COORDINATION OF TORQUE INTERVENTIONS IN MPC-BASED POWERTRAIN CONTROL

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ABSTRACT
A propulsion system, control system, and method are provided for optimizing fuel economy, which use model predictive control systems to generate a plurality of sets of possible command values and determine a cost for each set of possible command values of based on a first predetermined weighting value, a second predetermined weighting value, a plurality of predicted values, and a plurality of requested values. The set of possible command values having the lowest cost is determined and defined as a set of selected command values. Arbitration is performed including at least one of the following: A) determining at least one requested value based on arbitrating between a driver requested value and an intervention requested value; and B) determining a desired command value by arbitrating between a selected command value of the set of selected command values and a command intervention value.

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COORDINATION OF TORQUE INTERVENTIONS IN MPC-BASED POWERTRAIN CONTROL

TECHNICAL FIELD

The disclosure relates to a control system and method for a propulsion system of a motor vehicle having an engine and a transmission, and more particularly to a control system and method that uses a multivariable controller.

INTRODUCTION

Propulsion system control in a motor vehicle generally involves reading driver and vehicle inputs, such as accelerator pedal position, vehicle sensor data, and torque requests, and communicating these inputs to an Engine Control Module (ECM) and a Transmission Control Module (TCM). The ECM may calculate a driver axle torque requested from the driver and vehicle inputs. The driver axle torque requested may then be communicated to the engine and to the ECM. The engine is controlled based on the desired driver axle torque to produce an actual axle torque. Meanwhile, and typically concurrently with calculating desired engine and axle torques, a desired speed or gear ratio is calculated from the desired axle torque and the vehicle speed. The desired gear ratio, or CVT pulley ratio, is then communicated to the transmission. The transmission is controlled based on the desired gear ratio (or CVT pulley ratio) to produce an actual gear or pulley ratio. The actual axle torque and the actual gear or pulley ratio define the operating conditions of the motor vehicle.

While this system of propulsion system control is useful for its intended purpose, there is room in the art for improvements that provide dynamic control of the axle torque to balance drivability, performance, and fuel economy, especially in propulsion systems having a continuously variable transmission. Engine control systems have been developed to control engine output torque to achieve a desired torque. Traditional engine control systems, however, may not control the engine output torque as accurately as desired. In addition, no traditional mechanism is known to exist that incorporates interventions from torque request overrides into an MPC-based control system.

SUMMARY

A method and system are provided to control a parameter, such as a vehicle acceleration, in a vehicle propulsion system while optimizing fuel economy, through the use of model predictive control. In some forms, model predictive control is used to coordinate the engine and transmission to improve fuel economy and drivability. Axle torque interventions and engine output torque interventions are incorporated into the MPC control system.

In one form, which may be combined with or separate from other forms disclosed herein, a method for controlling a propulsion system of a motor vehicle is provided. The method includes generating a plurality of sets of possible command values and determining a cost for each set of possible command values based on a first predetermined weighting value, a second predetermined weighting value, a plurality of predicted values, and a plurality of requested values. The method also includes determining which set of possible command values has the lowest cost and selecting the set of possible command values that has the lowest cost to define a set of selected command values. The method further includes performing an arbitration step comprising at least one of the following arbitration procedures A and B: A) determining at least one requested value based on arbitrating between a driver requested value and an intervention requested value; and B) determining a desired command value by arbitrating between a selected command value of the set of selected command values and a command intervention value.

In another form, which may be combined with or separate from the other forms disclosed herein, a control system for a motor vehicle propulsion system having a transmission and an engine is provided. The control system includes a command generator module configured to generate a plurality of sets of possible command values and a cost module. The cost module is configured to determine a cost for each set of possible command values of the plurality of sets of possible command values based on a first predetermined weighting value, a second predetermined weighting value, a plurality of predicted values, and a plurality of requested values. The cost module is further configured to determine which set of possible command values of the plurality of sets of possible command values has the lowest cost. A selection module is configured to select the set of possible command values that has the lowest cost to define a set of selected command values. An arbitration module is included that is configured to perform at least one of the following arbitration procedures A and B: A) determine at least one requested value of the plurality of requested values based on arbitrating between a driver requested value and an intervention requested value; and B) determine a desired command value by arbitrating between a selected command value of the set of selected command values and a command intervention value.

In yet another form, which may be combined with or separate from the other forms disclosed herein, a propulsion system for a motor vehicle is provided. The propulsion system includes an engine operable to power the motor vehicle, an engine having an engine output shaft configured to transfer engine output torque. The propulsion system also includes a continuously variable transmission having a variator assembly including a first pulley and a second pulley. The first and second pulleys are rotatably coupled by a rotatable member, and at least one of the first and second pulleys includes a movable sheave translatable along an axis to selectively change a transmission ratio between the engine output shaft and a transmission output shaft. A drive axle is configured to be driven via the transmission output shaft, the drive axle being configured to output axle torque to a set of wheels. A control system is included, which has a prediction module configured to generate a plurality of predicted actual axle torque values and a plurality of predicted actual fuel consumption rate values based on a plurality of sets of possible command values. The plurality of sets of possible command values include a plurality of possible commanded transmission ratio values and a plurality of possible commanded engine torque values. A cost module is configured to determine a cost for each set of possible command values based on a predicted actual axle torque value of the plurality of predicted axle torque values, a predicted actual fuel consumption rate value of the plurality of predicted actual fuel consumption rate values, a first predetermined weighting value, a second predetermined weighting value, a plurality of predicted values, and a plurality of requested values. The plurality of requested values includes the driver axle torque requested, an engine output torque requested, a transmission ratio requested, and a fuel consumption rate requested. The cost module is further configured to determine which set of
possible command values has a lowest cost. A selection
module is configured to select the set of possible command
values that has the lowest cost to define a set of selected
command value. An axi force arbitration module is pro-
vided, which is configured to determine at least one
requested value of the plurality of requested values based
on arbitrating between a driver axle torque requested and an
axle intervention requested value. An engine torque arbi-
tration module is provided, which is configured to determine a
desired command value by arbitrating between a selected
engine torque command value of the set of selected com-
mand values and an engine torque command intervention
value.

Additional features may be provided, including but not
limited to the following: wherein performing the arbitration
step comprises performing both of the arbitration procedures
A and B wherein the plurality of sets of possible command
values includes a plurality of commanded engine output
torque values; wherein the set of selected command values
includes a selected engine output torque value; wherein the
command intervention value includes an engine torque
intervention value; the method further comprising generat-
ing a plurality of predicted actual axle torque values and a
plurality of predicted actual fuel consumption rate values
based on the plurality of sets of possible command values;
the plurality of sets of possible command values including a
plurality of possible transmission ratio command values;
the method further comprising determining the cost for each set
of possible command values further based on a predicted
actual axle torque value of the plurality of predicted axe
torque values and a predicted actual fuel consumption rate
value of the plurality of predicted actual fuel consumption
rate values; the plurality of requested values including a
driver axle torque requested, an engine output torque
requested, a transmission ratio requested, and a fuel con-
sumption rate requested.

In addition, the method/control system may be configured
to determine the plurality of predicted actual axle torque values
and the plurality of predicted actual fuel consumption rate
values with the following set of equations:

\[
\begin{align*}
x_{i+k+1} &= A \cdot x_i + B \cdot \left( \begin{array}{c}
Te_{c,arb} \\
Rat_{c,j}
\end{array} \right) + v + K_{K \theta} \cdot \left( \begin{array}{c}
Te_{c,arb} \\
Rat_{c,j}
\end{array} \right) \\
\begin{bmatrix}
Te_{a,j} \\
FR_{a,j}
\end{bmatrix} &= C \cdot x_{i+k+1} + w
\end{align*}
\]

where,
\(x_i, x_{i+1}\) = state estimates at time steps \(k\) and \(k+1\) respectively;
\(A\) = a state (or transition) matrix;
\(B\) = an input matrix;
\(Te_{c,arb}\) = one of: engine output torque commanded at the
prediction step \(k\) and an engine torque intervention
value;
\(Rat_{c,j}\) = transmission ratio commanded at the prediction
step \(k\);
\(K_{K \theta}\) = a Kalman filter gain;
\(Te_{a,j}\) = predicted actual engine output torque at the predic-
tion step \(k\);
\(FR_{a,j}\) = predicted actual fuel consumption rate at the predic-
tion step \(k\);
\(Rat_{a,j}\) = predicted actual transmission ratio at the prediction
step \(k\);
\(Ta_{a,j}\) = predicted actual axle torque at the prediction step \(k\);
\(Te_{m,f}\) = measured engine output torque at the prediction step
\(k\);
\(FR_{m,f}\) = measured fuel consumption rate at the prediction step
\(k\);
\(Rat_{m,f}\) = measured transmission ratio at the prediction step
\(k\);
\(Ta_{m,f}\) = measured axle torque at the prediction step \(k\);
\(Ta_{a,k}\) = predicted actual axle torque at the prediction step
\(k+1\);
\(FR_{a,k}\) = predicted actual fuel consumption rate at the predic-
tion step \(k+1\);
\(C\) = an output (or measured) matrix;
\(v\) = process noise; and
\(w\) = measurement noise.

Further additional features may include the following:
the method/control system being configured to determine that
the desired command value is the engine torque intervention
value after arbitrating between the selected command value
of the plurality of selected command values and the engine
torque intervention value; the method/control system being
configured to determine that \(Te_{c,arb}\) is equal to the engine
torque intervention value; the method torque intervention
value from among the following: a transmission
torque reduction request, an engine overspeed request, a
boost request, a speed control request, an engine crank
shutdown ring request, a power take off ring request, an
exhaust O2 sensor ring request, a torque cut off ring request,
a hybrid torque request, and a power take off control request;
the method and a steady state optimizer module of the
control system being configured to determine an acceler-
ator pedal position (PP), an engine speed (RPM), a vehicle speed
(V), an air-fuel ratio (AF); the method and the steady state
optimizer module being configured to determine the driver
axle torque requested (\(Ta_{dr}\)) based on the accelerator pedal
position (PP) and the vehicle speed (V), to determine an
arbitrated axle torque requested (\(Ta_{arb}\)) by selecting a
winner between the driver axle torque requested (\(Ta_{dr}\)) and
the intervention requested value (\(A_{i,j}\)), to determine a trans-
mision ratio requested (\(Rat_{r}\)) based on the arbitrated axle
torque requested (\(Ta_{arb}\)) and the vehicle speed (V), to
determine the engine output torque requested (\(Te_{r}\)) based
on the arbitrated axle torque requested (\(Ta_{arb}\)), the trans-
mision ratio requested (\(Rat_{r}\)), and a final drive ratio (FD),
and/or to determine the fuel consumption rate requested
(\(FR_{r}\)) based on the driver axle torque requested (\(Ta_{dr}\)),
the vehicle speed (V), the engine speed (RPM), and the air-fuel
ratio (AF); wherein the plurality of requested values includes
the driver axle torque requested (\(Ta_{dr}\)), the fuel consumption rate requested (\(FR_{r}\)), the engine output torque
requested (\(Te_{r}\)), and the transmission ratio requested
(\(Rat_{r}\)).

Further additional features may also be provided, includ-
ing by not limited to the following: the method/control
system being configured to determine the (axle) interven-
tion requested value from among the following: a brake
torque management request, a vehicle overspeed condition
request, a traction control request, a deceleration fuel cut off
request, a shaping request, a chassis system request, a
performance launch request, a four wheel drive request, and
an emergency autonomous braking request; the plurality of
selected command values including a selected transmission
ratio command value; the method/control system being
configured to control a vehicle parameter based on at least
one of the desired command values; the arbitration module
being an engine torque arbitration module configured to
determine the desired command value by selecting a winner
between the selected engine output torque value and the
engine torque intervention value; the control system further comprising an axle torque arbitration module configured to determine an arbitrated axle torque request based on the differences between predicted and actual axle torque values and the intervention requested value, the control system further comprising a prediction module configured to generate a plurality of predicted actual axle torque values and a plurality of predicted actual fuel consumption rate values based on predictions of the plurality of sets of possible command values; the plurality of sets of possible command values including a plurality of possible transmission ratio command values and a plurality of possible engine torque command values; wherein the cost module is configured to determine the cost for each set of possible command values further based on a predicted actual axle torque value of the plurality of predicted actual axle torque values and a predicted actual fuel consumption rate value of the plurality of predicted actual fuel consumption rate values; the plurality of requested values including the driver axle torque requested, an engine output torque requested, a transmission ratio requested, and a fuel consumption rate requested; wherein the engine torque arbitration module is configured to feed the arbitrated engine output torque command \( T_{e\_arb} \) back to the prediction module.

Additional features, aspects and advantages will become apparent by reference to the following description and appended drawings wherein like reference numbers refer to the same component, element or feature.

**BRIEF DESCRIPTION OF THE DRAWINGS**

The drawings described herein are for illustration purposes only and are not intended to limit the scope of the present disclosure in any way.

**FIG. 1** is a schematic diagram of a motor vehicle having an exemplary propulsion system, in accordance with the principles of the present disclosure;

**FIG. 2** is a schematic diagram showing a propulsion control system for use with the propulsion system shown in **FIG. 1**, according to the principles of the present disclosure;

**FIG. 3** is a schematic diagram of a control system for use with the propulsion control system shown in **FIG. 2**, in accordance with the principles of the present disclosure;

**FIG. 4** is a schematic diagram illustrating additional details of the control system shown in **FIG. 3**, according to the principles of the present disclosure;

**FIG. 5** is a schematic diagram illustrating additional details of a multivariable controller of the control system shown in **FIGS. 3-4**, in accordance with the principles of the present disclosure; and

**FIG. 6** is a block diagram illustrating a method for controlling a vehicle propulsion system, according to the principles of the present disclosure.

**DESCRIPTION**

With reference to **FIG. 1**, an exemplary motor vehicle is shown and generally indicated by reference number **89**. The motor vehicle **89** is illustrated as a passenger car, but it should be appreciated that the motor vehicle **89** may be any type of vehicle, such as a truck, van, sport-utility vehicle, etc. The motor vehicle **89** includes an exemplary propulsion system **10**. It should be appreciated at the outset that while a rear-wheel drive propulsion system **10** has been illustrated, the motor vehicle **89** may have a front-wheel drive propulsion system without departing from the scope of the present disclosure.

The propulsion system **10** generally includes an engine **12** interconnected with a transmission **14** and a final drive unit **16**. The engine **12** may be a conventional internal combustion engine or an electric engine, hybrid engine, or any other type of prime mover, without departing from the present disclosure. The engine **12** supplies a driving engine output torque to the transmission **14** via a crankshaft or engine output shaft **18**. The driving engine output torque may be transmitted through a flexplate and/or starting device **20** to the transmission **14**. The starting device **20** may be a hydrodynamic device, such as a fluid coupling or torque converter, a wet dual clutch, or an electric motor, by way of example. Torque is then transmitted from the starting device **20** to at least one transmission input shaft **22**.

The transmission **14** may be a stepped transmission having planetary gears, a countershaft transmission, a continuously variable transmission, or an infinitely variable transmission. Torque from the transmission input shaft **22** is communicated through a ratio control unit **24** to a transmission output shaft **26**. Generally, the ratio control unit **24** provides a plurality of forward or reverse speeds or gear ratios, or an infinite number of forward or reverse speeds or gear ratios, between the transmission input shaft **22** and the transmission output shaft **26**.

Where the transmission **14** is a continuously variable transmission, the ratio control unit **24** may include a variator assembly **24a** including having first and second pulleys **24b, 24c** that are rotatably coupled by an endless rotate member **24d** wrapped around the variable diameter pulleys **24b, 24c**. At least one of the first and second pulleys **24b, 24c** includes a movable sheave **24e** translatable along an axis to selectively change a gear ratio between the engine output shaft **18** and the transmission output shaft **26**.

The transmission output shaft **26** communicates output torque to the final drive unit **16**. The final drive unit **16** generally includes a differential **28** that transfers axle torque through drive axles **30** to drive wheels **32**.

Turning now to **FIG. 2**, a vehicle propulsion control system for use with the exemplary propulsion system **10** is generally indicated by reference number **34**. The vehicle propulsion control system **34** includes a supervisory control module **36** in electronic communication with an engine control module **38** and a transmission control module **40**. The modules **36**, **38**, and **40** may communicate through a vehicle network or cable area network (CAN) bus. The vehicle propulsion control system **34** may include or communicate with various other control modules, such as a body control module or infotainment control module. Alternatively, the supervisory control module **36** may be subsumed within the engine control module **38** or transmission control module **40**.

The supervisory control module **36** is a non-generalized, electronic control device having a preprogrammed digital computer or processor **42**, memory or non-transitory computer readable medium **44** used to store data such as control logic, instructions, image data, lookup tables, etc., and a plurality of input/output peripherals or ports **46**. The processor **42** is configured to execute the control logic or instructions.

The engine control module **38** is a non-generalized, electronic control device having a preprogrammed digital computer or processor **48**, memory or non-transitory computer readable medium **50** used to store data such as control logic, instructions, image data, lookup tables, etc., and a plurality of input/output peripherals or ports **52**. The processor **48** is configured to execute the control logic or
instructions. The engine control module 38 communicates with, and controls, the engine 12.

The transmission control module 40 is a non-generalized, electronic control device having a preprogrammed digital computer or processor 54, memory or non-transitory computer readable medium 56 used to store data such as control logic, instructions, image data, lookup tables, etc., and a plurality of input/output peripherals or ports 58. The processor 54 is configured to execute the control logic or instructions. The transmission control module 40 communicates with, and controls, the transmission 14.

The vehicle propulsion control system 34 communicates with a plurality of sensors connected to the propulsion system 10 including an air flow sensor S2 in the engine 12, an engine speed sensor S4, a transmission input shaft speed sensor S6, a transmission output shaft speed sensor S8, a vehicle speed sensor S10, and a pedal position sensor S12. The air flow sensor S2 and the engine speed sensor S4 communicate with the engine control module 38. The transmission input shaft speed sensor S6 and the transmission output shaft speed sensor S8 communicate with the transmission control module 40. The vehicle speed sensor S10 and the pedal position sensor S12 communicate with both the engine control module 38 and the transmission control module 40.

With reference to FIG. 3 and continued reference to FIGS. 1 and 2, a control diagram for the vehicle propulsion control system 34 is illustrated. The control diagram illustrates a control system or method 100 for controlling a parameter, such as vehicle acceleration, while optimizing fuel economy, which utilizes a multivariable controller. The control system 100 includes a multivariable controller 102 and a plant 103 that is controlled by the multivariable controller 102. The multivariable controller 102 may iteratively control an engine output torque Te 104 and a transmission ratio Rat 106 to optimize a fuel consumption rate FR and to achieve a desired axle torque Ta. The axle torque Ta is the amount of torque at the vehicle axle 30. Inputs to the multivariable controller 102 include a measured actual axle torque Ta_m, a measured fuel consumption rate FR_m, and an axle torque requested Ta_r which may be based on driver vehicle inputs, and/or an axle torque intervention, which will be discussed in further detail below.

The control system 100 may include an engine torque controller 108, a transmission ratio controller 110 (which may be a variator controller for CVTs), a vehicle dynamics module 112, and an engine mode control module 114 that receives an engine mode signal 116 from the multivariable controller 102. The engine mode control module 114 may be used to control active fuel management, such as cylinder deactivation, or variable valve lift, by way of example. In some examples, the multivariable controller 102 is stored and executed by the supervisory control module 36, the engine torque controller 108 and the engine mode control module 114 are stored and executed by the engine control module 38, and the transmission ratio controller 110 is stored and executed by the transmission control module 40. The vehicle dynamics module 112 may be stored and executed by the engine control module 38, the transmission control module 40, or any other control module or a combination of control modules.

The multivariable controller 102 may optionally receive system limitations 105 from the engine controller 108 including a maximum engine output torque Te_{max}, a minimum engine output torque Te_{min}, a maximum rate of change of engine output torque ΔTe_{max}, and a minimum rate of change of engine output torque ΔTe_{min}. The multivariable controller 102 may also optionally receive system limitations 107 from the transmission ratio controller 110 including a maximum transmission ratio Rat_{max}, a minimum transmission ratio Rat_{min}, a maximum rate of change of transmission ratio ΔRat_{max}, and a minimum rate of change of transmission ratio ΔRat_{min}.

Referring now to FIG. 4, another representation of the control system 100 is illustrated, showing inputs and outputs to the multivariable controller 102 and the plant 103 controlled by the multivariable controller 102. For example, inputs to the multivariable controller 102 may include a driver axle torque requested Ta_dr, an arbitrator axle torque requested Ta_arb (which may or may not be equal to Ta_dr, as explained in further detail below) as well as vehicle speed V. Feedback inputs of axle torque measured Ta_m, fuel consumption rate measured FR_m, and arbitrator engine torque commanded Te_c_arb may also be input to the multivariable controller 102. Outputs of the multivariable controller 102 may include an engine output torque commanded Te_c and a transmission ratio commanded Rat_c. These controlled outputs, or “u” variables (Te_c and Rat_c), of the multivariable controller 102 may be inputs to the plant 103, which includes the engine 12 and transmission 14.

However, in some variations, the engine output torque commanded Te_c is first output to an engine arbitration module 120 to determine whether any other engine torque interventions F_i, j is allowed to control the engine output torque commanded. As is explained in further detail below, the engine arbitration module 120 outputs an arbitrator engine output torque commanded Te_c_arb to the plant 103, which may be equal to the selected engine output torque commanded Te_c or another engine output torque based on one of the intervention inputs F_i, j.

The arbitrator engine output torque commanded Te_c_arb is used to control the engine 12 to result in an actual engine output torque, which is the measured engine output torque Te_m. The transmission ratio commanded Rat_c is used to control the transmission 14 to provide an actual measured gear ratio or pulley ratio Rat_m between the transmission input shaft 22 and the transmission output shaft 26. Thus, the plant 103 outputs the “y” variables, the values that may be tracked, which may include actual measured engine torque Te_m, actual measured fuel consumption rate FR_m, actual measured transmission ratio (or pulley ratio) Rat_m, and actual measured axle torque Ta_m.

Referring now to FIG. 5, additional details of the multivariable controller 102 are illustrated, as well as details regarding an axle arbitration module 122 and the engine torque arbitration module 120. The axle arbitration module 122 may be included as part of the multivariable controller 102 or as part of another controller. The axle arbitration module 122 is configured to consider multiple axle torque requests from driver and vehicle sources, and decide which of the axle torque requests should be input to the multivariable controller 102 to be used as the axle torque requested Ta. For example, one input to the axle arbitration module 122 is a driver axle torque requested Ta_dr, which is a function of accelerator pedal position and vehicle speed. More particularly, the driver axle torque requested Ta_dr may be determined based on the accelerator pedal position PP and the vehicle speed V with a relationship such as

\[ Ta_{dr} = f(PP, V) \]

(1)In some examples, the driver axle torque requested Ta_dr may be determined from a lookup table or 2D map from a
vehicle speed V sensed by vehicle speed sensor S10 and an accelerometer position PP sensed by the pedal position sensor S12.

A plurality of axle torque intervention requested values A_i are also input to the axle arbitration module 122, as they occur under certain driving conditions. These axle torque intervention requested values A_i may include, for example, a brake torque management (BTM) request, a vehicle overspeed condition request, a traction control (TC) request, a deceleration fuel cut-off request, a shaping request, a chassis system request, a performance launch request, a four wheel drive request, and an emergency autonomous braking request. The axle arbitration module 122 is configured to determine whether any of the intervention requested values A_i may be allowed to override the driver axle torque requested Ta_dr. The axle arbitration module 122 chooses a "winner" by selecting between, or arbitrating between, the driver axle torque requested Ta_dr and any intervention requested values A_i. The winner of the arbitration is output from the axle arbitration module 122 as the arbitrated axle torque value Ta_arb to a steady state optimizer module 200. The axle arbitration module 122 also outputs the driver axle torque requested Ta_dr to the steady state optimizer module 200.

The steady state observer module 200 is a reference generator that is included as part of the multivariable controller 102. The steady state observer module 200 determines reference values (desired or requested values) for the "u" variables (controlled variables) and the "y" variables (the optimized output variables that may be tracked). For example, the steady state optimizer module 200 is configured to determine an engine output torque requested Te_r, a transmission ratio requested Rat_r, a fuel consumption rate requested FR_r, and to output the driver axle torque requested Ta_dr. The steady state optimizer module 200 may also output the arbitrated axle torque requested Ta_arb, in some example. (In some forms, the axle arbitration module 122 may be included as part of the steady state optimizer module 200).

The u_ref values include the engine output torque requested Te_r and the transmission ratio requested Rat_r, while the y_ref values may include all four of the engine output torque requested Te_r, the transmission ratio requested Rat_r, the fuel consumption rate requested FR_r, and the driver axle torque requested Ta_dr (and in some cases, the arbitrated axle torque requested Ta_arb). The u_ref and the y_ref values are values that are desirable during a steady state. The MPC module 202, described below, optimizes the trajectory, particularly of the fuel consumption rate FR, during the transient from one steady state to another.

The fuel consumption rate requested FR_r may be determined based on the driver axle torque requested Ta_dr, the vehicle speed V, the engine speed RPM, and the air-fuel ratio AF. For example,

\[ \text{FR}_r = f(Ta_dr, \text{RPM}, \text{AF}) \]  

(2)

The engine speed RPM may be determined from the engine speed sensor S4. The air-fuel ratio AF is the ratio of the mass of air to the mass of fuel, which may be reported by a fuel control module, by way of example.

The transmission ratio requested Rat_r may be determined based on the arbitrated axle torque requested Ta_arb and the vehicle speed V. For example,

\[ \text{Rat}_r = f(Ta_arb, V) \]  

(3)

Again, Ta_arb may be equal to the driver axle torque requested Ta_dr if the axle arbitration module 122 determines that the driver axle torque requested Ta_dr should win the arbitration or if the axle arbitration module 122 determines that, even though one of the intervention requested values A_i should win the arbitration, the intervention requested value A_i should not be allowed to alter the axle torque requested Ta used to determine the transmission ratio requested Rat_r, and therefore, the axle torque requested should remain as the driver axle torque requested Ta_dr to be used to determine the transmission ratio requested Rat_r.

The engine output torque requested Te_r may be determined based on the arbitrated axle torque requested Ta_arb, the transmission ratio requested Rat_r, and the final drive ratio FR (which is constant for a given vehicle). For example,

\[ Te_r = \frac{Ta_arb + \text{Loss}}{Rat_r \times FR \times FD}. \]  

(4)

The "loss" factor may encompass mechanical losses, such as friction and pulley clamping losses, by way of example. As with computing the transmission ratio requested Rat_r, when computing the engine output torque requested Te_r, Ta_arb may equal to the driver axle torque requested Ta_dr if the axle arbitration module 122 determines that the driver axle torque requested Ta_dr should win the arbitration or if the axle arbitration module 122 determines that, even though one of the intervention requested values A_i should win the arbitration, the intervention requested value A_i should not be allowed to alter the axle torque requested Ta used to determine the engine output torque requested Te_r, and therefore, the axle torque requested should remain as the driver axle torque requested Ta_dr to be used to determine the engine output torque requested Te_r.

Once the requested values, or reference values, are determined, the steady state optimizer module 200 outputs them (the u_ref and the y_ref) to the MPC module 202. The MPC module 202 uses model predictive control and may also be referred to as a quadratic programming solver, such as a Dantzig LP solver.

A prediction module 204 is configured to predict at least an actual axle torque and an actual fuel consumption rate for use in the MPC module 202. The prediction module 204 may also be referred to as a state observer, which uses a Kalman filter. The predicted actual values 206 are output from the prediction module 204 to the MPC module 202.

The prediction module 204 is configured to generate a plurality of predicted actual axle torques and fuel consumption rates. For example, the prediction module generates at least a first predicted actual axle torque and a first predicted actual fuel consumption rate based on a first set of possible command values (which may be generated, for example, by a command generator module formed as part of the prediction module 204 or as part of the MPC module 202), where the first set of possible command values includes a first commanded engine output torque Te_c and a first commanded transmission ratio Rat_c. The prediction module 204 is further configured to generate at least a second predicted actual axle torque and a second predicted actual fuel consumption rate based on a second set of possible command values, where the second set of possible command values includes a second commanded engine output torque Te_c and a second commanded transmission ratio Rat_c. In practice, a much larger number of predicted values may be generated based on additional sets of possible command values (third, fourth,
The predicted actual values \( 206 \) are output to the MPC module \( 202 \).

As will be discussed in further detail below, the engine torque commanded \( \text{Te}_c \) that is input to the prediction module \( 204 \) may, in some circumstances, include an arbitrated engine torque commanded \( \text{Te}_{c\text{ arb}} \).

The MPC module \( 202 \) contains a cost module \( 208 \) that is configured to determine a first cost for the first set of possible command values \( \text{Te}_c, \text{Rat}_c \) based on at least first and second predetermined weighting values, the first predicted actual axle torque, the first predicted actual fuel consumption rate, the driver axle torque requested \( \text{Ta}_\text{dr} \), the engine output torque requested \( \text{Te}_r \), the transmission ratio requested \( \text{Fr}_r \), and the fuel consumption rate requested \( \text{Fr}_r \). Similarly, the cost module \( 208 \) is configured to determine a second cost for the second set of possible command values \( \text{Te}_c, \text{Rat}_c \) based on at least the first and second predetermined weighting values, the second predicted actual axle torque, the second predicted actual fuel consumption rate, the driver axle torque requested \( \text{Ta}_\text{dr} \), the engine output torque requested \( \text{Te}_r \), the transmission ratio requested \( \text{Fr}_r \), and the fuel consumption rate requested \( \text{Fr}_r \). Likewise, many more additional costs may be determined based on additional sets of predicted values and command values, in order to optimize for the lowest cost.

The MPC module \( 202 \) may also include a selection module \( 210 \) configured to select one of the plurality of sets of possible command values \( \text{Te}_c, \text{Rat}_c \) based on the lowest of the determined costs and set a selected engine output torque \( \text{Te}_c \) and a selected transmission ratio \( \text{Rat}_c \) equal to, or based on, the possible command values \( \text{Te}_c, \text{Rat}_c \) of the selected one of the plurality of possible sets.

The cost module \( 202 \) may be configured to determine the plurality of costs, with the following cost equation (5):

\[
\text{Cost} = \sum \left( y(k) - y_{ref} \right)^T Q_k \left( y(k) - y_{ref} \right) + \left( u(k) - u_{ref} \right)^T R_k \left( u(k) - u_{ref} \right) + \Delta u(k)^T \Delta Q_{1\text{a}} \Delta u(k)
\]

where \( \text{Te}_a = \) predicted actual engine output torque; \( \text{FR}_a = \) predicted actual fuel consumption rate; \( \text{Rat}_a = \) predicted actual transmission ratio; \( \text{Ta}_a = \) predicted actual axle torque; \( \text{Te}_r = \) engine output torque requested; \( \text{FR}_r = \) fuel consumption rate requested; \( \text{Rat}_r = \) transmission ratio requested; \( \text{Ta}_\text{dr} = \) driver axle torque requested; \( \text{Te}_c = \) commanded engine output torque; \( \text{Rat}_c = \) commanded transmission ratio; \( Q_{1\text{a}} = \) first predetermined weighting value; \( Q_{2\text{a}} = \) second predetermined weighting value; \( Q_{3\text{a}} = \) third predetermined weighting value; \( i = \) index value; \( k = \) prediction step; and \( T = \) transposed vector.

In this case, there are two values for the "u" variables, \( u_1 \) and \( u_2 \), such that \( i = 1, 2, 3, 4 \). As explained above, the \( y_{ref} \) and \( u_{ref} \) values may be determined by the steady state optimizer module \( 200 \).

The plurality of costs may be determined even more particularly with the following equation (6), which is an MPC equation having a prediction horizon of three and a control horizon of two:

\[
\text{Cost} = \left[ \left( y_a - y_{ref} \right)^T R_k \left( y_a - y_{ref} \right) + \left( u_a - u_{ref} \right)^T R_k \left( u_a - u_{ref} \right) + \Delta u_a^T \Delta Q_1 \Delta u_a \right] + \lambda_a^T \left( \text{Fr}_r - \text{Fr}_{ref} \right) + \lambda_a^T \left( \text{Rat}_r - \text{Rat}_{ref} \right) + \lambda_a^T \left( \text{Ta}_\text{dr} - \text{Ta}_\text{dr}_{ref} \right) + \lambda_a^T \left( \text{Te}_c - \text{Te}_{c \text{ ref}} \right) + \lambda_a^T \left( \text{Rat}_c - \text{Rat}_{c \text{ ref}} \right) + \lambda_a^T \left( \text{Te}_r - \text{Te}_{r \text{ ref}} \right) + \lambda_a^T \left( \text{Fr}_r - \text{Fr}_{r \text{ ref}} \right) + \lambda_a^T \left( \text{Rat}_r - \text{Rat}_{r \text{ ref}} \right) + \lambda_a^T \left( \text{Te}_c - \text{Te}_{c \text{ ref}} \right) + \lambda_a^T \left( \text{Rat}_c - \text{Rat}_{c \text{ ref}} \right)
\]

where \( \lambda_a = \) a first predetermined weighting value; \( y_a = \) a predicted actual axle torque at a prediction step \( k \); \( u_a = \) a predicted driver axle torque requested; \( \text{Ta}_{a1} = \) a predicted actual axle torque at a prediction step \( k+1 \); \( \text{Ta}_{a2} = \) a second predetermined weighting value; \( \text{FR}_{a1} = \) a predicted actual fuel consumption rate at a prediction step \( k \); \( \text{FR}_{a2} = \) a second predetermined weighting value at a prediction step \( k+2 \); \( \lambda_a = \) a third predetermined weighting value; \( \text{Te}_{c1} = \) engine output torque commanded at the prediction step \( k \); \( \text{Te}_{c2} = \) engine output torque requested; \( \text{Te}_{c3} = \) engine output torque commanded at the prediction step \( k+1 \); \( \lambda_a = \) a fourth predetermined weighting value; \( \text{Rat}_{c1} = \) transmission ratio commanded at the prediction step \( k \); \( \text{Rat}_{c2} = \) transmission ratio requested; \( \text{Rat}_{c3} = \) transmission ratio commanded at the prediction step \( k+1 \); \( \lambda_a = \) a fifth predetermined weighting value; \( \Delta \text{Te}_{c1} = \) change in engine output torque commanded at the prediction step \( k \); and \( \Delta \text{Te}_{c2} = \) change in engine output torque commanded at the prediction step \( k+1 \). The prediction step \( k \) is the prediction at a current step, the prediction step \( k+1 \) is a prediction one step ahead, and the prediction step \( k+2 \) is a prediction two steps ahead. As explained above, the \( y_{ref} \) and \( u_{ref} \) values may be determined by the steady state optimizer module \( 200 \).

The cost equation (e.g., equation (5) or (6)) may be applied iteratively to arrive at the lowest cost for a plurality of sets of possible command values \( \text{Te}_c, \text{Rat}_c \), where the plurality of sets of possible command values \( \text{Te}_c, \text{Rat}_c \) include the first and second sets of possible command values as well as a number of other possible sets of command values for \( \text{Te}_c, \text{Rat}_c \). Then, the selection module \( 210 \) may select the set of possible command values \( \text{Te}_c, \text{Rat}_c \) of the plurality of command values having the lowest cost, where the set of possible command values \( \text{Te}_c, \text{Rat}_c \) having the lowest cost may be defined as the selected set, including the selected transmission ratio \( \text{Rat}_c \) and the selected engine output torque \( \text{Te}_c \). Similarly, the cost module \( 208 \) may generate a surface representing the cost of possible sets of command values \( \text{Te}_c, \text{Rat}_c \). The cost module \( 208 \) and/or the selection module \( 210 \) may then identify the possible set that has the lowest cost based on the slope of the cost surface.

The prediction module \( 204 \) may provide a number of predicted actual values \( 206 \) to the MPC module \( 202 \) for use
in the cost equation (e.g., equation (5) or (6)) by the cost module 208. The prediction module 204 may use equations such as the following to determine the predicted actual values 206:

\[ x_t = C x_{t-1} \]
\[ y_{t+1} = C_x y_{t+1} + w \]
\[ x_{t+1} = A_t x_t + B_t u_t + v + K e_t (y_t - y_{t+1}) \]
\[ y = \begin{bmatrix} \text{Te}_{-x} \\ \text{te}_{-y} \end{bmatrix} \]
\[ u = \begin{bmatrix} \text{Te}_{-u} \\ \text{te}_{-u} \end{bmatrix} \]

where \( A \) is a state (or transition) matrix; \( B \) is an input matrix; \( C \) is an output (or measured) matrix; \( \text{Te}_{-x} \) is the predicted actual engine output torque at the prediction step \( k \); \( \text{FR}_{-x} \) is the predicted actual fuel consumption rate at the prediction step \( k \); \( \text{Rat}_{-x} \) is the predicted actual transmission ratio at the prediction step \( k \); \( \text{Ta}_{-x} \) is the predicted actual axle torque at the prediction step \( k \); \( x_{t+1} \) is the state variable at a prediction step \( k \); \( \text{Te}_{-x} \) is the predicted actual engine output torque at the prediction step \( k+1 \); \( \text{FR}_{-x} \) is the predicted actual fuel consumption rate at the prediction step \( k+1 \); \( \text{Rat}_{-x} \) is the predicted actual transmission ratio at the prediction step \( k+1 \); \( \text{Ta}_{-x} \) is the predicted actual axle torque at the prediction step \( k+1 \); \( x_{t+1} \) is the state variable at a prediction step \( k+1 \); \( \text{Te}_{-c} \) is the engine output torque commanded at the prediction step \( k \), which could be substituted by \( \text{Te}_{-c} \), \( \text{FR}_{-c} \), \( \text{Rat}_{-c} \) and \( \text{Ta}_{-c} \) as transmission ratio commanded at the prediction step \( k \), \( K e_t \) is a Kalman filter gain; \( \text{Te}_{-a} \) is the measured engine output torque at the prediction step \( k \); \( \text{FR}_{-a} \) is the measured fuel consumption rate at the prediction step \( k \); \( \text{Rat}_{-a} \) is the measured transmission ratio at the prediction step \( k \); \( \text{Ta}_{-a} \) is the measured axle torque at the prediction step \( k \); \( v \) is process noise; and \( w \) is measurement noise. The prediction step \( k \) is a prediction at the current time (e.g., now), and the prediction step \( k+1 \) is a prediction one step ahead.

Measured engine output torque \( \text{Te}_{-a} \) may be sensed from the engine torque sensor S4. The measured transmission ratio, or pulley ratio, \( \text{Rat}_{-a} \) may be determined from the speed of the transmission input shaft 22 sensed by the transmission input shaft speed sensor S6 and the speed of the transmission output shaft 26 sensed by the transmission output shaft speed sensor S8, and may be provided by the TCM 40.

\( \text{Te}_{-x} \) and \( \text{FR}_{-x} \) may be defined as or equal to the first predicted actual axle torque and the first predicted actual fuel consumption rate, respectively, when generated based on the first set of possible command values for \( \text{Te}_{-x} \) and \( \text{Rat}_{-x} \) and \( \text{Ta}_{-x} \) and \( \text{FR}_{-x} \) may be defined as or equal to the second predicted actual axle torque and the second predicted actual fuel consumption rate, respectively, when generated based on the second set of possible command values for \( \text{Te}_{-x} \) and \( \text{Rat}_{-x} \) and \( \text{Ta}_{-x} \) and \( \text{FR}_{-x} \) may be defined as or equal to the third predicted actual axle torque and the third predicted actual fuel consumption rate, respectively, when generated based on the third set of possible command values for \( \text{Te}_{-x} \) and \( \text{Rat}_{-x} \) and \( \text{Ta}_{-x} \) and so on.

The cost equation (e.g., equation (5) or (6)) may be subject to the following constraints 105, 107:

\[ \begin{align*}
\text{Te}_{-x} &\geq \text{Te}_{-x} \text{min} \\
\text{te}_{-x} &\geq \text{te}_{-x} \text{min} \\
\text{Rat}_{-x} &\leq \text{Rat}_{-x} \text{max} \\
\text{Ta}_{-x} &\leq \text{Ta}_{-x} \text{max} \\
\text{FR}_{-x} &\leq \text{FR}_{-x} \text{max} \end{align*} \]

where \( \text{Te}_{-x} \text{min} \) is the minimum possible engine output torque, \( \text{te}_{-x} \text{min} \) is the minimum possible engine output torque, \( \text{Rat}_{-x} \text{max} \) is the maximum possible transmission ratio, \( \text{Ta}_{-x} \text{max} \) is the maximum possible transmission ratio, \( \text{FR}_{-x} \text{max} \) is the maximum possible rate of change in transmission ratio, \( \Delta \text{Rat}_{-x} \text{max} \) is the maximum possible rate of change in transmission ratio, \( \Delta \text{Te}_{-x} \text{max} \) is the maximum possible rate of change in engine output torque, and \( \Delta \text{te}_{-x} \text{max} \) is the maximum possible rate of change in engine output torque, where the constraints 105, 107 may be provided by the ECM 38 and the TCM 40, by way of example.

The constants, matrices, and gain referred to above, including \( A, B, C, K e_t, Q_a, Q_e, Q_{a_sa}, \lambda_a, \lambda_e, \lambda_{a_sa}, \lambda_{e_sa}, \lambda_{e_{a_sa}}, \lambda_{a_{e_{a_sa}}}, \) are parameters of the system determined through testing, physical models, or other means. In some variations, a system identification procedure is run offline, for example, during a calibration, to identify the constants, matrices, and gain, and also to define \( u_t \) and \( y_t \). Once \( u_t \) and \( y_t \) are known, then \( x_t \) can be computed from the prediction module equations (e.g., equations (7)-(9) or a subset thereof). Thereafter, each of the prediction module 204 and MPC module 202 equations (e.g., equations (5)-(9) or a subset thereof) can be run to obtain initial values offline. Then, the control system 102 can be run online to constantly optimize the controlled parameters \( \text{Te}_{-c} \) and \( \text{Rat}_{-c} \) as the vehicle 9 is running through steady state and transient states. The constants allow the cost to be determined based on the relationship between and relative importance of each of the commanded values \( \text{Te}_{-c} \), \( \text{Rat}_{-c} \) and tracked values (e.g., \( \text{FR}_{-a}, \text{Ta}_{-a}, \text{Rat}_{-a}, \text{Te}_{-a} \)). The relationships are weighted to control the effect that each relationship has on the cost.

In some forms, the MPC module 202 may generate the possible sets of command values \( \text{Te}_{-c}, \text{Rat}_{-c} \) by determining possible sequences, sets, or a surface containing the command values \( \text{Te}_{-c}, \text{Rat}_{-c} \) that could be used for \( N \) future control loops. The prediction module 204 may determine predicted responses to the possible sets of the command values \( \text{Te}_{-c}, \text{Rat}_{-c} \) using the prediction module equations (e.g., equations (7)-(9) or a subset thereof). For example, the prediction module 204 may determine a set of predicted actual axle torques \( \text{Ta}_{-a} \) and a set of predicted actual fuel consumption rates \( \text{FR}_{-a} \) for \( N \) control loops.

More particularly, a set of \( N \) values for each command value \( \text{Te}_{-c}, \text{Rat}_{-c} \) may be determined, and a set of \( M \) values...
for each predicted actual value \( T_a \), \( FR_a \) may be determined based on the N command values \( T_e \), \( Rat_c \). The cost module 208 may then determine the cost value for each of the possible sets of command values \( T_e \), \( Rat_c \) based on the predicted actual parameters \( T_a \), \( FR_a \) which may include \( T_a \), \( T_a+1 \), \( T_a+2 \), \( FR_a \), \( FR_a+1 \), and \( FR_a+2 \), depending on the particular cost equation (5), (6) used. The selection module 210 may then select one of the possible sets of the command values \( T_e \), \( Rat_c \) based on the costs of the possible sets, respectively. For example, the selection module 210 may select the possible set of command values \( T_e \), \( Rat_c \) having the lowest cost while satisfying the system constraints 105, 107 (e.g., \( T_e \), \( Rat_c \), \( T_e \), \( Rat_c \)).

5 In some forms, satisfaction of the constraints 105, 107 may be considered in the cost determination. For example, the cost module 208 may determine the cost values further based on the constraints 105, 107, and the selection module 210 may select the possible set of command values \( T_e \), \( Rat_c \) that best achieves the axle torque request \( T_a \) while minimizing fuel consumption rate \( FR \) that has been determined to comply with the constraints 105, 107.

During steady-state operation, the command values \( T_e \), \( Rat_c \) may settle at or near reference, or requested, values \( T_e \), \( Rat_c \). During transient operation, however, the MPC module 202 may adjust the command values \( T_e \), \( Rat_c \) away from the reference values \( T_e \), \( Rat_c \) in order to best achieve the torque request \( T_arb \), while minimizing the fuel consumption rate \( FR \) and satisfying the constraints 105, 107.

In operation, the MPC module 202 may determine the cost values for the possible sets of controlled and predicted values \( u \), \( y \). The MPC module 202 may then select the one of the possible sets having the lowest cost. The MPC module 202 may next determine whether the selected possible set satisfies the constraints 105, 107. If so, the possible set may be defined as the selected set. If not, the MPC module 202 determines the set with the lowest cost that satisfies the constraints 105, 107 and defines that set as the selected set.

The selected \( Rat_c \) command value is output from the MPC module 202 to the plant 103 (see FIG. 4) in the selected engine output torque \( T_e \), however, may undergo another procedure before being output to the plant 103.

More particularly, the MPC module 202 outputs the selected engine output torque \( T_e \) from the selection module 210 to the engine torque arbitration module 120. The engine torque arbitration module 120 may be included as part of the multivariable controller 102, if desired.

The engine output torque arbitration module 120 is configured to consider multiple engine torque requests from driver and vehicle sources, and decide which of the engine output torque requests should be used for a final engine torque request signal. For example, one input to the engine torque arbitration module 120 is the selected engine output torque \( T_e \) from the MPC module of the controller 102.

A plurality of engine torque intervention commanded values \( E_i \) are also input to the engine torque arbitration module 120. These engine output torque intervention commanded values \( E_i \) may include, for example, a transmission torque reduction request, an engine overspeed request, a boost request, a speed control request, an engine crank shutdown ring request, a power take off ring request, an exhaust O2 sensor ring request, a torque cut off ring request, a hybrid torque request, and a power take off control request.

The engine output torque arbitration module 120 is configured to determine whether any of the engine output torque intervention commanded values \( E_i \) may be allowed to override the MPC-selected engine output torque commanded \( T_e \). The engine output torque arbitration module 120 chooses a “winner” by selecting between, or arbitrating between, the MPC-selected engine output torque commanded \( T_e \) and any engine output torque intervention commanded values \( E_i \). The winner of the arbitration is output from the engine torque arbitration module 120 as the arbitrated engine output torque commanded \( T_e \) to control the final engine output torque request signal.

The engine torque arbitration module 120 also outputs the arbitrated engine output torque commanded \( T_e \) back to the multivariable controller 102, and more particularly, to the prediction module 204 to affect future iterations of the MPC module 202. As explained above, \( T_e \) is input in place of \( T_e \) k to the prediction module 204.

Even if \( T_e \) arb is chosen by the engine torque arbitration module 120 from one of the engine output torque intervention commanded values \( E_i \), the MPC module 202 continues to run, but is overwritten by the \( T_e \) arb value selected by the engine torque arbitration module 120 to send to an actuator for controlling engine output torque. \( T_e \) arb may be the MPC-selected engine output torque \( T_e \) if the engine torque arbitration module 120 determines that the MPC-selected engine output torque \( T_e \) should win the arbitration or if the engine torque arbitration module 120 determines that, even though one of the engine output torque intervention commanded values \( E_i \) should win the arbitration, the engine output torque intervention commanded values \( E_i \) should not be allowed to alter the engine output torque request \( T_e \) used to control actual engine output torque, and the engine output torque commanded should remain as the MPC-selected engine output torque \( T_e \).

The MPC module 202 may then output the arbitrated engine output torque value \( T_e \) arb and the selected transmission ratio commanded \( Rat_c \) to the plant 103. The multivariable controller 102 or the plant 103 may contain an actuation module configured to control a vehicle parameter based on at least one of the command values \( T_e \), \( Rat_c \). For example, acceleration of the vehicle 9 may be controlled to optimize the fuel consumption rate. In some forms, the actuation module may be contained within the vehicle dynamics module 112 shown in FIG. 3. Any vehicle system that varies an engine or transmission parameter may be referred to as an actuation module. In some forms, for example, the actuation module may vary the engine spark timing or the throttle, in order to control vehicle acceleration and/or axle torque.

Referring now to FIG. 6, a flowchart depicting an example method for controlling the propulsion system 10 of the motor vehicle 9 is presented and generally designated at 300. The method 300 may optionally begin with a step 302 of determining at least one requested value based on arbitrating between a driver requested value and an intervention requested value. For example, as explained above, with respect to the axle arbitration module 122, a requested value, such as \( T \) arb, may be determined by arbitrating between a driver axle torque requested \( T_{dr} \) and a plurality of axle intervention requested values \( A_i \), such as \( A_{i-1}, A_{i-2}, A_{i-3} \).

The method 300 may further include a step 304 of generating a plurality of sets of possible command values. The possible command values may be generated based in
part on the requested value or values, such as $T_a_{dr}$ and $T_a_{arb}$, by an MPC module 202 or prediction module 204 of a multivariable controller 102, as explained above.

The method 300 may also include a step 306 of determining a cost for each set of possible command values of the plurality of sets of possible command values, based on a first predetermined weighting value, a second predetermined weighting value, a plurality of predicted values, and a plurality of requested values (such as $T_a_{dr}$, $T_a_{arb}$, etc.), such as by a cost module 208.

The method 300 may further include a step 308 of determining which set of possible command values of the plurality of sets of possible command values has the lowest cost, and a step 310 of selecting the set of possible command values that has the lowest cost to define a set of selected command values. The set of selected command values may include, for example, a selected transmission ratio command $R_{at_c}$ and a selected engine output torque command $T_e_{c,arb}$, as explained above.

The method 300 may then proceed to an optional step 312 of performing an arbitration step comprising determining a desired command value by arbitrating between a selected command value of the set of selected command values and a command intervention value. For example, the step 312 may include arbitrating between the selected command value $T_e_{c,arb}$ (selected engine output torque value) and a plurality of engine torque intervention values $E_i$, such as $E_{i,1}$, $E_{i,2}$, $E_{i,3}$, to arrive at the desired command value $T_e_{c,arb}$. In some outcomes, the method 300 will include determining that the desired command value $T_e_{c,arb}$ is equal to one of the engine torque intervention values $E_i$ after arbitrating between the selected command value $T_e_{c}$ of the plurality of selected command values and the engine torque intervention value $E_i$. Thus, the method 300 will include, in those circumstances, determining that $T_e_{c,arb}$ is equal to the engine torque intervention value $E_i$.

In some forms, the method 300 may also include generating a plurality of predicted actual axle torque values and a plurality of predicted actual fuel consumption rate values based on the plurality of sets of possible command values, such as $T_e_{c}$, $R_{at,c}$, and determining the cost for each set of possible command values further based on a predicted actual axle torque value of the plurality of predicted axle torque values and a predicted actual fuel consumption rate value of the plurality of predicted actual fuel consumption rate values. The plurality of requested values used to generate the predicted values may include the driver axle torque requested $T_a_{dr}$, the engine output torque requested $T_e_{r}$, the $R_{at,r}$ transmission ratio requested, and the fuel consumption rate requested $FR_{r}$, which may be computed using the equations (1)-(4), as explained above. Furthermore, the method 300 may include determining the predicted values using the equations (7)-(9) above.

The method 300 may determine the engine torque intervention values $E_{i}$ and the axle intervention requested values $A_{i}$ similarly as explained above with respect to the control system 100. For example, the method 300 may include determining the engine torque intervention value $E_{i}$ from among the following: a transmission torque reduction request, an engine overspeed request, a boost request, a speed control request, an engine crank shutdown ring request, a power take off ring request, an exhaust O2 sensor ring request, a torque cut off ring request, a hybrid torque request, and a power take off control request; and the method 300 may include determining the axle intervention requested value $A_{i}$ from among the following: a brake torque management request, a vehicle overspeed condition request, a traction control request, a deceleration fuel cut off request, a shaping request, a chassis system request, a performance launch request, a four wheel drive request, and an emergency autonomous braking request.

The method 300 may include a step 314 of controlling a vehicle parameter based on at least one of the desired command values $T_e_{c,arb}$, $R_{at,c}$. In some forms, the method 300 may also include using the arbitrated engine torque command value $T_e_{c,arb}$ to determine the predicted values.

The method 300 may include additional steps to determine the selected engine output torque value $T_e_{c}$, which are explained in more detail above, such as generating a number of predicted actual axle torques (at least first and second predicted actual axle torques) and a number of predicted actual fuel consumption rates (at least first and second predicted actual fuel consumption rates) based on a number of sets (at least two) of possible command values. For example, the first set of possible command values includes a first commanded engine output torque and a first commanded transmission ratio, the second set of possible command values includes a second commanded engine output torque and a second commanded transmission ratio, and so on as desired. These initial steps may be performed, for example, by the prediction module 204 shown in FIG. 5.

The method 300 may accomplish the steps 302, 304, 306, 308, 310, 314 in any of the ways described above, such as by applying any of the equations (1)-(9).

The terms controller, control module, module, control, control unit, processor and similar terms refer to any one or various combinations of Application Specific Integrated Circuit(s) (ASIC), electronic circuit(s), central processing unit(s), e.g., microprocessor(s) and associated non-transitory memory component in the form of memory and storage devices (read only, programmable read only, random access, hard drive, etc.). The non-transitory memory component may be capable of storing machine readable instructions in the form of one or more software or firmware programs or routines, combinational logic circuit(s), input/output circuit(s) and devices, signal conditioning and buffer circuitry and other components that can be accessed by one or more processors to provide a desired functionality.

Input/output circuit(s) and devices include analog/digital converters and related devices that monitor inputs from sensors, with such inputs monitored at a preset sampling frequency or in response to a triggering event. Software, firmware, programs, instructions, control routines, code, algorithms and similar terms can include any controller-executable instruction sets including calibrations and look-up tables. Each controller executes control routine(s) to provide desired functions, including monitoring inputs from sensors and other logic devices and executing control and diagnostic instructions to control operation of actuators. Routines may be executed at regular intervals, for example each 100 microseconds during ongoing operation. Alternatively, routines may be executed in response to occurrence of a triggering event.

Communication between controllers, and communication between controllers, actuators and/or sensors may be accomplished using a direct wired link, a networked communication link, a wireless link or any another suitable communication link. Communication includes exchanging data signals in any suitable form, including, for example, electrical signals via a conductive medium, electromagnetic signals via air, optical signals via optical waveguides, and the like.

Data signals may include signals representing inputs from sensors, signals representing actuator commands, and com-
communication signals between controllers. The term ‘model’ refers to a processor-based or processor-executable code and associated calibration that simulates a physical existence of a device or a physical process. As used herein, the terms “dynamic” and “dynamically” describe steps or processes that are executed in real-time and are characterized by monitoring or otherwise determining states of parameters and regularly or periodically updating the states of the parameters during execution of a routine or between iterations of execution of the routine.

The control system 100 may be configured to execute each of the steps of the method 300. Thus, the entire description with respect to FIGS. 1-6 may be applied by the control system 100 to effectuate the method 300 shown in FIG. 6. Furthermore, the control system 100 may be or include a controller that includes a number of control logics that are configured to execute the steps of the method 300.

The controller(s) of the control system 100 may include a computer-readable medium (also referred to as a processor-readable medium), including any non-transitory (e.g., tangible) medium that participates in providing data (e.g., instructions) that may be read by a computer (e.g., by a processor of a computer). Such a medium may take many forms, including, but not limited to, non-volatile media and volatile media. Non-volatile media may include, for example, optical or magnetic disks and other persistent memory. Volatile media may include, for example, dynamic random access memory (DRAM), which may constitute a main memory. Such instructions may be transmitted by one or more transmission media, including coaxial cables, copper wire and fiber optics, including the wires that comprise a system bus coupled to a processor of a computer. Some forms of computer-readable medium include, for example, a floppy disk, a flexible disk, hard disk, magnetic tape, any other magnetic medium, a CD-ROM, DVD, any other optical medium, punch cards, paper tape, any other physical medium with patterns of holes, a RAM, a PROM, an EPROM, a FLASH-EEPROM, any other memory chip or cartridge, or any other medium from which a computer can read.

Look-up tables, databases, data repositories or other data stores described herein may include various kinds of mechanisms for storing, accessing, and retrieving various kinds of data, including a hierarchical database, a set of files in a file system, an application database in a proprietary format, a relational database management system (RDBMS), etc. Each such data store may be included within a computer device employing a computer operating system such as one of those mentioned above, and may be accessed via a network in any one or more of a variety of manners. A file system may be accessible from a computer operating system, and may include files stored in various formats. An RDBMS may employ the Structured Query Language (SQL) in addition to a language for creating, storing, editing, and executing stored procedures, such as the PL/SQL language mentioned above.

The detailed description and the drawings or figures are supportive and descriptive of the many aspects of the present disclosure. While certain aspects have been described in detail, various alternative aspects exist for practicing the disclosure as defined in the appended claims.

What is claimed is:
1. A method for controlling a propulsion system of a motor vehicle, the method comprising:
   generating a plurality of sets of possible command values;
   determining a cost for each set of possible command values of the plurality of sets of possible command values based on a first predetermined weighting value, a second predetermined weighting value, a plurality of predicted values, and a plurality of requested values; determining which set of possible command values of the plurality of sets of possible command values has a lowest cost; and selecting the set of possible command values that has the lowest cost to define a set of selected command values; performing an arbitration step comprising at least one of the following arbitration procedures A and B:
   A) determining at least one requested value of the plurality of requested values based on arbitrating between a driver requested value and an intervention requested value; and
   B) determining a desired command value by arbitrating between a selected command value of the set of selected command values and a command intervention value; and
   controlling a vehicle parameter based on the desired command value;
2. The method of claim 1, wherein performing the arbitration step comprises performing both of the arbitration procedures A and B.
3. The method of claim 2, wherein:
   the plurality of sets of possible command values includes a plurality of commanded engine output torque values; the set of selected command values includes a selected engine output torque value; and the command intervention value includes an engine torque intervention value.
4. The method of claim 3, further comprising:
   generating a plurality of predicted actual axle torque values and a plurality of predicted actual fuel consumption rate values based on the plurality of sets of possible command values, the plurality of sets of possible command values including a plurality of possible commanded transmission ratio values; and
determining the cost for each set of possible command values further based on a predicted actual axle torque value of the plurality of predicted axle torque values and a predicted actual fuel consumption rate value of the plurality of predicted actual fuel consumption rate values, the plurality of requested values including a driver axle torque requested, an engine output torque requested, a transmission ratio requested, and a fuel consumption rate requested.
5. The method of claim 4, further comprising determining the plurality of predicted actual axle torque values and the plurality of predicted actual fuel consumption rate values with the following set of equations:

\[
x_{k+1} = A x_k + B \left[ \frac{Te_{c,m,b}}{Ra_{c,v}} \right] + v + K_F \left[ \frac{Te_{c,n_b}}{Fr_{c,n_b}} - \frac{Te_{c,n_a}}{Fr_{c,n_a}} \right] + \frac{Fr_{a,n_a}}{Ra_{a,n_a}}
\]

\[
T_{a,x_{k+1}} = C x_{k+1} + w
\]

where

- \(x_{k+1}\) = state variable at a prediction step \(k+1\);
- \(x_k\) = state variable at a prediction step \(k\);
- \(A\) = a state matrix;
- \(B\) = an input matrix;
The method of claim 8, further comprising determining the intervention requested value from among the following: a brake torque management request, a vehicle overspeed condition request, a traction control request, a deceleration fuel cut off request, a shaping request, a chassis system request, a performance launch request, a four wheel drive request, and an emergency autonomous braking request.

10. The method of claim 9, the plurality of selected command values including a selected transmission ratio command value, the method further comprising controlling a vehicle parameter based on at least one of the desired command values.

11. A control system for a propulsion system of a motor vehicle having a transmission and an engine, the control system comprising:

A command generator module configured to generate a plurality of sets of possible command values; a cost module configured to:

determine a cost for each set of possible command values of the plurality of sets of possible command values based on a first predetermined weighting value, a second predetermined weighting value, a plurality of predicted values, and a plurality of requested values; and
determining which set of possible command values of the plurality of sets of possible command values has a lowest cost; and

a selection module configured to select the set of possible command values that has the lowest cost to define a set of selected command values;
an arbitration module configured to perform at least one of the following arbitration procedures A and B:

A) determine at least one requested value of the plurality of requested values based on arbitrating between a driver requested value and an intervention requested value; and

B) determine a desired command value by arbitrating between a selected command value of the set of selected command values and a command intervention value; and

an actuation module configured to control a vehicle parameter based on the desired command value.

12. The control system of claim 11, at least one selected command value of the set of selected command values being a selected engine output torque value, the command intervention value being an engine torque intervention value, the driver requested value being an driver axle torque requested, the arbitration module being an engine torque arbitration module configured to determine the desired command value by selecting a winner between the selected engine output torque value and the engine torque intervention value, the control system further comprising an axle torque arbitration module configured to determine an arbitrated axle torque requested by selecting a winner between the driver axle torque requested and the intervention requested value.

13. The control system of claim 12, further comprising a prediction module configured to generate a plurality of predicted actual axle torque values and a plurality of predicted actual fuel consumption rate values based on the plurality of sets of possible command values, the plurality of sets of possible command values including a plurality of possible commanded transmission ratio values and a plural-
ity of possible commanded engine torque values, wherein the cost module is configured to determine the cost for each set of possible command values further based on a predicted actual axle torque value of the plurality of predicted axle torque values and a predicted actual fuel consumption rate value of the plurality of predicted actual fuel consumption rate values, the plurality of requested values including the driver axle torque requested, an engine output torque requested, a transmission ratio requested, and a fuel consumption rate requested.

14. The control system of claim 13, wherein the prediction module is configured to determine the plurality of predicted actual axle torque values and the plurality of predicted actual fuel consumption rate values with the following set of equations:

\[
\begin{align*}
    x_{k+1} &= \begin{bmatrix} A \times x_k + B \times \begin{bmatrix} Te_{c-arb} \\ Rat_c \end{bmatrix} + w + K_{KF} \\ x_k 
    \end{bmatrix}
    \\
    Te_{a, k+1} &= C \times x_{k+1} + w
\end{align*}
\]

where

- \( x_{k+1} \): state variable at a prediction step \( k+1 \);
- \( x_k \): state variable at a prediction step \( k \);
- \( A \): state matrix;
- \( B \): an input matrix;
- \( Te_{c-arb} \): one of: engine output torque commanded at the prediction step \( k \) and the engine torque intervention value;
- \( Rat_c \): transmission ratio commanded at the prediction step \( k \);
- \( K_{KF} \): Kalman filter gain;
- \( Te_{a, k} \): predicted actual engine output torque at the prediction step \( k \);
- \( FR_{a, k} \): predicted actual fuel consumption rate at the prediction step \( k \);
- \( Rat_{a, k} \): predicted actual transmission ratio at the prediction step \( k \);
- \( Te_{m, k} \): measured engine output torque at the prediction step \( k \);
- \( FR_{m, k} \): measured fuel consumption rate at the prediction step \( k \);
- \( Rat_{m, k} \): measured transmission ratio at the prediction step \( k \);
- \( Ta_{m, k} \): measured axle torque at the prediction step \( k \);
- \( FR_{a,k+1} \): predicted actual fuel consumption rate at the prediction step \( k+1 \);
- \( C \): an output matrix;
- \( w \): process noise; and
- \( w \): measurement noise.

15. The control system of claim 14, wherein the engine torque arbitration module is configured to feed \( Te_{c-arb} \) back to the prediction module, the engine torque arbitration module being configured to determine the engine torque intervention value from among the following: a transmission torque reduction request, an engine overspeed request, a boost request, a speed control request, an engine crank shutdown ring request, a power take off ring request, an exhaust \( \text{O}_2 \) sensor ring request, a torque cut off ring request, a hybrid torque request, and a power take off control request.

16. The control system of claim 15, further comprising a steady state optimizer module configured to:
    - determine an accelerator pedal position (PP);
    - determine an engine speed (RPM);
    - determine a vehicle speed (V);
    - determine an air-fuel ratio (AF);
    - determine the driver axle torque requested (\( Ta_{dr} \)) based on the accelerator pedal position (PP) and the vehicle speed (V);
    - determine a transmission ratio requested (\( Rat_r \)) based on the arbitrated axle torque requested (\( Ta_{arb} \)) and the vehicle speed (V);
    - determine the engine output torque requested (\( Te_{r} \)) based on the arbitrated axle torque requested (\( Ta_{arb} \)), the transmission ratio requested (\( Rat_{r} \)), and a final drive ratio (FD); and
    - determine the fuel consumption rate requested (\( FR_{r} \)) based on the driver axle torque requested (\( Ta_{dr} \)), the vehicle speed (V), the engine speed (RPM), and the air-fuel ratio (AF).

17. The control system of claim 16, wherein the axle torque arbitration module is configured to determine the intervention requested value from among the following: a brake torque management request, a vehicle overspeed condition request, a traction control request, a deceleration fuel cut off request, a shifting request, a chassis system request, a performance launch request, a four wheel drive request, and an emergency autonomous braking request.

18. The control system of claim 17, the plurality of selected command values including a selected transmission ratio command value, the control system further comprising an actuation module configured to control a vehicle parameter based on at least one of the desired command values.

19. A propulsion system for a motor vehicle, comprising:
    - an engine operable to power the motor vehicle, the engine having an engine output shaft configured to transfer engine output torque;
    - a continuously variable transmission having a variator assembly including a first pulley and a second pulley, the first and second pulleys being rotatably coupled by a rotatable member, at least one of the first and second pulleys including a movable sheave translatable along an axis to selectively change a transmission ratio between the engine output shaft and a transmission output shaft;
    - a drive axle configured to be driven via the transmission output shaft, the drive axle being configured to output axle torque to a set of wheels; and
    - a control system comprising:
        - a prediction module configured to generate a plurality of predicted actual axle torque values and a plurality of predicted actual fuel consumption rate values based on a plurality of sets of possible command values, the plurality of sets of possible command values including a plurality of possible commanded transmission ratio values and a plurality of possible commanded engine torque values;
        - a cost module configured to:
            - determine a cost for each set of possible command values of the plurality of sets of possible command values based on a predicted actual axle torque value of the plurality of predicted axle torque values, a predicted actual fuel consumption rate value of the plurality of predicted actual fuel consumption rate values, a first predetermined...
weighting value, a second predetermined weighting value, and a plurality of requested values, the plurality of requested values including the driver axle torque requested, an engine output torque requested, a transmission ratio requested, and a fuel consumption rate requested; and determine which set of possible command values of the plurality of sets of possible command values has a lowest cost;

a selection module configured to select the set of possible command values that has the lowest cost to define a set of selected command values;
an axle torque arbitration module configured to determine at least one requested value of the plurality of requested values based on arbitrating between a driver axle torque requested and an axle intervention requested value;
an engine torque arbitration module configured to determine a desired command value by arbitrating between a selected engine torque command value of the set of selected command values and an engine torque command intervention value; and

an actuation module configured to control a vehicle parameter based on the desired command value.

The propulsion system of claim 19, wherein the prediction module is configured to determine the plurality of predicted actual axle torque values and the plurality of predicted actual fuel consumption rate values with the following set of equations:

\[
\begin{align*}
x_{k+1} &= \begin{bmatrix} A \times x_k + B \times \begin{bmatrix} T_e,\text{arb} \\ \text{Rat}_e \end{bmatrix} + v \end{bmatrix} + K_{KF} \times \begin{bmatrix} T_e, m_k \\ \text{FR}_m \end{bmatrix} + K_{KF} \times \begin{bmatrix} T_e, a_k \\ \text{FR}_a \end{bmatrix} \\
&= C \times x_k + v
\end{align*}
\]

where

- \( x_{k+1} \) = state variable at a prediction step \( k+1 \);
- \( x_k \) = state variable at a prediction step \( k \);
- \( A \) = a state matrix;
- \( B \) = an input matrix;
- \( T_e,\text{arb} \) = one of: engine output torque commanded at the prediction step \( k \) and the engine torque intervention value;
- \( \text{Rat}_e \) = transmission ratio commanded at the prediction step \( k \);
- \( K_{KF} \) = a Kalman filter gain;
- \( T_e, a_k \) = predicted actual engine output torque at the prediction step \( k \);
- \( \text{FR}_m \) = predicted actual fuel consumption rate at the prediction step \( k \);
- \( \text{Rat}_a \) = predicted actual transmission ratio at the prediction step \( k \);
- \( T_a, m_k \) = measured engine output torque at the prediction step \( k \);
- \( \text{FR}_a \) = measured fuel consumption rate at the prediction step \( k \);
- \( \text{Rat}_a \) = measured transmission ratio at the prediction step \( k \);
- \( T_a, m_0 \) = measured axle torque at the prediction step \( k \);
- \( T_a, a_{k+1} \) = predicted actual axle torque at the prediction step \( k+1 \);
- \( \text{FR}_{a_{k+1}} \) = predicted actual fuel consumption rate at the prediction step \( k+1 \);
- \( C \) = an output matrix;
- \( v \) = process noise; and
- \( w \) = measurement noise,

wherein the engine torque arbitration module is configured to feed \( T_e,\text{arb} \) back to the prediction module, request a hybrid torque request, and a power take off control request, the control system further comprising a steady state optimizer module configured to:

- determine an accelerator pedal position (PP); determine an engine speed (RPM);
determine a vehicle speed (V); determine an air-fuel ratio (AF);
determine the driver axle torque requested (Ta,dr) based on the accelerator pedal position (PP) and the vehicle speed (V);
determine a transmission ratio requested (Rat) based on the arbitrated axle torque requested (Ta,arb) and the vehicle speed (V);
determine the engine output torque requested (Te,r) based on the arbitrated axle torque requested (Ta,arb), the transmission ratio requested (Rat), and a final drive ratio (FD); and determine the fuel consumption rate requested (FR,r) based on the driver axle torque requested (Ta,dr), the vehicle speed (V), the engine speed (RPM), and the air-fuel ratio (AF).