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Decentralized model predictive control of dynamically coupled linear systems☆

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ABSTRACT

This paper proposes a decentralized model predictive control (DMPC) scheme for large-scale dynamical processes subject to input constraints. The global model of the process is approximated as the decomposition of several (possibly overlapping) smaller models used for local predictions. The degree of decoupling among submodels represents a tuning knob of the approach: the less coupled are the submodels, the lighter the computational burden and the load for transmission of shared information; but the smaller is the degree of cooperativeness of the decentralized controllers and the overall performance of the control system. Sufficient criteria for analyzing asymptotic closed-loop stability are provided for input constrained open-loop asymptotically stable systems and for unconstrained open-loop unstable systems, under possible intermittent lack of communication of measurement data between controllers. The DMPC approach is also extended to asymptotic tracking of output set-points and rejection of constant measured disturbances. The effectiveness of the approach is shown on a relatively large-scale simulation example on decentralized temperature control based on wireless sensor feedback.

1. Introduction

Large-scale systems such as power distribution grids, water networks, urban traffic networks, supply chains, formations of cooperating vehicles, mechanical and civil engineering structures, and many others, are often hard to control in a *centralized* way. The spatial distribution of the process impedes collecting all the measurements at a single location, where complex calculations based on all such information are executed, and redistributing the control decision to all actuators; moreover constructing and maintaining a full dynamical model of the system for control design is a time consuming task. Hence the current trend for *decentralized* decision making, distributed computations, and hierarchical control.

In a decentralized control scheme several local control stations only acquire local output measurements and decide local control inputs, possibly under the supervision of an upper hierarchical control layer improving their coordination. Consequently, the main advantages in controller implementation are the reduced and parallel computations, and reduced communications. However, all the controllers are involved in controlling the same large-scale process, and is therefore of paramount importance to determine conditions under which there exists a set of appropriate local feedback control laws stabilizing the entire system.

Ideas for decentralizing and hierarchically organizing the control actions in industrial automation systems date back to the 70's [1], but were mainly limited to the analysis of stability of decentralized linear control of interconnected subsystems. The interest in decentralized control raised again since the late 90's because of the advances in computation techniques like convex optimization [2]. Decentralized control and estimation schemes based on distributed convex optimization ideas have been proposed recently in [3,4] based on Lagrangean relaxations. Here global solutions can be achieved after iterating a series of local computations and inter-agent communications.

Large-scale multi-variable control problems, such as those arising in the process industries, are often dealt with model predictive control (MPC) techniques. In MPC the control problem is formulated as an optimization one, where many different (and possibly conflicting) goals are easily formalized and constraints on state and control variables can be included [5,6]. However, centralized MPC is often unsuitable for control of large-scale and networked systems, mainly due to lack of scalability and to maintenance issues of global models.

In view of the above considerations, it is then natural to look for *decentralized* or for *distributed* MPC (DMPC) algorithms, in which the original large-size optimization problem is replaced by a number of smaller and easily tractable ones that work iteratively and cooperatively towards achieving a common, system-wide control objective. In a typical DMPC framework at each sample

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instant each local controller measures local variables and update state estimates, solves the local receding-horizon control problem, applies the control signal for the current instant, and exchanges information with other controllers. Besides some benefits, the decentralized design also introduces some issues: how to ensure the asymptotic stability of the overall system, the feasibility of global constraints, the loss of performance with respect to a centralized MPC design. We briefly review the main contributions addressing those issues in the following paragraphs, the reader is referred to [7] for a more detailed survey.

In [8] the system under control is composed by a number of unconstrained linear discrete-time subsystems with decoupled input signals. The effect of dynamical coupling between neighboring states is modeled in prediction through a disturbance signal, while the information exchanged between control agents at the end of each sample step is the entire prediction of the local state vector. Under certain assumptions of the model state matrix, closed-loop stability is proved by introducing a contractive constraint on the state prediction norm in each local MPC problem, which the authors prove to be a recursively feasible constraint.

In [9–11] the authors propose a distributed MPC algorithm based on negotiations among DMPC agents. The effect of the inputs of a subsystem on another subsystem is modeled by using an "interaction model". All interaction models are assumed stable, and constraints on inputs are assumed decoupled (e.g., input saturation). Starting from a multiobjective formulation, the authors distinguish between a "communication-based" control scheme, in which each controller is optimizing his own local performance index, and a "cooperation-based" control scheme, in which each controller is optimizing a weighted sum of all performance indices. At each time step a sequence of iterations is taken before computing and implementing the input vector. With the communication-based approach, the authors show that if the sequence of iterations converges, it converges to a Nash equilibrium. With the cooperation-based approach, convergence to the optimal (centralized) control performance is established. The stability guarantees are not compromised by stopping the iterations before convergence, as only the optimality is affected in that case.

In [12] the authors consider the control of a special class of dynamically decoupled continuous-time nonlinear subsystems where the local states of each model represent a position and a velocity signal. State vectors are only coupled by a global performance objective under local input constraints, and the overall integrated cost is decomposed in distributed integrated cost functions. Before computing DMPC actions, neighboring subsystems broadcast in a synchronous way their states, and each agent transmits and receives an "assumed" control trajectory. Closed-loop stability is ensured by constraining the state trajectory predicted by each agent to stay close enough to the trajectory predicted at the previous time step that has been broadcasted, which introduces some conservativeness.

Dynamically decoupled submodels are also considered in [13], where a special nonlinear discrete-time system structure is assumed, subject to local input and state constraints. Subsystems are coupled by the cost function and by global constraints. Stability is analyzed for the problem without coupling constraints under some technical assumptions.

Distributed MPC *and* estimation problems are considered in [14] for square plants perturbed by noise. A distributed Kalman filter based on the local submodels is used for state estimation. The DMPC approach is similar to Venkat et al.'s "communication-based" approach, although only first moves are transmitted and assumed frozen in prediction, instead of the entire optimal sequences. Only constraints on local inputs are handled by the approach. Experimental results on a four-tank system are reported to show the effectiveness of the approach.

Another approach to decentralized MPC for nonlinear systems has been formulated in [15]. Under some technical assumptions of regularity of the dynamics and of boundedness of the disturbances, closed-loop stability is ensured by the inclusion in the optimization problem of a contractive constraint. The absence of information exchange between controllers leads to some conservativeness of the approach. Distributed Lyapunov-based MPC of nonlinear processes was also addressed in [16], and sufficient conditions for ultimately boundedness of the closed-loop system are derived in [17] in the presence of delays and asynchronous measurements.

Finally, very recently in [18] the authors introduced a robust DMPC for multiple dynamically decoupled subsystems in which distributed control agents exchange plans to achieve satisfaction of coupling constraints. The local controllers rely on the concept of "tubes" rather than single trajectories, to achieve robust feasibility and stability despite the presence of persistent, bounded disturbances.

This paper proposes a decentralized MPC design approach for large-scale processes that are possibly dynamically coupled and that are subject to input constraints. The paper extends preliminary work appeared in the conference papers [19–21]. A (partial) decoupling assumption only appears in the *prediction* models used by different MPC controllers. The chosen degree of decoupling represents a tuning knob of the approach. Sufficient criteria for analyzing the asymptotic stability of the process model in closed loop with the set of decentralized MPC controllers are provided. If such conditions are not verified, then the structure of decentralization should be modified by augmenting the level of dynamical coupling of the prediction submodels, increasing consequently the number and type of exchanged information about state measurements among the MPC controllers. To cope with the case of a non-ideal communication channel among neighboring MPC controllers, sufficient conditions for ensuring closed-loop stability of the overall closed-loop system are also given when packets containing state measurements are lost intermittently.

The paper is organized as follows. In Section 2 we propose a model decentralization scheme and the associated decentralized MPC formulation, whose closed-loop convergence properties are analyzed in Section 3, under both ideal and lossy feedback channels. The proposed DMPC approach is tested in Section 4 for decentralized wireless control of the temperature in different passenger areas in a railcar. The DMPC design and analysis tools and the simulation example proposed in the paper are included in the WIDE Toolbox for MATLAB [22].

2. Decentralized model predictive control setup

Consider the problem of regulating the discrete-time linear time-invariant system

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

$$u_{\min} \leq u(t) \leq u_{\max} \quad (2)$$

to the origin while fulfilling the constraints (2) at all time instants $t \in \mathbb{Z}_{0+}$, where $\mathbb{Z}_{0+}$ is the set of nonnegative integers, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are the state, input, and output vectors, respectively, and the pair (A, B) is stabilizable. In (2) the constraints should be interpreted component-wise and we assume $u_{min} < 0 < u_{max}$. A centralized MPC scheme would approach such a constrained regulation problem by solving at each time t, given the state vector $x(t)$, the following finite-horizon optimal control

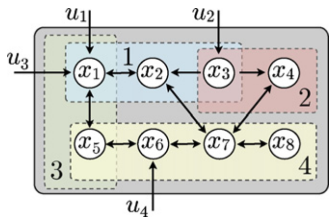

Fig. 1. Example of decomposition of a global model into four submodels. Each colored rectangle identifies the states belonging to the corresponding submodel.

problem

$$\min_{U} \quad x_N' P x_N + \sum_{k=0}^{N-1} x_k' Q x_k + u_k' R u_k \tag{3a}$$

$$s.t. \quad x_{k+1} = A x_k + B u_k,\ k = 0, \ldots, N-1 \tag{3b}$$

$$y = C x_k,\ k = 0, \ldots, N \tag{3c}$$

$$x_0 = x(t) \tag{3d}$$

$$u_{\min} \leq u_k \leq u_{\max},\ k = 0, \ldots, N_u - 1 \tag{3e}$$

$$u_k = K x_k,\ k = N_u, \ldots, N-1 \tag{3f}$$

where $U \triangleq \{u_0, \ldots, u_{N_u-1}\}$ is the sequence of future input moves, x_k denotes the predicted state vector at time $t+k$, obtained by applying the input sequence $u_0, \ldots, u_{k-1}$ to model (1), starting from $x(t)$. In (3) $N > 0$ is the prediction horizon, $N_u \leq N-1$ is the control horizon, $Q = Q' \succcurlyeq 0$, $R = R' \succ 0$, $P = P' \succcurlyeq 0$ are square weight matrices defining the performance index, and K is some terminal feedback gain. Matrices P and K are usually chosen to ensure closed-loop stability of the overall process [23].

2.1. Decentralized prediction models

We construct a set of prediction submodels based on the observation that, typically, in large-scale systems matrices A, B have a certain number of zero or negligible components, corresponding to a partial dynamical decoupling of the process. Fig. 1 depicts a dynamically coupled system decomposed into four partially overlapping subsystems.

For all $i \in \{1, \ldots, M\}$ we define $x^i \in \mathbb{R}^{n_i}$ as the vector collecting a subset $\mathcal{I}_{xi} \subseteq \{1, \ldots, n\}$ of the state components,

$$x^i = W_i' x = \left[x_1^i \cdots x_{n_i}^i\right]' \in \mathbb{R}^{n_i} \tag{4a}$$

where $W_i \in \mathbb{R}^{n \times n_i}$ collects the n_i columns of the identity matrix of order n corresponding to the indices in $\mathcal{I}_{xi}$, and, similarly,

$$u^i = Z_i' u = \left[u_1^i \cdots u_{m_i}^i\right]' \in \mathbb{R}^{m_i} \tag{4b}$$

as the vector of input signals tackled by the i-th controller, where $Z_i \in \mathbb{R}^{m \times m_i}$ collects m_i columns of the identity matrix of order m corresponding to the set of indices $\mathcal{I}_{ui} \subseteq \{1, \ldots, m\}$. Note that

$$W_i' W_i = I_{n_i}, Z_i' Z_i = I_{m_i}, \forall i \in \{1, \ldots, M\} \tag{5}$$

where $I_{(\cdot)}$ denotes the identity matrix of order $(\cdot)$. By definition of x^i in (4a) we obtain

$$x^i(t+1) = W_i' x(t+1) = W_i' A x(t) + W_i' B u(t) \tag{6}$$

An *approximation* of (1) is obtained by changing $W_i' A$ in (6) into $W_i' A W_i W_i'$ and $W_i' B$ into $W_i' B Z_i Z_i'$, therefore getting the new prediction model of reduced order

$$x^i(t+1) = A_i x^i(t) + B_i u^i(t) \tag{7}$$

where matrices $A_i = W_i' A W_i \in \mathbb{R}^{n_i \times n_i}$ and $B_i = W_i' B Z_i \in \mathbb{R}^{n_i \times m_i}$ are submatrices of the original A and B matrices, respectively, describing in a possibly approximate way the evolution of the states of subsystem #i.

The size (n_i, m_i) of model (7) in general will be much smaller than the size (n, m) of the overall process model (1). The choice of the *decoupling matrices* (W_i, Z_i) (and, consequently, of the dimensions n_i, m_i of each submodel) is a tuning knob of the DMPC procedure proposed in this paper.

We want to design a controller for each set of moves u^i according to prediction model (7) and based on feedback from x^i, for all $i \in \{1, \ldots, M\}$. Note that in general different input vectors u^i, u^j may share common components. To avoid ambiguities on the control action to be commanded to the process, we impose in the design that only a subset $\mathcal{I}_{ui}^{\#} \subseteq \mathcal{I}_{ui}$ of input signals computed by controller #i is actually applied to the process, with the following conditions

$$\bigcup_{i=1}^{M} I_{ui}^{\#} = \{1, \ldots, m\} \tag{8a}$$

$$I_{ui}^{\#} \cap I_{uj}^{\#} = \varnothing,\ \forall i, j = 1, \ldots, M,\ i \neq j \tag{8b}$$

Condition (8a) ensures that all actuators are commanded, condition (8b) that each actuator is commanded by only one controller. For the sake of simplicity of notation, since now on we assume that $M = m$ and that $I_{ui}^{\#} = i$, $i = 1, \ldots, m$, i.e., that each controller #i only controls the i-th input signal. As observed earlier, in general $\mathcal{I}_{xi} \cap \mathcal{I}_{xj} \neq \varnothing$, meaning that controller #i may partially share the same feedback information with controller #j, and $\mathcal{I}_{ui} \cap \mathcal{I}_{uj} \neq \varnothing$. This means that controller #i may model the *effect* of control actions that are actually decided by another controller #j, $i \neq j$, $i, j = 1, \ldots, M$, which ensures a certain degree of cooperation.

The designer has the flexibility of choosing the pairs (W_i, Z_i) of decoupling matrices, $i = 1, \ldots, M$. A first guess of the decoupling matrices can be inspired by the intensity of the dynamical interactions existing in the model. The larger (n_i, m_i) the smaller the model mismatch and hence the better the performance of the overall-closed loop system. On the other hand, the larger (n_i, m_i) the larger is the communication and computation efforts of the controllers, and hence the larger the sampling time of the controllers. An example of model decomposition is given in Section 4.

2.2. Decentralized optimal control problems

In order to exploit submodels (7) for formulating local finite-horizon optimal control problems that lead to an overall closed-loop stable DMPC system, let the following assumptions be satisfied.

Assumption 1. Matrix A has all its eigenvalues inside the unit circle.

Assumption 1 restricts the strategy and stability results of DMPC to processes that are open-loop asymptotically stable, leaving to the controller the mere role of optimizing the performance of the closed-loop system.

Assumption 2. Matrix A_i has all its eigenvalues inside the unit circle, $\forall i \in \{1, \ldots, M\}$.

Assumption 2 restricts the degrees of freedom in choosing the decentralized models. Note that if $A_i = A$ for all $i \in \{1, \ldots, M\}$ is the only choice satisfying Assumption 2, then no decentralized MPC can be formulated within this framework. For all $i \in \{1, \ldots, M\}$ consider the following infinite-horizon constrained optimal control problems

$$V_i(x(t)) = \min_{\{u_k^i\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} (x_k^i)' Q_i x_k^i + (u_k^i)' R_i u_k^i = \tag{9a}$$

$$= \min_{u_0^i} (x_1^i)' P_i x_1^i + x^i(t)' Q_i x^i(t) + (u_0^i)' R_i u_0^i \tag{9b}$$

$$s.t.\, x_1^i = A_i x^i(t) + B_i u_0^i \tag{9c}$$

$$x_0^i = W_i' x(t) = x^i(t) \tag{9d}$$

$$u_{\min}^i \le u_0^i \le u_{\max}^i \tag{9e}$$

$$u_k^i = 0,\ \forall k \ge 1 \tag{9f}$$

where $P_i = P_i' \succcurlyeq 0$ is the solution of the Lyapunov equation

$$A_i' P_i A_i - P_i = -Q_i \tag{10}$$

such that $x' P_i x = \sum_{k=0}^{\infty} (A_i^k x)' Q_i (A_i^k x)$ and that exists by virtue of Assumption 2, $Q_i = W_i' Q W_i$ and $R_i = Z_i' R Z_i$. Problem (9) corresponds to a finite-horizon constrained problem with control horizon $N_u = 1$ and linear stable prediction model. Note that only the local state vector $x^i(t)$ is needed to solve Problem (9).

At time t, each controller MPC #i measures the state $x^i(t)$ (usually corresponding to local and neighboring states), solves problem (9) and obtains the optimizer

$$u_0^{*i} = [u_0^{*i1}, \ldots, u_0^{*ii}, \ldots, u_0^{*im_i}]' \in \mathbb{R}^{m_i} \tag{11}$$

In the simplified case $M = m$ and $I_{ui}^{\#} = i$, only the i-th sample of u_0^{*i}

$$u_i(t) = u_0^{*ii} \tag{12}$$

will determine the i-th component $u_i(t)$ of the input vector actually commanded to the process at time t. The inputs $u_0^{*ij}, j \neq i, j \in \mathcal{I}_{ui}$ to the neighbors are discarded, their only role is to provide a better prediction of the state trajectories x_k^i, and therefore a possibly better performance of the overall closed-loop system.

The collection of the optimal inputs of all the M MPC controllers,

$$u(t) = [u_0^{*11}\ \ldots\ u_0^{*ii}\ \ldots\ u_0^{*mm}]' \tag{13}$$

is the actual input commanded to process (1), whose components $u_i(t)$ are broadcasted at time $t+1$ to the interested local controllers j such that $i \in \mathcal{I}_{uj}$.

The optimizations (9) are repeated at time $t+1$ based on the new states $x^i(t+1) = W_i' x(t+1)$, $\forall i \in \{1, \ldots, M\}$, according to the usual receding horizon control paradigm in MPC. The degree of coupling between the DMPC controllers is dictated by the choice of the decoupling matrices (W_i, Z_i). Clearly, the larger the number of interactions captured by the submodels, the more complex the formulation (and, in general, better the solution) of the optimization problems (9) and hence the computations performed by each control agent.

3. Convergence properties

As mentioned in the introduction, one of the major issues in decentralized MPC is to ensure the stability of the overall closed-loop system. The non-triviality of this issue is due to the fact that the prediction of the state trajectory made by MPC #i about state $x^i(t)$ is in general not correct, because of partial state and input information and of the mismatch between u^{*ij} (desired by controller MPC #i) and u^{*jj} (computed and applied to the process by controller MPC #j).

The following theorem summarizes the closed-loop convergence results of the proposed DMPC scheme using the cost function $V(x(t)) \triangleq \sum_{i=1}^{M} V_i(W_i' x(t))$ as a Lyapunov function for the overall system.

Theorem 1. *Let* Assumptions 1 and 2 *hold and define* P_i *as in* (10), $\forall i \in \{1, \ldots, M\}$. *Define*

$$\begin{array}{ll} \Delta u^i(t) \triangleq u(t) - Z_i u_0^{*i}(t), & \Delta x^i(t) \triangleq (I - W_i W_i') x(t) \\ \Delta A^i \triangleq (I - W_i W_i') A, & \Delta B^i \triangleq B - W_i W_i' B Z_i Z_i' \end{array} \tag{14a}$$

$$\begin{aligned} \Delta Y^i(x(t)) \triangleq\ & W_i W_i' (A \Delta x^i(t) + B Z_i Z_i' \Delta u^i(t)) \\ & + \Delta A^i x(t) + \Delta B^i u(t) \end{aligned} \tag{14b}$$

$$\begin{aligned} \Delta S^i(x(t)) \triangleq\ & (2(A_i W_i' x(t) + B_i u_0^{*i}(t))' \\ & + \Delta Y^i(x(t))' W_i) P_i W_i' \Delta Y^i(x(t)) \end{aligned} \tag{14c}$$

If at least one of the conditions

$$(i)\ x' \bar{Q} x - \sum_{i=1}^{M} \Delta S^i(x) \ge 0 \tag{15a}$$

$$(ii)\ x' \bar{Q} x - \sum_{i=1}^{M} \Delta S^i(x) + \sum_{i=1}^{M} u_0^{*i}(x)' R_i u_0^{*i}(x) \ge \alpha x' x \tag{15b}$$

is satisfied for some scalar $\alpha > 0$ *and* $\forall x \in \mathbb{R}^n$, *where* $\bar{Q} = \left(\sum_{i=1}^{M} W_i W_i' Q W_i W_i' \right)$ *and* $R_i = Z_i' R Z_i$, *then the decentralized MPC scheme defined in* (9)–(13) *in closed loop with* (1) *is globally asymptotically stable.*

Proof. Since $x^i(t) = W_i' x(t)$, by exploiting (10) at time t the optimal cost $V_i(x(t))$ of subproblem (9) can be rewritten as

$$\begin{aligned} V_i(x(t)) = &\ (W_i' x(t))' (W_i' Q W_i) W_i' x(t) + u_0^{*i}(t)' Z_i' R Z_i u_0^{*i}(t) \\ & + (A_i W_i' x(t) + B_i u_0^{*i}(t))' P_i (A_i W_i' x(t) + B_i u_0^{*i}(t)) \end{aligned} \tag{16}$$

where P_i is defined as in (10). As the input $u_0^i = 0$ satisfies the constraints $u_{\min} \le u_0^i \le u_{\max}$, by (10) the optimal cost at time $t+1$ satisfies the following inequality

$$\begin{aligned} V_i(x(t+1)) \le &\ (W_i' x(t+1))' (W_i' Q W_i) W_i' x(t+1) \\ & + (W_i' x(t+1))' A_i' P_i A_i W_i' x(t+1) \\ = &\ (W_i' x(t+1))' (A_i' P_i A_i + W_i' Q W_i) W_i' x(t+1) \\ = &\ x(t+1)' W_i P_i W_i' x(t+1) \end{aligned} \tag{17}$$

By rewriting

$$\begin{aligned} x(t+1) = &\ Ax(t) + Bu(t) = (A \pm W_i W_i' A)(x(t) \pm W_i W_i' x(t)) \\ & + (B \pm W_i W_i' B Z_i Z_i')(u(t) \pm Z_i u_0^{*i}(t)) \\ = &\ W_i (A_i W_i' x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t)) \end{aligned} \tag{18}$$

where $\pm[\,\cdot\,]$ means that the same quantity $[\,\cdot\,]$ is added and subtracted, from (17) and recalling (5) we obtain

$$\begin{aligned} V_i(x(t+1)) \le &\ (W_i (A_i W_i' x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t)))' W_i P_i \cdot \\ & \cdot W_i' (W_i (A_i W_i' x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t))) = (A_i W_i' x(t) \\ & + B_i u_0^{*i}(t))' P_i (A_i W_i' x(t) + + B_i u_0^{*i}(t)) + \Delta S^i(x(t)) \end{aligned}$$

By (16), we obtain

$$\begin{aligned} V_i(x(t+1)) \le &\ V_i(x(t)) - x'(t) W_i W_i' Q W_i W_i' x(t) + \\ & - u_0^{*i}(t)' Z_i' R Z_i u_0^{*i}(t) + \Delta S^i(x(t)) \end{aligned} \tag{19}$$

Consider first condition (15a). By positive definiteness of R and full column rank of all matrices Z_i it follows that $Z_i' R Z_i \succ 0$

and hence that the function $V(x(t)) \triangleq \sum_{i=1}^{M} V_i(W_i' x(t))$ is non-increasing. Since $V(x(t)) \geq 0$, $\forall t \geq 0$, it follows that there exists $\lim_{t\to\infty} V(x(t)) = \lim_{t\to\infty} V(x(t+1))$ Hence, by (19) it also follows that

$$\lim_{t\to\infty} x'(t)\bar{Q}x(t) - \sum_{i=1}^{M} \Delta S^i(x(t)) + \sum_{i=1}^{M} u_0^{*i}(x(t))' R_i u_0^{*i}(x(t)) = 0$$

Because of (15a), it follows that $\lim_{t\to\infty} \sum_{i=1}^{M} u_0^{*i}(x(t))' R_i u_0^{*i}(x(t)) = 0$, and by positive definiteness of $Z_i' R Z_i$, that $\lim_{t\to\infty} u_0^{*i}(x(t)) = 0$, and hence that $\lim_{t\to\infty} u_0^{*ii}(x(t)) = 0$, $\forall i \in \{1, \ldots, M\}$, which in turn implies $\lim_{t\to\infty} u(t) = 0$. As by Assumption 1 the open-loop process (1) is linear and asymptotically stable, it is also input-to-state stable [24], and hence $\lim_{t\to\infty} x(t) = 0$.

The same follows under condition (15b), as $\lim_{t\to\infty} \alpha x'(t)x(t) = 0$ and hence $\lim_{t\to\infty} x(t) = 0$. □

Theorem 1 provides two alternative conditions for verifying closed-loop stability. Condition (15a) amounts to testing that the cumulated effect of model mismatch $\sum_{i=1}^{M} \Delta S_i(x)$ is dominated by the global decreasing rate $x'\bar{Q}x$, therefore advising the designer to choose a weight Q large enough to dominate the influence of the prediction error due to unmodeled dynamics. Condition (15b) attempts to exploit also the nonnegative term $\sum_{i=1}^{M} u_0^{*i}(x)' R_i u_0^{*i}(x)$ to dominate model mismatch, provided that the slightly more stringent condition "$\geq \alpha x'x$" instead of "≥ 0" is satisfied for some $\alpha > 0$.

3.1. Stability tests

By using the explicit MPC results of [23], each optimizer function $u_0^{*i} : \mathbb{R}^n \mapsto \mathbb{R}^{m_i}$, $i = 1, \ldots, M$, can be expressed as a piecewise affine function of x:

$$u_0^{*i}(x) = F_{ij}x + G_{ij} \quad \text{if}\, H_{ij}x \leq K_{ij}, j = 1, \ldots, n_{ri} \tag{20}$$

Theorem 2. *Let the pairs (A_i, B_i) be stabilizable, $\forall i \in \{1, \ldots, M\}$. Let* Problem (9) *be replaced by*

$$V_i(x(t)) = \min_{\{u_k^i\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} (x_k^i)' Q_i x_k^i + (u_k^i)' R_i u_k^i = \tag{21a}$$

$$= \min_{u_0^i} (x_1^i)' P_i x_1^i + x^i(t)' Q_i x^i(t) + (u_0^i)' R_i u_0^i \tag{21b}$$

$$s.t.\, x_1^i = A_i x^i(t) + B_i u_0^i \tag{21c}$$

$$x_0^i = W_i' x(t) = x^i(t) \tag{21d}$$

$$u_k^i = K_{LQ_i} x_k^i, \ \forall k \geq 1 \tag{21e}$$

where $P_i = P_i' \succcurlyeq 0$ is the solution of the Riccati equation

$$Q_i + K_{LQ_i}' R_i K_{LQ_i} + (A_i + B_i K_{LQ_i})' P_i (A_i + B_i K_{LQ_i}) = P_i \tag{22}$$

$K_{LQ_i} = -(Z_i' R Z_i + B_i' P_i B_i)^{-1} B_i' P_i A_i$ is the corresponding local LQR feedback, $Q_i = W_i' Q W_i$ and $R_i = Z_i' R Z_i$. *Let $\Delta Y^i(x(t))$ and let $\Delta S^i(x(t))$ be defined as in* (14), *in which P_i is defined as in* (22). *If* (15a) *is satisfied, or* (15b) *is satisfied for some scalar $\alpha > 0$, then the decentralized MPC scheme defined in* (21), (13) *in closed-loop with* (1) *is globally asymptotically stable.*

Proof. By recalling that $x^i(t) = W_i' x(t)$ and exploiting (22), at time t the optimal cost $V_i(x(t))$ of subproblem (21) can be rewritten as

$$\begin{aligned} V_i(x(t)) = {} & (W_i' x(t))' (W_i' Q W_i) W_i' x(t) + (A_i W_i' x(t) \\ & + B_i u_0^{*i}(t))' P_i (A_i W_i' x(t) + B_i u_0^{*i}(t)) \\ & + u_0^{*i}(t)' Z_i' R Z_i u_0^{*i}(t) \end{aligned} \tag{23}$$

Now choose $u_0^i(t+1) = K_{LQ_i}(W_i' x(t+1))$, with P_i, as solution of the Riccati equation (22), and K_{LQ_i} as the correspond local LQR feedback. By feasibility of Problem (21) at time t, the optimal cost at time $t+1$ satisfies the following equality

$$\begin{aligned} V_i(x(t+1)) & = (W_i' x(t+1))' (W_i' Q W_i) W_i' x(t+1) + (A_i W_i' \cdot x(t+1) + B_i u_0^{*i}(t+1)) + B_i u_0^{*i}(t+1)' P_i (A_i \cdot W_i' x(t+1) + u_0^{*i}(t+1))' R_i u_0^{*i}(t+1) \\ & = (W_i' x(t+1))' (A_i + B_i K_{LQ_i})' P_i (A_i + B_i K_{LQ_i}) (W_i' \cdot x(t+1)) + x(t+1)' W_i (W_i' Q W_i)(W_i' x(t+1)) \\ & + (K_{LQ_i} W_i' x(t+1))' R_i (K_{LQ_i} W_i' x(t+1)) \\ & = x(t+1)' W_i ((A_i + B_i K_{LQ_i})' P_i (A_i + B_i K_{LQ_i}) + W_i' Q W_i + K_{LQ_i}' Z_i R Z_i' K_{LQ_i})(W_i' x(t+1)) \\ & = x(t+1)' W_i P_i W_i' x(t+1) \end{aligned} \tag{24}$$

Hence, both condition (15a) and condition (15b) are a composition of quadratic and piecewise affine functions, so that *global stability* can be tested through linear matrix inequality relaxations [25] (cf. the approach of [26]).

As $u_{\min} < 0 < u_{\max}$, there exists a ball around the origin $x = 0$ contained in one of the regions, say $\{x \in \mathbb{R}^n : H_{i1}x \leq K_{i1}\}$, such that $G_{i1} = 0$. Therefore, around the origin both (15a) and (15b) become a quadratic form $x'(\sum_{i=1}^{M} E_i)x$ of x, where matrices E_i can be easily derived from (14) and (15). Hence, *local stability* of (9)–(13) in closed loop with (1) can be simply tested by checking the positive semidefiniteness of the square $n \times n$ matrix $\sum_{i=1}^{M} E_i$. Note that, depending on the degree of decentralization, in order to satisfy the sufficient stability criterion one may need to set $Q \succ 0$ in order to dominate the unmodeled dynamics arising from the terms ΔS^i.

3.2. Open-loop unstable subsystems

In the absence of input constraints, Assumptions 1, 2 can be removed to extend the previous DMPC scheme to the case where (A, B) is not an asymptotically stable system, although stabilizable.

By rewriting $x(t+1) = Ax(t) + Bu(t) = W_i(A_i W_i' x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t))$ as in (18), from (24) and recalling (5) we obtain $V_i(x(t+1)) = (W_i(A_i W_i' x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t)))' W_i P_i W_i' (W_i(A_i W_i' x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t))) = (A_i W_i' x(t) + B_i u_0^{*i}(t))' P_i (A_i W_i' x(t) + B_i u_0^{*i}(t)) + \Delta S^i(x(t))$, where $\Delta S^i(x(t))$ is defined as in (14c). By (23), we obtain

$$\begin{aligned} V_i(x(t+1)) & \leq V_i(x(t)) - x'(t) W_i W_i' Q W_i W_i' + x(t) + \\ & - u_0^{*i}(t)' Z_i' R Z_i u_0^{*i}(t) + \Delta S^i(x(t)) \end{aligned}$$

for which the same reasoning as in the proof of Theorem 1 can be repeated. □

3.3. Decentralized MPC under arbitrary packet loss

So far we assumed that the communication model among neighboring MPC controllers is faultless, so that each MPC agent successfully receives the information about the states of its corresponding submodel. However, one of the main issues in networked control systems is the unreliability of communication channels, especially in wireless automation systems, which may result in data packet dropout. In this section we prove a sufficient condition for ensuring convergence of the DMPC closed-loop in the case packets containing measurements are lost for an arbitrary but upper-bounded number N of consecutive time steps. The underlying operating assumption is that if the actual number of lost packets exceeds the given N, the decentralized controllers are turned off and $u = 0$ is applied persistently, so that a number of packet drops larger than N is not considered. The results shown here are based on formulation (9) and rely on the open-loop asymptotic stability Assumptions 1 and 2.

Setting $u(t) = 0$ to an open-loop stable system is a natural backup choice when no state feedback is available because of a communication blackout. Because of (9f), setting $u^i(t) = 0$ also amounts to applying the prosecution of the most recent available optimal control sequence, a practice often used in MPC in case of failures of the QP solver. Different backup options may be considered, such as solving (9) by replacing $x^i(t)$ with an estimate obtained through model (7) and the available measurements, for instance by applying distributed Kalman filtering techniques [27]. Of course whether one or the other approach is better strongly depends on the amount of model mismatch introduced by the decentralized modeling.

The next theorem provides conditions for asymptotic closed-loop stability of decentralized MPC under packet loss, generalizing the result of Theorem 1.

Theorem 3. *Let N be a positive integer such that no more than N consecutive steps of channel transmission blackout can occur. Assume* $u(t) = 0$ *is applied when no packet is received. Let Assumptions* 1, 2 *hold and* $\forall i \in \{1, \ldots, M\}$ *define* P_i *as in* (10), $\Delta u^i(t)$, $\Delta x^i(t)$, ΔA^i, ΔB^i as in (14a), $\Delta Y^i(x(t))$ as in (14b), *let* $\xi_i(x) \triangleq A_i W_i' x + B_i u_0^{*i}(x)$, *and for* $j = 1, \ldots, N$ *let*

$$\Delta S_j^i(x) \triangleq [2(A_i W_i' x + B_i u_0^{*i}(x))' W_i' + \Delta Y^i(x)'] \cdot (A^{j-1})' W_i P_i W_i' A^{j-1} \Delta Y^i(x) \tag{25}$$

If at least one of the conditions

$$(i)\ x'\bar{Q}x + \sum_{i=1}^{M} \xi_i(x)' \bar{P}_{ij} \xi_i(x) - \Delta S_j^i(x) \geq 0 \tag{26}$$

$$(ii)\ x'\bar{Q}x - \alpha x'x + \sum_{i=1}^{M} \xi_i(x)' \bar{P}_{ij} \xi_i(x) - \Delta S_j^i(x) + u_0^{*i}(x)' Z_i' R \cdot Z_i u_0^{*i}(x) \geq 0 \tag{27}$$

is satisfied for some scalar $\alpha > 0$ *and* $\forall x \in \mathbb{R}^n$, $\forall j \in \{1, \ldots, N\}$, *where* $\bar{Q} = (\sum_{i=1}^{M} W_i W_i' Q W_i W_i')$ *and* $\bar{P}_{ij} = P_i - W_i'(A^{j-1})' W_i P_i W_i' A^{j-1} W_i$, *then the decentralized MPC scheme defined in* (9)–(13) *in closed loop with* (1) *is globally asymptotically stable under packet loss.*

Proof. Let $\{t_k\}_{k=0}^{\infty}$ be the sequence of sampling steps at which packet information is received, and let $j_k = t_{k+1} - t_k$ the corresponding number of consecutive packet drops, $1 \leq j_k \leq N$. We want to examine the difference $V_i(x(t_{k+1})) - V_i(x(t_k))$, where $V_i(x(t))$ is the optimal cost of subproblem (9) at time t. As the backup input $u(t_k + h) = 0$ is applied from time t_k to $t_{k+1} - 1$ ($h = 0, \ldots, j_k - 1$), we have

$$\begin{aligned} x(t_{k+1}) &= A^{j_k-1}(Ax(t_k) + Bu(t_k)) \\ &= A^{j_k-1}(\Delta Y^i(x(t_k)) + W_i \xi(x(t_k))) \end{aligned}$$

Since $x^i(t_{k+1}) = W_i' x(t_{k+1})$, at time t_{k+1} the optimal cost $V_i(x(t_{k+1}))$ of subproblem (9) can be rewritten as

$$\begin{aligned} V_i(x(t_{k+1})) = {} & (W_i' x(t_{k+1}))' W_i' Q W_i W_i' x(t_{k+1}) \\ & + [A_i W_i' x(t_{k+1}) + (B_i u_0^{*i}(t_{k+1}))' P_i (A_i W_i' x(t_{k+1}) \\ & + B_i u_0^{*i}(t_{k+1}))] + u_0^{*i}(t_{k+1})' Z_i' R Z_i u_0^{*i}(t_{k+1}) \end{aligned}$$

where P_i is defined as in (10) and is such that $(x_0^i)' P_i x_0^i = \sum_{k=0}^{\infty} (x_k^i)' W_i' Q W_i x_k^i$ with $x_{k+1}^i = A_i x_k^i$. Hence, considering that $u_0^i(t_{k+1}) = 0$ is a feasible suboptimal choice for problem (9), we obtain the following inequality

$$\begin{aligned} V_i(x(t_{k+1})) &\leq x'(t_{k+1}) W_i P_i W_i' x(t_{k+1}) \\ &\leq (A^{j_k-1}[\Delta Y^i(x(t_k)) + W_i \xi(x(t_k))])' W_i P_i \cdot \\ &\quad W_i' A^{j_k-1}(\Delta Y^i(x(t_k)) + W_i \xi(x(t_k))) \\ &= \Delta Y(x(t_k))' (A^{j_k-1})' W_i P_i W_i' A^{j_k-1} \Delta Y(x(t_k)) + \\ &\quad 2\xi(x(t_k))' W_i' (A^{j_k-1})' W_i P_i W_i' A^{j_k-1} \Delta Y(x(t_k)) + \\ &\quad \xi(x(t_k))' W_i' (A^{j_k-1})' W_i P_i W_i' A^{j_k-1} W_i \xi(x(t_k)) \\ &= \Delta S_{j_k}^i (x(t_k)) + \xi(x(t_k))' W_i' (A^{j_k-1})' \cdot \\ &\quad \cdot W_i P_i W_i' A^{j_k-1} W_i \xi(x(t_k)) \end{aligned}$$

Since $V_i(x_i(t_k)) = x'(t_k W_i' W_i' Q W_i W_i' x(t_k)) + \xi(x(t_k))' P_i \xi(x(t_k)) + u_0^{*i}(t_k)' Z_i' R Z_i u_0^{*i}(t_k)$ we get

$$\begin{aligned} & V_i(x(t_{k+1})) - V_i(x_i(t_k)) \leq \\ & \Delta S_{j_k}^i (x(t_k)) + \xi(x(t_k))' W_i' (A^{j_k-1})' W_i P_i W_i' A^{j_k-1} \cdot \\ & \cdot W_i \xi(x(t_k)) - ((W_i' x(t_k))' (W_i' Q W_i) W_i' x(t_k) + \xi \cdot (x(t_k))' P_i \xi(x(t_k)) \\ & + u_0^{*i}(t_k)' Z_i' R Z_i u_0^{*i}(t_k)) \leq \Delta S_{j_k}^i (x(t_k)) - u_0^{*i}(t_k)' Z_i' R Z_i u_0^{*i}(t_k) + \\ & x(t_k)' W_i (W_i' Q W_i) W_i' x(t_k) + -\xi(x(t_k))' (P_i + \\ & -W_i' (A^{j_k-1})' W_i P_i W_i' A^{j_k-1} W_i) \xi(x(t_k)) \end{aligned}$$

Let $V(x(t)) \triangleq \sum_{i=1}^{M} V_i(W_i' x(t))$. If (26) holds, then by (19) it follows that $V(x(t_k))$ is a decreasing function of k lower-bounded by zero, and therefore converges as $k \to \infty$, which proves $\lim_{k\to\infty} V(x(t_{k+1})) - V(x(t_k)) = 0$. This in turn implies by (19) that

$$\lim_{k\to\infty} u_0^{i*}(t_k)' Z_i' R Z_i u_0^{i*}(t_k) = 0$$

As $R \succ 0$ and Z_i are full-column-rank matrices, it follows that $Z_i' R Z_i \succ 0$ and hence that $\lim_{k\to\infty} u(t_k) = 0$. If (27) holds, then in a similar way it is immediate to see that $\lim_{k\to\infty} x(t_k) = 0$ which again implies $\lim_{k\to\infty} u(t_k) = 0$, as around the origin $u(t_k)$ is a linear function of $x(t_k)$ (corresponding to the unconstrained solution of problem (9)). Since in the presence of packet drop $u(t) = 0$, the input sequence $\{\ldots, 0, 0, u(t_k), 0, \ldots, 0, u(t_{k+1}), 0, \ldots, 0, u(t_{k+2}), \ldots\}$ is actually applied to the process, and clearly $\lim_{t\to\infty} u(t) = 0$. As asymptotically stable linear systems are also input-to-state stable [24], it immediately follows that $\lim_{t\to\infty} x(t) = 0$.

Note that $V(x(t))$ is not a common Lyapunov function for the switched system under consideration since $V(x(t_k)) \leq V(x(t_k - 1))$ for $t_k - 1 \notin \{t_k\}_{k=0}^{\infty}$ does not hold for the general case. However, Assumptions 1, 2, implies $V(x(t))$ to be decreasing when the plant evolve unforced, i.e. $u(t) = 0$, condition that applies for $t \notin \{t_k\}_{k=0}^{\infty}$. Therefore $V(x(t))$ is upperbounded by

$$\bar{V}(x(t)) = \begin{cases} V(x(t)) & t \in \{t_k\}_{k=0}^{\infty} \\ V(x(t_k)) & t_k \leq t < t_{k+1} \end{cases} \tag{28}$$

that is a common Lyapunov function for each mode of the switched system. It is trivial to verify that $\bar{V}(x(t) \geq V(x(t)))$, $\forall t \in \mathbb{Z}_{0+}$ and that $\bar{V}(x(t))$ converge to 0 by virtue of measured instants. □

Note again that around the origin the conditions in (27) become a quadratic form, so local stability of (9)–(13) in closed loop with (1) under packet loss can be easily tested for any arbitrary fixed N. Note also that conditions (27) are a generalization of (15), as for $j = 1$ (no packet drop) matrix $P_i - W_i'(A^{j-1})' W_i P_i W_i' A^{j-1} W_i = P_i - P_i = 0$.

3.4. Extension to set-point tracking

Consider the following discrete-time linear global process model

$$\begin{cases} z(t+1) = Az(t) + Bv(t) + F_d(t) \\ h(t) = Cz(t) \end{cases} \tag{29}$$

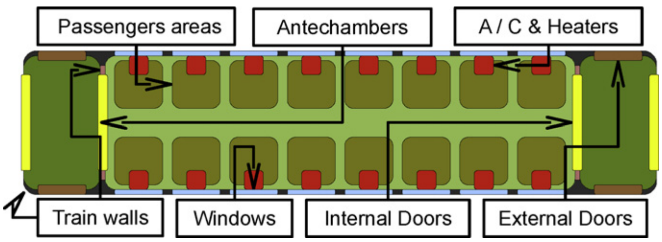

Fig. 2. Physical structure of the railcar.

where $z\in\mathbb{R}^n$ is the state vector, $v\in\mathbb{R}^m$ is the input vector, $h\in\mathbb{R}^p$ is the output vector, $F_d\in\mathbb{R}^d$ is a vector of measured disturbances. Let A satisfy Assumption 1 and assume F_d is constant. The considered set-point tracking problem is that of h tracking a given reference value $r\in\mathbb{R}^p$, despite the presence of F_d. In order to recast the problem as a regulation problem, assume steady-state vectors $z_r\in\mathbb{R}^n$ and $v_r\in\mathbb{R}^m$ exist solving the static problem

$$\begin{cases} (I-A)z_r = Bv_r + F_d \\ r = Cz_r \end{cases} \tag{30}$$

and let $x=z-z_r$, $y=h-r$, and $u=v-v_r$. Input constraints $v_{\min}\le v\le v_{\max}$ are mapped into constraints $v_{\min}-v_r\le u\le v_{\max}-v_r$. Note that in case $v_r\notin[v_{\min}, v_{\max}]$, perfect tracking under constraints is not possible, and an alternative is to set

$$\begin{bmatrix} z_r \\ v_r \end{bmatrix} = \arg\min \left\| \begin{bmatrix} I-A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} z_r \\ v_r \end{bmatrix} - \begin{bmatrix} F_d \\ r \end{bmatrix} \right\| \quad \text{s.t.} \quad v_{\min}\le v_r\le v_{\max}$$

Proposition 1. *Under the global coordinate transformation (30), the process (29) under the decentralized MPC law (9)–(13) is such that $h(t)$ converges asymptotically to the set-point r, either under the assumption of* Theorem 1 *or, in the presence of packet sdrops, of* Theorem 3.

Note that problem (30) is solved in a *centralized* way. Defining local coordinate transformations v_{ir}, z_{ir} based on submodels (7) would not lead, in general, to offset-free tracking, due to the mismatch between global and local models. This is a general observation one needs to take into account in decentralized tracking. Note also that both v_r and z_r depend on F_d as well as r, so problem (30) should be solved each time the value of F_d or r change and retransmitted to each controller.

4. Decentralized temperature control in a railcar

In this section we test the proposed DMPC approach for decentralized control of the temperature in different passenger areas in a railcar. The system is schematized in Fig. 2. Each passenger area has its own heater and air conditioner but its thermal dynamics interacts with surrounding areas (neighboring passenger areas, external environment, antechambers) directly or through windows, walls and doors. Passenger areas are composed by a table and the associated four seats. Temperature sensors are located in each four-seat area, in each antechamber, and along the corridor. The goal of the controller is to adjust each passenger area to its own temperature set-point to maximize passenger comfort. Temperature sensors may be wired or wireless, in the latter case we assume that information packets may be dropped, because of very low power transmission, simplified transmission protocols, no acknowledgement and retransmission, and because of interferences from passengers' electronic equipment.

Let $2N_a$ be the number of four-seat areas ($N_a=8$ in Fig. 2), N_a the number of corridor partitions, and 2 the number of antechambers. Under the assumption of perfectly mixed fluids in each j th volume, $j=1,\ldots,n$ where $n=3N_a+2$, the heat transmission equations by conduction lead to the linear model

$$\frac{dT_j(\tau)}{d\tau} = \sum_{i=0}^{n} Q_{ij}(\tau) + Q_{uj},\quad Q_{ij}(\tau) = \frac{S_{ij}K_{ij}(T_i(\tau)-T_j(\tau))}{C_jL_{ij}} \tag{31}$$

$j=1,\ldots,n$, where $T_j(\tau)$ is the temperature of volume #j at time $\tau\in\mathbb{R}$, $T_0(\tau)$ is the ambient temperature outside the railcar, $Q_{ij}(\tau)$ is heat flow due to the temperature difference $T_i(\tau)-T_j(\tau)$ with the neighboring volume #i, S_{ij} is the contact surface area, Q_{uj} is the heat flow of heater #j, K_{ij} is the thermal coefficient that depends on the materials, $C_j=K_c^jV_j$ is the (material dependent) heat capacity coefficient K_c^j times the fluid volume V_j, and L_{ij} is the thickness of the separator between the two volumes #i and #j. We assume that $Q_{ij}(\tau)=0$ for all volumes i, j that are not adjacent, $\forall\tau\in\mathbb{R}$. Hence, the process can be modeled as a linear time-invariant continuous-time system with state vector $z\in\mathbb{R}^{3N_a+2}$ and input vector $v\in\mathbb{R}^{2N_a}$

$$\begin{cases} \dot z(\tau) = A_cz(\tau) + B_cv(\tau) + FT_0(\tau) \\ h(\tau) = Cz(\tau) \end{cases} \tag{32}$$

where $F\in\mathbb{R}^n$ is a constant matrix, $T_0(\tau)$ is treated as a piecewise constant measured disturbance, and $C\in\mathbb{R}^{p\times n}$ is such that $h\in\mathbb{R}^p$ contains the components of z corresponding to the temperatures of the passenger seat areas, $p=2N_a$. Since we assume that the thermal dynamics are relatively slow compared to the sampling time T_s of the decentralized controller we are going to synthesize, we use first-order Euler approximation to discretize dynamics (32) without introducing excessive errors:

$$\begin{cases} z(t+1) = Az(t) + Bv(t) + F_dT_0(t) \\ h(t) = Cz(t) \end{cases} \tag{33}$$

where $A=I+A_cT_s$, $B=B_cT_s$, and $F_d=FT_s$. We assume that A is asymptotically stable, as an inheritance of the asymptotic stability of matrix A_c.

In order to track generic temperature references $r(t)$, we adopt the coordinate shift defined by (30). The next step is to decentralize the resulting global model. The particular topology of the railcar suggests a decomposition of model (1) as the cascade of four-seat areas. There are two kinds of four-seat areas, namely (*i*) the ones next to the antechambers, and (*ii*) the remaining ones. Besides interacting with the external environment, the areas of type (*i*) interact with another four-seat-area, an antechamber, and a section of the corridor, while the areas of type (*ii*) only with the four-seat areas at both sides and the corresponding section of the corridor. Note that the decentralized models overlap, as they share common states and inputs. The decoupling matrices Z_i are chosen so that in each subsystem only the first component of the computed optimal input vector is actually applied to the process.

As a result, each submodel has 5 states and 2 or 3 inputs, depending whether it describes a seat area of type (*i*) or (*ii*), which is considerably simpler than the centralized model (1) with 26 states and 16 inputs. We apply the DMPC approach (9) with

$$Q = \begin{bmatrix} 200I_{16} & 0 \\ 0 & 2I_{10} \end{bmatrix} \quad R = 10^5I_{16}$$
$$v_{\max} = -v_{\min} = 0.03\,\text{W} \quad T_s = 540\,\text{s} \tag{34}$$

where $v_{\min}$ is the lower bound on the heat flow subtracted by the air-conditioners, and $v_{\max}$ is the maximum heating power of the heaters (with a slight abuse of notation we denoted by $v_{\min}$, $v_{\max}$ the entries of the corresponding lower and upper bound vectors of $\mathbb{R}^{16}$). Note that the first sixteen diagonal elements of matrix Q correspond to the temperatures of the four-seat areas. It is easy to check that with the parameters in (34) condition (15a) for local stability is satisfied. For comparison, a centralized MPC approach (3)

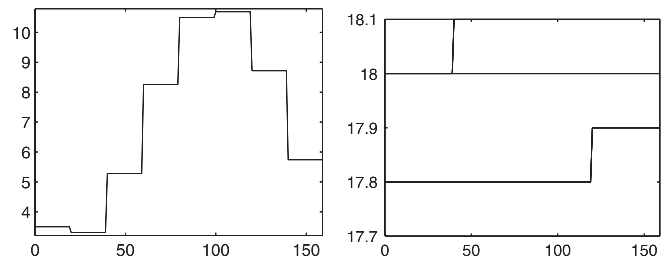

Fig. 3. Exogenous signals used in the reported simulations.

with the same weights, horizon, and sampling time as in (34) is also designed. The associated QP problem has 16 optimization variables and 32 constraints, while the complexity of each DMPC controller is either 2 (or 3) variables and 4 (or 6) constraints. The DMPC approach is in fact largely scalable: for longer railcars the complexity of the DMPC controllers remains the same, while the complexity of the centralized MPC problem grows with the increased model size. Note also that, even if a centralized computation is taken, the DMPC approach can be immediately parallelized.

4.1. Simulation results

We investigate different simulation outcomes depending on four ingredients: *i*) type of controller (centralized / decentralized), *ii*) packet-loss probability, *iii*) change in reference values, *iv*) changes of external temperature (acting as a measured disturbance). Fig. 3 shows the external temperature and reference scenarios, respectively, used in all simulations.

In order to compare closed-loop performances in different simulation scenarios, define the following performance index

$$J=\sum_{t=1}^{N_{sim}} e_z'(t)Qe_z(t)+e_v'(t)Re_v(t) \tag{35}$$

where $e_z(t)=h(t)-r(t)$, $e_v(t)=v(t)-v_r$ and $N_{sim}=160$ (one day) is the total number of simulation steps.

The initial condition is 17° C for all seat-area temperatures, except for the antechamber, which is 15°C. Note that the steady-state value of antechamber temperatures is not relevant for the posed control goals. The closed-loop trajectories of centralized MPC feedback vs. decentralized MPC with no packet-loss are shown in Fig. 4 (we only show the first state and input for clarity). In both cases the temperatures of the four-seat areas converge to the set-point asymptotically. Fig. 5 shows the temperature vector $h(t)$ tracking the time-varying reference $r(t)$ in the absence of packet-loss, where the coordinate transformation (30) is repeated after each set-point and external temperature change.

To simulate packet loss, we assume that the probability of losing a packet depends on the state of a Markov chain with N states (see Fig. 6).

We parameterize with the probability parameter p, $0\leq p\leq 1$ the probabilities associated with the Markov chain: the Markov chain is in the j th state if $j-1$ consecutive packets have been lost. The probability of losing a further packet is $1-p$ in every state of the chain, except for the $(N+1)$ th state where no packet can be lost any more.

Let π be the stationary probability vector of the Markov chain of Fig. 6, obtained through the one-step probability matrix P, obtained

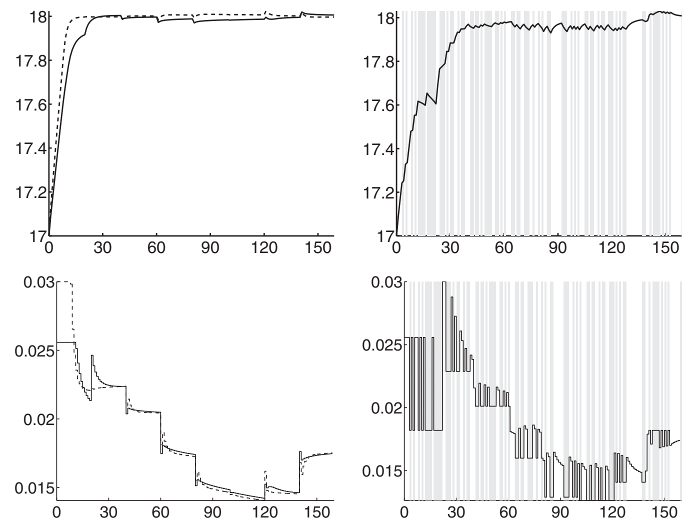

Fig. 4. Comparison between centralized MPC (dashed lines) and decentralized MPC (continuous lines): output h_1 (upper plots) and input v_1 (lower plots). Gray areas denote packet drop intervals.

by solving

$$\begin{cases}\pi'=\pi'P\\ \sum_{i=1}^{N}\pi_i=1\end{cases}\Rightarrow P=\begin{bmatrix}p & 1-p & 0 & \cdots & 0\\ p & 0 & 1-p & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ p & 0 & 0 & \cdots & 1-p\\ 1 & 0 & 0 & \cdots & 0\end{bmatrix}$$

Recalling the meaning of the nodes of the Markov chain, the steady-state probability π_i of the ith state is the probability of losing consecutively exactly $i-1$ packets. The packet-loss discrete probability is shown in Fig. 7 when $N=10$ maximum consecutive packet-losses are possible and $p=0.7$.

Fig. 7 highlights the exponential decrease of the stationary probabilities as a function of consecutive packets lost. Such a probability model is confirmed by the experimental results on relative frequencies of packet failure burst length observed in [28]. Note that our model assumes that the probability of losing a packet is null after N packets, hence satisfying the assumption of an upper-bound on the number of consecutive drops (as mentioned earlier, we can assume

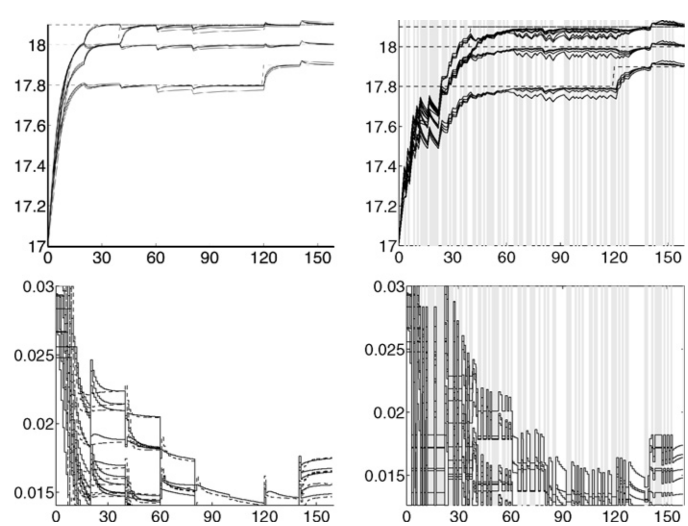

Fig. 5. Decentralized MPC results. Upper plots: output variables h (continuous lines) and references r (dashed lines). Lower plots: command inputs v. Gray areas denote packet drop intervals.

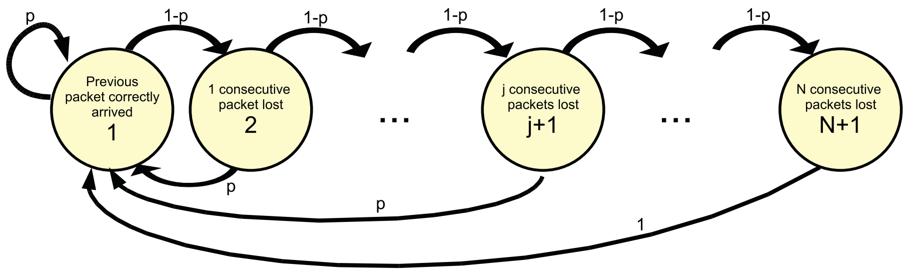

Fig. 6. Markov chain modeling packet-loss probability: the network is in state i if the last $i-1$ packets have been lost.

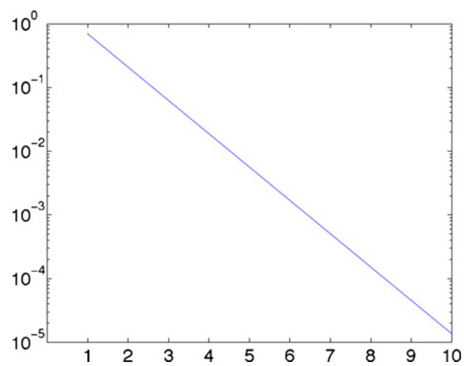

Fig. 7. Markov chain packet-loss probability with $N=10$ and $p=0.7$.

for instance that if $k>N$ consecutive packets are lost, the control loops are shut down). The simulation results obtained with $p=0.5$ are shown in Section 4.1. In case of packet loss, we also compare the performance of centralized vs. decentralized MPC. Note that in case packet loss occurs also on the communication channel between the point computing the coordinate shift and the decentralized controllers, the last received coordinate shift is kept. The stability condition (26) of Theorem 3 was tested and proved satisfied for values of j up to 160.

Fig. 8 shows that the performance index J defined in Eq. (35) increases as the packet-loss probability grows, implying performance to deteriorate due to the conservativeness of the backup control action $u=0$ (that is, $v=v_r$). The results of Fig. 8 are averaged over 10 simulations per probability sample. As a general consideration, centralized MPC dominates over the decentralized, although for certain values of p the average performance of decentralized MPC is slightly better, probably due to the particular packet loss sequences that have realized. However, the loss of performance due to decentralization, with regard to the present example, is largely negligible.

The simulations were run on a MacBook Air 1.86 GHz running Matlab R2008a under OS X 10.5.6 and the Hybrid Toolbox for Matlab [29]. The average CPU time for solving the centralized QP problem associated with (3) is 6.0 ms (11.9 ms in the worst case). For the decentralized case, the average CPU time for solving the QP problem associated with (9) is 3.3 ms (7.4 ms in the worst case). Although the decrease of CPU time is only a few milliseconds, we remark that for increasing N_a the complexity of DMPC remains constant, while the complexity of centralized MPC would grow with N_a. To quantify this aspect consider that, if one thinks to the explicit form of the MPC controllers [23], the number of regions of the centralized MPC is upper bounded by 3^{16}, while in decentralized case by 3^2 for submodels with two inputs and by 3^3 for submodels with three inputs, respectively.

Note that in all simulations the reference vectors v_r, z_r are computed globally by a higher-level control layer in the hierarchical setting. In this example the complexity of such a static calculation is negligible with respect to solving the QP problems. Moreover, the communication burden is also negligible, as new reference vectors are transmitted individually to each MPC agent only when set-point and disturbances change.

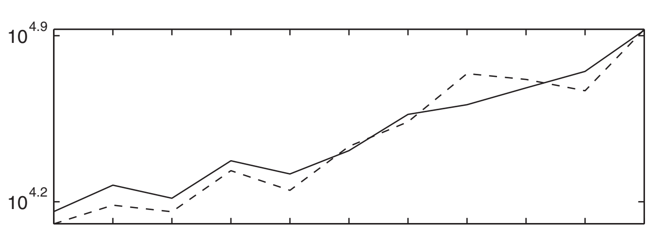

Fig. 8. Performance indices of Centralized MPC (dashed line) and Decentralized MPC (solid line).

5. Conclusions

In this paper we have proposed an approach for controlling large-scale processes subject to input constraints using the cooperation of multiple decentralized model predictive controllers. Each controller is based on a submodel of the overall process, and different submodels may share common states and inputs, to possibly decrease modeling errors in case of dynamical coupling, and to increase the level of cooperativeness of the controllers. The possible loss of global optimal performance is compensated by the gain in controller scalability, reconfigurability, and maintenance. Open research issues remain to be further explored to extend the proposed DMPC scheme, such as: systematic ways to decompose the model into local submodels, when this is not obvious from the physics of the process, determining the optimal model decomposition (i.e., the best achievable closed-loop performance) for a given channel capacity and computer power available to the control agents; hierarchical MPC schemes, in which the DMPC controllers are supervised by a centralized (possibly hybrid) MPC controller running at a slower sampling frequency to enforce global constraints.

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