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Decentralized Model Predictive Control of Dynamically-Coupled Linear Systems: Tracking under Packet Loss

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Abstract: For large-scale processes whose dynamics can be represented as the interaction of several dynamically-coupled linear subsystems, this paper proposes a decentralized model predictive control (MPC) design approach for set-point tracking under input constraints and possible loss of information packets. Following earlier results in (Alessio and Bemporad, 2007, 2008), the global model of the process is approximated as the decomposition of several (possibly overlapping) smaller models used for local predictions. We present sufficient criteria for asymptotic tracking of output set-points and rejection of constant measured disturbances when the overall process is in closed loop with the set of decentralized MPC controllers, under possible intermittent lack of communication of measurement data between controllers. The effectiveness of the approach is shown in a simulation example on distributed temperature control in the passenger area of a railcar.

Keywords: Model predictive control, decentralized control, networked control, packet loss, tracking

1. INTRODUCTION

Ideas for decentralizing and hierarchically organizing the control actions in industrial automation systems date back to the 70's (Sandell et al., 1978), but were mainly limited to the analysis of stability of decentralized linear control of interconnected subsystems, so the interest faded. Decentralized control and estimation schemes based on distributed convex optimization ideas based on Lagrangean relaxations have been proposed recently by various authors, see e.g. (Johansson et al., 2008). Here global solutions can be achieved after iterating a series of local computations and inter-agent communications.

Large-scale multi-variable control problems, such as those arising in the process industries, are often dealt with model predictive control (MPC) techniques. In MPC the control problem is formulated as an optimization one, where many different (and possibly conflicting) goals are easily formalized and state and control constraints can be included. Many results are nowadays available concerning stability and robustness of MPC, see e.g. (Bemporad et al., 2002) and references therein. However, centralized MPC is often unsuitable for control of large-scale networked systems, mainly due to lack of scalability and to maintenance issues of global models. The idea of decentralized MPC (DMPC) is to replace the original large size optimization problem by a number of smaller and easily tractable ones that work iteratively and cooperatively towards achieving a common, system-wide control objective. The goal of the decomposition is twofold: first, each subproblem is much smaller than the overall problem, and second, each subproblem is coupled to only a few other subproblems. Along with the benefits of a decentralized design, inherent issues in ensuring stability and feasibility of the system must be faced due to the mismatch of predictions that neighboring subsystems make about each other.

A few contributions have appeared in recent years in the context of DMPC. In (Camponogara et al., 2002) the system under control is composed by a number of unconstrained linear discrete-time subsystems with decoupled input signals and closed-loop stability is enforced through contractive constraints. In (Venkat et al., 2005) the authors propose a cooperation-based distributed MPC algorithm based on a process of negotiations among DMPC agents, where the model considered for the subsystems only admits coupling through the control inputs. In (Dunbar and Murray, 2006) the authors consider the control of dynamically decoupled subsystems, whose state vectors are only coupled by a global performance objective. Closed-loop stability is ensured by constraining the state trajectory predicted by each agent close enough to the trajectory predicted at the previous time step that has been broadcasted. Dynamically decoupled submodels are also considered in (Keviczky et al., 2006). Closed-loop stability is achieved by including sufficient stability conditions based on prediction errors in each DMPC subproblem.

A DMPC design approach was also proposed in (Alessio and Bemporad, 2007) for *process* models that are not necessarily dynamically decoupled. The decoupling assumption only appears in the *prediction* models used by

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different MPC controllers. The degree of chosen decoupling represents a tuning knob of the approach. Sufficient criteria for the asymptotic stability of the process model in closed loop with the set of decentralized MPC controllers were given. A comparison of the above strategy with other decentralized strategy is reported in (Damoiseaux et al., 2008) on a problem of distributed power network control. Moreover, to cope with the case of a communication channel among neighboring MPC controllers which is faulty, a sufficient condition for ensuring closed-loop stability of the overall closed-loop system when a certain fixed number of packets containing state measurements may be lost was suggested in (Alessio and Bemporad, 2008).

This paper extends and generalizes the latter results and handles the case of tracking of constant output references, proposing a strategy to achieve offset-free tracking despite the decentralization. More general stability conditions are given for decentralized MPC in the presence of packet drops. The above are tested on a realistic simulation example of decentralized temperature control in a railcar.

2. PROBLEM SETUP

We recall the MPC problem setup of (Alessio and Bemporad, 2007).

2.1 Centralized model predictive control

Consider the problem of regulating the discrete-time linear time-invariant system

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

to the origin while fulfilling the constraints

$$u_{\min} \leq u(t) \leq u_{\max} \tag{2}$$

at all time instants $t \in \mathbb{Z}_{0+}$, where $\mathbb{Z}_{0+}$ is the set of nonnegative integers, $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are the state, input, and output vectors, and the pair (A, B) is stabilizable.

Assumption 1. Matrix A in (1) has all its eigenvalues strictly inside the unit disc.

Assumption 1 restricts the strategy and stability results to processes that are open-loop asymptotically stable, leaving to the controller the mere role of optimizing the performance of the closed-loop system.

Consider the following finite-horizon optimal control problem

$$V(x(t)) = \min_U \; x'_{t+N} P x_{t+N} + \sum_{k=0}^{N-1} x'_k Q x_k + u'_k R u_k \tag{3a}$$

$$\text{s.t. } x_{k+1} = Ax_k + Bu_k,\; k = 0, \ldots, N-1 \tag{3b}$$

$$y_k = Cx_k,\; k = 0, \ldots, N \tag{3c}$$

$$x_0 = x(t) \tag{3d}$$

$$u_{\min} \leq u_k \leq u_{\max},\; k = 0, \ldots, N_u - 1 \tag{3e}$$

$$u_k = 0,\; k = N_u, \ldots, N-1 \tag{3f}$$

where, at each time t, $U \triangleq \{u_0, \ldots, u_{N_u-1}\}$ is the sequence of future input moves, x_k denotes the predicted state vector at time $t+k$, obtained by applying the input sequence $u_0, \ldots, u_{k-1}$ to model (1), starting from $x(t)$. In (3) $N > 0$ is the prediction horizon, $N_u \leq N$ is the input horizon, and "$\leq$" denotes component-wise inequalities. In (3) we assume that $Q = Q' \geq 0$, $R = R' > 0$, $P = P' \geq 0$ are square weight matrices defining the performance index, and P solves the Lyapunov equation $P = A'PA + Q$.

Problem (3) can be recast as a quadratic programming (QP) problem (see e.g. (Mayne et al., 2000; Bemporad et al., 2002)), whose solution $U^*(x(t)) \triangleq \{u_0^* \ldots u_{N_u-1}^*\} \in \mathbb{R}^{m \times N_u}$ is a sequence of optimal control inputs. Only the first input $u(t) = u_0^*$ is actually applied to system (1), as the optimization problem (3) is repeated at time $t+1$, based on the new state $x(t+1)$.

2.2 Decentralized prediction models

In general the matrices A, B in (1) have a certain number of zero or negligible components corresponding to a partial dynamical decoupling of the process, or are even block diagonal in case of total dynamical decoupling.

Let M be the number of decentralized control actions that we want to design, for example $M = m$ in case each individual actuator is governed by its own controller. For all $i = 1, \ldots, M$, we define $x^i \in \mathbb{R}^{n_i}$ as the vector collecting a subset $\mathcal{I}_{xi} \subseteq \{1, \ldots, n\}$ of the state components,

$$x^i = W_i' x = [\, x_1^i \cdots x_{n_i}^i \,]' \in \mathbb{R}^{n_i}$$

where $W_i \in \mathbb{R}^{n \times n_i}$ collects the n_i columns of the identity matrix of order n corresponding to the indices in $\mathcal{I}_{xi}$, and, similarly, $u^i = Z_i' u = [\, u_1^i \cdots u_{m_i}^i \,]' \in \mathbb{R}^{m_i}$ as the vector of input signals tackled by the i-th controller, where $Z_i \in \mathbb{R}^{m \times m_i}$ collects m_i columns of the identity matrix of order m corresponding to the set of indices $\mathcal{I}_{ui} \subseteq \{1, \ldots, m\}$. Note that

$$W_i' W_i = I_{n_i},\; Z_i' Z_i = I_{m_i},\; \forall i = 1, \ldots, M \tag{4}$$

where $I_{(\cdot)}$ denotes the identity matrix of order $(\cdot)$. By definition of x^i we obtain

$$x^i(t+1) = W_i' x(t+1) = W_i' Ax(t) + W_i' Bu(t) \tag{5}$$

An *approximation* of (1) is obtained by changing $W_i'A$ in (5) into $W_i'AW_iW_i'$ and $W_i'B$ into $W_i'BZ_iZ_i'$, therefore getting the new prediction model of reduced order

$$x^i(t+1) = A_i x^i(t) + B_i u^i(t) \tag{6}$$

where matrices $A_i = W_i' A W_i \in \mathbb{R}^{n_i \times n_i}$ and $B_i = W_i' B Z_i \in \mathbb{R}^{m_i \times m_i}$ are submatrices of the original A and B matrices, respectively, describing in a possibly approximate way the evolution of the states of subsystem $\#i$.

The choice of the pair (W_i, Z_i) of *decoupling matrices* (and, consequently, of the dimensions n_i, m_i of each submodel) is a tuning knob of the DMPC procedure.

We want to design a controller for each set of moves u^i according to the prediction model (6) and based on feedback from x^i, for all $i = 1, \ldots, M$. Note that in general different states x^i, x^j and different u^i, u^j may share common components. To avoid ambiguities on the control action provided to the process, we impose that only a subset $\mathcal{I}_{ui}^{\#} \subseteq \mathcal{I}_{ui}$ of input signals computed by controller $\#i$ is actually applied to the process without ambiguity, and for the sake of simplicity of notation since now on we assume that $M = m$, i.e., that controller $\#i$ only controls the ith input signal.

2.3 Decentralized optimal control problems

Let the following assumption be satisfied:

Assumption 2. Matrix A_i has all its eigenvalues strictly inside the unit disc, $\forall i = 1, \ldots, M$.

Assumption 2 restricts the degrees of freedom in choosing the decentralized models (if $A_i = A$ for all $i = 1, \ldots, M$ is the only choice satisfying Assumption 2, then no decentralized MPC can be formulated within this framework). For all $i = 1, \ldots, M$ consider the following infinite-time constrained optimal control problems

$$V_i(x(t)) = \min_{\{u_k^i\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} (x_k^i)' W_i' Q W_i x_k^i + (u_k^i)' Z_i' R Z_i u_k^i =$$

$$\min_{u_0^i} (x_1^i)' P_i x_1^i + x^i(t)' W_i' Q W_i x^i(t) + (u_0^i)' Z_i' R Z_i u_0^i \quad (7a)$$

$$\text{s.t. } x_1^i = A_i x^i(t) + B_i u_0^i \qquad x_0^i = W_i' x(t) = x^i(t) \quad (7b)$$

$$u_{\min}^i \leq u_0^i \leq u_{\max}^i \qquad u_k^i = 0,\ \forall k \geq 1 \quad (7c)$$

where $P_i = P_i' \geq 0$ is the solution of the Lyapunov equation

$$A_i' P_i A_i - P_i = -W_i' Q W_i \quad (8)$$

that exists by virtue of Assumption 2. Problem (7) corresponds to a finite-horizon constrained problem with control horizon $N_u = 1$ and linear stable prediction models. Note that only the local state vector $x^i(t)$ is needed to solve Problem (7).

At time t, each controller MPC #i measures (or estimates) the state $x^i(t)$ (usually corresponding to local and neighboring states), solves problem (7) and obtains the optimizer $u_0^{*i} = [u_0^{*i1}, \ldots, u_0^{*ii}, \ldots, u_0^{*im_i}]' \in \mathbb{R}^{m_i}$. In the simplified case $M = m$ and $I_{ui}^{\#} = i$, only the i-th sample of u_0^{*i}, $u_i(t) = u_0^{*ii}$ will determine the i-th component $u_i(t)$ of the input vector actually implemented to the process at time t. The inputs u_0^{*ij}, $j \neq i$, $j \in \mathcal{I}_{ui}$ to the neighbors are discarded, their only role is to provide a better prediction of the state trajectories x_k^i, and therefore a possibly better performance of the overall closed-loop system.

The collection of the optimal inputs of all the M MPC controllers,

$$u(t) = [u_0^{*11} \ \ldots \ u_0^{*ii} \ \ldots \ u_0^{*mm}]' \quad (9)$$

is the actual input commanded to process (1). The optimizations (7) are repeated at time $t+1$, based on the new states $x^i(t+1) = W_i' x(t+1)$, according to the usual receding horizon control paradigm.

The degree of coupling between the DMPC controllers is dictated by the choice of the decoupling matrices (W_i, Z_i). Clearly, the larger the number of interactions captured by the submodels, the more complex the formulation (and, in general, the solution) of the optimization problems (7) and hence the computations performed by each control agent. Note also that a higher level of information exchange between control agents requires a higher communication overhead.

2.4 Convergence properties of decentralized MPC

The stability theorem proved in (Alessio and Bemporad, 2007) provides closed-loop convergence results of the proposed DMPC scheme using the cost function $V(x(t)) \triangleq \sum_{i=1}^{M} V_i(W_i' x(t))$ as a Lyapunov function for the overall system. It is useful to recall here some quantities introduced in (Alessio and Bemporad, 2007)

$$\begin{aligned} \Delta u^i(t) &\triangleq u(t) - Z_i u_0^{*i}(t), & \Delta x^i(t) &\triangleq (I - W_i W_i')x(t) \\ \Delta A^i &\triangleq (I - W_i W_i')A, & \Delta B^i &\triangleq B - W_i W_i' B Z_i Z_i' \end{aligned} \quad (10)$$

and

$$\begin{aligned} \Delta Y^i(x(t)) \triangleq\ & W_i W_i' (A \Delta x^i(t) + B Z_i Z_i' \Delta u^i(t)) \\ & + \Delta A^i x(t) + \Delta B^i u(t) \end{aligned} \quad (11)$$

$$\begin{aligned} \Delta S^i(x(t)) \triangleq\ & \big(2(A_i W_i' x(t) + B_i u_0^{*i}(t))' + \\ & \Delta Y^i(x(t))' W_i\big) P_i W_i' \Delta Y^i(x(t)) \end{aligned} \quad (12)$$

3. DECENTRALIZED MPC UNDER ARBITRARY PACKET LOSS

In the previous section we assumed that the communication model among neighboring MPC controllers was faultless, so that each MPC agent could successfully receive the information about the states of its corresponding submodel. However, one of the main issues in networked control systems is the unreliability of communication channels, which may result in data packet dropout. In this section we derive a sufficient condition for ensuring convergence of the DMPC closed-loop in the case packets containing measurements are lost for an arbitrary but upper-bounded number N of consecutive time steps. The results shown here are based on formulation (7) and rely on the open-loop asymptotic stability Assumptions 1 and 2. The issue is still non-trivial, as if a set of measures for subsystem i is lost, this would affect not only the trajectory of subsystem i because of the improper control action u^i, but, due to the dynamical coupling, also the trajectories of subsystems $j \in J$, where $J = \{j \mid i \in \mathcal{I}_{xj} \cup \mathcal{I}_{uj}\}$, and thus the closed-loop stability of the overall system may be endangered.

By relying on open-loop stability, setting $u(t) = 0$ is a natural choice for backup input moves when no state measurements are available because of a communication blackout. The next theorem proves asymptotic closed-loop stability of decentralized MPC under packet loss. The proof of the theorem generalizes and unifies the results of (Alessio and Bemporad, 2007, 2008).

Theorem 1. Let N be a positive integer such that no more than N consecutive steps of channel transmission blackout can occur. Assume $u(t) = 0$ is applied when no packet is received. Let Assumptions 1, 2 hold and $\forall i = 1, \ldots, M$ define P_i as in (8), $\Delta u^i(t)$, $\Delta x^i(t)$, ΔA^i, ΔB^i as in (10), $\Delta Y^i(x(t))$ as in (11),

$$\begin{aligned} \Delta S_j^i(x) \triangleq\ & [2(A_i W_i' x + B_i u_0^{*i}(x))' W_i' + \Delta Y^i(x)'] \cdot \\ & (A^{j-1})' W_i P_i W_i' A^{j-1} \Delta Y^i(x) \end{aligned} \quad (13)$$

and let $\xi_i(x) \triangleq A_i W_i' x + B_i u_0^{*i}(x)$. If the condition

$$(i)\ \sum_{i=1}^{M} \big(x' W_i W_i' Q W_i W_i' x + \xi_i(x)'(P_i - W_i'(A^{j-1})' W_i P_i W_i' A^{j-1} W_i)\xi_i(x) - \Delta S_j^i(x)\big) \geq 0, \qquad \forall x \in \mathbb{R}^n,\ \forall j = 1, \ldots, N \quad (14a)$$

is satisfied, or the condition

$$(ii)\ \sum_{i=1}^{M}\Big(x'W_iW_i'QW_iW_i'x+\xi_i(x)'(P_i-W_i'(A^{j-1})'\cdot$$
$$W_iP_iW_i'A^{j-1}W_i)\xi_i(x)-\Delta S_j^i(x)+u_0^{*i}(x)'Z_i'RZ_iu_0^{*i}(x)\Big)$$
$$-\alpha x'x\geq 0,\qquad \forall x\in\mathbb{R}^n,\ \forall j=1,...,N \tag{14b}$$

is satisfied for some scalar $\alpha > 0$, then the decentralized MPC scheme defined in (7)–(9) in closed loop with (1) is globally asymptotically stable under packet loss.

Proof Let $\{t_k\}_{k=0}^{\infty}$ be the sequence of sampling steps at which packet information is received, and let $j_k = t_{k+1}-t_k$ the corresponding number of consecutive packet drops, $1 \leq j_k \leq N$. We want to examine the difference $V_i(x(t_{k+1})) - V_i(x(t_k))$, where $V_i(x(t))$ is the optimal cost of subproblem (7) at time t. As the backup input $u(t_k + h) = 0$ is applied from time t_k to $t_{k+1}-1$ ($h = 0, \ldots, j_k-1$), we have $x(t_{k+1}) = A^{j_k-1}(Ax(t_k)+Bu(t_k)\ ) = A^{j_k-1}(\Delta Y^i(x(t_k))+W_i\xi(x(t_k)))$. Since $x^i(t_{k+1}) = W_i'x(t_{k+1})$, at time t_{k+1} the optimal cost $V_i(x(t_{k+1}))$ of subproblem (7) can be rewritten as $V_i(x(t_{k+1})) = (W_i'x(t_{k+1}))'W_i'QW_iW_i'x(t_{k+1}) + (A_iW_i'x(t_{k+1}) + B_iu_0^{*i}(t_{k+1}))'P_i(A_iW_i'x(t_{k+1}) + B_iu_0^{*i}(t_{k+1})) + u_0^{*i}(t_{k+1})'Z_i'RZ_iu_0^{*i}(t_{k+1})$, where P_i is defined as in (8) and is such that $x_0^iP_ix_0^i = \sum_{k=0}^{\infty}(x_k^i)'W_i'QW_ix_k^i$ with $x_{k+1}^i = A_ix_k$. Hence, considering that $u_0^i(t_{k+1}) = 0$ is a feasible suboptimal choice for problem (7), we obtain the following inequality

$$V_i(x(t_{k+1}))\leq x'(t_{k+1})W_iP_iW_i'x(t_{k+1})\leq(A^{j_k-1}(\Delta Y^i(x(t_k))+$$
$$W_i\xi(x(t_k))))'W_iP_iW_i'A^{j_k-1}(\Delta Y^i(x(t_k))+W_i\xi(x(t_k)))=$$
$$\Delta Y(x(t_k))'(A^{j_k-1})'W_iP_iW_i'A^{j_k-1}\Delta Y(x(t_k))+2\xi(x(t_k))'\cdot$$
$$W_i'(A^{j_k-1})'W_iP_iW_i'A^{j_k-1}\Delta Y(x(t_k))+\xi(x(t_k))'W_i'(A^{j_k-1})'$$
$$W_iP_iW_i'A^{j_k-1}W_i\xi(x(t_k))=\Delta S_{j_k}^i(x(t_k))+\xi(x(t_k))'\cdot$$
$$W_i'(A^{j_k-1})'W_iP_iW_i'A^{j_k-1}W_i\xi(x(t_k)) \tag{15}$$

Since $V_i(x_i(t_k)) = (W_i'x(t_k))'(W_i'QW_i)W_i'x(t_k) + \xi(x(t_k))'P_i\cdot \xi(x(t_k)) + u_0^{*i}(t_k)'Z_i'RZ_iu_0^{*i}(t_k)$ we get

$$V_i(x(t_{k+1}))-V_i(x_i(t_k))\leq\Delta S_{j_k}^i(x(t_k))+\xi(x(t_k))'W_i'(A^{j_k-1})'W_i\cdot$$
$$P_iW_i'A^{j_k-1}W_i\xi(x(t_k))-((W_i'x(t_k))'(W_i'QW_i)W_i'x(t_k)+\xi(x(t_k))'\cdot$$
$$P_i\xi(x(t_k))u_0^{*i}(t_k)'Z_i'RZ_iu_0^{*i}(t_k))\leq\Delta S_{j_k}^i(x(t_k))-u_0^{*i}(t_k)'Z_i'RZ_i\cdot$$
$$u_0^{*i}(t_k)-x(t_k)'W_i(W_i'QW_i)W_i'x(t_k)-\xi(x(t_k))'(P_i-W_i'(A^{j_k-1})'\cdot$$
$$W_iP_iW_i'A^{j_k-1}W_i)\xi(x(t_k)) \tag{16}$$

Let $V(x(t)) \triangleq \sum_{i=1}^{M} V_i(W_i'x(t))$. If (14a) holds, then it follows that $V(x(t_k))$ is a decreasing function of k lower-bounded by zero, and therefore converges as $k \to \infty$, which proves $\lim_{k\to\infty} V(x(t_{k+1}))-V(x(t_k)) = 0$. This in turn implies that $\lim_{k\to\infty} u_0^{i*}(t_k)'Z_i'RZ_iu_0^{i*}(t_k) = 0$. As $R > 0$ and Z_i are full-column-rank matrices, it follows that $Z_i'RZ_i > 0$ and hence that $\lim_{k\to\infty} u(t_k) = 0$. If (14b) holds, then in a similar way it is immediate to see that $\lim_{k\to\infty} x(t_k) = 0$ which again implies $\lim_{k\to\infty} u(t_k) = 0$, as around the origin $u(t_k)$ is a linear function of $x(t_k)$ (corresponding to the unconstrained solution of problem (7)). Since in the presence of packet drop $u(t) = 0$, the input sequence $\ldots, 0, 0, u(t_k), 0, \ldots, 0, u(t_{k+1}), 0, \ldots, 0, u(t_{k+2}), \ldots$ is actually applied to the process, clearly $\lim_{t\to\infty} u(t) = 0$. As asymptotically stable linear systems are also input-to-state stable (Jiang and Wang, 2001), it immediately follows that $\lim_{t\to\infty} x(t) = 0$. □

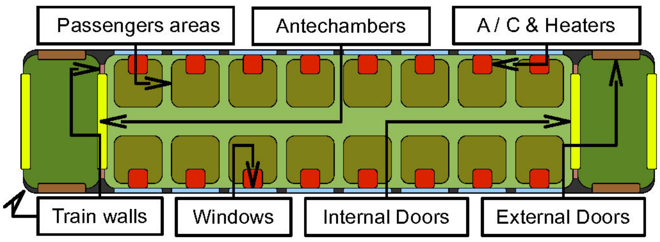

Fig. 1. Physical structure of the railcar

Note again that around the origin the conditions in (14) become a quadratic form to be checked positive semidefinite, so local stability of (7)–(9) in closed loop with (1) under packet loss can be tested for any arbitrary fixed N.

Note also that conditions (14) are a generalization of the result of (Alessio and Bemporad, 2007), as for $j = 1$ (no packet drop) matrix $P_i - W_i'(A^{j-1})'W_iP_iW_i'A^{j-1}W_i = P_i - P_i = 0$.

4. EXTENSION TO SET-POINT TRACKING

Consider the following discrete-time linear global process model

$$\begin{cases} z(t+1) = Az(t) + Bv(t) + F_d(t) \\ h(t) = Cz(t) \end{cases} \tag{17}$$

where $z \in \mathbb{R}^n$ is the state vector, $v \in \mathbb{R}^m$ is the input vector, $y \in \mathbb{R}^p$ is the output vector, $F_d \in \mathbb{R}^d$ is a vector of measured disturbances. Let A satisfy Assumption 1 and assume F_d is constant. We aim at solving a set-point tracking problem so that h tracks a given reference value $r \in \mathbb{R}^p$, despite the presence of F_d. In order to recast the problem as a regulation problem, assume steady-state vectors $z_r \in \mathbb{R}^n$ and $v_r \in \mathbb{R}^m$ exist solving the static problem

$$\begin{cases} (I-A)z_r = Bv_r + F_d \\ r = Cz_r \end{cases} \tag{18}$$

and let $x = z - z_r$ and $u = v - v_r$. Input constraints $v_{\min} \leq v \leq v_{\max}$ are mapped into constraints $v_{\min} - v_r \leq u \leq v_{\max} - v_r$ [1].

Proposition 1. Under the global coordinate transformation (18), the process (17) under the decentralized MPC law (7)–(9) is such that $h(t)$ converges asymptotically to the set-point r, under the assumption of Theorem in (Alessio and Bemporad, 2007) or, in the presence of packet drops, of Theorem 1.

Note that problem (18) is solved in a *centralized* way. Defining local coordinate transformations v_{ir}, z_{ir} based on submodels (6) would not lead, in general, to offset-free tracking, due to the mismatch between global and local models. This is a general observation one needs to take into account in decentralized tracking. Note also that both v_r and z_r depend on F_d as well as r, so problem (18) should be solved each time F_d or r change.

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[1] In case $v_r \notin [v_{\min}, v_{\max}]$, perfect tracking under constraints is not possible, and an alternative is to set

$$\begin{bmatrix} z_r \\ v_r \end{bmatrix} = \begin{array}{ll} \arg\min & \left\| \begin{bmatrix} I-A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} z_r \\ v_r \end{bmatrix} - \begin{bmatrix} F_d \\ r \end{bmatrix} \right\| \\ \text{s.t.} & v_{\min} \leq v_r \leq v_{\max} \end{array}$$

5. A DECENTRALIZED TEMPERATURE CONTROL PROBLEM

In this section we test the proposed DMPC approach for decentralized control of the temperature in different passenger areas in a railcar. The system is schematized in Figure 1. Each passenger area has its own heater and air conditioner but its thermal dynamics interacts with surrounding areas (neighboring passenger areas, external environment, antechambers) directly or through windows/walls/doors. The internal doors can be opened by passengers, external doors automatically open at train stops. Passenger areas are composed by a table and the associated four seats. Temperature sensors are located in each four-seat area, in each antechamber, and along the corridor. The goal of the controller is to adjust each passenger area to its own temperature set-point to maximize passenger comfort.

Let $2N$ be the number of four-seat areas ($N = 8$ in Figure 1), N the number of corridor partitions, and 2 the number of antechambers. Under the assumption of perfectly mixed fluids in each jth volume, $j = 1, \ldots, n$ where $n = 3N+2$, the heat transmission equations by conduction lead to the linear model $\frac{dT_j(\tau)}{d\tau} = \sum_{i=0}^{n} Q_{ij}(\tau) + Q_{uj}$, $Q_{ij}(\tau) = \frac{S_{ij}K_{ij}(T_i(\tau)-T_j(\tau))}{C_j L_{ij}}$, $j = 1, \ldots, n$, where $T_j(\tau)$ is the temperature of volume $\#j$ at time $\tau \in \mathbb{R}$, $T_0(\tau)$ is the ambient temperature outside the railcar, $Q_{ij}(\tau)$ is heat flow due to the temperature difference $T_i(\tau) - T_j(\tau)$ with the neighboring volume $\#i$, S_{ij} is the contact surface area, Q_{uj} is the heat flow of heater $\#j$, K_{ij} is the thermal coefficient that depends on the materials, $C_j = K_c^j V_j$ is the the (material dependent) heat capacity coefficient K_c^j times the fluid volume V_j, and L_{ij} is the thickness of the separator between the two volumes $\#i$ and $\#j$. We assume that $Q_{ij}(\tau) = 0$ for all volumes i, j that are not adjacent, $\forall \tau \in \mathbb{R}$. Hence, the process can be modeled as a linear time-invariant continuous-time system with state vector $z \in \mathbb{R}^{3N+2}$ and input vector $v \in \mathbb{R}^{2N}$

$$\begin{cases} \dot{z}(\tau) = A_c z(\tau) + B_c v(\tau) + F T_0(\tau) \\ h(\tau) = C z(\tau) \end{cases} \tag{19}$$

where $F \in \mathbb{R}^n$ is a constant matrix, $T_0(\tau)$ is treated as a piecewise constant measured disturbance, and $C \in \mathbb{R}^{p \times n}$ is such that $h \in \mathbb{R}^p$ contains the components of z corresponding to the temperatures of the passenger seat areas, $p = 2N$. Since we assume that the thermal dynamics are relatively slow compared to the sampling time T_s of the decentralized controller we are going to synthesize, we use first-order Euler approximation to discretize dynamics (19) without introducing excessive errors:

$$\begin{cases} z(t+1) = Az(t) + Bv(t) + F_d T_0(t) \\ h(t) = Cz(t) \end{cases} \tag{20}$$

where $A = I + A_c T_s$, $B = B_c T_s$, and $F_d = F T_s$. We assume that A is asymptotically stable, as an inheritance of the asymptotic stability of matrix A_c.

In order to track generic temperature references $r(t)$, we adopt the coordinate shift defined by (18). The next step is to decentralize the resulting global model. The particular topology of the railcar suggests a decomposition of model (1) as the cascade of four-seat areas. There are two kinds of four-seat areas, namely (i) the ones next to the antechambers, and (ii) the remaining ones. Besides interacting with the external environment, the areas of type (i) interact with another four-seat-area, an antechamber, and a section of the corridor, while the areas of type (ii) only with the four-seat areas at both sides and the corresponding section of the corridor. Note that the decentralized models overlap, as they share common states and inputs. The decoupling matrices Z_i are chosen so that in each subsystem only the first component of the computed optimal input vector is actually applied to the process.

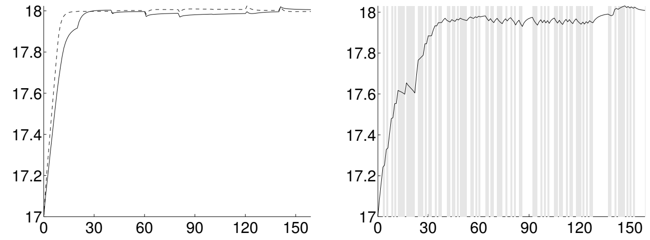


Fig. 2. Comparison between centralized MPC (dashed lines) and decentralized MPC (continuous lines): output h_1 (upper plots) and input v_1 (lower plots). Gray areas denote packet drop intervals

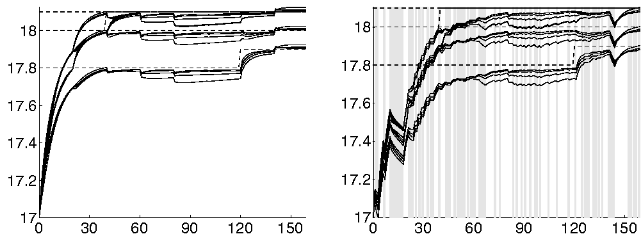
As a result, each submodel has 5 states and 2 or 3 inputs, depending whether it describes a seat area of type (i) or (ii), which is considerably simpler than the centralized model (1) with 26 states and 16 inputs.

We apply the DMPC approach (7) with $Q = \left[\begin{smallmatrix} 200 I_{16} & 0 \\ 0 & 0.2 I_{10} \end{smallmatrix}\right]$, $R = 10^5 I_{16}$, $\left[\begin{smallmatrix} v_{\min} = -0.03 \\ v_{\max} = 0.03 \end{smallmatrix}\right]$ W, $T_s = 9$ min, where $v_{\min}$ is the lower bound on the heat flow subtracted by the air-conditioners, and $v_{\max}$ is the maximum heating power of the heaters (with a slight abuse of notation we denoted by $v_{\min}$, $v_{\max}$ the entries of the corresponding lower and upper bound vectors of $\mathbb{R}^{16}$). Note that the first sixteen diagonal elements of matrix Q correspond to the temperatures of the four-seat areas. It is easy to check that with the above parameters condition in (Alessio and Bemporad, 2007) for local stability is satisfied. For comparison, a centralized MPC approach (3) with the same weights, horizon, and sampling time is also designed. The associated QP problem has 16 optimization variables and 32 constraints, while the complexity of each DMPC controller is either 2 (or 3) variables and 4 (or 6) constraints. The DMPC approach is in fact largely scalable: for longer railcars the complexity of the DMPC controllers remains the same, while the complexity of the centralized MPC problem grows with the increased model size. Note also that, if a multiple cores computation is taken, the DMPC approach can be immediately parallelized.

5.1 Simulation results

We investigate different simulation outcomes depending on four ingredients: *i)* type of controller (centralized/decentralized), *ii)* packet-loss probability, *iii)* change in reference values, *iv)* changes of external temperature (acting as a measured disturbance).

The initial condition is 17°C for all seat-area temperatures, except for the antechamber, which is 15°C. Note that



(a) no packet drop (b) packet drop

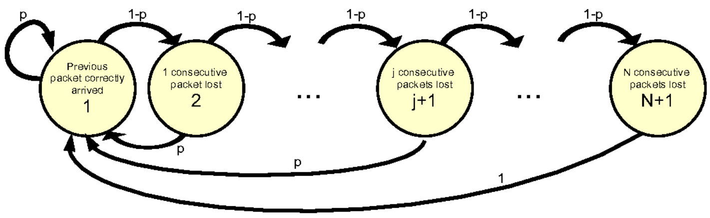
Fig. 3. Decentralized MPC results. Upper plots: output variables h (continuous lines) and references r (dashed lines). Lower plots: command inputs v. Gray areas denote packet drop intervals

the steady-state value of antechamber temperatures is not relevant for the posed control goals.

The closed-loop trajectories of centralized MPC feedback vs. decentralized MPC with no packet-loss are shown in Figure 2(a) (we only show the first state and input for clarity). In both cases the temperatures of the four-seat areas converge to the set-point asymptotically.

Figure 3(a) shows the temperature vector $h(t)$ tracking the time-varying reference $r(t)$ in the absence of packet-loss, where the coordinate transformation (18) is repeated after each set-point and external temperature change.

To simulate packet loss, we assume that the probability of losing a packet depends on the state of the Markov chain depicted below.



The Markov chain is in the jth state if $j-1$ consecutive packets have been lost. The probability of losing a further packet is $1-p$, $0 \leq p \leq 1$, except for the $(N+1)$th state where no packet can be lost any more. Such a probability model is partially confirmed by the experimental results on relative frequencies of packet failure burst length observed in Willig and Mitschke (2006). The simulation results obtained with $p = 0.5$ are shown in Figure 2(b) and Figure 3(b). The stability condition (14a) of Theorem 1 was tested and proved satisfied for values of j up to 160.

The simulations were run on a MacBook Air 1.86 GHz running Matlab R2008a under OS X 10.5.6 and the Hybrid Toolbox for Matlab (Bemporad, 2003). The average CPU time for solving the centralized QP problem associated with (3) is 6.0 ms (11.9 ms in the worst case). For the decentralized case, the average CPU time for solving the QP problem associated with (7) is 3.3 ms (7.4 ms in the worst case).

6. CONCLUSIONS

In this paper we have addressed the problem of controlling a distributed process through the cooperation of multiple decentralized model predictive controllers for set-point tracking under possible packet loss. Each controller is based on a submodel of the overall process, and different submodels may share common states and inputs, to decrease modeling errors and increase the level of cooperativeness of the controllers. The degree of decoupling of the submodels represents a tuning knob of the DMPC design.

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