Model-Varying Predictive Control of a Nonlinear System

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Abstract

Model Predictive Control (MPC) can be used for nonlinear systems if they are working *around* an operating point. If the operating point is moved away from the nominal working point the controller is less effective due to model mismatch. This situation can be tackled by using a Model-Varying Predictive Controller (MVPC), which changes its internal model, switching among a set of liner models, according to the working point.

1 Introduction

Model Predictive Controller (MPC) can be applied to a nonlinear system but, when the operating point is moved away from the nominal working point, the internal model of the predictive controller (linearized at the nominal working point) is not valid and the control is ineffective. A Model-Varying Predictive Controller (MVPC) is, as it is understood in this report, a set of linear MPC with different internal linear models, switching among them according to the actual working point. This work applies the MPVC to a steam turbine exemplifying how to control nonlinear systems.

To control a steam turbine of a thermal power plant is a well known problem in industry. It is generally done with PI-like controllers handling the problem around a local operating point without guarantees of satisfying input constrains (e.g. opening and closing speed of the valves) and constrains on outputs (e.g. internal pressures variation) over the entire range of operating power. More advanced control strategies can be applied with the increasing performances of actual computers. MPC is probably the best, mainly due to the advantage of dealing with the control problem in a global manner considering constrains on the manipulated and output variables.

These work is mainly based on [9], where one can find, in addition, a PI control of the steam turbine and an economical study of MPC benefits.

The report is organized as follows: Section 2 describes the steam turbine and Section 3 presents the equations describing the model. Section 4 explains process requirements. Section 5 provides background material on Model Predictive Control and Section 7 presents simulations of Model-Varying Predictive Control of a steam turbine. The report finishes in Section 8 with some conclusions.

2 Plant Description

The steam turbine is divided into three sections: high pressure (HPT), intermediate pressure (IPT) and low pressure (LPT). The steam turbine and its surroundings are shown in Figure 1. The total steam supply is divided irregularly into the three mass flows $\dot{m}_{HP}$, $\dot{m}_{IP}$ and $\dot{m}_{LP}$. While $\dot{m}_{HP}$ provides 80% of the total steam mass flow the other two just provide 10% of the total steam mass flow each. All circles in Figure 1 are volume elements and therefore the storage elements of the process: three boilers with the pressures p_{HP}, p_{IP} and p_{LP}, the reheater with the pressure p_{RHTR}, three wheel chambers with the pressures p_{HPWC}, p_{IPWC} and p_{LPWC} and the condenser with the pressure p_{COND}. The boiler is modeled as pure steam storage. The process of steam production is not modeled. The condenser is modeled as an infinitely large volume, i.e., the condenser pressure always remains constant. Six valves are available to control the steam turbine. One distinguishes between control valves (e.g. u_{HPCT}) and bypass valves (e.g. u_{HPBP}). The valve after the IP-boiler is always fully open and it is not controllable. It is used only for modelling purposes.

Consider first how the steam flows at full power of the turbine. All control valves are fully opened ($u_{CT} = 1$), all bypass valves are closed ($u_{BP} = 0$). The steam flows out of the HP boiler into the high pressure section of the turbine and merges with the steam from the IP boiler. Then, both are heated in the reheater (the process of the heating is not modeled and the reheater is a plain volume element). The steam then flows through the IP turbine

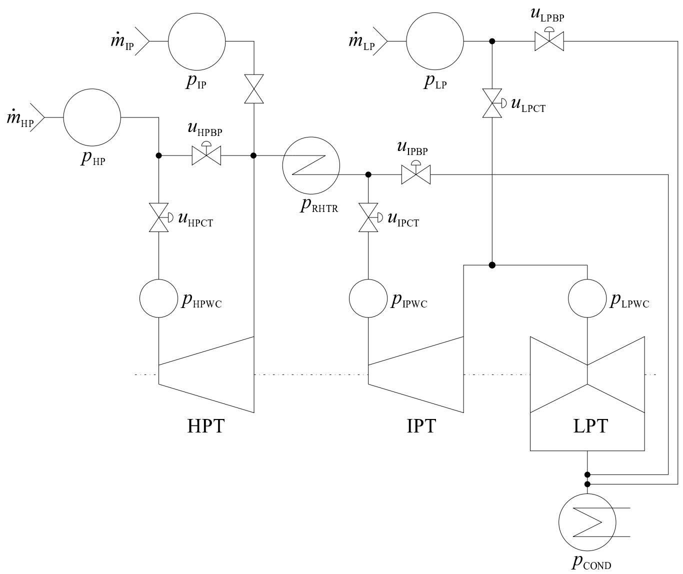

Figure 1: Process model.

section and merges with the steam from the LP-boiler. This steam flows through the LP section of the turbine and ends in the condenser. The condenser feeds back the steam to the boilers but it is not modeled.

If the power of the turbine is lower than 100% , the steam partially flows through the bypass valves without producing any contribution to the power. The control valves are mainly used to regulate the power, the bypass valves to regulate the three pressures p_{HP}, p_{RHTR} and p_{LP} on a specified setpoint. We therefore have six manipulated variables and four variables to be controlled. Not all manipulated variables can be used to perform the control task of the four controlled variables because of some requirements explained in section 4.

The power delivered by the turbine is equal to the sum of the powers of the three turbine sections. The HP and the LP turbine sections provide approximately 30% of the total power each, the IP turbine section approximately 40%.

3 The Nonlinear Model

Modeling the individual process subsystems requires some simplifications so that the model is not unnecessarily complicated. The main simplification arises from the assumption for the inlet temperatures to be constant. The notation used in this report is summarized in Table 1 and the physical constants are listed in Table 2. The following equations are to describe the nonlinear model.

Notation	Meaning	Unit
m	mass	kg
$\dot{m}$	mass flow	kg s^{-1}
p	pressure	kg m^{-1} s^{-2}
V	volume	m^3
v	specific volume	m^3 kg^{-1}
ϱ	density	kg m^{-3}
T	temperature	K
τ	time constant	s
P	power	kg m^2 s^{-3}
h	specific enthalpy	J kg^{-1}
η	isentropic efficiency	$-$
$\Pi(t) = \frac{p_\omega(t)}{p_\alpha(t)}$	valve pressure ratio	$-$

Table 1: Physical variables.

Constant	Meaning	Value
$R_{\mathrm{H_2O}}$	steam gas constant	461.52 J kg^{-1} K^{-1}
κ	isentropic exponent	1.3
Π_{kr}	critical pressure ratio	0.5

Table 2: Constants.

The state equation for the volume element is:

$$\frac{d}{dt}\frac{p(t)}{p_{\mathrm{R}}} = \frac{1}{\tau_{\mathrm{s}}}\Big(\frac{\dot{m}_{\mathrm{in}}(t)}{\dot{m}_{\mathrm{R}}} - \frac{\dot{m}_{\mathrm{out}}(t)}{\dot{m}_{\mathrm{R}}}\Big). \tag{1}$$

where $\tau_{\mathrm{s}} = \frac{m_0 p_{\mathrm{R}}}{p_0 \dot{m}_{\mathrm{R}}}$ describes the filling time of the volume element and has the unit s.

The equation for the valve is:

$$\frac{\dot{m}(t)}{\dot{m}_{\mathrm{R}}} = k_{\mathrm{mv}}\frac{p_\alpha(t)}{p_{\mathrm{R}}}\frac{A(t)}{A_0}\xi(t). \tag{2}$$

by introducing $k_{\mathrm{mv}} = \frac{p_{\mathrm{R}}\dot{m}_0}{\dot{m}_{\mathrm{R}} p_{\alpha 0}\xi_0}$. The area of the section $A(t)$ has been related to $A_0 = A_{\max}$ so that $A(t)/A_0 = A(t)/A_{\max}$ had a value between 0 and 1 and the expression for $\xi(t)$ is to be approximated as follows:

$$\xi(t) = \sqrt{1 - \Pi(t)^4} \qquad 0 \le \Pi(t) \le 1. \tag{3}$$

The output of a servo drive $u(t)$ actuating the valve determines its normalized cross-section:

$$\frac{A(t)}{A_0} = u(t) + \frac{1}{4}sin(\pi u(t)). \tag{4}$$

This relation can vary considerably between different valves.

The mass flow through the turbine section is:

$$\frac{\dot{m}(t)}{\dot{m}_{\mathrm{R}}} = k_t \frac{p_\alpha(t)}{p_{\mathrm{R}}} \sqrt{1 - \Pi(t)^2} \tag{5}$$

with the constant $k_t = \frac{\dot{m}_0 p_{\mathrm{R}}}{p_{\alpha 0} \dot{m}_{\mathrm{R}} \sqrt{1-\Pi_0{}^2}}$. The normalized relation for the power is

$$\frac{P(t)}{P_{\mathrm{R}}} = k_{\mathrm{tP}} \frac{\dot{m}(t)}{\dot{m}_{\mathrm{R}}} \left(1 - \Pi(t)^{\frac{\kappa-1}{\kappa}}\right) \tag{6}$$

with the constant $k_{\mathrm{tP}} = \eta_0 \frac{\kappa}{\kappa-1} \frac{R_{\mathrm{H_2O}} T}{v_{\mathrm{R}} p_{\mathrm{R}}}$.

4 Control Requirements

Controlling a steam turbine is a well known problem in industry. There are certain rules on how the valves should be actuated in order to avoid undesirable effects on the process such as ventilation or valve trimming.

The valves have two input constrains: an upper and lower physical bound, and the velocity of the opening and closing. In a normal operating state, a valve should fully open in 5s and fully close in 2.5s. In case of a security shutdown the valve must close in 0.2s. The constrains are listed in Table 3.

elements	Normal state	Security shutdown
all valves	$0 \leq u \leq 1$	$0 \leq u \leq 1$
all valves	$-0.4 \leq \dot{u} \leq 0.2$	$-5 \leq \dot{u} \leq 0.2$

Table 3: Input constrains.

If the pressure in a boiler drops too quickly, its content gets overheated and the water begins to boil. One part of the water evaporates and skims up. It is possible that water drops enter the pipes and even the turbine. The blades can be damaged or the shaft distorted. The gradients of the pressures p_{HP}, p_{RHTR} and p_{LP} must therefore be lower-bounded. The pressure p_{RHTR} is nearly equal to p_{IP} and therefore its gradient is chosen to be bounded. The lower bound of the three pressures is -3% per minute of their nominal value (see Table 4).

Constrained elements	Values
HP-pressure gradient	$\dot{p}_{HP} \geqslant -0.05$
RHTR-pressure gradient	$\dot{p}_{RHTR} \geqslant -0.0125$
LP-pressure gradient	$\dot{p}_{LP} \geqslant -0.003125$

Table 4: Output constraints.

5 Model Predictive Control

Model Predictive Control is an open loop control design procedure based on obtaining plant measurements and predicting future outputs by means of a model of the process. This is done at each sampling time k. These predictions are used to compute m control moves $u(k+i|k), i = 0, 1, \ldots, m-1$ by minimizing an objective function $J_p(k)$, defined over a prediction horizon p as follows:

$$\min_{u(k+i|k), i=0,1,\ldots,m-1} J_p(k), \tag{7}$$

subject to constraints on the control input $u(k+i|k), i = 0, 1, \ldots, m-1$ and possibly also on the state $x(k+i|k), i = 0, 1, \ldots, p$ and the output $y(k+i|k), i = 1, 2, \ldots, p$. Here

$x(k+i|k)$, $y(k+i|k)$	:	state and output respectively, at time $k+i$, predicted based on the measurements at time k; $y(k|k)$ and $x(k|k)$ refer respectively to the output measured and the state estimated at time k, i.e., $y(k|k) = \hat{y}(k),\ x(k|k) = \hat{x}(k)$.
$u(k+i|k)$	:	control move at time $k+i$, computed by the optimization problem (7) at time k; $u(k|k)$ is the control move to be implemented at time k.
m	:	*input* or *control* horizon.
p	:	*output* or *prediction* horizon.

Although more than one optimal control input is computed, only the first computed control move $u(k|k)$ is implemented. At the next sampling time $k+1$, new measurements are obtained from the plant and the optimization problem (7) is solved again. Both the control horizon m and the prediction horizon p *move* or *recede* ahead by one step as time moves ahead by one step. This is the reason why MPC is also sometimes referred to as Receding Horizon Control (RHC) or Moving Horizon Control (MHC). The purpose of taking new measurements at each time step is to compensate for unmeasured disturbances and model inaccuracy both of which cause the system output to be different from the one predicted by the model.

The objective function $J_p(k)$ in the optimization problem (7) can take numerous forms to reflect the desired behavior of the predicted output $y(k+i|k), i = 1, 2, \ldots, p$. The most common among them is the following quadratic form:

$$\begin{aligned} J_p(k) = &\sum_{i=1}^{p}(r - y(k+i|k))^T\Gamma_y(r - y(k+i|k)) + \sum_{i=0}^{m-1} u(k+i|k)^T\Gamma_u u(k+i|k) \\ &+ \sum_{i=0}^{m-1} \Delta u(k+i|k)^T\Gamma_{\Delta u}\Delta u(k+i|k) \end{aligned} \tag{8}$$

where r is reference for y, $\Delta u(k+i|k) = u(k+i|k) - u(k+i-1|k)$ and $\Gamma_y, \Gamma_u, \Gamma_{\Delta u} \geq 0$ are weighting matrices.

The following constrains on the manipulated input and the state and output variables can be enforced in the framework of MPC:

Component-wise constraint on u:

$$u_{j,\min} \leq u_j(k+i|k) \leq u_{j,\max}, \quad k \geq 0, \quad i = 0, 1, \ldots, m-1, \quad j = 1, 2, \ldots, n_u.$$

Component-wise constrains on the state and output:

$$\begin{aligned} y_{j,\min} \leq y_j(k+i|k) \leq y_{j,\max}, \quad k \geq 0, \quad i = 1, 2, \ldots, p, \quad j = 1, 2, \ldots, n_y \\ x_{j,\min} \leq x_j(k+i|k) \leq x_{j,\max}, \quad k \geq 0, \quad i = 1, 2, \ldots, p, \quad j = 1, 2, \ldots, n_x. \end{aligned}$$

Component-wise constraint on the rate of change of u:

$$|\Delta u_j(k+i|k)| \leq \Delta u_{j,\max}, \quad k \geq 0, \quad i = 0, 1, \ldots, m-1, \quad j = 1, 2, \ldots, n_u$$

where $|\Delta u_j(k+i|k)| \triangleq u(k+i|k) - u(k+i-1|k)$. Again, as in the case of the weighting matrices and the set-point, the values of $u_{j,\min}$, $u_{j,\max}$, $\Delta u_{j,\max}$, $y_{j,\min}$, $y_{j,\max}$, $x_{j,\min}$, $x_{j,\max}$ can all be allowed to vary with time.

It can be shown that the optimization problem (7), with $J_p(k)$ given by (8), in the presence of any or all of the above mentioned constraints can be reduced to a quadratic program [1, 4] of moderate size. Moreover, commercial software [5] is available to solve such MPC-based optimization/simulation problems.

6 Model-Varying Predictive Control

The accuracy of the future predictions is critical for an optimal control sequence to be computed at each sampling time. Thus, as a linear model is used to predict such future outputs, it is desirable for this model to describe as well as possible the process to be controlled.

It is possible to sub-divide the operating power range into sub-ranges and use a single model within each sub-range as the prediction model in MPC. This results in a family of linear models obtained by linearizing the nonlinear equations in several operating points. The *Model-Varying Predictive Controller* identifies the operating point at each sample time and uses the model corresponding to the operating point as the internal model in the MPC computation. Table 5 shows the family of models used by the MVPC and the linearization point where they are obtained.

Linear models	Power
Model-A	80%
Model-B	70%
Model-C	60%
Model-D	50%
Model-E	40%

Table 5: Linear models used by the Model-Varying Predictive Controller (MVPC) and the linearization point.

Applying the MVPC to a nonlinear system involves the following steps:

Step 1 At sampling time k, identify the output power level $\frac{P(k)}{P_R}$.

Step 2 Choose the appropriate linear model corresponding to this operating power level (Table 5).

Step 3 Obtain plant measurements with the model chosen in **Step 2**.

Step 4 Solve the MPC optimization problem (7) using the model from **Step 2** as the prediction model.

Step 5 Implement the first computed control move $u(k|k)$.

Step 6 $k = k + 1$. Go to **Step 1**.

The switch in **Step 2** will occur if the operating point of the system goes in a different sub-range.

There exist other approaches [3] based on the linearization of the model of the process at each sampling time in order to be used as the internal model when solving the optimization problem (7). Our approach looks like the others but using the same model during several sampling time. This eliminates unnecessary model switching and simplifies the computation costs.

7 Simulations

The equation describing the model of the steam turbine are built in Simulink and the performance achieved with the Model-Varying Predictive Control is shown below. See [9] for details.

The main task of the controller is to force the power to track a reference trajectory and to limit the pressure gradients. Here, we evaluate the MVPC in the following case: while the system is operating at 80% of the power, a negative step of 40% is applied during 10 seconds.

The parameters used in the simulation are listed in Table 6. With that tuning, the control action isn't to much aggressive. This is due to the large prediction horizon (compared to the control horizon) and the weighting matrix on the control signal changes. As a result, the closed loop system is expected to be more robust to model-plant mismatch [4]. It can be seen that the elements of the output matrix are not equal: putting the same weights on the gradients of the pressure and on the total power, the response was excessively slow. The output matrix chosen sets to zero the weight on the gradients and increases the weight on the total power. With that choice, the pressures move as free as the controller allows to satisfy the constrains and the total power follows the set-point better. Some guidelines about MPC tuning can be found in [4].

When applying a ramp-down in power from 80% to 40%, the Model-A is active while the output is higher than 75%. In that point, the MVPC chooses Model-B and so on (See Table 6).

parameters	value
p	20
m	2
T_{MPC}	0.05s
Γ_u	[0 0 0 0 0]
$\Gamma_{\Delta u}$	[0.5 0.5 0.5 0.5 0.5]
Γ_y	[1 1 1 0 0 0 10]

Power ranges	Active Model
$[80\%, 75\%)$	Model-A
$[75\%, 65\%)$	Model-B
$[65\%, 55\%)$	Model-C
$[55\%, 45\%)$	Model-D
$[45\%, 40\%)$	Model-E

Table 6: Simulation parameters (left) and ranges in which internal models used by the MVPC are active (right)

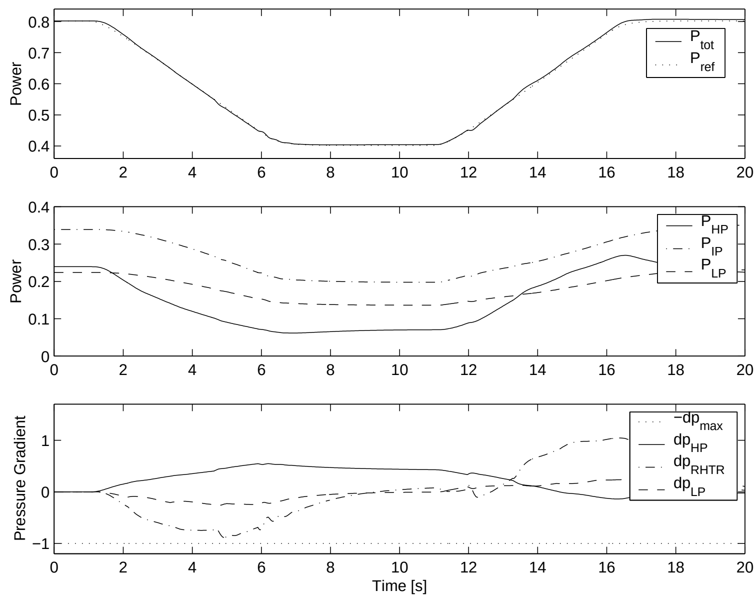

Figure 2: 80%-40%-80% set-point change of Power.

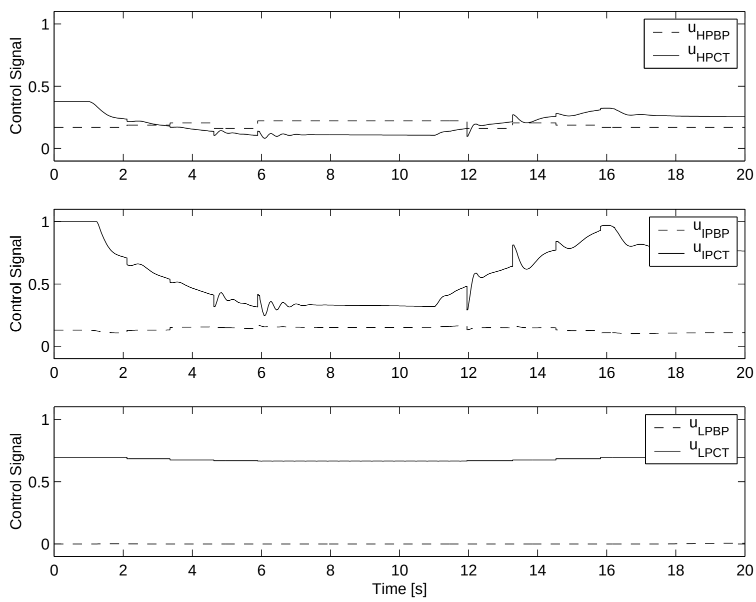

Figure 3: Control signals.

Figure 2 shows how the Power follows reasonably well the set-point and how the pressure gradients don't reach the limit. Figure 3 shows the behaviour of the Control signals. Here, any constraint on the velocity of the opening and closing won't be satisfied because the switching controller changes suddenly the control signal when changing the internal model.

8 Conclusions

A Model Predictive Controller can be used to control nonlinear systems if the the system's output is not moved so far away from the nominal point or if the system isn't *too much* nonlinear. If it is not the case, varying the model used by the controller seems to be a good solution. This is exemplified in this work controlling a steam turbine.

Controlling a steam turbine is generally addressed with PI controllers obtaining quite good results. Nevertheless, other control strategies as the one presented here, give significant benefits.

However, as a consequence of switching among different internal models, control signal changes instantaneously and, for that reason, it doesn't fulfil any constraint on the opening and closing velocity. This is the known *Bumpless* problem and some existing techniques could be applied to solve it.

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