A NONLINEAR COMMAND GOVERNOR FOR CONSTRAINED CONTROL SYSTEMS*

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Abstract: A method based on conceptual tools of predictive control is described for solving tracking problems wherein pointwise-in-time input and/or state inequality constraints are present. It consists of adding to a primal compensated system a nonlinear device called command governor (CG) whose action is based on the current state, set-point and prescribed constraints. The overall system is proved to fulfill the constraints, be asymptotically stable, and exhibit an offset-free tracking behaviour, provided that an admissibility condition on the initial state is satisfied.

Keywords: Constraints, Saturation control, Nonlinear control, Predictive control, Quadratic programming.

1. INTRODUCTION

In recent years there have been substantial theoretical advancements in the field of feedback control of dynamic systems with input and/or state-related constraints. The main goal of the present paper is to address this issue by laying down guidelines for synthesizing *command governors* (CG) based on predictive control ideas (Keerthi and Gilbert, 1988). A CG is a nonlinear device which is added to a primal compensated control system. The latter, in the absence of the CG, is designed so as to perform satisfactorily in the absence of constraints. Whenever necessary, the CG modifies the input to the primal control system so as to avoid violation of the constraints. Hence, the CG action is finalized to let the primal control system operate linearly within a wider dynamic range than that which would result with no CG. Preliminary studies along these lines have already appeared in (Bemporad and Mosca, 1994, 1995). For CGs approached from different perspectives see (Kapasouris, Athans and Stein, 1990) and (Gilbert, Kolmanovsky and Tin Tan, 1995) Simulation examples are presented so as to exhibit the results achievable by the method in comparison with other CG strategies.

2. COMMAND GOVERNOR DESIGN

Consider the following linear time-invariant system

$$\begin{cases} x(t+1) &= \Phi x(t) + Gg(t) \\ y(t) &= Hx(t) \\ c(t) &= H_c x(t) + Dg(t) \end{cases} \tag{1}$$

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In (1): $t \in \mathbb{Z}_+ := \{0, 1, \ldots\}$; $x(t) \in \mathbb{R}^n$ is the state vector; $g(t) \in \mathbb{R}^p$ the manipulable command input which, if no constraints were present, would essentially coincide with the output reference $r(t) \in \mathbb{R}^p$; $y(t) \in \mathbb{R}^p$ the output which is required to track $r(t)$; and $c(t) \in \mathbb{R}^{n_c}$ the constrained vector which has to fulfill the pointwise-in-time set-membership constraint

$$c(t) \in \mathcal{C}, \ \forall t \in \mathbb{Z}_+ \tag{2}$$

with $\mathcal{C} \subset \mathbb{R}^{n_c}$ a prescribed constraint set. The problem is to design a memoryless device

$$g(t) := \underline{g}(x(t), r(t)) \tag{3}$$

in such a way that, under suitable conditions, the constraints (2) are fulfilled and possibly $y(t) \approx r(t)$. It is assumed that

$$(\mathbf{A.1.}) \begin{cases} \mathbf{1}. & \Phi \text{ is a stability matrix, i.e. all its eigenvalues are in the open unit disk;} \\ \mathbf{2}. & \text{System (1) is offset-free, i.e. } H(I-\Phi)^{-1}G = I_p. \end{cases}$$

One important instance of (1) consists of a linear plant under stabilizing linear state-feedback control. In this way, the system is compensated so as to satisfy stability and performance requirements, regardless of the prescribed constraints. In order to enforce the constraints, the CG (3) is added to the primal compensated system (1).

It is convenient to adopt the following notations for the equilibrium solution of (1) to a constant command

$g(t) \equiv w$

$$\left\{\begin{array}{lcl} x_w & := & (I-\Phi)^{-1}Gw \\ y_w & := & Hx_w \\ c_w & := & H_c x_w + Dw = \left[H_c(I-\Phi)^{-1}G + D\right] w \end{array}\right. \tag{4}$$

It is further assumed that:

$$(\mathbf{A.2.}) \left\{ \begin{array}{ll} \mathbf{1}. & \mathcal{C} \text{ is bounded;} \\ \mathbf{2}. & \mathcal{C} = \{c \in \mathbb{R}^{n_c} : q_j(c) \leq 0,\ j \in \underline{n}_q\}, \text{ with} \\ & \underline{n}_q := \{1,2,...,n_q\} \text{ and } q_j : \mathbb{R}^{n_c} \to \mathbb{R} \\ & \text{continuous and convex;} \\ \mathbf{3}. & \mathcal{C} \text{ has a non-empty interior.} \end{array} \right.$$

(A.2) implies that $\mathcal{C}$ is compact and convex.

Consider a θ-parameterized family $\mathcal{V}_\Theta$ of sequences

$$\mathcal{V}_\Theta = \{v(\cdot,\theta) :\ \theta \in \Theta \subset \mathbb{R}^{n_\theta}\},\quad v(\cdot,\theta) := \{v(k,\theta)\}_{k=0}^{\infty} \tag{5}$$

with the property of closure w.r.t. left time-shifts, viz. $\forall \theta \in \Theta$, $\forall k \in \mathbb{Z}_+$, there exist $\bar\theta \in \Theta$ such that

$$v(k+1,\theta) = v(k,\bar\theta) \tag{6}$$

Suppose temporarily that $v(\cdot,\theta)$ is used as an input to (1) from the state $x(t)$ at time 0. The latter will be referred to as the event $(0,x(t))$. Assume that

$$c(\cdot,x(t),\theta) := \{c(k,x(t),\theta)\}_{k=0}^{\infty} \subset \mathcal{C} \tag{7}$$

In (7), $c(k,x(t),\theta)$ denotes the c-response at time k to $v(\cdot,\theta)$ from the event $(0,x(t))$. If the inclusion (7) is satisfied for some $\theta \in \Theta$, $x(t)$ is said to be *admissible*, $(x(t),\theta)$ an *executable* pair, and $v(\cdot,\theta)$ a *virtual* command sequence for the state $x(t)$. Notice that (6) ensures that $(x(t),\theta)$ is executable $\Longrightarrow \exists \bar\theta \in \Theta : (x(t+1),\bar\theta)$ is executable, provided that $x(t+1) = \Phi x(t) + Gv(0,\theta)$. In fact, from (6) it follows that $c(k+1,x(t),\theta) = c(k,x(t+1),\bar\theta)$. Then, any state is admissible along the trajectory corresponding to a virtual command sequence $v(\cdot,\theta)$. Consequently, no danger occurs of being trapped into a blind alley if (1) is driven by a virtual command sequence or its input switched from one to another virtual command sequence.

For reasons which will appear clear soon, it is convenient to introduce the following sets for a given $\delta > 0$:

$$\mathcal{C}_\delta := \{c \in \mathcal{C} : \mathcal{B}_\delta(c) \subset \mathcal{C}\}, \tag{8}$$

$$\text{with } \mathcal{B}_\delta(c) := \{\bar c \in \mathbb{R}^{n_c} : \|c - \bar c\| \leq \delta\}$$

$$\mathcal{W}_\delta := \{w \in \mathbb{R}^p : c_w \in \mathcal{C}_\delta\} \tag{9}$$

We shall assume that for a possibly vanishingly small $\delta > 0$

$$(\mathbf{A.3.}) \left\{ \quad \mathcal{W}_\delta \text{ is non-empty} \right.$$

From the foregoing definitions and (A.3), it follows that $\mathcal{W}_\delta$ is closed and convex. In the developments that follow we shall consider the family $\mathcal{V}_\Theta$ where

$$v(k,\theta) = \gamma^k \mu + w, \tag{10}$$

$$\theta := [\mu' \ \ w']' \in \Theta \ \ := \mathbb{R}^p \times \mathcal{W}_\delta \tag{11}$$

where $\gamma \in [0,1)$ and the prime denotes transpose. In such a case, (6) is satisfied with

$$\bar\theta = [\gamma\mu' \ \ w']'. \tag{12}$$

We consider next the c-response $c(\cdot,x,\theta)$ to the command sequence (10)-(11). By straightforward manipulations we find

$$c(k) := c(k,x,\theta) \tag{13}$$

$$= \hat c(k) + H_c\Phi^k\left[x - x_{\mu+w}\right] + \tilde c(k) \tag{14}$$

$$\hat c(k) := \gamma^k c_{\mu+w} + (1-\gamma^k)c_w \tag{15}$$

$$\tilde c(k) := (1-\gamma)H_c\sum_{i=0}^{k-1}\Phi^i\gamma^{k-1-i}x_\mu \tag{16}$$

If in (13)-(16) $x = x_{\bar w}$, $\bar w \in \mathcal{W}_\delta$, $w \in \mathcal{W}_\delta$ and $\mu = \bar w - w$ we get $c(k) = \hat c(k) + \tilde c(k)$ with $\hat c(k) = \gamma^k c_{\bar w} + (1-\gamma^k)c_w$. Thus, being $\mathcal{C}$ convex, $\hat c(k) \in \mathcal{C}$. Hence, there are $\mu \in \mathbb{R}^p$, $\|\mu\| > 0$, such that $c(k) \in \mathcal{C}$, $\forall k \in \mathbb{Z}_+$. It follows that $(x_{\bar w}, [(\bar w - w)' \ \ w']')$ is executable for $\bar w, w \in \mathcal{W}_\delta$, $\|w - \bar w\| > 0$, and $\lim_{k\to\infty} x(k) = x_w$. These conclusions can be shown to hold true if the initial state $x_{\bar w}$ is perturbed by any $\tilde x$, with $\|\tilde x\| \leq \varepsilon$, for $\varepsilon > 0$ sufficiently small. From the foregoing considerations the following viability property, as will be hereafter referred to by adopting the terminology of Aubin (1991), easily follows.

Proposition 1 (Viability property). *Consider the system (1) along with the family of command sequences (10)-(11). Let the assumptions (A.1)-(A.3) be fulfilled and the initial state $x(0)$ of (1) be admissible. Then, there exists a concatenation of a finite number of virtual command sequences $v(\cdot,\theta_i)$, $\theta_i = [\mu_i' \ \ w_i']'$, $\theta_i \in \Theta$, with finite switching times, capable of asymptotically driving the system state from $x(0)$ to x_w, any $w \in \mathcal{W}_\delta$.* □

Hereafter, we shall address the problem of how to select appropriate virtual command sequences, and when to switch from one to another. To this end, consider the quadratic selection index

$$J(x(t),r(t),\theta) := \|\mu\|^2_{\Psi_\mu} + \|w - r(t)\|^2_{\Psi_w} + \sum_{k=0}^{\infty}\|y(k,x(t),\theta) - w\|^2_{\Psi_y} \tag{17}$$

where θ is as in (11), $\|x\|^2_\Psi := x'\Psi x$, $\Psi_\mu = \Psi_\mu' > 0$, $\Psi_w = \Psi_w' > 0$, $\Psi_y = \Psi_y' \geq 0$, and $y(k,x(t),\theta)$ the output response at time k to the command $v(k,\theta) = \gamma^k\mu + w$ from the event $(0,x(t))$. It is easy to see that (17) has a unique unconstrained minimum $\theta(t) \in \mathbb{R}^{2p}$ for every $x(t) \in \mathbb{R}^n$ and $r(t) \in \mathbb{R}^p$. Let $\mathcal{V}(x)$ be the set of all $\theta \in \Theta$ such that (x,θ) is executable

$$\mathcal{V}(x) := \{\theta \in \Theta :\ c(\cdot,x,\theta) \subset \mathcal{C}\} \tag{18}$$

Assume that, for every $t \in \mathbb{Z}_+$, $\mathcal{V}(x(t))$ is non-empty, closed and convex. This implies that the following minimizer exists unique

$$\begin{aligned} \theta(t) &:= \arg\min_{\theta\in\Theta} \{J(x(t),r(t),\theta) : \; c(\cdot,x(t),\theta) \subset \mathcal{C}\} \\ &= \arg\min_{\theta\in\mathcal{V}(x(t))} J(x(t),r(t),\theta) \end{aligned} \tag{19}$$

Proposition 1 ensures that $\mathcal{V}(x(t))$ is non-empty and implies that $\mathcal{V}(x(t+1))$ is non-empty too if $(x(t),\theta)$ is executable and $x(t+1) = \Phi x(t) + Gv(0,\theta)$. Further, the concatenation mechanism embedded in the viability property of Proposition 1 naturally suggests that we can select the actual CG action according to the following receding horizon control strategy if $\theta(t)$ is as in (19):

$$\begin{aligned} g(t) &= v(0,\theta(t)) \\ &= \begin{cases} \mu(t) + w(t), & \gamma \in (0,1) \\ \qquad w(t) & \gamma = 0 \end{cases} \end{aligned} \tag{20}$$

We defer the proof that $\mathcal{V}(x(t))$ is closed and convex to Sect. 3. A question we wish to address now is whether the foregoing CG yields an overall stable offset-free control system. Assume that the reference is kept constant, $r(t) \equiv r$ for all $t \geq t^*$, and $\mathcal{V}(x(t))$ is non-empty, closed and convex at each $t \in \mathbb{Z}_+$. Consider the following candidate Lyapunov function

$$V(t) := J(x(t), r, \theta(t)) \tag{21}$$

If $x(t+1) = \Phi x(t) + Gv(0,\theta(t))$, it results that $J(x(t+1), r, [\gamma\mu'(t) \;\; w'(t)]') \geq V(t+1)$. In fact, $(x(t+1), [\gamma\mu'(t) \;\; w'(t)]')$ is executable, but $[\gamma\mu'(t) \;\; w'(t)]'$ need not be the minimizer for $J(x(t+1), r, \theta)$. It follows that along the trajectories of the system

$$V(t) - V(t+1) \geq (1-\gamma^2)\|\mu(t)\|^2_{\Psi_\mu} + \|y(t) - w(t)\|^2_{\Psi_y} \geq 0 \tag{22}$$

Hence, $V(t)$, being nonnegative monotonically non increasing, has a finite limit $V(\infty)$ as $t \to \infty$. This implies $\lim_{t\to\infty} [V(t) - V(t+1)] = 0$, and by (22)

$$\lim_{t\to\infty} \mu(t) = 0_p \tag{23}$$

$$\lim_{t\to\infty} \|y(t) - w(t)\|_{\Psi_y} = 0 \tag{24}$$

The asymptotic properties of $w(t)$ and $y(t)$ can be further characterized by taking into account the uniform boundedness of $x(t)$ and assuming the existence and uniqueness of $\theta(t)$, $\forall t\, \mathbb{Z}_+$. In such a case, the output of the system controlled by the CG converges to the closest admissible approximation to the reference.

Proposition 2 *Consider the system (1) controlled by the CG (19)-(20). Assume that (A.1)-(A.3) are satisfied. Let $x(0)$ be admissible and $\mathcal{V}(x(t))$ closed and convex at each $t \in \mathbb{Z}_+$. Let $r(t) \equiv r$, $\forall t \geq t^* \in \mathbb{Z}_+$. Then, the prescribed constraints are satisfied at every $t \in \mathbb{Z}_+$, and*

$$\lim_{t\to\infty} w(t) = \lim_{t\to\infty} y(t) = \lim_{t\to\infty} g(t) = w_r \tag{25}$$

$$w_r := \arg\min_{w\in\mathcal{W}_\delta} \|w - r\|^2_{\Psi_w} \tag{26}$$

Proof. See (Bemporad, Casavola and Mosca, 1995). □

3. SOLVABILITY AND COMPUTABILITY

It remains to find existence conditions for the minimizer (19). Further, even if solvability is guaranteed, (19) embodies an infinite number of constraints. For practical implementation, we must find out if and how these constraints can be reduced to a finite number of constraints whose time locations be determinable off-line. To this end, it is convenient to introduce some extra notation. We express the response of (1) from an event $(0,x)$ to the command sequence (10)-(11) as follows

$$\begin{cases} z(k+1) &= Az(k), \text{ with } z(0) = \begin{bmatrix} x \\ \theta \end{bmatrix} \in \mathbb{R}^n \times \Theta, \\ \quad c(k) &:= c(k,x,\theta) \\ &= E_c z(k) \end{cases} \tag{27}$$

where

$$A = \begin{bmatrix} \Phi & G & G \\ 0_{p\times n} & \gamma I_p & 0_{p\times p} \\ 0_{p\times n} & 0_{p\times p} & I_p \end{bmatrix}, \quad E_c = [H_c \;\; 0_{p\times p} \;\; D] \tag{28}$$

For $i \in \mathbb{Z}_1 := \{1,2,3,...\}$, consider the following sets

$$\begin{aligned} \mathcal{Z}_i &:= \{z \in \mathbb{R}^n \times \Theta : \; q_j(E_c A^{k-1} z) \leq 0, \\ &\qquad j \in \underline{n}_q, \; k \in \underline{i}\}. \end{aligned} \tag{29}$$

$$\mathcal{Z} := \bigcap_{i=0}^{\infty} \mathcal{Z}_i \tag{30}$$

$\mathcal{Z}_i$ are the sets of initial states z with $w \in \mathcal{W}_\delta$ which give rise to evolutions fulfilling the constraints over the first i-th time steps $k = 0,1,...,i-1$, while $\mathcal{Z}$ is the set of all executable pairs (x,θ). $\mathcal{Z}_{i+1} \subset \mathcal{Z}_i$, $\forall i \in \mathbb{Z}_1$, and under (A.2), all the $\mathcal{Z}_i$'s, and hence $\mathcal{Z}$, are closed and convex. Moreover, $\mathcal{Z}_i = \mathcal{Z}_{i+1} \Longrightarrow \mathcal{Z}_i = \mathcal{Z}$ and, by the viability property of Proposition 1, $\mathcal{Z}$ is non-empty.

Consider next the "slice" of $\mathcal{Z}$ along x introduced in (18)

$$\mathcal{V}(x) := \{\theta \in \Theta : \begin{bmatrix} x \\ \theta \end{bmatrix} \in \mathcal{Z}\}. \tag{31}$$

If x is admissible for some $\theta \in \Theta$, $\mathcal{V}(x)$ is non-empty. In addition, it is closed being the intersection of two closed sets, $\mathcal{V}(x) = \mathcal{Z} \bigcap \{\{x\} \times \Theta\}$. $\mathcal{V}(x)$ is also convex because the "slicer" operator is convexity-preserving. Then, existence and uniqueness of the minimizer (19) follows, provided that the initial state of (1) be admissible.

Proposition 3 *Let (A.1)-(A.4) be fulfilled and $(x(0), \theta)$ executable for some $\theta \in \Theta$. Thus, the optimization problem (19) is equivalent to the following convex constrained optimization problem*

$$\theta(t) = \begin{bmatrix} \mu(t) \\ w(t) \end{bmatrix} := \arg \min_{\theta \in \mathcal{V}(x(t))} J(x(t), r(t), \theta), \ \forall t \in \mathbb{Z}_+ \tag{32}$$

This is uniquely solvable at each $t \in \mathbb{Z}_+$, being $\mathcal{V}(x)$ non-empty, closed and convex.

Proof. The viability property of Proposition 1 ensures that $\mathcal{V}(x(t))$ is non-empty. Existence and uniqueness of $\theta(t)$ follow because J is quadratic in θ, and $\mathcal{V}(x(t))$ is also closed and convex. □

Practical implementation of the CG requires an effective way to solve the optimization problem (32). Notice in fact that there might be no algorithmic procedure capable of computing the exact minimizer, unless $\mathcal{Z}$ is finitely determinable. In what follows, we shall show that only a finite number of pointwise-in-time constraints suffices to determine $\mathcal{Z}$. To this end, let (A_o, E_{co}), with $A_o \in \mathbb{R}^{n_o \times n_o}$, $n_o \leq n + 2p$, be an observable subsystem obtained via a canonical observability decomposition of (A, E_c). Then

$$c(k) = E_{c_o} A_o^k z_o(0) \tag{33}$$

with $z_o = P_o z$, P_o defined by the observability decomposition. Consequently, define the following sets

$$\mathcal{Z}_i^o := \{P_o z \in \mathbb{R}^{n_o} : z \in \mathcal{Z}_i\}, \quad \mathcal{Z}^o := \bigcap_{i=0}^{\infty} \mathcal{Z}_i^o \tag{34}$$

It is easy to see that $\mathcal{Z}_i^o$ and $\mathcal{Z}^o$ own the same properties shown to hold for $\mathcal{Z}_i$ and, respectively, $\mathcal{Z}$. In particular, they are non-empty, closed and convex. Moreover, the following result holds.

Proposition 4 *Let (A.1)-(A.4) be fulfilled. Then, $\mathcal{Z}_i^o$, $\forall i \geq n_o$ is compact and convex. Moreover, there exists an integer $i_o \geq n_o$ such that $\mathcal{Z}_{i_o} = \mathcal{Z}$.*

Proof. See (Bemporad, Casavola and Mosca). □

It follows that $\mathcal{Z}^o$, and hence $\mathcal{Z}$ as well, is finitely determinable, that is it suffices to check the constraints over the initial i_o-th time steps in order to ensure constraint fulfillment over $\mathbb{Z}_+$. Consequently, problem (32) is equivalent to the following finite dimensional convex constrained optimization problem at each $t \in \mathbb{Z}_+$:

$$\begin{array}{rcl} \theta(t) & := & \arg\min_{\theta \in \Theta} J(x(t), r(t), \theta) \\ & & \text{subject to } q_j(c(i-1, x(t), \theta) \leq 0, \ j \in \underline{n}_q, \ i \in \underline{i}_o \end{array} \tag{35}$$

A modification of the Gilbert and Tin Tan algorithm (1991), as detailed in (Bemporad, Casavola and Mosca, 1995) can be used to find $i_o = \min_{i \geq n_o}\{i \mid \mathcal{Z}_i^o = \mathcal{Z}^o\}$.

In conclusion, we have found that our initial optimization problem having an infinite number of constraints is equivalent to a convex constrained optimization problem with a finite number of constraints.

Theorem 1 (Main result) *Let (A.1)-(A.3) be fulfilled. Consider the system (1) with the CG (19)-(20), and let $x(0)$ be admissible. Then:*

i. *The J-minimizer (32) uniquely exists at each $t \in \mathbb{Z}_+$ and can be obtained by solving a convex constrained optimization problem with inequality constraints $q_j(c(i-1, x(t), \theta)) \leq 0$, $j \in \underline{n}_q$, limited to a finite number i_o of time-steps, viz. $i = 1, ..., i_o$;*

ii. *The integer i_o can be computed off-line from the outset;*

iii. *The overall system satisfies the constraints, is asymptotically stable and off-set free in that the conclusions of Proposition 2 hold.*

4. COMPARISON WITH OTHER CG SCHEMES

The present CG design methodology, hereafter referred to as BCM96, is general enough to encompass other CG strategies as special cases, even if obtained from different perspectives. In particular, in this section we compare the BCM96 and the GKT95 (Gilbert, Kolmanovsky and Tin Tan, 1995) governors, in terms of available design knobs, computational complexity and robustness of the constraint enforcement mechanisms against state disturbances.

In GKT95 the actual command $g(t)$ is obtained as follows

$$g(t) = g(t-1) + K\left[r(t-1) - g(t-1)\right] \tag{36}$$

where $K \in [0, 1]$ is maximized under the constraint that the state remains within a specific admissible set. This scheme can be embedded in BCM96. In fact, at each time $t \in \mathbb{Z}_+$, (36) can be reformulated as finding the smallest $\beta = 1 - K \in [0, 1]$ such that the constant command sequence ($\gamma = 0$)

$$\underline{v}(k, \beta) = \beta\left[w(t-1) - r(t-1)\right] + r(t-1) \tag{37}$$

is virtual. This is equivalent to minimize the functional

$$J(\mu, w) = \beta^2 \tag{38}$$

and imposing the additional specifications that w is as the R.H.S. of (37). In contrast with BCM96, this formulation does not take into account the tracking performance, and there are no design knobs for modifying the dynamics of the overall closed-loop system. Further, each command trajectory increases toward its desired value by the same fraction β. Consequently, especially in a MIMO context, slower transients can be expected.

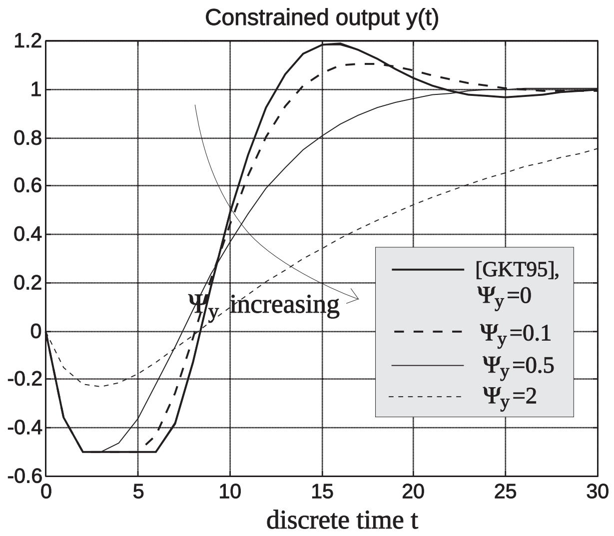

Figure 1: Example 1. Response for different Ψ_y

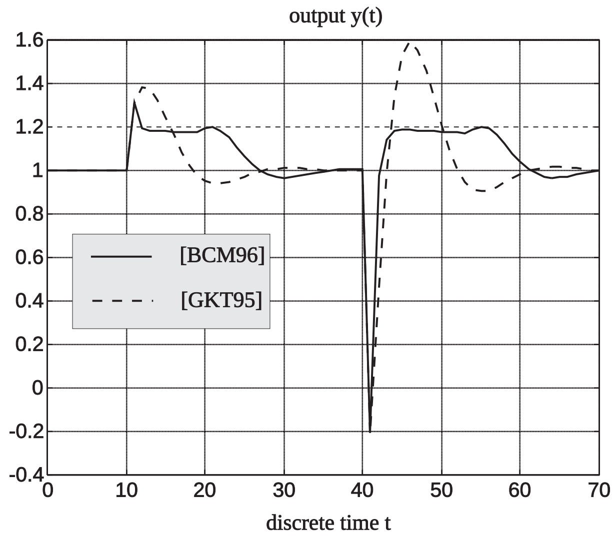

Figure 2: Example 1. Robustness against state perturbations.

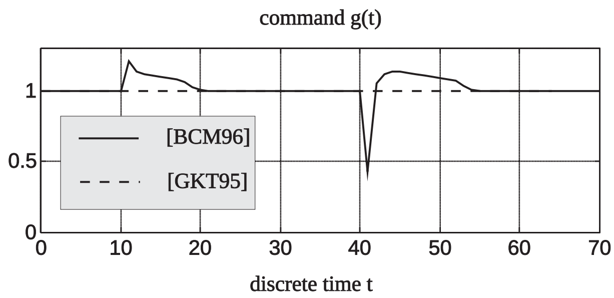

Figure 3: Example 1. Commands for Fig. 2.

On the other hand, only a scalar optimization has to be performed in GKT95, reducing remarkably the related computational burden.

Example 1 - Consider the following SISO non-minimum phase plant

$$y(t) = \frac{-0.8935z + 1.0237}{z^2 - 1.5402z + 0.6703} g(t) \qquad (39)$$

The task of the CG is to govern the command input so that $-0.5 \leq y(t) \leq 1.2$. Let $\delta = 0.05$. Accordingly, $c(t) = y(t)$, $\mathcal{C} = [-0.5, 1.2]$ and $\mathcal{W}_\delta = [-0.45, 1.15]$. The unit step responses of (39) subjects to the GKT95 and BCM96 governors are reported in Fig. 1, where $\gamma = 0.99$, $\Psi_\mu = 1$ and $\Psi_w = 10$ were chosen for BCM96 whereas GKT95 was implemented as previously described except for $r(t)$ instead of $r(t-1)$ in (37), for a better comparison. As expected because of $\gamma \approx 1$, for $\Psi_y = 0$ the BCM96 and GKT95 responses are identical. However, the BCM96 responses can be modified by acting on the design knobs, e.g. by modifying Ψ_y as in Fig. 1.

For values of γ close to 0, the features of BCM96 and GKT95 differ remarkably. In particular, from Figs. 2 and 3 the robustness of BCM96 against state perturbation results superior. Fig. 2 and Fig. 3 report respectively the plant output $y(t)$ and the command $g(t)$ for both governors, with $\gamma = 0.1$ and $\Psi_y = 0$, the remaining design knobs being as for the simulations reported in Fig. 1. In those figures, the equilibrium state $[7.6848\ 7.6848]'$, corresponding to the admissible constant reference $r = 1$, is perturbed at time instants 10 and 40 respectively by $[1.0198\ -0.2944]'$ and $[1.1551\ 1.3152]'$. Observe that, for both perturbations, GKT95 remains inactive whereas BCM96 is capable to compensate them, though after one time step at time instant 10. The reason results clear by examining Figs. 4 and 5, which report respectively the admissible state and the 1-step recoverable state sets for both governors, the latter being the set of all states that give rise to admissible c-sequences except possibly for the first sample $c(0)$. Notice that in the example at hand $c(0)$ does not depend on μ and w since there is one step delay between $c(t)$ and $g(t)$. In both those figures, the "cross" and the "dot" indicate the state before and, respectively, after the perturbations, with the perturbations of Figs. 5 and 4 corresponding respectively to the ones occurring at time instants 10 and 40. Then, the explanation is that the state after the perturbation at time 10 (40) remains 1-step recoverable (admissible) for BCM96 whereas it is not such for GKT95. It is worth pointing out that the above sets do not depend on Ψ_μ, Ψ_w and Ψ_y but are instead strongly dependent on γ and δ. In fact, in Figs. 4 and 5 the same sets are also depicted (dashed line) for $\gamma = 0.99$ that are approximately comprised within the other two sets.

Example 2 - In order to investigate the limit of the scalar optimization underlying the GKT95 scheme with respect to the vectorial one of the BCM96 governor, consider the following MIMO plant.

$$\Phi = \begin{bmatrix} -0.6 & -0.6 & 0 \\ 0.6 & 0.9 & 0 \\ -0.3 & -0.1 & 0.7 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0.4 \\ -1 & 0.5 \\ 2 & 1 \end{bmatrix},$$

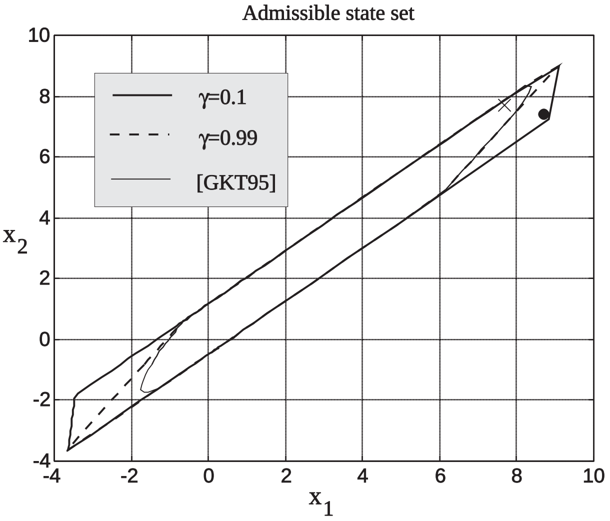

Figure 4: Example 1. Admissible state sets.

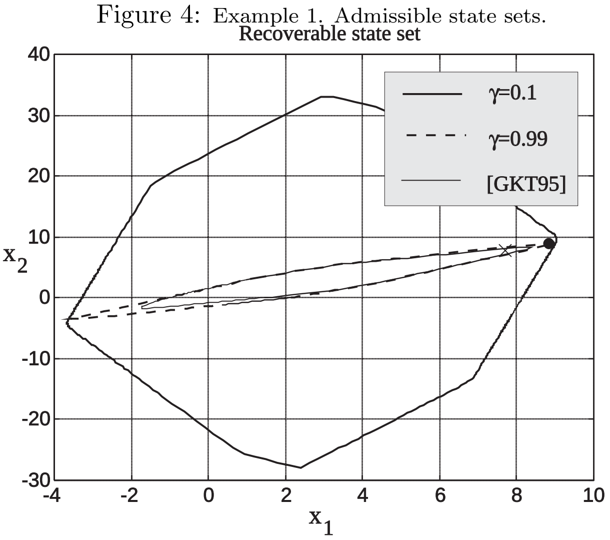

Figure 5: Example 1. One-step recoverable state sets.

$$H = \begin{bmatrix} -0.8244 & -0.5073 & 0.1902 \\ 2.6341 & 1.3902 & -0.1463 \end{bmatrix}$$

Also in this case the task of the CG is to bound the plant outputs so that $-0.05 \leq y_1(t), y_2(t) \leq 1.2$. Let $\delta = 0.001$ and $\gamma = 0.9$. Accordingly, $c(t) = y(t)$, $\mathcal{C} = [-0.05, 1.2]$, $\mathcal{W}_\delta = [-0.049, 1.199]$, $\Psi_y = 0_2$ and $\Psi_\mu = \Psi_w = I_2$. Fig. 6 depicts the output y_1 and y_2 corresponding to a step reference respectively for: (upper-left) the ungoverned case (no CG action) and (lower) the governed case where the continuous and the dashed lines represent the BCM96 and, respectively, the GKT95 actions. In the upper-right figure is represented the plot (g_1 vs g_2) of the command actions for both the BCM96 (continuous line) and the GKT95 (dashed line) governors. As expected, in the GKT95 schemes the commands g_1 and g_2 increase by the same quantity toward the desired set-point value $(1, 1)$ whereas this is

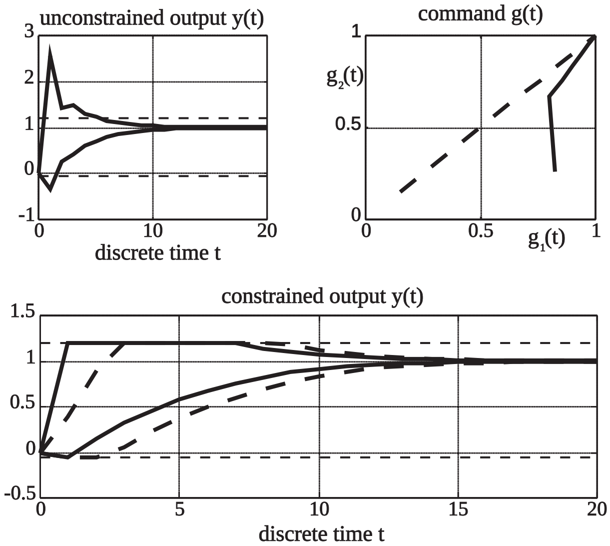

Figure 6: Example 2. Output and command responses.

not the case for BCM96.

5. CONCLUSIONS

The CG problem, viz. the one of on-line designing a command input in such a way that a primal compensated control system can operate in a stable way with satisfactory tracking performance and no constraint violation, has been addressed by exploiting some ideas originating from predictive control.

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