International Journal of Control
Vol. 85, No. 8, August 2012, 1162–1177

Assessment of non-centralised model predictive control techniques for electrical power networks

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(*Received 11 November 2011; final version received 23 March 2012*)

Model predictive control (MPC) is one of the few advanced control methodologies that have proven to be very successful in real-life applications. An attractive feature of MPC is its capability of explicitly taking state and input constraints into account. Recently, there has been an increasing interest in the usage of MPC schemes to control electrical power networks. The major obstacle for implementation lies in the large scale of these systems, which is prohibitive for a centralised approach. In this article, we therefore assess and compare the suitability of several non-centralised predictive control schemes for power balancing, to provide valuable insights that can contribute to the successful implementation of non-centralised MPC in the real-life electrical power system.

Keywords: model predictive control; decentralised control; distributed control; power systems

1. Introduction

Electrical power networks are among the largest and most complex-engineered systems ever created. In spite of their immense complexity, power systems have shown an impressive level of performance and robustness. One reason for their success is that traditional power systems are characterised by a highly repetitive daily pattern of power flows, with a relatively small amount of suddenly occurring, uncertain fluctuations on the demand side, and with controllable, large power plants on the supply side. As a consequence, in traditional power systems, a large portion of energy production can be efficiently scheduled in an open-loop manner, with automatic generation control (AGC) (Jaleeli, VanSlyck, Ewart, Fink, and Hoffmann 1992; Kundur 1994) providing efficient real-time power balancing of uncertain demands.

However, today, electrical power systems are undergoing two fundamental restructuring processes. Firstly, from a regulated, single-utility controlled operation, the system has been restructured to include many parties that compete for power production and consumption (see, e.g. Stoft (2002) and the references therein). With power markets as the central operational mechanism, competitive economic forces often push the system towards its operational boundaries. Secondly, there has been an increasing integration of small-scale distributed generation (DG), see, e.g. Borbely and Kreider (2001) and the references therein. Since large amounts of DG are expected to be based on renewable, intermittent energy sources, such as wind and sun, future power systems will be characterised by large and unpredictable power fluctuations on the power production side. These observations lead to the conclusion that the preservation of the high performance and robustness levels that were attained in the past will become a major challenge for power system control in the near future.

Various recent papers have observed that model predictive control (MPC) has the potential for solving the problems that will appear in future electrical power networks (Camponogara 2000; Camponogara, Jia, Krogh, and Talukdar 2002; Venkat 2006; Jokić, Lazar, and Van den Bosch 2007; Venkat, Hiskens, Rawlings, and Wright 2008). MPC is capable of handling control problems where off-line computation of a classical control law is difficult, particularly since MPC can explicitly take system constraints into account when computing the control action. Furthermore, MPC allows the use of disturbance models, which can be employed to counteract the fluctuations in power generation introduced by renewable energy sources. For a detailed survey of MPC and

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ISSN 0020–7179 print/ISSN 1366–5820 online

http://dx.doi.org/10.1080/00207179.2012.679972
http://www.tandfonline.com

constrained optimal control, the interested reader is referred to Mayne, Rawlings, Rao, and Scokaert (2000) and Goodwin, Seron, and De Dona (2005).

Yet, the fact that MPC is a centralised control mechanism is a major issue when considering power system operation. Centralised control implies that a single controller is able to measure all the system outputs, compute the optimal control solution, and apply that action to all actuators in the network, within one sampling period. As power networks are large-scale systems, both computationally and geographically, a centralised MPC controller is practically impossible to implement.

The difficulties with centralised predictive control for large-scale systems explain the increasing attention for non-centralised MPC implementations in the control literature (see, for example Camponogara 2000; Camponogara et al. 2002; Keviczky, Borrelli, and Balas 2006; Alessio and Bemporad 2007; Dunbar 2007; Venkat et al. 2008). Roughly speaking, non-centralised MPC schemes can be divided into two categories: *decentralised techniques*, which do not allow for communication between local controllers and *distributed techniques*, where communication between different controllers is exploited to improve the prediction accuracy. Distributed MPC methods can be further categorised as techniques that require communication between all the controllers in the network and techniques that require communication solely with directly neighbouring controllers.

In the literature on non-centralised predictive control, various power system implementations have been illustrated, (Camponogara 2000; Camponogara et al. 2002; Venkat et al. 2008). These methods differ in terms of computational complexity, the extent of communication and the size of the embedded prediction model, and, as a consequence, in terms of performance. In this article, we consider *decentralised MPC* (DMPC; Alessio and Bemporad 2007), *stability-constrained distributed MPC* (SC-DMPC; Camponogara et al. 2002) and *feasible cooperation-based MPC* (FC-MPC; Venkat 2006), all of which represent viable candidates for implementation in power systems. Alternative methods, such as strategies for enforcing constraints that involve the dynamics of multiple control areas (see for instance, Keviczky et al. (2006), Richards and How (2007)) are not discussed, as the literature does not yet give a solid solution for dealing with coupled constraints in a low-complexity and non-conservative fashion. For a discussion on the theoretical issues regarding non-centralised MPC in general, the interested reader is referred to (Camponogara 2000; Camponogara et al. 2002; Keviczky et al. 2006; Alessio and Bemporad 2007; Dunbar 2007; Richards and How 2007; Venkat et al. 2008) and the references therein.

The choice for DMPC, SC-DMPC and FC-MPC is further motivated by our main research goal, which is to study the correlation between the complexity and usefulness of non-centralised MPC schemes and their corresponding attainable performance. DMPC does not require communication and therefore belongs to the decentralised and simplest category of non-centralised MPC. Although, specific implementations of DMPC do exploit an exchange of information between controllers, we will only consider the completely decentralised version in this article, to give an indication of the performance that can be obtained without communication. SC-DMPC and FC-MPC are distributed MPC schemes, as they both employ communication to increase the accuracy of their state predictions. The FC-MPC technique requires communication between all local controllers and uses an iterative procedure to compute the control action, while the SC-DMPC scheme requires communication between directly neighbouring subsystems only. As such, SC-DMPC can be viewed as the outcome of a trade-off between the complexity and performance attainable by DMPC on the one hand, and FC-MPC on the other.

The remainder of this article is organised as follows. Section 2 introduces the (centralised) MPC methodology along with the conditions that are necessary to guarantee closed-loop stability. In Section 3, we describe the non-centralised MPC techniques from an engineering perspective, particularly focusing on the details relevant for controller implementation. Section 4 contains a simulation study of the MPC algorithms under consideration, which is based on a suitably constructed power network example. The non-centralised MPC schemes are compared with centralised MPC and with the classical AGC method that is currently employed in real-life power system control. Given the results of this benchmark test, we discuss the suitability of the considered methods for balancing/frequency control in power networks. We finish by listing the main conclusions in Section 5.

1.1 *Nomenclature*

Let $\mathbb{R}$, $\mathbb{Z}$ and $\mathbb{Z}_+$ denote the field of real numbers, the set of integers and the set of non-negative integers, respectively. For an arbitrary sequence $\mathbf{u} = (u(0), u(1), \ldots)$, we use the notation $\mathbf{u}_{[k]}$ to denote the truncation of $\mathbf{u}$ at $k \in \mathbb{Z}_+$, i.e. $\mathbf{u}_{[k]} := (u(0), u(1), \ldots, u(k))$ with $k \geq 1$. The operator $\mathrm{col}(\cdot, \ldots, \cdot)$ stacks its operands into a column vector and

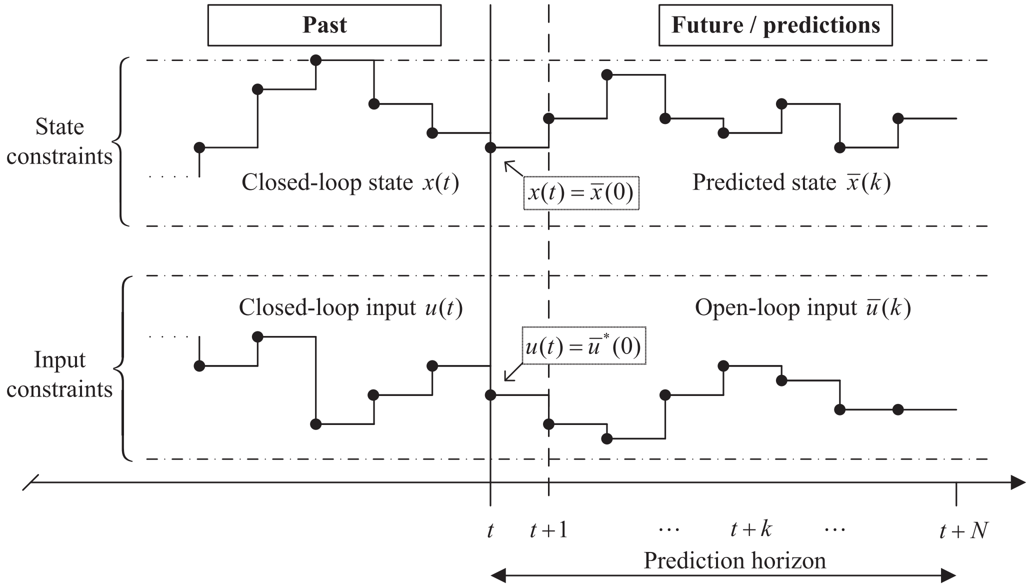

Figure 1. A schematic illustration of model predictive control.

$\mathrm{diag}(M_1, \ldots, M_n)$ denotes a block diagonal matrix with matrices M_i on the main diagonal. The notations $A \succ B$ and $A \succeq B$ denote that A and B are Hermitian and $A - B$ is positive definite or positive semi-definite, respectively.

2. Centralised MPC

The basic principles of MPC are illustrated in Figure 1. In MPC, the control action is computed by solving a finite-horizon open-loop optimal control problem at each sampling instant. The controller employs a model to obtain a prediction of the state evolution over time, given the current state of the controlled system. Only the first sample of the optimising input sequence is applied to the plant, after which the whole process is repeated at the next time instant. This is the main difference with classical control, which commonly uses a pre-computed and fixed feedback law. The unique, distinguishing feature of MPC lies in its ability to compute the control input while explicitly taking input and state constraints into account.

In this article, we consider systems that can be accurately modelled using linear discrete-time state-space representations of the form

$$x(t+1) = Ax(t) + Bu(t), \tag{1}$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $x(t) \in \mathbb{R}^n$ is the state and $u(t) \in \mathbb{R}^m$ is the control input at discrete-time instant $t \in \mathbb{Z}_+$. Note that the number of states n in power networks that consist of thousands of nodes/buses can be very large, which corresponds to high-dimensional A and B matrices.

Let the control input and the predicted state at time instants $t + k \in \mathbb{Z}_+$, given $x(t)$, be denoted by $\bar{u}(k)$ and $\bar{x}(k)$, respectively. Moreover, let $\bar{\mathbf{u}}_{[N-1]} = (\bar{u}(0), \ldots, \bar{u}(N-1))$ be a sequence of control moves, where $N \in \mathbb{Z}_+$ is the prediction horizon. The optimal control problem that the MPC controller solves each sampling instant, is formally defined as follows.

Problem 2.1: At discrete-time instant $t \in \mathbb{Z}_+$ let $x(t)$ and $N \geq 1$ be given, set $\bar{x}(0) := x(t)$ and solve

$$V_N^*(x) = \min_{\bar{\mathbf{u}}_{[N-1]}} \{V_N(x, \bar{\mathbf{u}}_{[N-1]}) \mid \bar{\mathbf{u}}_{[N-1]} \in \mathscr{U}_N(x)\}, \tag{2a}$$

where

$$\begin{aligned} V_N(x, \bar{\mathbf{u}}_{[N-1]}) &= F(\bar{x}(N)) + \sum_{k=0}^{N-1} \ell\,(\bar{x}(k), \bar{u}(k)) \\ &= \bar{x}^\top(N) P \bar{x}(N) \\ &\quad + \sum_{k=0}^{N-1} \bar{x}^\top(k) Q \bar{x}(k) + \bar{u}^\top(k) R \bar{u}(k) \end{aligned} \tag{2b}$$

$$\bar{x}(k+1) = A\bar{x}(k) + B\bar{u}(k), \tag{2c}$$

for $k = 0, \ldots, N-1$.

The matrices $Q = Q^\top \succeq 0$ and $R = R^\top \succ 0$ are suitably chosen performance weights, i.e. tuning parameters, whereas the matrix $P = P^\top \succ 0$ that weights the terminal state is usually computed off-line in such a way that closed-loop stability is guaranteed (Mayne et al. 2000).

The control problem defined by (2a) minimises the quadratic cost function $V_N(x, \bar{\mathbf{u}}_{[N-1]})$ over all input sequences $\bar{\mathbf{u}}_{[N-1]}$ in the set $\mathscr{U}_N(x)$. We assume that $\mathscr{U}_N(x)$ can be defined by a finite number of linear inequalities on u, such that the MPC optimisation problem can be formulated as a quadratic program (QP). The set of feasible input sequences is determined by the constraints on the states and inputs,

$$\mathscr{U}_N(x) := \{\bar{\mathbf{u}}_{[N-1]} \in \mathbb{U}^N \mid \bar{x}(k) \in \mathbb{X}, \\ k = 1, \dots, N-1, \ \bar{x}(N) \in \mathbb{X}_f\}, \quad (3)$$

where $\mathbb{U}^N := \mathbb{U} \times \cdots \times \mathbb{U}$ is the N-times Cartesian product of $\mathbb{U}$. The set of feasible inputs $\mathbb{U}$ is a compact subset of $\mathbb{R}^m$ and $\mathbb{X}$ is a closed subset of $\mathbb{R}^n$. Asymptotic stability of the MPC-controlled system can be guaranteed *a priori* by constraining the terminal state $\bar{x}(N)$ to an appropriately chosen terminal set $\mathbb{X}_f \subseteq \mathbb{X}$ and by using a specific terminal weight P (Mayne et al. 2000). The set $\mathbb{X}_f$ must be positively invariant (see Blanchini (1994)) and should satisfy the following property:

$$\mathbb{X}_f \subseteq \mathcal{O}_\infty := \{x \in \mathbb{R}^n \mid K(A+BK)^k x \in \mathbb{U} \\ \text{and } (A+BK)^k x \in \mathbb{X}, \ k = 0, \dots, \infty\}, \quad (4)$$

where the pair $\{P, K\}$ is obtained as the solution of the unconstrained infinite horizon LQR problem (Mayne et al. 2000), i.e.

$$P = (A+BK)^\top P (A+BK) + K^\top R K + Q, \quad (5a)$$

$$K = -(R + B^\top P B)^{-1} B^\top P A. \quad (5b)$$

After solving Problem 2.1, the controller applies the first element of the optimal input sequence $\bar{\mathbf{u}}^*_{[N-1]}$ to the system, i.e. $u(t) := \bar{u}^*(0)$, and discards the rest of the sequence. At the next time instant, the state of the system is measured and the procedure described above is repeated. This so-called receding horizon strategy introduces a closed-loop feedback mechanism to increase robustness.

3. Non-centralised MPC

As explained in Section 1, a centralised implementation of MPC is not well-suited for control of power networks due to the complexity and size of these systems. In this section, we describe three less complex non-centralised MPC techniques that are more appropriate for power system control. We start by introducing the basic notions and definitions used in the description of these algorithms.

Consider a power network that is represented by the directed graph $\mathcal{G} = (\mathcal{S}, \mathcal{E})$, with a finite number of vertices $\mathcal{S} = \{\varsigma_1, \dots, \varsigma_M\}$ and a set of directed edges $\mathcal{E} \subseteq \{(\varsigma_i, \varsigma_j) \in \mathcal{S} \times \mathcal{S} \mid i \neq j\}$. We can model the dynamics of the power network by assigning a dynamical system to each vertex $\varsigma_i \in \mathcal{S}$, with the dynamics governed by,

$$x_i(k+1) = f_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k))), \quad (6)$$

for $k \in \mathbb{Z}_+$ and $i \in \mathcal{I} := \{1, \dots, M\}$. Here, $x_i \in \mathbb{X}_i \subseteq \mathbb{R}^{n_i}$, $u_i \in \mathbb{U}_i \subseteq \mathbb{R}^{m_i}$ are the state and control input of the ith subsystem. We assume that the feasible input and state sets, $\mathbb{U}_i$ and $\mathbb{X}_i$ respectively, are polytopic, such that they can be described by a finite number of affine inequalities. With each edge $(\varsigma_i, \varsigma_j) \in \mathcal{E}$, we associate a function $v_{ij}: \mathbb{R}^{n_j} \to \mathbb{R}^{n_i}$ that defines the interconnection signal $v_{ij}(x_j(k))$ between subsystem j and i. We use $\mathcal{N}_i := \{j \mid (\varsigma_i, \varsigma_j) \in \mathcal{E}\}$ to denote the set of indices corresponding to the neighbours of subsystem i. The term *neighbour* of system i defines any system in the network whose dynamics appear explicitly (via the function $v_{ij}(\cdot)$) in the state equation that describes the dynamics of subsystem i. If system j is a neighbour of system i, in general this does not necessarily imply the reverse. Moreover, let $x_{\mathcal{N}_i}(k) := \mathrm{col}(\{x_j(k)\}_{j \in \mathcal{N}_i})$ be the vector that collects all the state vectors of the neighbours of system i and $v_i(x_{\mathcal{N}_i}) := \mathrm{col}(\{v_{ij}(x_j(k))\}_{j \in \mathcal{N}_i})$ be the vector-valued interconnection signals that enter system i.

3.1 *Decentralised MPC*

The DMPC technique (Alessio and Bemporad 2007) exploits the fact that many large-scale systems, such as power networks, consist of several subsystems (or control areas) that are only loosely coupled. As a consequence, these systems can be modelled by sparse state-space representations. In DMPC, the global state-space model is approximated via a state and input matrix partitioning that defines a set of M *decoupled* prediction models. Correspondingly, the DMPC controller equals the ensemble of M local MPC controllers that are independently designed for each subsystem.

Let the large-scale system that is to be controlled be described by the discrete-time state-space model given in (1). The division into M subsystems employed in DMPC is based on an explicit transformation via suitably defined matrices W_i and Z_i, $i \in \mathcal{I} := \{1, \dots, M\}$. These matrices collect the states and inputs assigned to subsystem i:

$$x_i = W_i^\top x, \quad u_i = Z_i^\top u, \quad (7a)$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{m_i}$. The corresponding local, decoupled prediction models are given by

$$\bar{x}_i(k+1) = A_i \bar{x}_i(k) + B_i \bar{u}_i(k) \quad (7b)$$

$$A_i = W_i^\top A W_i, \quad B_i = W_i^\top B Z_i, \tag{7c}$$

for $i \in \mathcal{I}$, where $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m_i}$.

Note that W_i and Z_i are such that, by (7a), each element of x is assigned to one or more x_i and each element of u is assigned to one or more u_i. This means that overlapping subsystems are allowed. However, in this article, we will consider DMPC with non-overlapping partitions only, i.e. we restrict our attention to the cases where each element of x is assigned to a unique x_i and each element of u is assigned to a unique u_i. Although DMPC performance is expected to improve with increasing subsystem overlap, the use of non-overlapping partitions is attractive as this requires no communication between subsystems. For more information about the construction of the partitioning matrices, and for details on handling overlapping inputs in particular, the reader is referred to Alessio and Bemporad (2007).

In contrast to centralised MPC, the DMPC control scheme assigns a controller to each subsystem i, which solves the following finite-horizon problem, at each sampling instant:

Problem 3.1 (DMPC): At discrete-time instant $t \in \mathbb{Z}_+$ let $x_i(t)$ and $N \geq 1$ be given, set $\bar{x}_i(0) := x_i(t)$ and solve

$$V_{i,N}^*(x_i) = \min_{\bar{\mathbf{u}}_{i,[N-1]}} \{V_{i,N}(x_i, \bar{\mathbf{u}}_{i,[N-1]}) \mid \bar{\mathbf{u}}_{i,[N-1]} \in \mathcal{U}_{i,N}(x_i)\}, \tag{8a}$$

where

$$\begin{aligned} V_{i,N}&(x_i, \bar{\mathbf{u}}_{i,[N-1]}) \\ &= F_i(\bar{x}_i(N)) + \sum_{k=0}^{N-1} \ell_i(\bar{x}_i(k), \bar{u}_i(k)) \\ &= \bar{x}_i^\top(N) P_i \bar{x}_i(N) \\ &\quad + \sum_{k=0}^{N-1} \bar{x}_i^\top(k) Q_i \bar{x}_i(k) + \bar{u}_i^\top(k) R_i \bar{u}_i(k) \end{aligned} \tag{8b}$$

$$\bar{x}_i(k+1) = A_i \bar{x}_i(k) + B_i \bar{u}_i(k), \quad k = 0, \ldots, N-1, \tag{8c}$$

where the penalty matrices used in each cost function, given the weights of a centralised controller, are $Q_i = W_i^\top Q W_i = Q_i^\top \succeq 0$, $R_i = Z_i^\top R Z_i = R_i^\top \succ 0$ and P_i, which will be specified below.

Problem 3.1 minimises the local quadratic cost over input sequences in the set

$$\mathcal{U}_{i,N}(x_i) := \{\bar{\mathbf{u}}_{i,[N-1]} \in \mathbb{U}_i^N\}, \tag{9}$$

where $\mathbb{U}_i^N := \mathbb{U}_i \times \cdots \times \mathbb{U}_i$ is the N-times Cartesian product of the set of feasible local inputs. We assume that $\mathcal{U}_{i,N}$ is a polytope, i.e. it can be described by a finite number of affine inequalities, such that we can formulate Problem 3.1 as a QP.

When all M controllers have calculated the optimal local control action sequence $\bar{\mathbf{u}}_{i,[N-1]}^*$, the ensemble of all local inputs, i.e.

$$u(t) = \mathrm{col}(\bar{u}_1^*(0), \ldots, \bar{u}_i^*(0), \ldots, \bar{u}_M^*(0)), \tag{10}$$

is applied to the global system (1) and the whole procedure is repeated at the next time instant.

Unlike centralised MPC, the DMPC algorithm does not take (coupled) state constraints into account. However, note that in the case of frequency control in power networks, state constraints such as bounds on tie-line flows can be essential to guarantee safe operation.

Moreover, as described in Section 2, one can specifically design centralised MPC to provide an *a priori* guarantee for closed-loop stability, based on a terminal penalty and terminal state conditions. However, these conditions only apply in the case of centralised MPC. Non-centralised predictive controllers, such as DMPC, exploit modified stabilisation conditions, that possibly yield a weaker guarantee for closed-loop stability. In DMPC, an *a priori* guarantee of stability for each *decoupled* subsystem can be obtained by defining the terminal penalty matrix P_i for each subsystem i as

$$P_i = (A_i + B_i K_i)^\top P_i (A_i + B_i K_i) + K_i^\top R_i K_i + Q_i, \tag{11a}$$

$$K_i = -(R_i + B_i^\top P_i B_i)^{-1} B_i^\top P_i A_i, \tag{11b}$$

and constraining the terminal state $\bar{x}_i(N)$ to an invariant (polytopic) terminal set

$$\begin{aligned} \mathbb{X}_{f_i} \subseteq \{x \in \mathbb{R}^{n_i} \mid K_i(A_i + B_i K_i)^k x \in \mathbb{U}_i \\ \text{and } (A_i + B_i K_i)^k x \in \mathbb{X}_i,\ k = 0, \ldots, \infty\}, \end{aligned} \tag{12}$$

where $\mathbb{X}_i$ is the set of feasible local states. If $K_i \equiv 0$, as in Alessio and Bemporad (2007), then (11a) reduces to the Lyapunov equation. Note that because $Q_i \succ 0$, this implies that each subsystem has to be open-loop stable, i.e. that all eigenvalues of A_i must be within the unit circle.

Nonetheless, observe that condition (11) only implies closed-loop stability under the assumption that the subsystems are indeed decoupled. Still, it is possible to provide *a posteriori* verifiable stability conditions for the network under coupled operation, as shown in Alessio and Bemporad (2007). More precisely, the proposed stability test checks stability of the entire system (1) in closed loop with (10), if the matrices P_i are chosen according to (11a). This *a posteriori* stability test checks whether the sum of all cost functions is a Lyapunov function for the overall system, and is based on the explicit form of each MPC controller (Bemporad, Morari, Dua, and Pistikopoulos 2002). Under certain conditions, this reduces to a positive semi-definiteness check of a square $n \times n$ matrix. However, this test has to be carried out on a centralised level, which partly cancels out the attractive features of DMPC's decentralised structure.

The main attractive feature of the DMPC scheme is that each local controller has to solve relatively small and simple optimisation problems, corresponding to low computational requirements per subsystem. However, it is important to observe that the DMPC cost function of subsystem i solely depends on the local states and inputs $x_i(t)$ and $\bar{\mathbf{u}}_{i,[N-1]}$, respectively. This is a consequence of the fact that the DMPC prediction model (7) approximates the real system by ignoring the dynamic coupling between subsystems and uses only local state information to initialise the optimisation problem. Therefore, (10) will, in general, not be optimal with respect to the centralised MPC optimisation problem (2) unless $x \equiv x_i$ and $u \equiv u_i$, for all i. If the dynamic coupling between the subsystems in (1) is strong, the prediction mismatch can be large, resulting in a significant loss of performance compared with that attained by a centralised controller.

3.2 *Stability-constrained distributed MPC*

The SC-DMPC scheme (Camponogara et al. 2002) is a distributed predictive control method, in which each local controller exploits communication with neighbouring subsystems to improve the accuracy of its local state predictions. As such, SC-DMPC is expected to outperform decentralised control schemes that neglect dynamic coupling and that do not exploit communication at all.

The SC-DMPC scheme requires that the dynamics of the system to be controlled are given by (1), with state-space matrices that have the structure:

$$A = \begin{bmatrix} A_{11} & \dots & A_{1M} \\ \vdots & \ddots & \vdots \\ A_{M1} & \dots & A_{MM} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_{MM} \end{bmatrix}, \tag{13}$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $A_{ii} \in \mathbb{R}^{n_i \times n_i}$, $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, $B_{ii} \in \mathbb{R}^{n_i \times m_i}$, $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$. Note that B is block diagonal, such that input u_i only affects subsystem i directly. Correspondingly, the neighbours of subsystem i are those systems for which $A_{ij} \neq 0$, $j \neq i$. The set of neighbours of system i is denoted by $\mathcal{N}_i = \{j \in \mathcal{I} \,|\, j \neq i, A_{ij} \neq 0\}$.

Let $N \geq 1$ be a fixed prediction horizon. At all discrete-time instants $t \in \mathbb{Z}_+$, each SC-DMPC controller solves the following optimisation problem:

Problem 3.2 (SC-DMPC)**:** At discrete-time instant $t \in \mathbb{Z}_+$, let $x_i(t)$ and $\bar{x}_j^N(k) := v_{ij}(x_j(k))$ for $k = 1, \dots, N-1$ and all $j \in \mathcal{N}_i$ be given. Set $\bar{x}_i(0) := x_i(t)$ and solve

$$V_{i,N}^*(x_i) = \min_{\bar{\mathbf{u}}_{i,[N-1]}} \{V_{i,N}(x_i, \bar{\mathbf{u}}_{i,[N-1]}) \mid \bar{\mathbf{u}}_{i,[N-1]} \in \mathcal{U}_{i,N}(x_i)\}, \tag{14a}$$

where

$$\begin{aligned} V_{i,N}(x_i, \bar{\mathbf{u}}_{i,[N-1]}) &= F_i(\bar{x}_i(N)) + \sum_{k=0}^{N-1} \ell_i(\bar{x}_i(k), \bar{u}_i(k)) \\ &= \bar{x}_i^\top(N) P_i \bar{x}_i(N) \\ &\quad + \sum_{k=0}^{N-1} \bar{x}_i^\top(k) Q_i \bar{x}_i(k) + \bar{u}_i^\top(k) R_i \bar{u}_i(k) \end{aligned} \tag{14b}$$

$$\bar{x}_i(k+1) = A_{ii}\bar{x}_i(k) + B_{ii}\bar{u}_i(k) + \sum_{j \in \mathcal{N}_i} A_{ij}\bar{x}_j^N(k), \tag{14c}$$

for $k = 0, \dots, N-1$.

The weight matrices used in the cost function, $Q_i \succeq 0$, $R_i \succ 0$ and $P_i \succ 0$, can be chosen based on a corresponding centralised problem, in a way that is analogous to the DMPC approach. Moreover, note that the SC-DMPC controllers take the dynamic coupling with their neighbours into account by including the state predictions of these subsystems, denoted by $\bar{x}_j^N(k)$, in their local model. However, as the state predictions of the neighbours are yet to be determined at instant t, the shifted predictions of the previous time instant $t-1$ are used instead:

$$\bar{x}_j^N(k) := \bar{x}_j^*(k+1|t-1), \quad k = 0, \dots, N-1, \tag{15}$$

where $\bar{x}_j^*(k+1|t-1)$ denotes the predicted state for time $t+k$, which is computed at subsystem j given the local state measurement $x_j(t-1)$.

Problem 3.2 minimises the cost over input sequences in the set

$$\mathcal{U}_{i,N}(x_i) := \{\bar{\mathbf{u}}_{i,[N-1]} \in \mathbb{U}_i^N \mid \ \|\bar{x}_i(1)\|_2^2 \leq \hat{l}_i\}, \tag{16}$$

where

$$\hat{l}_i := \max\left\{\|\bar{x}_i(1|t-1)\|_2^2, \|\bar{x}_i(0)\|_2^2\right\} - \beta_i \|x_i^1(0)\|_2^2, \tag{17}$$

with tuning parameter $0 < \beta_i < 1$, and

$$\bar{x}_i(1|t-1) := A_{ii}\bar{x}_i(0) + B_{ii}\bar{u}_i(0) + \sum_{j \in \mathcal{N}_i} A_{ij}\bar{x}_j^N(0), \tag{18a}$$

$$\bar{x}_i(0) := x_i(t-1), \quad \bar{u}_i(0) := \bar{u}_i^*(t-1). \tag{18b}$$

Here, $x_i^1(0)$ is obtained from $x(t)$ via a similarity transformation that is based on the controllable companion form (Camponogara et al. 2002). It is shown in Camponogara et al. (2002), that any $\mathbf{u}_{i,[N-1]} \in \mathscr{U}_{i,N}(x_i)$ stabilises the *local, decoupled* closed-loop system. This is a result of the contractive constraint on the state used in the definition of $\mathscr{U}_{i,N}(x_i)$.

When all M controllers have calculated their optimal local control input sequences $\bar{\mathbf{u}}^*_{i,[N-1]}$, the collection of all local inputs, i.e.

$$u(t) = \mathrm{col}(\bar{u}_1^*(0), \ldots, \bar{u}_i^*(0), \ldots, \bar{u}_M^*(0)), \tag{19}$$

is applied to the global system. Subsequently, all neighbouring controllers exchange their shifted state predictions, after which the whole procedure is repeated at the next time instant.

Except for local-state contraction constraint (16), the SC-DMPC scheme does not take state constraints into account. In Camponogara et al. (2002), it is proven that the construction of $\mathscr{U}_{i,N}(x_i)$, based on a controllable companion form, ensures the existence of control actions that satisfy (16). In addition, it is proven that (19) comprises a feasible solution for the overall system. This is the case even though the attained prediction mismatch of SC-DMPC will not be zero due to the delayed and possibly inaccurate coupling information. Still, the fact that SC-DMPC lacks the possibility to include physical constraints that span multiple subsystems limits its applicability for control of power networks.

As observed above, the contraction constraint guarantees stability if the subsystems are decoupled, since it enforces a strict decrease of the 2-norm of subsequent one-step-ahead subsystem state predictions. However, to conclude stability of the *overall* system, additional conditions on stability of a suitably defined full-state matrix A in a controllable companion form are required. More details on feasibility and stability of the SC-DMPC scheme can be found in Camponogara et al. (2002).

SC-DMPC relies on a communication network to exchange information between neighbouring controllers, in contrast to DMPC. However, note that certain large-scale systems, such as power networks, consist of subsystems that are only loosely coupled, such that the number of neighbours per subsystem is small and the extent of communication is limited. Because SC-DMPC controllers communicate with direct neighbours only, the graph of the required communication network coincides with the interconnection graph $\mathcal{G}$ of the underlying system. For control of power networks, this implies that control areas that are not directly physically coupled do not require a communication link. Because tie-lines are always equipped with a parallel communication link, this is an attractive feature of the SC-DMPC scheme compared to control methods that require global communication, such as centralised MPC.

3.3 *FC-based MPC*

The DMPC and SC-DMPC controllers described in the previous sections solve locally different optimisation problems. Such competitive strategies converge to Nash equilibria at best. Nash equilibria do not necessarily coincide with the global (Pareto) optimum attained by a centralised control scheme. Moreover, there are examples where these Nash equilibria are unstable, such that competitive optimisation algorithms are divergent (Camponogara 2000). The FC-MPC method (Venkat 2006; Venkat et al. 2008) on the other hand, cooperatively solves a *global* optimisation problem, thus, ensuring that the resulting equilibrium is stable and Pareto optimal. This is an attractive feature of FC-MPC over the DMPC and SC-DMPC schemes, although this comes at the cost of more extensive communication requirements.

Let the system to be controlled be of the form given in (1). In FC-MPC, a controller is assigned to each subsystem $i \in \mathcal{I}$. Because these controllers are able to optimise the global cost over their own local-manipulated variables (i.e. local control inputs) only, an iterative procedure that involves optimisation and communication is used to obtain the globally optimal solution. A convenient choice for a global objective that measures the systemwide impact of local control actions is a strict convex combination of local cost functions, i.e. $V_N^p(\cdot) = \sum_{i=1}^M w_i V_{i,N}(\cdot), w_1 > 0, \sum_{i=1}^M w_i = 1$. Now, we can define the FC-MPC optimisation problem of each controller i as:

Problem 3.3 (FC-MPC): At time $t \in \mathbb{Z}_+$ and iteration $p \in \mathbb{Z}_+$, let $\bar{\mathbf{u}}_{j,[N-1]}$ for $j \neq i$ and a fixed prediction horizon $N \geq 1$ be given, set $\bar{x}^p(0) := x(t)$ and solve

$$V_{i,N}^{p*}(x, \bar{\mathbf{u}}^p_{i,[N-1]}) = \min_{\bar{\mathbf{u}}^p_{i,[N-1]}} \{V_N^p(x, \bar{\mathbf{u}}^p_{i,[N-1]}) \mid \bar{\mathbf{u}}^p_{i,[N-1]} \in \mathscr{U}_{i,N}(x)\}, \tag{20a}$$

where

$$V_N^p(x, \bar{\mathbf{u}}_{i,[N-1]}^p) = F(\bar{x}^p(N)) + \sum_{k=0}^{N-1} \ell(\bar{x}^p(k), \bar{u}_i^p(k)) = \bar{x}^{p\top}(N) P \bar{x}^p(N) + \sum_{k=0}^{N-1} \bar{x}^{p\top}(k) Q \bar{x}^p(k) + \bar{u}_i^{p\top}(k) R_i \bar{u}_i^p(k), \quad (20b)$$

$$\bar{x}^p(k+1) = A\bar{x}^p(k) + B\,\mathrm{col}(\bar{u}_1^p(k), \ldots, \bar{u}_i^p(k), \ldots, \bar{u}_M^p(k)), \quad (20c)$$

for $k = 1, \ldots, N-1$.

The FC-MPC controller of subsystem i minimises the global cost function $V_N^p(\cdot)$ over the polytopic set of feasible local input sequences $\bar{\mathbf{u}}_{i,[N-1]}^p \in \mathcal{U}_{i,N}(x)$, which is defined as

$$\mathcal{U}_{i,N}(x) := \{\bar{\mathbf{u}}_{i,[N-1]}^p \in \mathbb{U}_i^N\}. \quad (21)$$

The terminal penalty matrix P used in (20b) is the solution of the unconstrained infinite horizon LQR problem,

$$P = (A + BK)^\top P(A + BK) + K^\top RK + Q, \quad (22a)$$

$$K = -(R + B^\top PB)^{-1} B^\top PA. \quad (22b)$$

Note that in Venkat et al. (2008), attention is restricted to open-loop stable systems, such that K is chosen equal to zero, yielding the stability condition $P = A^\top PA + Q$.

Given the parameters $\varepsilon > 0$, $w_i \in \mathbb{R}_{(0,1)}$ and $p_{\max} \in \mathbb{Z}_+$, at each discrete time instant t, the optimal control action is calculated in each controller via the following iterative procedure:

Algorithm 1 (FC-MPC)**:**

- Initialise the iteration counter $p := 0$.
- Measure the current local state $x_i(k)$ and exchange this information with all other controllers;
- Initialise the local input sequence $\bar{u}_{i,[N-1]}^0(k) := \bar{u}_{i,[N-1]}^{\bar{p}*}(k+1|t-1)$ for $k = 1, \ldots, N-1$ and $i = 1, \ldots, M$;

while $(\rho_i > \varepsilon\ \&\ p \le p_{\max})$

- Solve Problem 3.3 and let $\bar{\mathbf{u}}_{i,[N-1]}^{p,*}(k)$ be the local optimiser;
- Set $\mathbf{u}_{i,[N-1]}^{p,*}(k) := w_i \bar{\mathbf{u}}_{i,[N-1]}^{p,*}(k) + (1 - w_i)\bar{\mathbf{u}}_{i,[N-1]}^{p-1,*}(k)$.
- Set $\rho_i = \|\bar{\mathbf{u}}_{i,[N-1]}^{p,*} - \bar{\mathbf{u}}_{i,[N-1]}^{p-1,*}\|$;
- Exchange the local optimising input sequence $\mathbf{u}_{i,[N-1]}^{p,*}(k)$ with all other controllers;
- The iteration counter is increased by one: $p := p + 1$;

end

- Set $\bar{p}(t) := p$.

Whenever the stop criterion is satisfied in all nodes for some $p = \bar{p} \le p_{\max}$, the first element of the calculated control sequence is applied to the subsystem, i.e.

$$u(t) = \mathrm{col}(\bar{u}_1^{\bar{p}}(0), \ldots, \bar{u}_i^{\bar{p}}(0), \ldots, \bar{u}_M^{\bar{p}}(0)). \quad (23)$$

Then, the procedure is repeated at the next time instant $k+1$.

The FC-MPC algorithm starts by initialising the current state and the global input trajectory, using the shifted optimal input sequence of the previous time instant $t-1$ as the initial guess. Based on this information, each controller computes the new optimising control input. A weighted average of the current optimiser and the input computed at the previous iteration $p-1$ is used as the next estimate of the control input. This is required to ensure convergence over iterates (Venkat 2006).

FC-MPC takes only local input constraints, thus no state constraints, into account. As a consequence, existence of a feasible sequence for Problem 3.3 is guaranteed. It is possible to prove convergence of the iterative procedure, and to prove that FC-MPC control is globally stabilising. In fact, only a single iteration of the algorithm is required to guarantee closed-loop stability (Venkat 2006).

Both centralised MPC and FC-MPC require knowledge of the global state in order to guarantee Pareto optimal performance, implying reliance on extensive communication. However, note that in the FC-MPC scheme, this information has to be communicated to a possibly large number of local controllers, whereas in the case of centralised MPC, this information is required at one location only. In both cases, the communication distances can be very large, due to the large geographical scale of power systems. Moreover, note that the implementations of the centralised MPC, SC-DMPC and DMPC with overlapping subsystems utilise the communication network only once per discrete-time sample, whereas the FC-MPC scheme, in general, requires information exchange for each iteration. However, it is proven in Venkat et al. (2008) that the FC-MPC algorithm can be terminated prior to convergence, without compromising feasibility or closed-loop stability. We can therefore conclude that the iterative nature of the FC-MPC scheme is not necessarily a drawback compared to other, non-iterative communication-based algorithms.

4. Benchmark test

The balancing or load-frequency control problem in electrical power networks provides a suitable benchmark test for the assessment and comparison of the non-centralised MPC schemes studied in this article. Before presenting the simulation results, we describe the test setup in the next subsection.

4.1 *Test network and simulation scenario*

Our simulations were performed on the power network setup given in Venkat et al. (2008). A schematic representation of this test system is depicted in Figure 2. The system consists of four control areas, and the linearised dynamics of each area are given by the following standard model (Kundur 1994):

$$\frac{\mathrm{d}\Delta\omega_i}{\mathrm{d}t}=\frac{1}{J_i}\left(\Delta P_{M_i}-D_i\Delta\omega_i-\sum_{j\in\mathcal{N}_i}\Delta P^{ij}_{\mathrm{tie}}-\Delta P_{L_i}\right), \quad (24a)$$

$$\frac{\mathrm{d}\Delta P_{M_i}}{\mathrm{d}t}=\frac{1}{\tau_{T_i}}(\Delta P_{V_i}-\Delta P_{M_i}), \quad (24b)$$

$$\frac{\mathrm{d}\Delta P_{V_i}}{\mathrm{d}t}=\frac{1}{\tau_{G_i}}\left(\Delta P_{\mathrm{ref_i}}-\Delta P_{V_i}-\frac{1}{r_i}\Delta\omega_i\right), \quad (24c)$$

$$\frac{\mathrm{d}\Delta P^{ij}_{\mathrm{tie}}}{\mathrm{d}t}=b_{ij}(\Delta\omega_i-\Delta\omega_j), \quad (24d)$$

$$\Delta P^{ji}_{\mathrm{tie}}=-\Delta P^{ij}_{\mathrm{tie}}. \quad (24e)$$

Here, (24a)–(24c) describe the dynamics of a generator (or the lumped equivalent of multiple generators in a control area), whereas the dynamics of a transmission line connecting two generators/control areas are modelled by (24d) and (24e). These 'building blocks' are schematically depicted in Figure 3. Note that the control input to subsystem i is the signal $\Delta P_{\mathrm{ref_i}}$, which represents the change in the reference value for the power production in that area. The exogenous disturbance input ΔP_{L_i} represents the aggregated change of the power demand in control area i.

In our benchmark test, we compared the performance of the described non-centralised control schemes with the results attained using a conventional AGC controller. The classical AGC method used in the current power networks consists of local proportional-integral feedback controllers that drive the frequency $\Delta\omega_i$ and the transmission-line power flow deviations $\Delta P^{ij}_{\mathrm{tie}}$ to zero. The feedback controller for area i is described by

$$\frac{\mathrm{d}\Delta P_{\mathrm{ref_i}}}{\mathrm{d}t}=-K_i\left(B_i\Delta\omega_i+\sum_{j\in\mathcal{N}_i}\Delta P^{ij}_{\mathrm{tie}}\right), \quad (25)$$

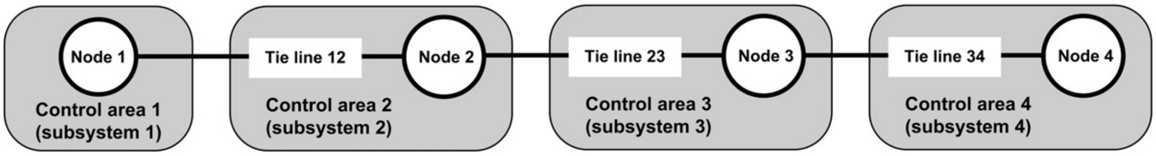

Figure 2. Schematic representation of a power network consisting of four control areas.

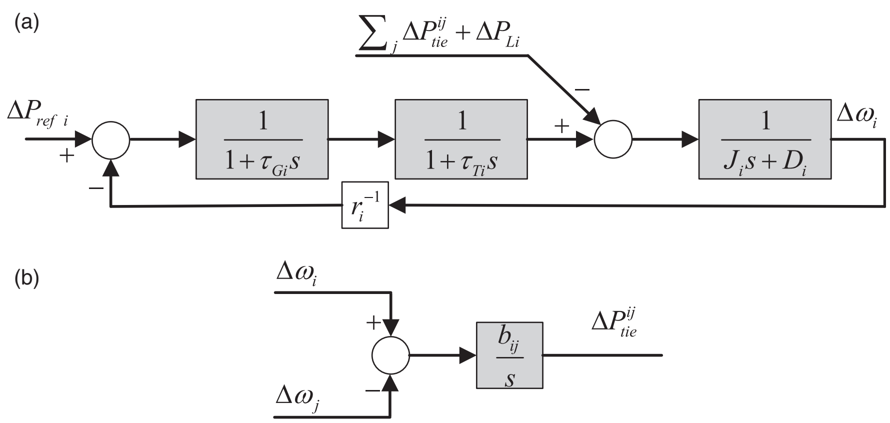

Figure 3. Block diagrams that model the linearised dynamics of a generator (a) and a tie-line (b).

with tuning parameters K_i and B_i. The interested reader is referred to Kundur (1994) and Jaleeli et al. (1992) for a more detailed discussion on classical AGC.

The simulation scenario used to assess the closed-loop performance of the described control methods was the following. For $t < 10$, the network was in steady-state with frequency and tie-line flow deviations equal to zero, and $\Delta P_{L_i} = 0$ for $i = 1, \ldots, 4$. For $t \geq 10$, control area 2 was subjected to a step disturbance of $\Delta P_{L_2} = 0.25$, while a simultaneous step disturbance of $\Delta P_{L_3} = -0.25$ affected control area 3.

For all control techniques, we used identical model and simulation parameter values, which are listed in the Appendix. The optimisation problems for all the assessed MPC schemes were formulated as QPs of the form

$$\min_v \; v^\top H v + f^\top v, \tag{26a}$$

$$\text{subject to } A_{\text{ineq}} v \leq B_{\text{ineq}}, \tag{26b}$$

with $v \in \mathbb{R}^{n_v}$, positive definite $H \in \mathbb{R}^{n_v \times n_v}$, $f \in \mathbb{R}^{n_v}$, $A_{\text{ineq}} \in \mathbb{R}^{n_c \times n_v}$ and $B_{\text{ineq}} \in \mathbb{R}^{n_c}$. All QPs were evaluated using Matlab's quadprog solver. Note that we employed the 1-norm in (16) to allow for a linear formulation of this contraction constraint, and thus, to enable a QP-based implementation of SC-DMPC. The number of iterations of the FC-MPC algorithm was fixed to 2. In all non-centralised schemes, the global prediction model was partitioned according to the physical control area structure, which is a natural choice to obtain a low extent of coupling between the local models.

Finally, note that so far, we assumed that the prediction models for the various methods do not explicitly account for exogenous disturbances, e.g. aggregated load changes ΔP_{L_i}. However, in the simulations, we used local state perturbed models of the form

$$\bar{x}(k+1) = A\bar{x}(k) + B\bar{u}(k) + \bar{d}_0, \quad k = 0, \ldots, N-1,$$

where $\bar{d}_0 := \bar{d}(t)$ is an estimate of a constant additive disturbance, e.g. the aggregated load ΔP_{L_i}, given the measured and predicted state for discrete-time instant t. The inclusion of this disturbance model makes the state predictions more accurate, as constant load disturbances can be compensated for, whereas the stability and feasibility properties of the non-centralised algorithms discussed in Section 3 are preserved. The interested reader is referred to Muske and Badgwell (2002) and Pannocchia and Rawlings (2003) for further details on disturbance estimation and zero-offset tracking in MPC.

4.2 *Simulation results*

The main simulation results are given in Figures 4 and 5. Figure 4 shows the closed-loop trajectories for centralised MPC and classical AGC control, whereas the results obtained with the non-centralised MPC schemes are given in Figure 5. Both figures show the trajectories of network frequency deviation $\Delta\omega_2$ and tie-line power flow deviation $\Delta P_{\text{tie}}^{23}$, together with the control inputs applied to subsystems 2 and 3, i.e. ΔP_{ref_2} and ΔP_{ref_3}, respectively.

Table 1 lists the settling times[1] of the penalised states, i.e. the states for which the corresponding elements in Q_i are nonzero (see the Appendix), and the global performance cost over 200 samples, namely the value of $\sum_{t=0}^{200} x(t)^\top Q x(t) + u(t)^\top R u(t)$.

The results show that for this particular scenario, the centralised MPC scheme outperforms all the other simulated control methods. By contrast, the classical AGC structure is characterised by the worst performance in terms of cost, settling time and overshoot; all the assessed non-centralised MPC schemes perform better than AGC. Moreover, note that the performance of the non-centralised control techniques appears to be directly correlated with the extent of inter-subsystem communication. The observed difference in the DMPC and SC-DMPC performance costs is relatively small, however, which is surprising given their significantly different communication requirements. Finally, note that the FC-MPC performance is almost identical to that of the centralised MPC controller, in spite of the fact that the number of FC-MPC iterations was fixed to only 2.

The computational complexity of each predictive control scheme can be expressed in terms of the dimensions of the corresponding local optimisation problems. The computational burden for the control hardware depends on the number of manipulated variables n_v and the number of inequality constraints n_c. These values are listed in Table 2, for the considered simulation and for the general case (with prediction horizon N, number of local control inputs m_i and number of local states n_i). Table 2 shows that the complexity of the local DMPC, SC-DMPC and FC-MPC controllers is *independent of the number of subsystems present in the network*, whereas this is not the case for the centralised MPC controller, where the optimisation problem scales quadratically with the total number of system inputs $\sum_i m_i$. This is a key motivation for research in the field of non-centralised predictive power network control, as scalability is an important aspect in light of the large and expanding character of today's power system.

Note that computational complexity can also be assessed by measuring the worst-case time that is

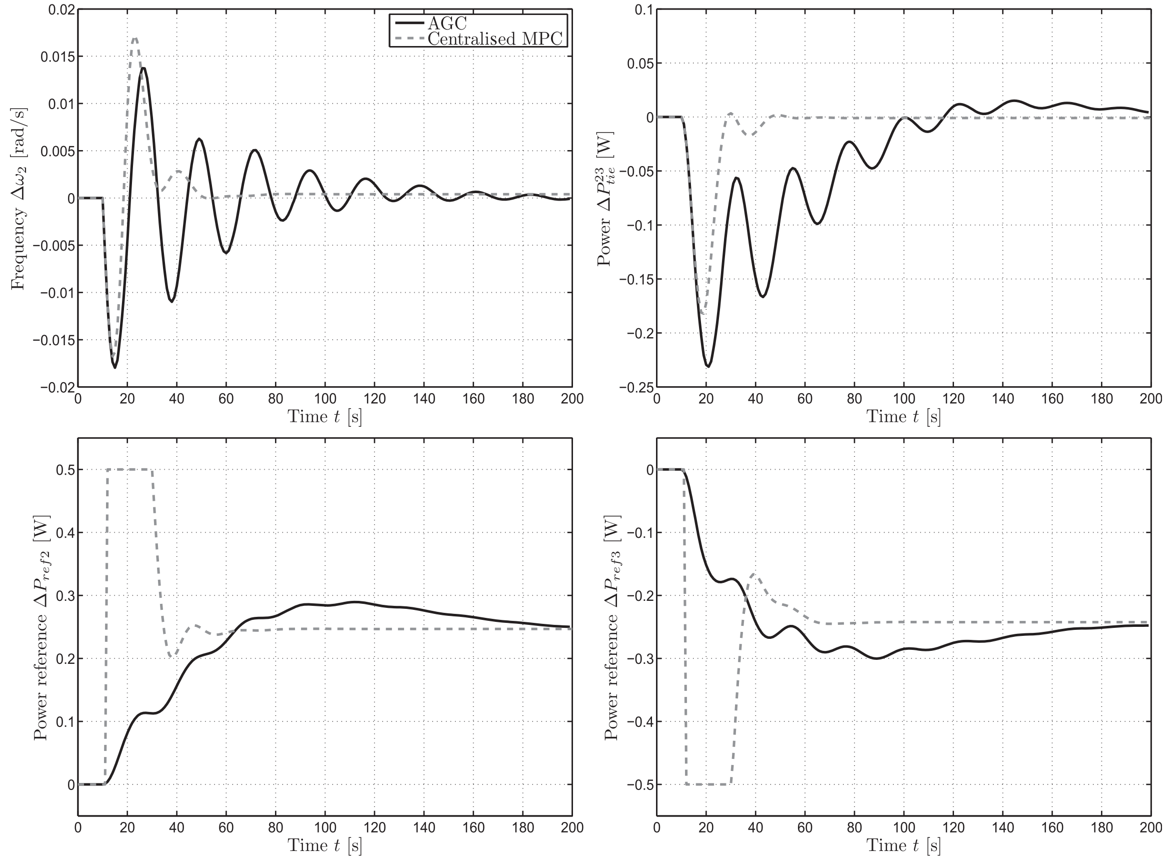

Figure 4. Simulation results of the centralised MPC control and classical AGC control schemes.

required for computing the optimal input sequence of controller *i*. These values, obtained for a simulation on a 3.48 GB RAM, 2.66 GHz Pentium-E PC, are shown in Figure 6.

The results show that DMPC and SC-DMPC are preferred from a computational point of view, because these techniques require significantly less computational effort compared to centralised MPC. The computational burdens of DMPC and SC-DMPC are comparable, as their optimisation problems are almost equally sized, except for the additional contraction constraints in SC-DMPC. The FC-MPC algorithm requires about twice as much computational time than SC-DMPC and DMPC if the maximum number of iterations is set to 2, because the local QPs that FC-MPC solves per iteration have dimensions that are comparable with those of the SC-DMPC and DMPC optimisation problems. Thus, although in this simulation, the FC-MPC controller needs less computational effort than centralised MPC to compute a control action, from a complexity point of view, FC-MPC is only advantageous as long as its number of iterations is small.

The simulation results are summarised in Figure 7, which indicates that performance is positively correlated with the extent of communication and the complexity of the prediction models/optimisation problems that underlie the control scheme.

4.3 *Assessment*

The results obtained in Section 3 and 4.2 indicate that there are two important aspects that determine the performance of a non-centralised MPC technique:

- *Prediction accuracy with respect to the centralised model*. A model that ignores the dynamic coupling between subsystems introduces a prediction error, i.e. a mismatch between the predicted (local) state trajectories and the state trajectories that would result from applying the ensemble of local inputs to the full network of interconnected systems. That is, the prediction error at time $k \in \mathbb{Z}_+$ of the prediction horizon is given by

$$\varepsilon(k) = A^k \bar{x}(0) + \sum_{\lambda=0}^{k-1} A^{k-1-\lambda} B \begin{bmatrix} \bar{u}_1(\lambda) \\ \vdots \\ \bar{u}_M(\lambda) \end{bmatrix} - \begin{bmatrix} \bar{x}_1(k) \\ \vdots \\ \bar{x}_M(k) \end{bmatrix}. \tag{27}$$

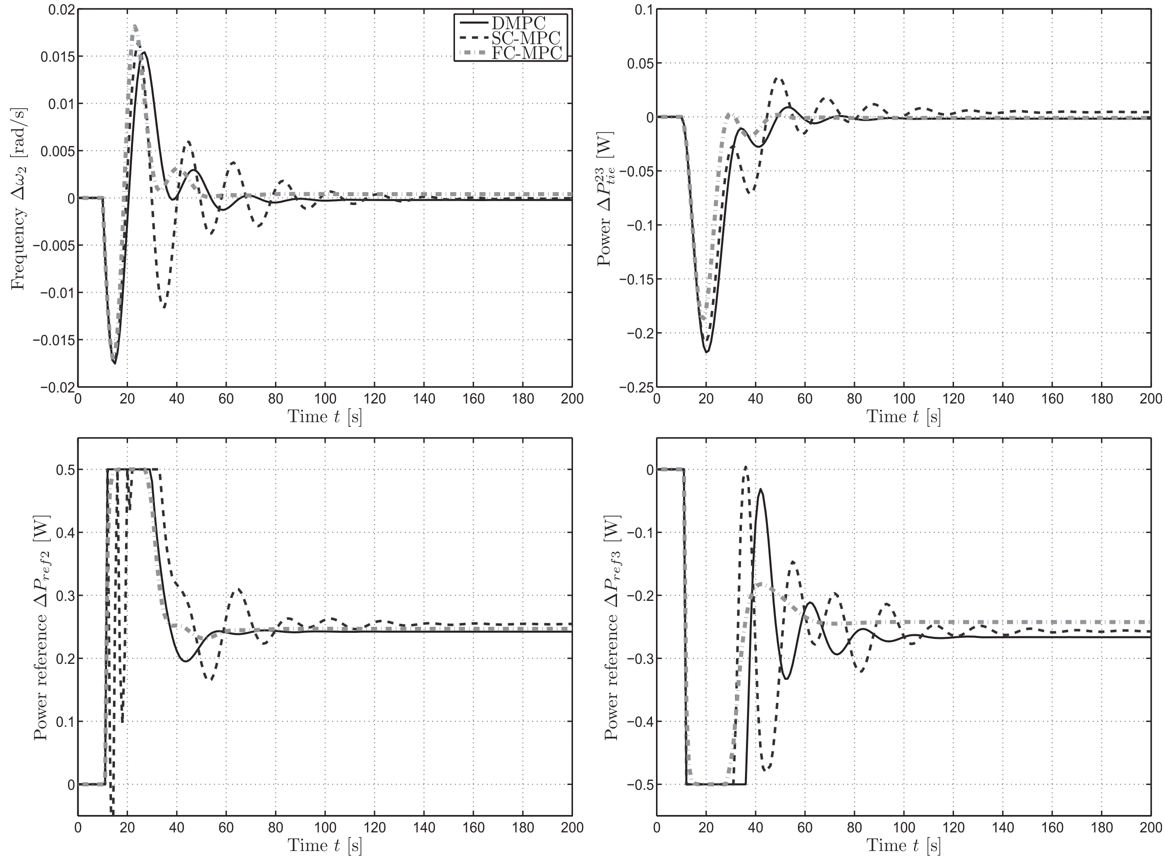

Figure 5. Simulation results of the non-centralised MPC control techniques.

Table 1. Performance in terms of settling time and cost.

	Settling time (s)							
	ω_1	ω_2	ω_3	ω_4	P_{tie}^{12}	P_{tie}^{23}	P_{tie}^{34}	Cost
AGC	164	165	175	175	235	353	187	530.62
MPC	58	48	45	43	56	55	66	176.59
FC-MPC (2 iterations)	50	48	45	44	56	55	65	182.44
DMPC	64	63	65	72	72	98	79	270.73
SC-DMPC	88	114	105	105	138	158	50	260.13

Table 2. Dimensions of the local quadratic programs.

	Size of A $(n_v \times n_c)$	
Technique	Example	General case
Centralised MPC	400×200	$2N \sum_i m_i \times N \sum_i m_i$
FC-MPC	100×50	$2N\ m_i \times N\ m_i$
SC-DMPC	108×50	$2(N\ m_i + n_i) \times N\ m_i$
DMPC	100×50	$2N\ m_i \times N\ m_i$

MPC controllers can exchange their local state predictions to use them as a measure for the dynamic coupling, and exploit this for increasing the prediction accuracy. Accurate predictions are important, as solving a control problem that is based on inexact predictions results in non-optimal closed-loop performance. Precise predictions are required also for state constraint handling, as a constrained optimal control action associated with inaccurate local predictions can be non-feasible for the actual coupled system.

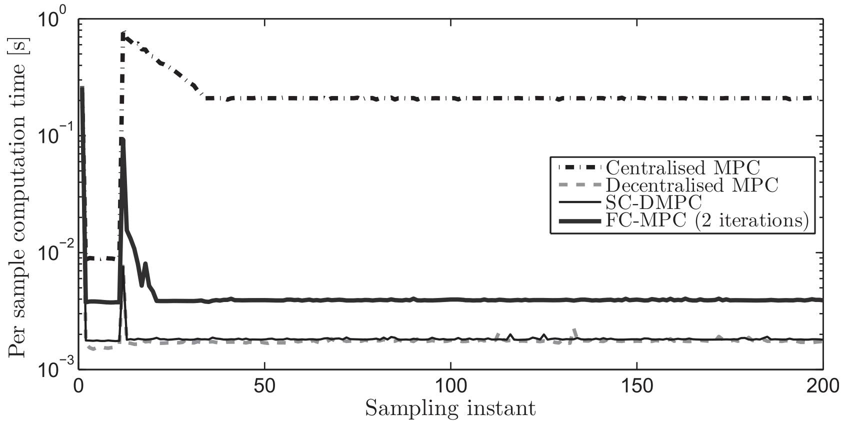

Figure 6. Computation times of all the assessed control algorithms.

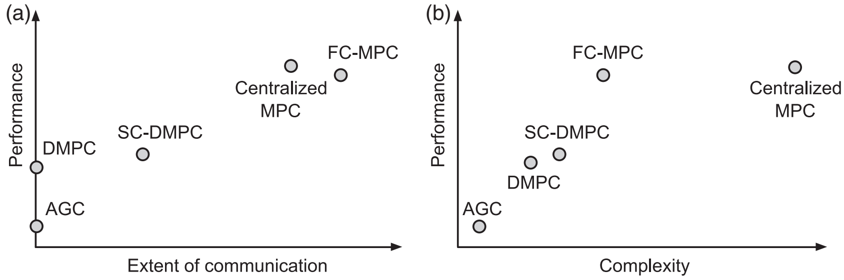

Figure 7. (a) Qualitative comparison of control performance versus communication requirements and (b) Qualitative comparison of control performance versus complexity.

- *Optimality with respect to the centralised solution*. The performance of a non-centralised control scheme depends on more than the prediction accuracy alone. Even if the prediction mismatch $\varepsilon(k)$ is zero for all k, the solution of a non-centralised control problem using local objectives will differ from the global optimiser. Schemes that rely on local control laws that seek to minimise their own objective function only, will induce a Nash equilibrium at best. Such an equilibrium is, in general, not equal to the centralised, i.e. Pareto optimal solution.

Hence, obtaining accurate predictions and globally optimal performance inevitably leads to the requirement of a full-prediction model that exploits global state information and objectives. However, the large scale of real-life power networks prohibits a non-centralised implementation of MPC that requires communication with a large number of subsystems in the network. It is therefore, questionable whether global or Pareto optimality of the non-centralised MPC control action is a feasible goal in current power networks. Future advances in communication technology and increasing processing power might bring this goal closer to realisation. Currently, however, communication with a small number of neighbouring subsystems is more realistic and relatively easy to implement, as transmission lines are usually equipped with communication links.

Moreover, although a completely decentralised implementation of MPC is usually outperformed by distributed methods, it may still perform better than conventional AGC. Hence, there is room for a tradeoff: one can use decentralised MPC if the corresponding performance is acceptable, and in this way avoid communication between neighbouring control areas, or use distributed MPC with limited communication to improve performance by attaining higher prediction accuracy.

Even if global performance is not a major concern in power networks, control schemes for supply-demand matching should always be able to ensure stable and reliable operation of the grid. Due to the liberalisation of the power market, efficient use of resources is becoming increasingly important and the system tends to be pushed towards its physical constraints and stability boundaries. In this respect, the non-centralised algorithms considered in this article all have the same flaw: they lack the ability of taking coupled state constraints into account, whereas ultimately, constraint handling for preventing outages was one of the most important reasons for using MPC. Coupled constraints can only be satisfied if the prediction of all the constrained states coincides with the predicted state trajectory of the centralised MPC scheme. This shows that network-wide communication among the subsystems is crucial to obtain globally feasible state predictions. Note, however, that this does not necessarily require the use of a global communication network, as state information can also be distributed via *iterative communication* among *neighbouring subsystems only*.

Other important issues that are not yet solved by non-centralised control algorithms for power network control include the following:

- Power networks are generally characterised by non-linear and hybrid dynamics. The state-space model (1) used in this article to represent the network dynamics is obtained via linearisation, such that it is only accurate for relatively small power and frequency fluctuations. The models used in the assessed algorithms do not capture saturation or switching effects that are often associated with larger disturbances. Moreover, due to the ongoing liberalisation of the electricity market, power generation control tends to become price-based (Jokić 2007; Jokić et al. 2007). This trend comes with the introduction of hybrid dynamics. These observations suggest a need for more advanced, nonlinear models, thus further complicating the application of non-centralised predictive control.
- Distributed control methods should be robust against the typical disturbances that are associated with communication over large networks, such as varying time delays and information loss. Stability must be guaranteed even in the presence of delayed or interrupted communications.

5. Conclusions and future research

MPC is a promising technique for real-time control of future power networks that are characterised by highly fluctuating power flows, tight-constraint margins and a strong demand for efficient, profitable operation. Since power networks are too large for centralised control to be feasible, this article assessed a number of non-centralised MPC schemes that differ in the level of decentralisation, communication requirements and complexity. Based on our investigations and analysis of the non-centralised algorithms, the following conclusions can be drawn.

The large scale of power networks appears to prohibit the application of non-centralised MPC schemes that require extensive communication among large number of subsystems. Thus, globally optimal performing non-centralised MPC may not be a feasible goal for power networks. Other implementations that rely on short-distance communication among neighbouring subsystems only are more realistic, however.

A completely decentralised implementation of MPC seems to outperform conventional AGC, whereas it offers lower complexity and a higher extent of decentralisation in comparison to alternative non-centralised predictive methods. Decentralised MPC is appropriate when acceptable performance can be achieved without any knowledge of the state of neighbouring subsystems, which is typically the case when the physical coupling between subsystems is weak. Distributed MPC can be employed to increase prediction accuracy, and thus, to improve closed-loop performance, in networks with strong system interactions, but only if the corresponding extent of communication is feasible in practice.

An important unsolved problem for existing non-centralised MPC schemes originates from coupled state constraints. It is challenging to enforce such constraints, in closed-loop, based on incomplete, local state measurements and predictions only. This issue is of paramount significance to power systems, where coupled state constraints, such as bounds on transmission-line power flows, are inherent and of growing importance.

To summarise, non-centralised MPC techniques are viable for power system control if they are scalable, can exploit communication among a small number of subsystems to improve state trajectory predictions, are able to deal with coupled state constraints, and guarantee closed-loop stability. Currently, the control systems community is actively searching for ways of achieving these objectives. In terms of guaranteeing stability under local state feedback, we refer the interested reader to refer recent publications on dissipativity-based, decentralised stabilisation (see, e.g. Jokić and Lazar (2009), Hermans, Lazar, Jokić, and Gielen (2011) and the references therein). Promising directions for state-constraint satisfaction under non-centralised predictive control are considered in, e.g. Doan, Keviczky, and Schutter (2011a,b), where methods are studied for improving convergence of distributed, iterative optimisation schemes. Even so, this article showed that there are still many obstacles that need to be overcome before non-centralised MPC can be successfully applied for frequency control in practice.

Acknowledgements

The authors would like to thank Armand Damoiseaux for his contributions to the case studies performed in this article. This work was supported by the EOS-LT Regelduurzaam project, funded by Agentschap NL, an agency of the Dutch Ministry of Economic Affairs, and the European Commission Research Project FP7-ICT-249096 (E-Price).

Note

1. With 'settling time', we mean the time required for a state transient to settle within an error band of $\pm 5.10^{-4}$ around the steady-state value.

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Appendix: List of parameter values

Table A1 lists the parameter values used in our simulation.

Table A1. Simulation parameters.

Sampling period	1 s
Simulation time	200 s
Prediction horizon N	50
Iterations (FC-MPC)	2
State of subsystem 1	$x_1 = \text{col}(\Delta P_{V1}, \Delta P_{M1}, \Delta\omega_1)$
State of subsystem 2	$x_2 = \text{col}(\Delta\delta_{12}, \Delta P_{V2}, \Delta P_{M2}, \Delta\omega_2)$
State of subsystem 3	$x_3 = \text{col}(\Delta\delta_{23}, \Delta P_{V3}, \Delta P_{M3}, \Delta\omega_3)$
State of subsystem 4	$x_4 = \text{col}(\Delta\delta_{34}, \Delta P_{V4}, \Delta P_{M4}, \Delta\omega_4)$
Disturbance ΔP_{L_1}	$0, \forall t$
Disturbance ΔP_{L_2}	0 for $t < 10$, $+0.25$ for $t \geq 10$
Disturbance ΔP_{L_3}	0 for $t < 10$, -0.25 for $t \geq 10$
Disturbance ΔP_{L_4}	$0, \forall t$
Constraint on ΔP_{ref_i}, $i = 1, \ldots, 4$	$-0.5 \leq \Delta P_{\text{ref}_i} \leq 0.5$
Generator damping: D_1, D_2, D_3, D_4	3, 0.275, 2, 2.75
Generator inertia: J_1, J_2, J_3, J_4	4, 40, 35, 10
Speed regulation: r_1, r_2, r_3, r_4	0.12, 0.28, 0.16, 0.12
Governor time constant: $\tau_{G1}, \tau_{G2}, \tau_{G3}, \tau_{G4}$	4, 25, 15, 5
Turbine time constant: $\tau_{T1}, \tau_{T2}, \tau_{T3}, \tau_{T4}$	5, 10, 20, 10
Tie-line gain: b_{12}, b_{23}, b_{34}	2.54, 1.5, 2.5
AGC gain 1: K_1, K_2, K_3, K_4	0.01, 0.02, 0.03, 0.01
AGC gain 2: B_1, B_2, B_3, B_4	36.33, 14.56, 27.00, 36.08
Q_1, Q_2	$100 \cdot \text{diag}(0, 0, 5)$, $100 \cdot \text{diag}(5, 0, 0, 5)$
Q_3, Q_4	$100 \cdot \text{diag}(5, 0, 0, 5)$, $100 \cdot \text{diag}(5, 0, 0, 5)$
R_1, R_2, R_3, R_4	1, 1, 1, 1