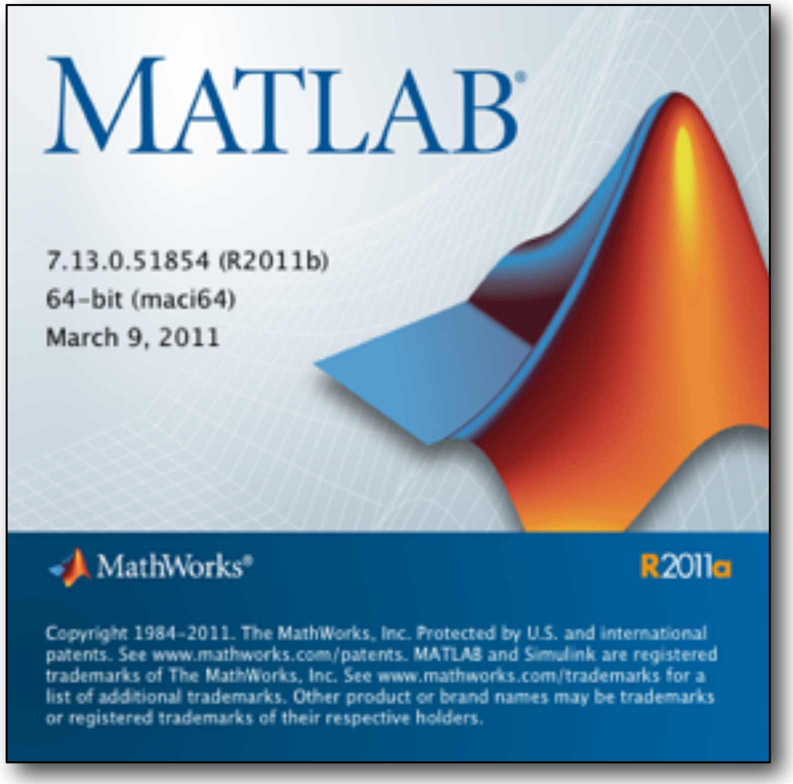


Model Predictive Control Toolbox

(Bemporad, Ricker, Morari, 1998-2011)

- **MPC Toolbox 4.0** (The Mathworks, Inc.)
  - Object-oriented implementation (MPC object)
  - MPC Simulink Library
  - MPC Graphical User Interface
  - Code generation [RTW, xPC Target, dSpace, etc.]
  - Linked to OPC Toolbox v2.0.1, SYS-ID Toolbox



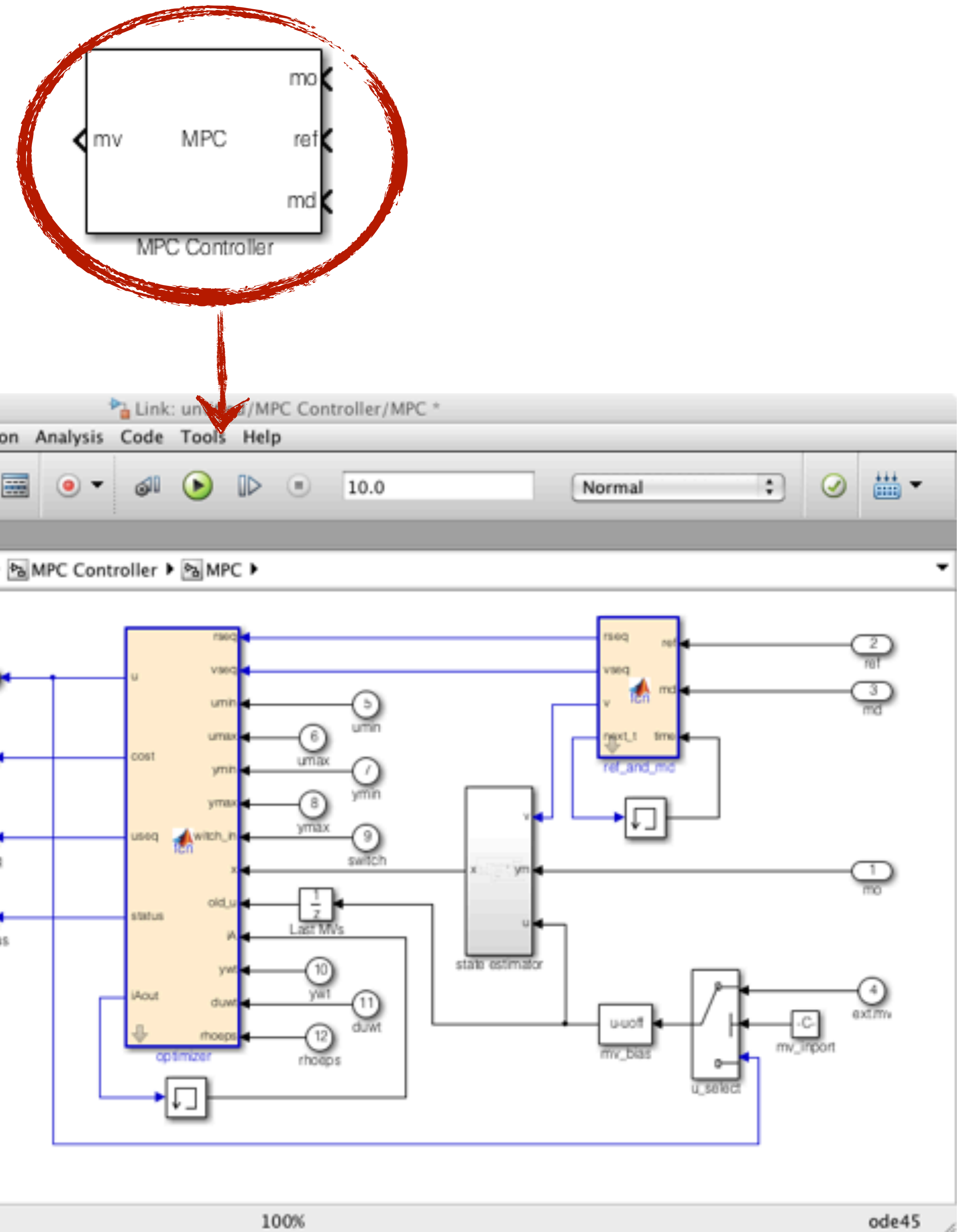
Complete solution for linear MPC design based on on-line QP

http://www.mathworks.com/products/mpc/

MPC Toolbox for MATLAB

(Bemporad, Ricker, Morari, 1998-2011)

- New features coming in next version:
  - MPC Simulink block coded in EML
  - New QP solver (EML)
  - On-line tuning
  - …

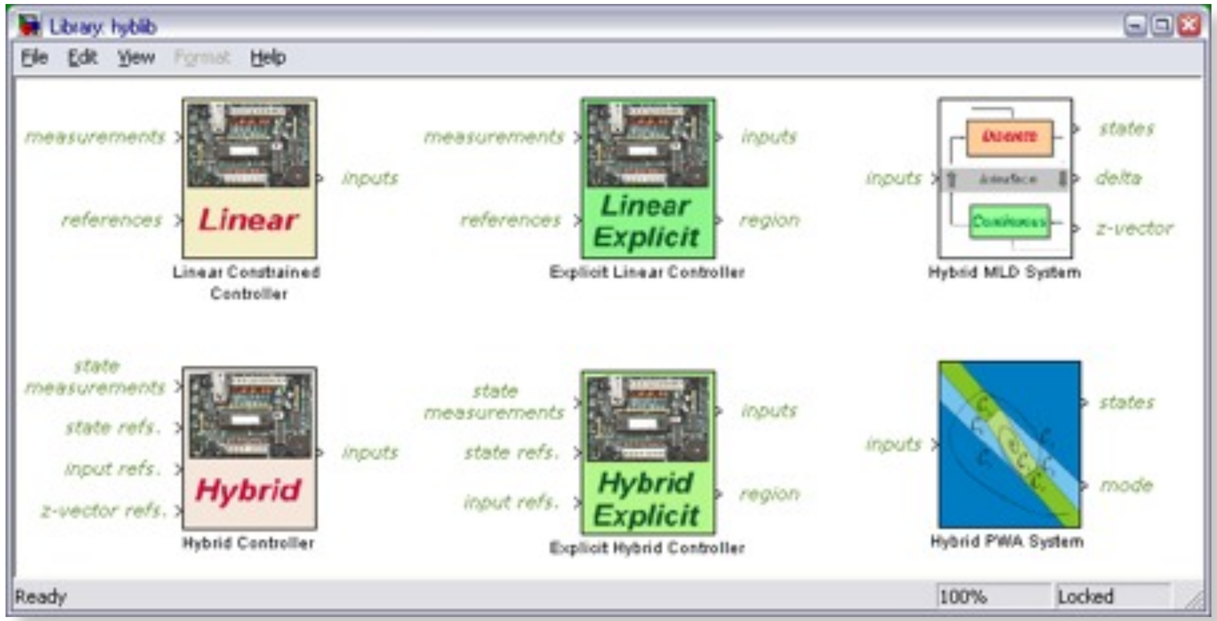


Hybrid Toolbox for MATLAB

(Bemporad, 2003-2011)

Features:

- **Hybrid** models: design, simulation, verification
- Control design for linear systems w/ constraints and hybrid systems (on-line optimization via QP/MILP/MIQP)
- **Explicit** MPC control (via multi-parametric programming)
- C-code generation
- Simulink library



3500+ download requests since October 2004

http://www.ing.unitn.it/~bemporad/hybrid/toolbox

MPCTOOL - An MPC Toolbox extension for ESA

(Bemporad, 2009-'11)

- Developed within the **ORCSAT project** funded by ESA (2009-2011)
- Large emphasis on ***real-time implementation*** capabilities of MPC
- MPC applications: orbit synchronization, impulsive hopping
- Toolbox features:
  - Set ***terminal*** weights and constraints in linear MPC (and ∞-horizon MPC)
  - MPC with discrete-valued inputs (***quantized***, ***variable horizon***)
  - Return the optimal sequence in Simulink for higher-level safety checks
  - MPC with ***PWA stage costs*** on inputs and outputs
  - MPC with ***mixed input and output constraints***
  - MPC for ***linear time-varying (LTV) models*** (EML)
  - ***LP and QP*** solvers for LTV-MPC (EML)

ORCSAT project
On-line Reconfiguration Control System and Avionics Technologies

MPCTOOL - An MPC Toolbox extension for ESA

- Hardware requirements for **LTV-MPC** based on LP tested by ThalesAleniaSpace (I) and Univ. Bristol (UK)

- AT697F processor not adequate, co-processor is needed

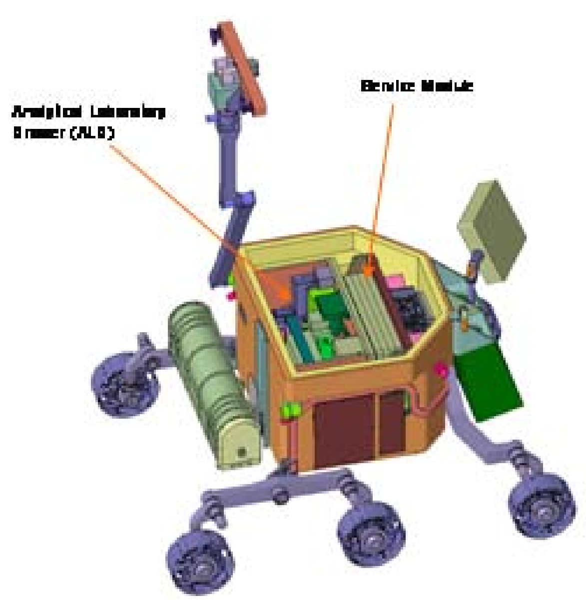
- FPGA implementation based on converting EML code to fixed-point + HDL code generation is not adequate (only ok for small MPC problems, due to fixed-point precision)

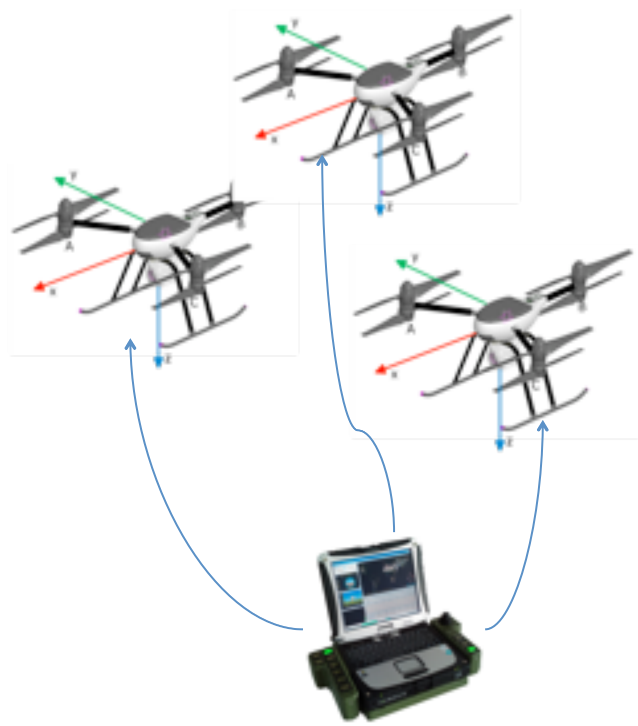
- Proposed solution: PowerPC750FX fully dedicated to solve MPC problem

MPCSoft - A new ESA toolbox for LTV-MPC

(Bemporad, 2010-'11)

- Developed within the **ROBMPC project** funded by ESA (2010-2012)
- Large emphasis on exploring MPC capabilities in new space apps
- Selected applications for LTV-MPC:
  - Planetary rover locomotion - wheel slip control
  - Planetary rover locomotion - path planning
  - Cooperative UAV navigation - formation forming and flying

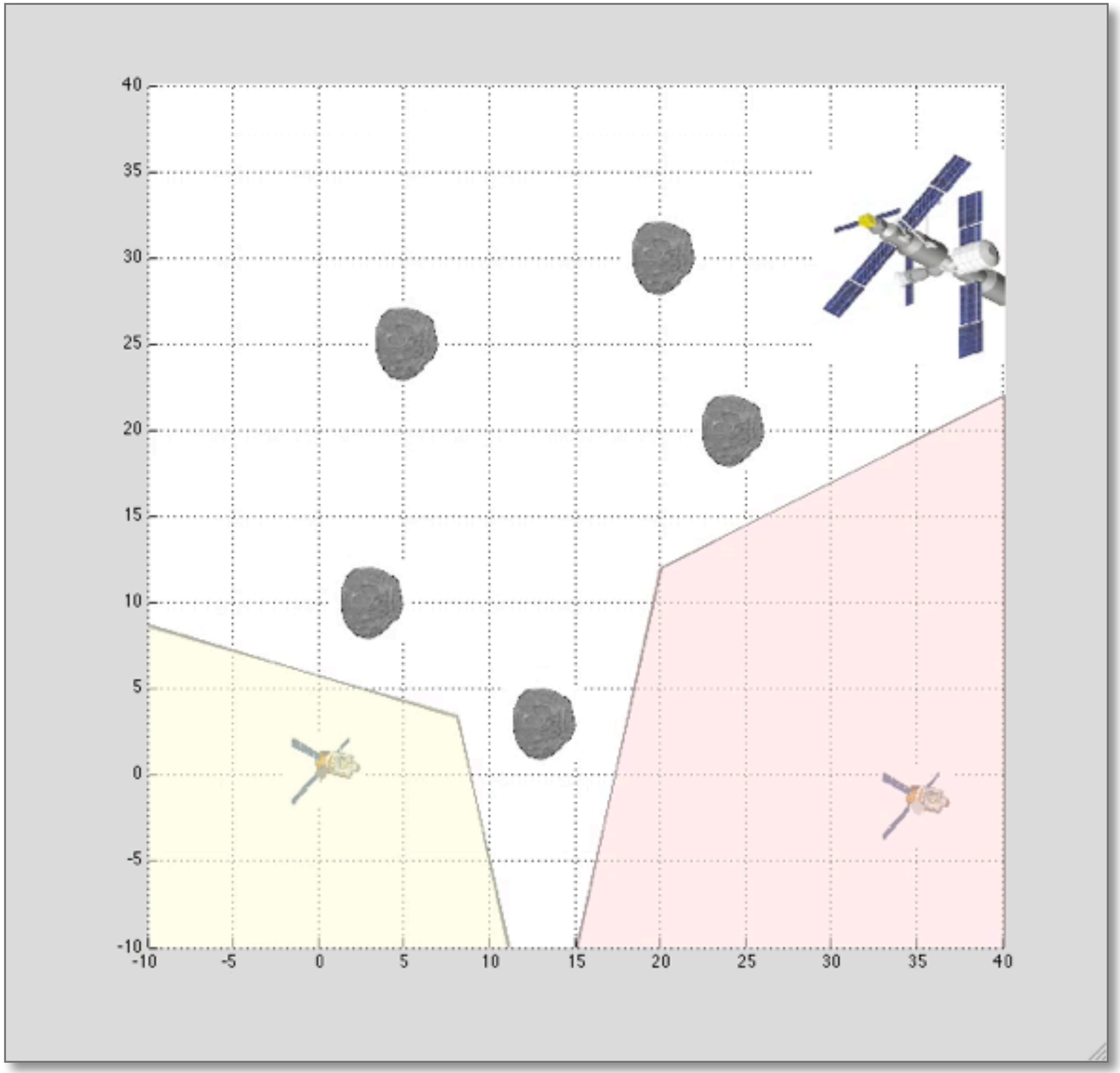




ROBMPC project
Robust Model Predictive Control (MPC) for Space Constrained Systems

Example: navigation demo using LTV-MPC

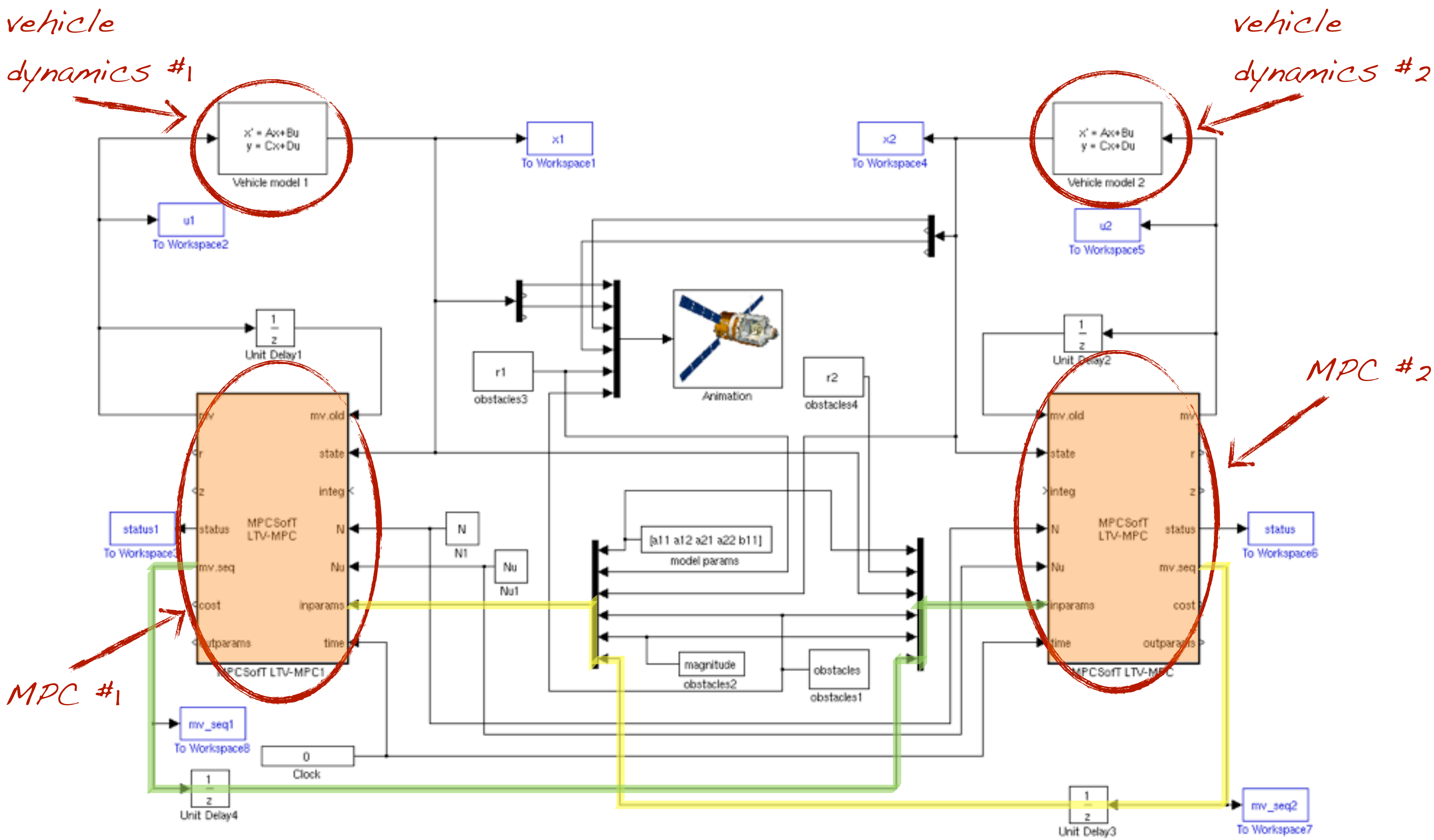
- Two vehicles avoiding each other and obstacles towards their targets



- Initial position #1 = (0,0)
- Target position #1 = (35,30)

- Initial position #2 = (35,-3)
- Target position #2 = (0,20)

Example: navigation demo using LTV-MPC



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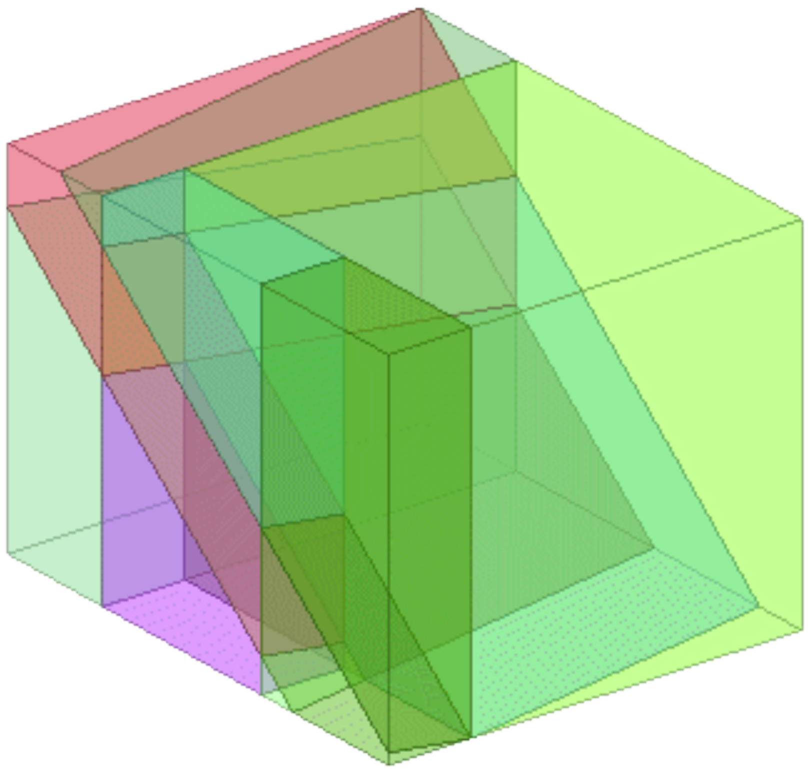


Industrial applications of MPC

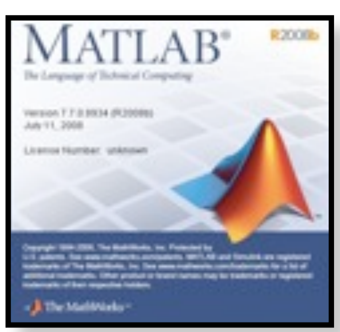
Area	Aspen Technology	Honeywell Hi-Spec	Adersa[b]	Invensys	SGS[c]	Total
Refining	1200	480	280	25		1985
Petrochemicals	450	80	—	20		550
Chemicals	100	20	3	21		144
Pulp and paper	18	50	—	—		68
Air & Gas	—	10	—	—		10
Utility	—	10	—	4		14
Mining/Metallurgy	8	6	7	16		37
Food Processing	—	—	41	10		51
Polymer	17	—	—	—		17
Furnaces	—	—	42	3		45
Aerospace/Defense	—	—	13	—		13
Automotive	—	—	7	—		7
Unclassified	40	40	1045	26	450	1601
Total	1833	696	1438	125	450	4542
First App.	DMC:1985 IDCOM-M:1987 OPC:1987	PCT:1984 RMPCT:1991	IDCOM:1973 HIECON:1986	1984	1985	
Largest App.	603 × 283	225 × 85	—	31 × 12	—	

(snapshot survey conducted in mid-1999)

(Qin, Badgewell, 2003)

"For us multivariable control is predictive control"

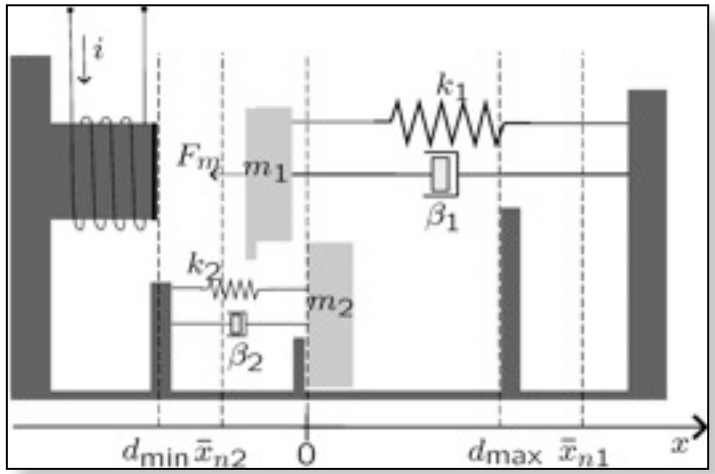
Tariq Samad, *Honeywell* (past President of IEEE Control System Society) (1997)



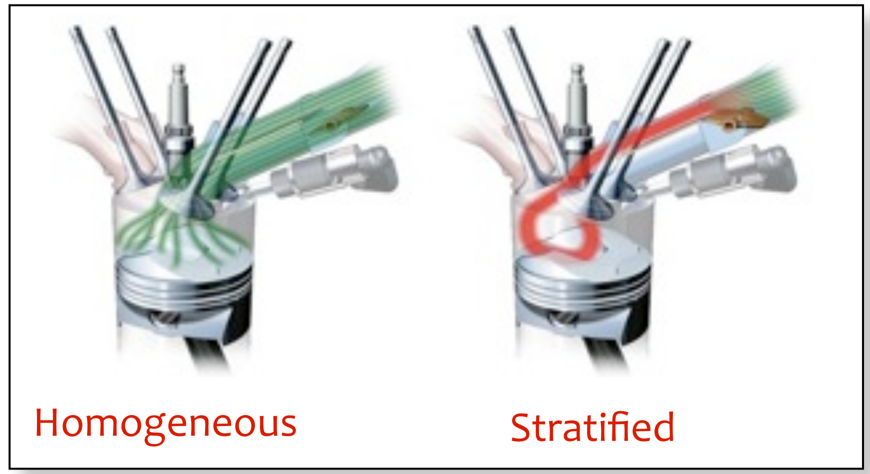
- MPC Toolbox 3.0 **most successful webinar** in 2009 !

Automotive applications of MPC

PhD students: Bernardini, Borrelli, Di Cairano, Giorgetti, Ripaccioli, Trimboli (2001-2011)
& Hrovat, Kolmanovsky, Tseng (Ford)



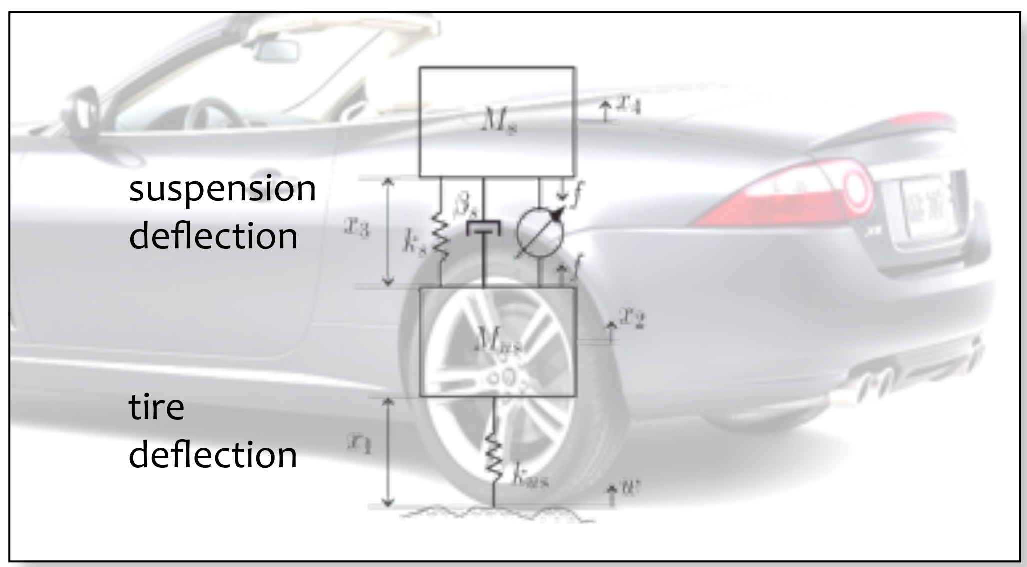
magnetic actuators



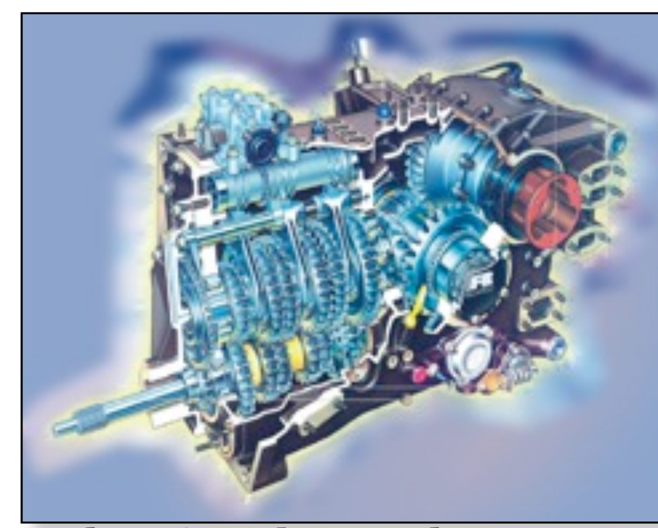
engine control

active steering

traction control



semiactive suspensions



robotized gearbox

Ford, JAGUAR, CENTRO RICERCHE FIAT, DENSO

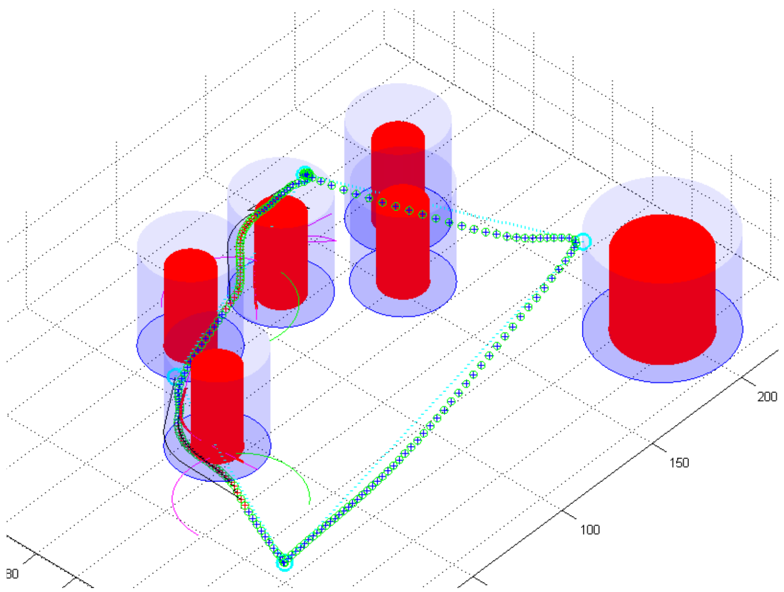
NICE project - European Defence Agency

- **NICE project** funded by EDA (2010-2012)
  (project coordinator: Onera, France)

- Univ. Trento responsible for "**MPC-based guidance design**"
  (UniTN, Bertin, MBDA)

- Develop hybrid models of UAV + environment
- ***Hybrid MPC design***
- Tuning and validation in provided realistic simulation environment
- Evaluation criteria: time to complete the mission, fuel consumption, ability to avoid obstacles, smoothness of manoeuvres.

LAAS
Univ. Roma TV
TU Munich
E&Q
IRIDA
DLR
BERTIN
DASSAULT
LFK
MBDA-France
MBDA-Italy



EDA

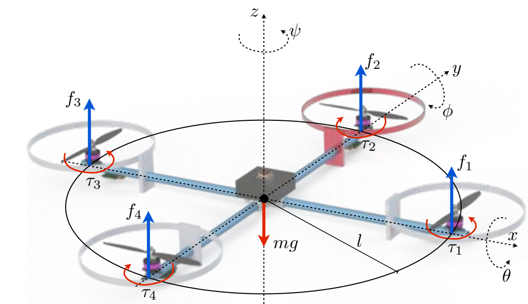
NICE project
Nonlinear Guidance Design & Assessment

MPC applications in aerospace (universities)

- **Linear Time-Varying MPC** for wheel momentum damping by thrust orientation mechanism

(Bemporad, Losa, Piliego, Ramirez-Prado, 2009)

- **LTI MPC** for stabilization, **Hybrid MPC** for navigation of small UAVs

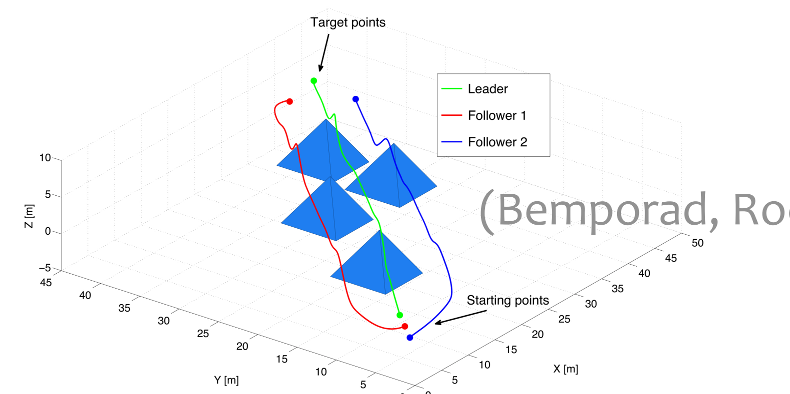


(Bemporad, Pascucci, Rocchi, 2009)

- **Hybrid MPC** for formation flying of small UAVs

(Bemporad, Rocchi, IFAC 2011)

- Decentralized **LTV-MPC** for formation flying



(Bemporad, Rocchi, CDC 2011 ?)

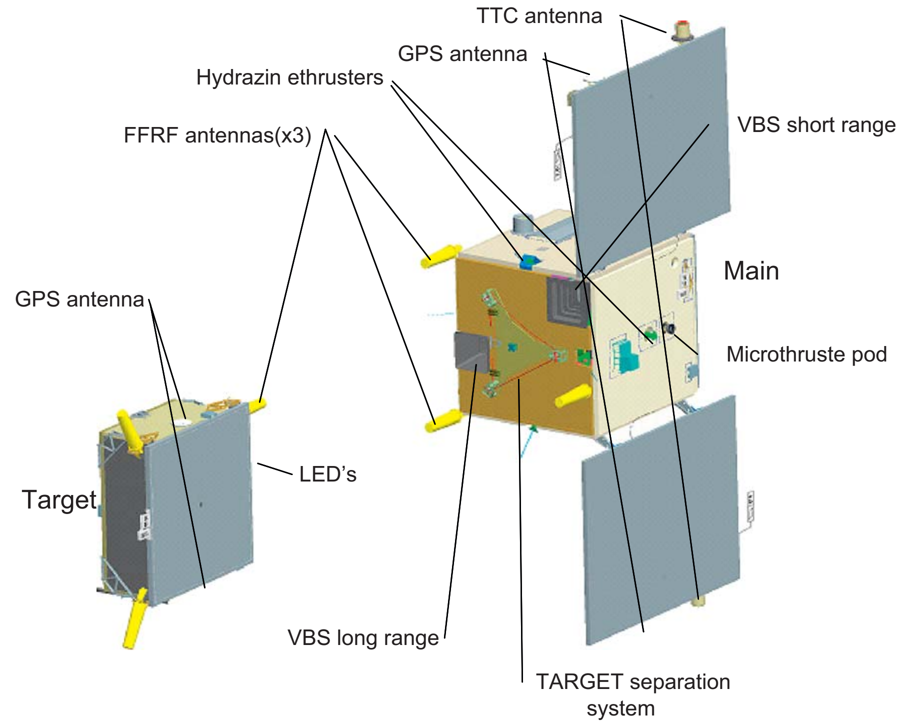
- Many other contributors:

  - A. Richards, P. Trodden (Bristol, UK)
  - J. Maciejowski, E.N. Hartley (Cambridge, UK)
  - G. Balas, F. Borrelli, T. Keviczky (Minnesota)
  - R. Murray, W.B. Dunbar (Caltech)
  - J. How, L. Breger, M. Tillerson (MIT)
  - (...)

Embedded optimization in aerospace

- **PRISMA** project for autonomous formation flying (S. Persson, S. Veldman, P. Bodin, 2009)

http://www.prismasatellites.se/



Swedish Space Corporation

DTU

cnes
CENTRE NATIONAL D'ÉTUDES SPATIALES

DLR

- Objective function: **minimize fuel consumption** subject to keeping motion within a **box constraint** (solved by **linear programming**)

$$\begin{array}{ll} \min_{dV(t)} & \sum_{t=0}^{\infty} \|dV(t)\|_1 \\ \text{s.t.} & \|r - y_r\|_1 \leq y_{\text{box}} \end{array}$$

embedded optimization **operational in space !**

Conclusions

- MPC is a very versatile technique for solving rather complex control problems:
  - ***Nonlinear/switching/multivariable*** dynamics and ***constraints*** on inputs and outputs
  - Performance is ***optimized***
  - Systematic design approach, MPC designs are ***easy to maintain***
  - ***MATLAB tools*** exist to assist the design and for code generation

- MPC is constantly spreading in industry, due to advances in:
  - **Science**: more efficient numerical algorithms
  - **Technology**: control units are more powerful and cheaper
  - **Market**: increasing performance requirements and complexity

- Started in the 80's in the *process industries*, reached *automotive* in 2000. Now spreading to *aeronautics* and *aerospace* !

- Europe is ahead of North America and Asia (also thanks to ESA). But US companies are currently investigating MPC for aero applications ...

- ***Is MPC a mature technology for guidance and navigation ?***

Announcements

4th HYCON2 PhD School on Control of Networked and Large-Scale Systems

Trento, June 21-24, 2011

Wireless control systems	K.H. Johansson	(Sweden)
Stability of NCS	M. Heemels	(NL)
Distributed DES	C. Cassandras	(USA)
Quantization in NCS	H. Ishii	(Japan)
Event-triggered control	J. Lunze	(Germany)
Decentralized control	S. Stankovic	(Serbia)
Real-time control	L. Palopoli	(Italy)
Consensus & estimation	S. Zampieri	(Italy)
Distributed optimization	S. Boyd	(USA)
Model predictive control	A. Bemporad	(Italy)
Traffic networks	C. Canudas-de-Wit	(France)
Smart grids	K. Poolla	(USA)

HYCON2 network of excellence (ICT-FP7, 2010-2015)

"**H**ighly **co**mplex **n**etworked **co**ntrol systems"

http://control.ing.unitn.it/4hycon2

- **Spin-off company** starting soon … (consulting & software customization for development of optimization-based control solutions)