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IFAC PapersOnLine 51-25 (2018) 42–47

Simple Interpolating Control ⋆

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Abstract: We present a new simple approach to extend the admissible region of a saturated control law for discrete-time linear systems under state and input constraints. The approach is based on the interpolation of a given, performance driven, control law and a low-gain controller. The latter is designed such that its admissible set has maximum volume. The interpolation does not require on-line numerical optimization, and an analytical formula for the controller is given. Several numerical examples are presented to illustrate the control technique.

Keywords: Constrained control; uncertain linear systems; interpolation algorithms; predictive control; state feedback; robust stability.

1. INTRODUCTION

A great deal of literature exists on the control of discrete-time constrained plants. Relevant approaches include Model Predictive Control (MPC), described in e.g. Mayne (2014), and Vertex Control, described in e.g. Gutman and Cwikel (1986). In recent decades, MPC has become a leading control method in industry, and has received large academic interest; see e.g. Maciejowski (2002), Rossiter (2004), and Borrelli et al. (2017). MPC offers a systematic approach to control multivariate constrained systems, however it requires online optimization at each time step. This limitation can be overcome by explicit MPC methods where an optimal control law at each state-space region is precomputed off-line (Bemporad et al., 2002), and by specialized optimization tools (Wang and Boyd, 2010). These algorithms, combined with tailored hardware (Jerez et al., 2010), can decrease computation times and allow the implementation of MPC in fast dynamical systems.

The above *nominal* MPC algorithms are unable to guarantee robust stability and performance for uncertain systems. There exist *robust* MPC methods, such as the the linear matrix inequalities (LMIs) based MPC in Kothare et al. (1996), the tube-based MPC in Langson et al. (2004), and the min-max MPC approach in Bemporad et al. (2003). These may demand the on-line solution of large optimization problems, for the first two methods, or solving a point location problem in a large polyhedral partition, for the last mentioned method. Robust MPC methods thus require excessive computational load, and therefore are impractical for implementation.

Interpolation based approaches allow the use of simpler algorithms, applicable for the same type of problems as MPC. Although no optimality of some cost function is guaranteed, these algorithms provide a good compromise between computational load, feasible region, and performance. A variety of interpolation based methods, closely related to MPC, are presented in Rossiter and Ding (2010). An alternative is the recently developed *Interpolating Control* (IC) method by Nguyen et al. (2013); Nguyen (2014).

In IC, a high performance local controller is blended with low-gain global controllers, which are used to enlarge the domain of attraction. The interpolation makes use of the constraint-admissible invariant sets given by the local and global control laws. If polyhedral sets are used to describe the invariant sets, IC requires the solution of a single linear program (LP) at each time instant to derive a robustly stabilizing solution. However, the computation of such invariant sets for high-order systems and for systems with polytopic uncertainty is sometimes impractical from a computational perspective. The same issue goes with other polyhedral invariant set based interpolating methods such as the one in Pluymers et al. (2005). The complexity of polyhedral invariant sets may motivate the use of the easily computed and stored ellipsoidal sets of the interpolation instead, as presented in Nguyen et al. (2011). However, in that case the solution of a semidefinite program (SDP) at each time instant is required.

This work presents a remarkably simple alternative for interpolation between two saturated control laws: a high-

⋆ The first and second authors acknowledge the support from the Israel Ministry of Transport, and General Motors Company, respectively.

10.1016/j.ifacol.2018.11.079

gain performance-driven control law and a low-gain control law. The interpolation makes use of the properties of the corresponding invariant sets for each law.

2. PRELIMINARIES

Consider the uncertain and/or time-varying linear discrete-time system modeled by

$$x(k+1) = A(k)x(k) + B(k)u(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state and $u(k) \in \mathbb{R}^m$ is the control input. These signals are constrained by polytopes, assumed symmetrical w.r.t. the origin,

$$x(k) \in \mathcal{X} = \{x \in \mathbb{R}^n : \|Lx\|_\infty \le 1\}, \tag{2a}$$

$$u(k) \in \mathcal{U} = \{u \in \mathbb{R}^m : \|u\|_\infty \le u_{\max}\}, \tag{2b}$$

where $L \in \mathbb{R}^{n_c \times n}$ and $u_{\max} \in \mathbb{R}^m$.

The matrices $A(k) \in \mathbb{R}^{n\times n}$ and $B(k) \in \mathbb{R}^{n\times m}$ are given with polytopic uncertainty as follows

$$A(k) = \sum_{i=1}^{s} \alpha_i(k) A_i, \tag{3a}$$

$$B(k) = \sum_{i=1}^{s} \alpha_i(k) B_i, \tag{3b}$$

$$\sum_{i=1}^{s} \alpha_i(k) = 1,\ \alpha_i(k) \ge 0,\ \forall i = 1,\dots,s, \tag{3c}$$

where $\alpha_i(k)$ is unknown, and possibly time-varying. A more general form of polytopic uncertainty with different summations and indices in (3a) and (3b) can be brought to the form (3) (Nguyen et al., 2013).

Before presenting the main results, notation and preliminary results related to invariant theory are given. The function $\mathrm{sat}(\cdot)$ is used to denote the standard vector valued saturation function, for $u \in \mathbb{R}^m$, the i-th component of $\mathrm{sat}(u)$ is $\mathrm{sign}(u_i)\min(u_{\max}, |u_i|)$. An ellipsoid defined by a positive definite matrix P is denoted as $\mathcal{E}(P) = \{x : x^\top P^{-1} x \le 1\}$.

Definition 1. An ellipsoid $\mathcal{E}(P)$ is said to be *robustly invariant* with respect to the constrained system (1) and a given feedback control law $u(k) = \mathrm{sat}(Kx(k))$ if

$$x(k+1)^\top P^{-1} x(k+1) - x(k)^\top P^{-1} x(k) \le 0 \tag{4}$$

for all $x(k) \in \mathcal{E}(P)$ and

$$\mathcal{E}(P) \subset \mathcal{X}.$$

If inequality (4) is strict, and is satisfied for any $x(k) \in \mathcal{E}(P)$ with $x(k) \ne 0$, the ellipsoid $\mathcal{E}(P)$ is called a *robustly contractive* ellipsoid.

Several methods for the computation of ellipsoidal invariant sets for saturated control laws exist in the literature. One example is the approach in Hu et al. (2002), where the saturation non-linearity is modeled by linear difference inclusions (LDIs), and computation is done by the solution of an LMI problem. We note that these methods can be adapted to synthesize the saturated control law such that the associated ellipsoidal invariant sets is maximized (Hu et al., 2002).

3. MAIN RESULTS

The method proposed in this work relies on two saturating linear control laws, and two robustly contractive ellipsoids associated to these control laws. We consider a given robustly stabilizing controller, designed to satisfy a given performance requirement, $u(k) = \mathrm{sat}(K_a x(k))$. This control law is associated with a robustly contractive ellipsoid $\mathcal{E}(P_a)$. To extend the feasible region of the control law to be admissible outside $\mathcal{E}(P_a)$, a second controller $u(k) = \mathrm{sat}(K_b x(k))$ is synthesized such that the volume of its associated robustly contractive ellipsoid, $\mathcal{E}(P_b)$, is maximized. It is assumed that $\mathcal{E}(P_a) \subset \mathcal{E}(P_b)$, as depicted in Fig. 1. Note that this maximization depends on the level of conservatism of the method used to estimate the robustly contractive ellipsoids. These two controllers are denoted as *inner* and *outer*, respectively.

The problem at hand is how to blend these two controllers in a way that the resulting controller can be applied for any state in $\mathcal{E}(P_b) \setminus \mathcal{E}(P_a)$, with superior performance to the outer controller alone, and the inner controller in $\mathcal{E}(P_a)$, while avoiding chattering that often occurs when switching between controllers in the presence of noisy measurements. An Interpolating Controller that addresses all these issues, including proof of recursively feasibility and asymptotic stability, was presented in Nguyen et al. (2011); it however requires the solution of an SDP at every time step. That limits the applicability of the method and calls for a more computationally efficient alternative.

The controller proposed here is equal to the inner when the state belongs to $\mathcal{E}(P_a)$; otherwise, it is based on interpolation between the inner and the outer controller. Any state $x(k) \in \mathcal{E}(P_b) \setminus \mathcal{E}(P_a)$ can be decomposed as

$$x(k) = c(k)x_b(k) + (1 - c(k))x_a(k), \tag{5}$$

where $x_a(k) \in \mathcal{E}(P_a)$, $x_b(k) \in \mathcal{E}(P_b)$ and $0 \le c(k) \le 1$. Applying the same decomposition on the control signal yields

$$u(k) = c(k)u_b(k) + (1 - c(k))u_a(k), \tag{6}$$

with $u_a(k) = \mathrm{sat}(K_a x_a(k))$ and $u_b(k) = \mathrm{sat}(K_b x_b(k))$. In terms of performance, it is desirable to have $c(k)$ as small as possible which makes the inner (high-performance) controller dominant. Unfortunately, the problem of finding the minimum interpolation coefficient $c(k)$ as well as the corresponding states $x_a(k)$ and $x_b(k)$ requires the solution of an SDP at every time step (Nguyen et al., 2011). With the aim of reducing the computational burden, we resort to a sub-optimal approach to perform the interpolation.

Under the assumption that $\mathcal{E}(P_a) \subset \mathcal{E}(P_b)$, it is possible to select $x_a(k)$ and $x_b(k)$ on the same ray from the origin to $x(k)$, as illustrated in Fig. 1.Thus $x_a(k) = x(k)/a(k)$ where $a(k) > 1$ is given by

$$a(k)^2 = x(k)^\top P_a^{-1} x(k). \tag{7}$$

Similarly, $x_b(k) = x(k)/b(k)$ where $b(k) \le 1$ is obtained as

$$b(k)^2 = x(k)^\top P_b^{-1} x(k). \tag{8}$$

Then, from (5) the interpolation coefficient is found as

$$c(k) = \frac{b(k)(a(k) - 1)}{a(k) - b(k)}. \tag{9}$$

It is easily checkable that $c(k) \in (0,1]$ for any $x(k) \in \mathcal{E}(P_b) \setminus \mathcal{E}(P_a)$. Note also that $c(k) = 0\ \forall x(k) \in \mathcal{E}(P_a)$.

Theorem 2. The control law (6)−(9) is feasible for all $x(k) \in \mathcal{E}(P_b)$, i.e., $\|u(k)\|_\infty \le u_{\max}$ and $x(k+1) \in \mathcal{E}(P_b)$.

Proof. The proposed control action satisfies

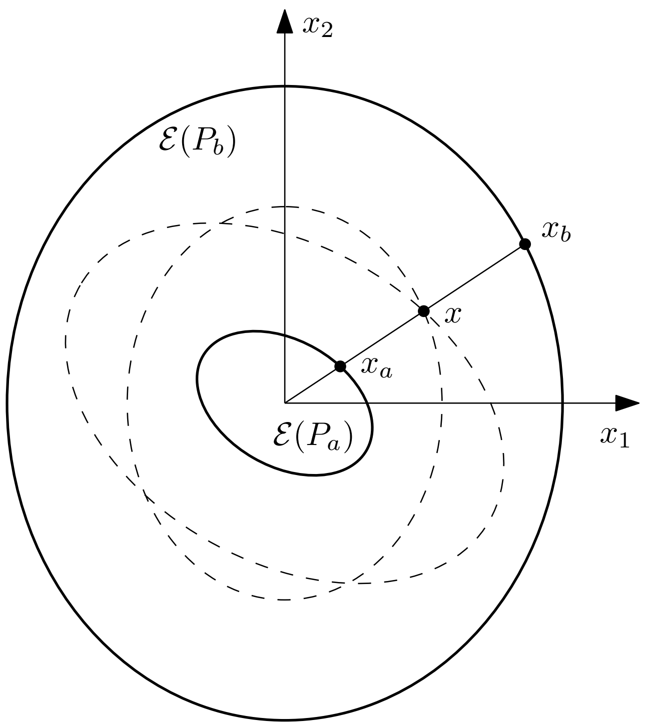

Fig. 1. Illustration of the interpolation concept.

$$\begin{aligned}\|u(k)\|_\infty &= \|c(k)\mathrm{sat}(K_b x_b(k)) + (1-c(k))\mathrm{sat}(K_a x_a(k))\|_\infty \\ &\le c(k)\|\mathrm{sat}(K_b x_b(k))\|_\infty + (1-c(k))\|\mathrm{sat}(K_a x_a(k))\|_\infty \\ &= u_{\max}.\end{aligned}$$

The state evolves under the proposed controller as

$$\begin{aligned}x(k+1) = c(k)\left(A(k)x_b(k) + B(k)u_b(k)\right) \\ + (1-c(k))\left(A(k)x_a(k) + B(k)u_a(k)\right).\end{aligned}$$

We also have that

$$\begin{gathered}A(k)x_b(k) + B(k)u_b(k) \in \mathcal{E}(P_b), \\ A(k)x_a(k) + B(k)u_a(k) \in \mathcal{E}(P_a), \\ \mathcal{E}(P_a) \subset \mathcal{E}(P_b).\end{gathered}$$

Therefore, it follows easily that $x(k+1) \in \mathcal{E}(P_b)$. □

Unfortunately, the proposed controller is not guaranteed to be asymptotically stabilizing. We present an example where the closed-loop system exhibits a limit cycle, see Example 1 below. The following result states that it is possible to check the asymptotic stability of the resulting closed-loop system by analyzing an uncertain discrete-time systems with convex polytopic uncertainty,

$$x(k+1) = A_c(k)x(k), \tag{10}$$

where $A_c(k)$ belongs to a polytopic set defined by the $2^{m+1}s$ vertices

$$A_i + B_i\left(D_j K_a + D_j^- H_a\right), \tag{11a}$$

$$A_i + B_i\left(D_j K_b + D_j^- H_b\right), \tag{11b}$$

$$i = 1, 2, \ldots, s, \quad j = 1, 2, \ldots, 2^m. \tag{11c}$$

Here, D_j denotes the j-th element of the set of diagonal matrices of dimension m whose diagonal elements are either 1 or 0, $D_j^- = I_m - D_j$, $\|H_a x(k)\|_\infty \le u_{\max}$ for all $x(k) \in \mathcal{E}(P_a)$ and $\|H_b x(k)\|_\infty \le u_{\max}$ for all $x(k) \in \mathcal{E}(P_b)$.

Stability conditions for system with time-varying polytopic uncertainty have been presented in the literature, see e.g. Daafouz and Bernussou (2001); Geromel and Colaneri (2006). So, if (10), (11) is asymptomatically stable by e.g. Daafouz and Bernussou (2001), then the closed loop interpolating control system is asymptotically stable with any $c(k)$. Another sufficient stability result, based on Hu and Lin (2001) is given in Theorem 3 below.

Remark: An alternative way to enhance the closed-loop stability is to fix, e.g., an inner control law, then solve an LMI optimization problem to obtain an outer one.

Theorem 3. The closed-loop system composed by the constrained system (1) and the controller (6)–(9) is asymptotically stable if the uncertain discrete-time systems with convex polytopic uncertainty (10)–(11) is asymptotically stable.

Proof. The proof is a straightforward extension to the two-gain-case of (Hu and Lin, 2001, Proposition 7.5.1). The controller (6)–(9) is given by

$$u(k) = c(k)\,\mathrm{sat}\left(\frac{K_b x(k)}{b(k)}\right) + (1-c(k))\,\mathrm{sat}\left(\frac{K_a x(k)}{a(k)}\right)$$

A saturated linear controller can be modeled using LDIs (Hu et al., 2002), the inner controller is modeled as

$$\begin{gathered}\mathrm{sat}(K_a x(k)/a(k)) \in a(k)^{-1}\mathrm{conv}\{D_j K_a x(k) + D_j^- H_a x(k)\}, \\ j = 1, 2, \ldots, 2^m, \\ \|H_a x(k)\|_\infty \le u_{\max} \quad \forall x(k) \in \mathcal{E}(P_a).\end{gathered}$$

The outer controller is modeled as

$$\begin{gathered}\mathrm{sat}(K_b x(k)/b(k)) \in b(k)^{-1}\mathrm{conv}\{D_j K_b x(k) + D_j^- H_b x(k)\} \\ j = 1, 2, \ldots, 2^m \\ \|H_b x(k)\|_\infty \le u_{\max} \quad \forall x(k) \in \mathcal{E}(P_b).\end{gathered}$$

Noting that $c(k)/b(k) + (1-c(k))/a(k) = 1$, it is possible to write

$$u(k) = K(k)x(k)$$

where $K(k)$ belongs to a polytopic set defined by the 2^{m+1} vertices:

$$\begin{gathered}D_j K_a + D_j^- H_a \\ D_j K_b + D_j^- H_b \\ j = 1, 2, \ldots, 2^m\end{gathered}$$

Therefore, the resulting closed-loop system can be modeled as the uncertain discrete-time system with convex polytopic uncertainty given by (10)–(11). □

4. NUMERICAL EXAMPLES

This section presents three examples that serves to show the properties of the presented controller. A pathological case is presented in Example 1. The usage and the benefits of the presented algorithm is shown in the other examples.

4.1 Example 1

This example presents a case where the controller (6)–(9) exhibits a limit cycle, i.e. the closed-loop system is not asymptotically stable for all initial conditions in $\mathcal{E}(P_b)$. Let us consider the constrained system

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), \\ -10 \le x_1 \le 10, \quad -10 \le x_2 \le 10, \\ -1 \le u_1 \le 1, \quad -1 \le u_2 \le 1, \end{cases}$$

with

$$A = \begin{bmatrix} -0.2 & 0.2 \\ -2.0 & -0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The inner and outer controller are given as

$$K_a = \begin{bmatrix} 0.1 & -2.2 \\ 2.2 & 0.1 \end{bmatrix},\ K_b = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

and their associated ellipsoidal contractive sets are given by

$$P_a = \begin{bmatrix} 20.6631 & -0.1375 \\ -0.1375 & 5.0901 \end{bmatrix},\ P_b = \begin{bmatrix} 24.6440 & 0.0052 \\ 0.0052 & 99.9995 \end{bmatrix}.$$

For the sake of simplicity, these ellipsoidal contractive sets have been obtained for the linear controllers, i.e. $u_a(k) = K_a x_a(k)$ and $u_b(k) = K_b x_b(k)$.

We note that $A + B(K_a + K_b)/2$ has the eigenvalue -1 and that the evolution of the system is given by

$$x(k+1) = \left(A + B\left(\frac{1-c(k)}{a(k)}K_a + \frac{c(k)}{b(k)}K_b\right)\right)x(k)$$

Therefore, an initial condition placed along the eigenvector associated with the eigenvalue -1 in such a way that $\frac{1-c(0)}{a(0)} = \frac{c(0)}{b(0)} = 0.5$ will exhibit a limit cycle. The initial condition $x(0) = [2.7685\ 2.7685]^\top$ satisfies the above condition, and it exhibits a trajectory that alternates between the states $[2.7685\ 2.7685]^\top$ and $[-2.7685\ -2.7685]^\top$.

4.2 Example 2

Consider the following constrained system from Nguyen and Gutman (2016) to be regulated to the origin

$$\begin{cases} x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.42 & 0.90 \\ 0.38 & 0.67 \end{bmatrix} u(k), \\ -40 \le x_1 \le 40, \quad -10 \le x_2 \le 10, \\ -0.1 \le u_1 \le 0.1, \quad -0.1 \le u_2 \le 0.1. \end{cases} \tag{12}$$

The proposed control technique, defined by (6)−(9), is applied by considering a saturated LQR controller with weightings $Q = I_2$ and $R = 30I_2$ as the inner controller, and a saturated linear controller that maximizes the volume of the robustly contractive ellipsoid for the given constrained system (12) as outer controller. The outer controller is defined by the gain

$$K_b = \begin{bmatrix} -0.0430 & -1.7220 \\ -0.0271 & -0.9796 \end{bmatrix}.$$

For comparison purposes, a dual mode, infinite prediction horizon MPC controller (Mayne et al., 2000) was also simulated. The MPC was designed to achieve identical behavior as the LQR above within its terminal set by using the same weightings Q and R, and the corresponding terminal weight. The prediction horizon was set to 75 for it to have a feasible set equal to the maximum control invariant set that includes $\mathcal{E}(P_b)$. Both controllers were simulated for a set of 629 equiangular initial conditions along the boundary of $\mathcal{E}(P_b)$. Performance was measured in terms of the cost function $J = \sum_{k=1}^{\infty} x^\top(k)Qx(k) + u(k)Ru(k)$. The histogram for the ratio between the cost functions of the MPC and the proposed controller is shown in Fig. 3. Fig. 4 shows the states and control inputs corresponding to the initial condition $x(0) = [2.5429\ 2.5429]^\top$ for both controllers, i.e. MPC and controller (6)−(9).

We note that the proposed controller is (slightly) penalized in terms of feasible set (see Fig. 2) and performance (see Fig. 3 and 4). On the other hand, the computational burden of the new controller is practically negligible, since neither numerical optimization nor a point-location algorithm in a polyhedral partition is needed.

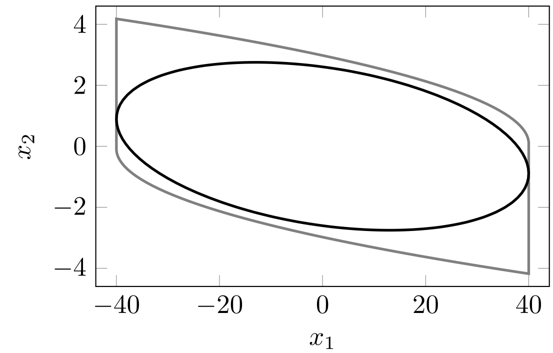

Fig. 2. Set of valid initial conditions for MPC (gray polyhedron) and proposed controller (black ellipsoid $\mathcal{E}(P_b)$) in Example 2.

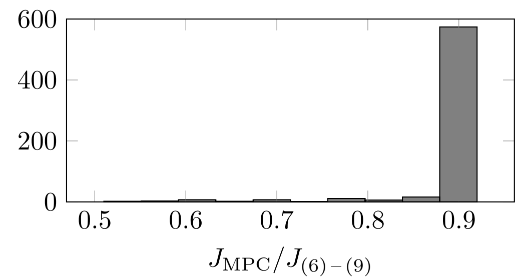

Fig. 3. Histogram ratio between cost function for MPC and proposed controller in Example 2.

Another benefit of the proposed controller is that the feasible set, $\mathcal{E}(P_b)$, is obtained a priori, off-line, by solving an LMI, while no efficient method is known to obtain this for MPC or standard polytopic IC, at least for high-dimensional and/or uncertain systems. In many practical such cases, the already designed MPC would require a check of convergence to the setpoint for any given initial condition. However, practitioners usually skip this check ("play and pray").

4.3 Example 3

Consider the following constrained uncertain system from Nguyen and Gutman (2015),

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k), \\ -10 \le x_1 \le 10, \quad -10 \le x_2 \le 10, \\ -1 \le u \le 1. \end{cases} \tag{13}$$

The matrices A and B are given as

$$A(k) = \alpha(k)A_1 + (1-\alpha(k))\,A_2,$$
$$B(k) = \alpha(k)B_1 + (1-\alpha(k))\,B_2,$$

with

$$A_1 = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}.$$

It is assumed that $\alpha(k) \in [0,1]$ is an unknown, time-varying, uniformly distributed random number. The high-gain control gain is chosen as

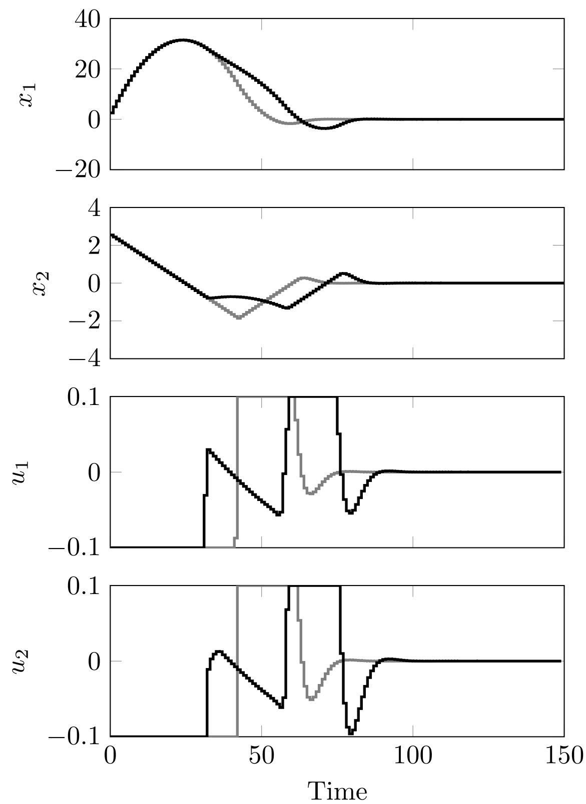

Fig. 4. States and control inputs for MPC (gray) and proposed controller (black) in Example 2.

$$K_a = [-1.1717\ \ -0.9542]\,.$$

The low-gain controller was computed such that $\mathcal{E}(P_b)$ has maximal volume, and its gain is

$$K_b = [-0.1525\ \ -0.7780]\,.$$

The presented approach is compared with the LMI based infinite-horizon robust MPC method in Kothare et al. (1996) with the weights $Q_1 = I_2$ and $R = 1$.

Fig. 5 shows the states and control inputs obtained from a numerical simulation for the initial condition $x(0) = [-3.95\ 9.80]^\top$ over 30 time samples. Fig. 6 shows the trajectory for the same simulation, plotted together with the ellipsoidal invariant sets $\mathcal{E}(P_a)$ and $\mathcal{E}(P_b)$, and the feasible set for the MPC denoted by Ω. The presented method is shown to give faster convergence times than MPC and share very similar feasible sets. The major difference is, obviously, the computational burden: The MPC algorithm requires the solution of an SDP at each computation time. Here the SPDs were computed using the Yalmip modeling package (Löfberg, 2004) and Mosek (Mosek ApS, 2017), and their worst-case computation time took approximately 93 ms (15 ms on average). Our approach requires no solvers, and was executed in under 0.16 ms (0.04 ms on average) on the same platform.

5. CONCLUSION

This paper presents a new control approach for discrete-time uncertain linear systems subjected to state constraints and actuator saturation. The presented approach is based on the interpolation of two saturated control laws. It is remarkably easy and simple to implement and

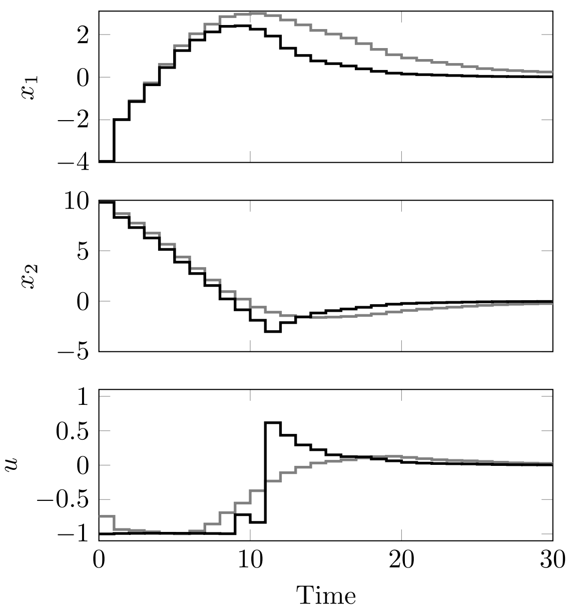

Fig. 5. States and control input results for Example 3. Proposed approach (black) vs. the approach in Kothare et al. (1996) (gray).

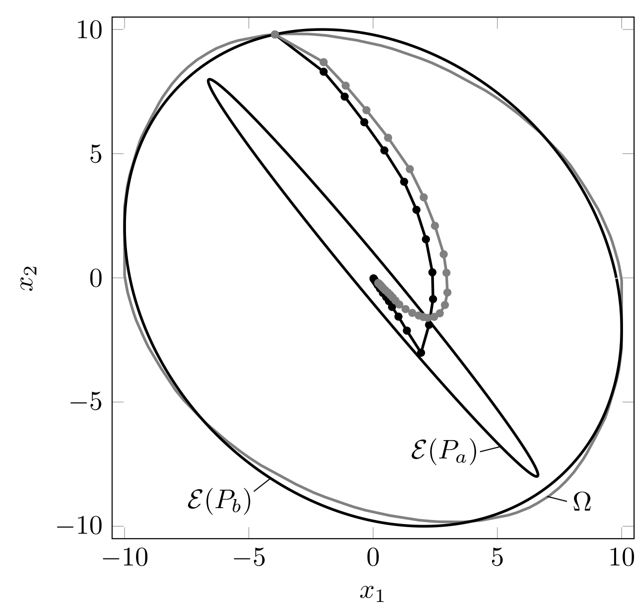

Fig. 6. Invariant sets and states trajectories for Example 3. Proposed approach (black) vs. the approach in Kothare et al. (1996) (gray).

gives reasonable trade-off between performance, feasible region size and computational load, in comparison to other interpolation based approaches and MPC. These benefits were demonstrated by two numerical examples.

In comparison with MPC, it should be noted that the presented method will obviously not give better performance in terms of the MPC criterion, but will always give a solution when the state is in the precomputed feasible ellipsoid. When using MPC, its feasible region is in general not precomputed since it is a highly demanding or even impossible task, see Rubin et al. (2018). Note that, in general, our ellipsoidal feasible region may not be a subset of the MPC feasible region. Therefore, with a given measured state, the MPC is not guaranteed to give a solution even if that state belongs to our ellipsoid. Moreover, the new *Simple Interpolating Control* demands less computational effort.

Future development of the proposed method will include disturbance attenuation, output feedback, and reference tracking. Currently under development is an extension that has a built-in closed loop asymptotic stability guarantee; Theorem 3 will be thrown out to the garbage heap of history.

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