Nonlinear Control of Constrained Linear Systems via Predictive Reference Management

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***Abstract*— A method based on conceptual tools of predictive control is described for solving set-point tracking problems wherein pointwise-in-time input and/or state inequality constraints are present. It consists of adding to a primal compensated system a nonlinear device, called command governor (CG), whose action is based on the current state, set-point, and prescribed constraints. The CG selects at any time a virtual sequence among a family of linearly parameterized command sequences, by solving a convex constrained quadratic optimization problem, and feeds the primal system according to a receding horizon control philosophy. The overall system is proved to fulfill the constraints, be asymptotically stable, and exhibit an offset-free tracking behavior, provided that an admissibility condition on the initial state is satisfied. Though the CG can be tailored for the application at hand by appropriately choosing the available design knobs, the required on-line computational load for the usual case of affine constraints is well tempered by the related relatively simple convex quadratic programming problem.**

***Index Terms*—Control under constraints, nonlinear feedback, predictive control, quadratic programming.**

I. Introduction

In recent years there have been substantial theoretical advancements in the field of feedback control of dynamic systems with input and/or state-related constraints. For an account of pertinent results see [1] and [2], which also include relevant references. Most of this work has addressed the pure regulation problem with *time-invariant* constraint sets, particularly input saturation constraints. This paper aims at studying constrained tracking problems, wherein the reference to be tracked is possibly time-varying. In some cases such problems can be recast as pure regulation problems subject to *time-varying* constraint sets. However, such a time dependence severely limits, in practice, the potential of many of the existing approaches. A convenient framework to deal with constrained tracking problems in the presence of time-varying references is the predictive control methodology [3]–[7]. Predictive control, wherein the receding horizon control philosophy is used, selects the control action by possibly taking into account the future evolution of the reference. Such an evolution can be 1) known in advance, as in applications where repetitive tasks are executed, e.g., industrial robots; 2) predicted, if a dynamic model for the reference is given; or 3) planned in real time. This last instance is a peculiar and important potential feature of predictive control. In fact, taking into account the current value of both the state vector and the reference, a potential or *virtual* reference evolution can be designed on-line so as to possibly make the related input and state responses fulfill pointwise-in-time inequality constraints. However, this mode of operation, whereby the reference is made state-dependent, introduces an extra feedback loop that complicates the stability analysis of the overall control system. This has been one of the reasons for which on-line reference design, though advocated for a long time as one of the key potential advantages of predictive control [6], [8]–[10], has received to date rare consideration in applications.

In most cases, predictive control computations amount to numerically solving on-line a high-dimensional convex quadratic programming problem. Though this can be tackled with existing software packages [11], it is a quite formidable computational burden if, as in predictive control, on-line solutions are required. In order to lighten computations, it is important to know when and how it is possible to borrow from predictive control the concept of on-line reference management so as to tackle constrained control problems by schemes requiring a lighter computational burden. The main goal of the present paper is to address this issue by laying down guidelines for synthesizing *command governors* (CG), based on predictive control ideas. A CG is a nonlinear device which is added to a primal compensated control system. The latter, in the absence of the CG, is designed so as to perform satisfactorily in the absence of constraints. Whenever necessary, the CG modifies the input to the primal control system so as to avoid violation of the constraints. Hence, the CG action is finalized to let the primal control system operate linearly within a wider dynamic range than that which would result with no CG. Preliminary studies along these lines have already appeared in [12] and [13]. For CG's approached from different perspectives, the reader is referred to [14]–[19].

The paper is organized as follows. Section II presents the problem formulation and defines the CG based on the concept of a virtual command sequence. Some of the CG stability and performance features are also considered in Section II. Section III discusses solvability aspects related to the CG optimization problem and addresses the important practical issue of reducing to a fixed and off-line computable finite prediction-horizon the infinite time-interval over which the fulfillment of constraints has to be checked. In particular, in Section III Theorem 1 summarizes the main properties of the CG. Simulation examples are presented in Section IV so as to exhibit the results achievable by the method.

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II. Problem Formulation and CG Design

Consider the following linear time-invariant system:

$$\begin{cases} x(t+1) = \Phi x(t) + Gg(t) \\ y(t) = Hx(t) \\ c(t) = H_c x(t) + Dg(t). \end{cases} \tag{1}$$

In (1) $t \in Z_+ := \{0, 1, \cdots\}$; $x(t) \in \mathbb{R}^n$ is the state vector; $g(t) \in \mathbb{R}^p$, the manipulable command input which, if no constraints were present, would essentially coincide with the output reference $r(t) \in \mathbb{R}^p$; $y(t) \in \mathbb{R}^p$, the output which is required to track $r(t)$; and $c(t) \in \mathbb{R}^{n_c}$, the constrained vector which has to fulfill the pointwise-in-time set-membership constraint

$$c(t) \in \mathcal{C}, \quad \forall t \in Z_+ \tag{2}$$

with $\mathcal{C} \subset \mathbb{R}^{n_c}$ a prescribed constraint set. The problem is to design a memoryless device

$$g(t) := \underline{g}(x(t), r(t)) \tag{3}$$

in such a way that under suitable conditions, the constraints (2) are fulfilled and possibly $y(t) \approx r(t)$. It is assumed that

$$\text{(A.1)} \begin{cases} \mathbf{1}) & \Phi \text{ is a stability matrix, i.e., all its eigenvalues} \\ & \text{are in the open unit disk;} \\ \mathbf{2}) & \text{System (1) is offset-free, i.e.,} \\ & H(I-\Phi)^{-1}G = I_p. \end{cases}$$

One important instance of (1) consists of a linear plant under stabilizing linear state-feedback control. In this way, the system is compensated so as to satisfy stability and performance requirements, regardless of the prescribed constraints. In order to enforce the constraints, the CG (3) is added to the primal compensated (1).

It is convenient to adopt the following notations for the equilibrium solution of (1) to a constant command $g(t) \equiv w$:

$$\begin{cases} x_w := (I-\Phi)^{-1}Gw \\ y_w := Hx_w \\ c_w := H_c x_w + Dw = [H_c(I-\Phi)^{-1}G + D]w. \end{cases} \tag{4}$$

It is further assumed that

$$\text{(A.2)} \begin{cases} \mathbf{1}) & \mathcal{C} \text{ is bounded;} \\ \mathbf{2}) & \mathcal{C} = \{c \in \mathbb{R}^{n_c} : q_j(c) \le 0, j \in \underline{n}_q\}, \text{ with} \\ & \underline{n}_q := \{1, 2, \cdots, n_q\} \text{ and } q_j : \mathbb{R}^{n_c} \to \mathbb{R} \\ & \text{continuous and convex;} \\ \mathbf{3}) & \mathcal{C} \text{ has a nonempty interior.} \end{cases}$$

(A.2) implies that $\mathcal{C}$ is compact and convex.

Consider a θ-parameterized family $\mathcal{V}_\Theta$ of sequences

$$\begin{aligned} \mathcal{V}_\Theta &= \{v(\cdot, \theta) : \theta \in \Theta \subset \mathbb{R}^{n_\theta}\} \\ v(\cdot, \theta) &:= \{v(k, \theta)\}_{k=0}^{\infty} \end{aligned} \tag{5}$$

with the property of closure w.r.t. left time-shifts, viz. $\forall \theta \in \Theta$, there exist $\bar{\theta} \in \Theta$ such that

$$v(k+1, \theta) = v(k, \bar{\theta}), \quad \forall k \in Z_+. \tag{6}$$

Suppose temporarily that $v(\cdot, \theta)$ is used as an input to (1) from the state x at time 0. The latter will be referred to as the event $(0, x)$. Assume that

$$c(\cdot, x, \theta) := \{c(k, x, \theta)\}_{k=0}^{\infty} \subset \mathcal{C}. \tag{7}$$

In (7), $c(k, x, \theta)$ denotes the c-response at time k to $v(\cdot, \theta)$ from the event $(0, x)$. If the inclusion (7) is satisfied for some $\theta \in \Theta$, x is said to be *admissible*, (x, θ) an *executable* pair, and $v(\cdot, \theta)$ a *virtual* command sequence for the state x. Notice that (6) ensures that

$$(x, \theta) \text{ is executable} \Rightarrow \exists \bar{\theta} \in \Theta : (\bar{x}, \bar{\theta}) \text{ is executable} \tag{8}$$

provided that $\bar{x} = \Phi x + Gv(0, \theta)$. In fact, from (6) it follows that $c(k+1, x, \theta) = c(k, \bar{x}, \bar{\theta})$. Then, any state is admissible along the trajectory corresponding to a virtual command sequence $v(\cdot, \theta)$. Consequently, no danger occurs of being trapped in a blind alley if (1) is driven by a virtual command sequence or its input switched from one to another virtual command sequence.

For reasons which will appear clear soon, it is convenient to introduce the following sets for a given $\delta > 0$:

$$\mathcal{C}_\delta := \{c \in \mathcal{C} : \mathcal{B}_\delta(c) \subset \mathcal{C}\}$$

with

$$\mathcal{B}_\delta(c) := \{\bar{c} \in \mathbb{R}^{n_c} : \|c - \bar{c}\| \le \delta\} \tag{9}$$

$$\mathcal{W}_\delta := \{w \in \mathbb{R}^p : c_w \in \mathcal{C}_\delta\}. \tag{10}$$

Henceforth, we shall assume that there exists a possibly vanishingly small $\delta > 0$ such that

(A.3) $\mathcal{W}_\delta$ is nonempty.

From the foregoing definitions and (A.3), it follows that $\mathcal{W}_\delta$ is closed and convex. In the developments that follow we shall consider the family $\mathcal{V}_\Theta$, where

$$v(k, \theta) = \gamma^k \mu + w \tag{11}$$

$$\theta := [\mu' \quad w']' \in \Theta := \mathbb{R}^p \times \mathcal{W}_\delta \tag{12}$$

where $\gamma \in [0, 1)$ and the prime denotes transpose. The rationale for (11), (12) hinges upon the requirement that $\bar{c}(k)$, as given next in (20), be in $\mathcal{C}, \forall k \in Z_+$, and the following result whose straightforward proof is omitted.

Lemma 1: The family of command sequences $v(\cdot, \theta) = \{\gamma(k)\mu + w\}_{k=0}^{\infty}$, with θ as in (11), (12) and $\gamma(\cdot)$, a real-valued asymptotically vanishing nonnegative sequence, owns the property of closure w.r.t. left time-shifts (6) if and only if

$$\gamma(k) = \gamma^k, \quad \gamma \in [0, 1). \tag{13}$$

In such a case, (6) is satisfied with

$$\bar{\theta} = [\gamma\mu' \quad w']'. \tag{14}$$

We consider next the c-response $c(\cdot, x, \theta)$ to the command sequence (11), (12). By straightforward manipulations we find

$$\begin{aligned} c(k) &:= c(k, x, \theta) && (15) \\ &= \hat{c}(k) + H_c \Phi^k [x - x_{\mu+w}] + \tilde{c}(k) && (16) \\ \hat{c}(k) &:= \gamma^k c_{\mu+w} + (1 - \gamma^k) c_w && (17) \\ \tilde{c}(k) &:= (1-\gamma) H_c \sum_{i=0}^{k-1} \Phi^i \gamma^{k-1-i} x_\mu. && (18) \end{aligned}$$

In order to establish the existence of admissible c-responses $c(\cdot, x, \theta)$, consider the special case $x = x_{\bar{w}}$ with $\bar{w} \in \mathcal{W}_\delta$.

Thus, we can make $x - x_{\mu+w} = 0$ by the choice $\mu = \bar{w} - w$. Accordingly

$$c(k) = \bar{c}(k) + \tilde{c}(k) \tag{19}$$

$$\bar{c}(k) = \gamma^k c_{\bar{w}} + (1-\gamma^k)c_w. \tag{20}$$

By the convexity of $\mathcal{C}_\delta$, it follows that $\bar{c}(k) \in \mathcal{C}_\delta,\ \forall k \in Z_+$. Then, $c(k)$ belongs to $\mathcal{C}$, provided that $\|\tilde{c}(k)\|$ is sufficiently small for all $k \in Z_+$. In this connection, by stability of (1) and given $\gamma \in [0,1)$, there are two positive reals M and λ, $\lambda \in [0,1)$ with $\lambda \neq \gamma$, such that for each $x \in \mathbb{R}^n$ one has that $\|\Phi^k x\| \le M\lambda^k\|x\|, \forall k \in Z_+$. Then, it is possible to show that there are $\mu = \bar{w} - w,\ \|\bar{w} - w\| > 0$ such that $c(\cdot, x, [(\bar{w}-w)' \quad w']') \subset \mathcal{C}$. In fact, the following inequality holds for all $k \in Z_+$:

$$\|\tilde{c}(k)\| \le \frac{(1-\gamma)}{|\gamma - \lambda|}\bar{\sigma}(H_c)M\|x_{\bar{w}} - x_w\| \tag{21}$$

with $\bar{\sigma}(H_c)$ the maximum singular value of H_c. Recalling that $x_w = (I-\Phi)^{-1}Gw$, from (21) it follows that $\|\tilde{c}(k)\| \le \delta,\ \forall k \in Z_+$, provided that $\|x_{\bar{w}} - x_w\| \le \delta|\gamma - \lambda|/[(1-\gamma)\bar{\sigma}(H_c)M]$, or $\|\bar{w} - w\| \le \delta(1-\lambda)|\gamma-\lambda|/[(1-\gamma)\bar{\sigma}(H_c)\bar{\sigma}(G)M^2]$. The foregoing analysis holds true if the initial state $x_{\bar{w}}$ is additively perturbed by $\tilde{x},\ 0 < \|\tilde{x}\| \le \varepsilon$, with ε sufficiently small. In this case, the perturbed constrained vector $c(k)$ is such that $c(k) - \bar{c}(k) = H_c\Phi^k\tilde{x} + \tilde{c}(k)$. The condition $\|c(k) - \bar{c}(k)\| \le \delta,\ \forall k \in Z_+$ can be ensured, e.g., by requiring that $\|x_{\bar{w}} - x_w\| \le \frac{1}{2}\delta|\gamma-\lambda|/[(1-\gamma)\bar{\sigma}(H_c)M]$, and $\|\tilde{x}\| \le \frac{1}{2}\delta/[\bar{\sigma}(H_c)M]$. The conclusion is that starting sufficiently close to an equilibrium state $x_{\bar{w}},\ \bar{w} \in \mathcal{W}_\delta$ in a finite time one can arrive as close as desired to any state x_w, $w \in \mathcal{W}_\delta$, at a nonzero, though possibly small, distance from $x_{\bar{w}}$. Then, we can move out from any admissible state $x(0)$ to reach asymptotically x_w, any $w \in \mathcal{W}_\delta$, by concatenating a finite number of virtual command sequences by switching from one to another, the last switching taking place at a finite, though possibly large, time. This result, which by adopting the terminology of [20] will be referred to as a viability property, is summarized in the following proposition.

Proposition 1 (Viability Property): Consider (1) along with the family of command sequences (11), (12). Let Assumptions (A.1)–(A.3) be fulfilled and the initial state $x(0)$ of (1) be admissible. Then, there exists a concatenation of a finite number of virtual command sequences $v(\cdot, \theta_i),\ \theta_i = [\mu_i' \quad w_i']'$, $\theta_i \in \Theta$, with finite switching times, capable of asymptotically driving the system state from $x(0)$ to x_w, any $w \in \mathcal{W}_\delta$. □

Remark 1: We leave to the reader the simple task of specializing the analysis and the results of this paper to the case of a family $\mathcal{V}_\Theta$ of constant sequences

$$v(k,\theta) = \theta \in \Theta := \mathcal{W}_\delta \subseteq \mathbb{R}^p. \qquad \square$$

Hereafter, we shall address the problem of how to select appropriate virtual command sequences and when to switch from one to another. To this end, consider the quadratic selection index

$$J(x(t), r(t), \theta) := \|\mu\|^2_{\Psi_\mu} + \|w - r(t)\|^2_{\Psi_w} + \sum_{k=0}^{\infty}\|y(k, x(t), \theta) - w\|^2_{\Psi_y} \tag{22}$$

where θ is as in (12), $\|x\|^2_\Psi := x'\Psi x,\ \Psi_\mu = \Psi_\mu' > 0$, $\Psi_w = \Psi_w' > 0,\ \Psi_y = \Psi_y' \ge 0$, and $y(k, x(t), \theta)$ the output response at time k to the command $v(k,\theta) = \gamma^k\mu + w$ from the event $(0, x(t))$. It is easy to see that (22) has a unique unconstrained minimum $\theta(t) \in \mathbb{R}^{2p}$ for every $x(t) \in \mathbb{R}^n$ and $r(t) \in \mathbb{R}^p$. Let $\mathcal{V}(x)$ be the set of all $\theta \in \Theta$ such that (x, θ) is executable

$$\mathcal{V}(x) := \{\theta \in \Theta:\ c(\cdot, x, \theta) \subset \mathcal{C}\}. \tag{23}$$

Assume that $\mathcal{V}(x(t))$ is nonempty, closed, and convex for every $t \in Z_+$. This implies that the following minimizer exists uniquely:

$$\begin{aligned}\theta(t) &:= \arg\min_{\theta\in\Theta}\{J(x(t), r(t), \theta):\ c(\cdot, x(t), \theta) \subset \mathcal{C}\}\\ &= \arg\min_{\theta \in \mathcal{V}(x(t))} J(x(t), r(t), \theta).\end{aligned} \tag{24}$$

Proposition 1 ensures that $\mathcal{V}(x(t))$ nonempty implies that $\mathcal{V}(x(t+1))$ nonempty if $(x(t), \theta)$ is executable and $x(t+1) = \Phi x(t) + Gv(0,\theta)$. Further, the concatenation mechanism embedded in the viability property of Proposition 1 naturally suggests that we can select the actual CG action according to the following receding horizon control strategy if $\theta(t)$ is as in (24):

$$g(t) = v(0, \theta(t)) = \mu(t) + w(t). \tag{25}$$

Remark 2: If the computational delay is not negligible w.r.t. the sampling interval, we can modify (24) as follows:

$$\theta((i+1)\tau) = \arg\min_{\theta \in \mathcal{V}(x(i\tau))} J(x(i\tau), \tau(i\tau), \theta)$$

$i \in Z_+$, and set for $k = 0, 1, \cdots, \tau - 1$

$$g((i+1)\tau + k) = v(k, \theta((i+1)\tau)).$$

This amounts to using an "open-loop" command sequence over intervals made up by τ steps. While the results proved in the remaining part of this section and in Section III can be easily extended to cover this case, a tracking performance degradation typically results from a significant computational delay. □

Remark 3: As elaborated in some detail in Example 2, the weighting matrices Ψ_μ and Ψ_w can be made $r(t)$-dependent so as to force the direction of the selected vector $g(t) = \mu(t) + w(t)$ to be as close as possible to that of $r(t)$, compatibly with the constraints. This can be a qualitatively important requirement in some multi-input/multi-output applications. □

We defer the proof that $\mathcal{V}(x(t))$ is closed and convex to Section III. A question we wish to address now is whether the foregoing CG yields an overall stable offset-free control system. Assume that the reference is kept constant, $r(t) \equiv r$ for all $t \ge t^*$, and $\mathcal{V}(x(t))$ is nonempty, closed, and convex at each $t \in Z_+$. Consider the following candidate Lyapunov function:

$$V(t) := J(x(t), r, \theta(t)). \tag{26}$$

If $x(t+1) = \Phi x(t) + Gv(0, \theta(t))$, it results that $J(x(t+1),\ r, [\gamma\mu'(t) \quad w'(t)]') \ge V(t+1)$. In fact, $(x(t+1), [\gamma\mu'(t) \quad w'(t)]')$ is executable, but $[\gamma\mu'(t) \quad w'(t)]'$ need not be the minimizer for $J(x(t+1), r, \theta)$. It follows that along the trajectories of the system:

$$V(t) - V(t+1) \geq (1-\gamma^2)\|\mu(t)\|^2_{\Psi_\mu} + \|y(t) - w(t)\|^2_{\Psi_y} \geq 0. \quad (27)$$

Hence $V(t)$, being nonnegative monotonically nonincreasing, has a finite limit $V(\infty)$ as $t \to \infty$. This implies $\lim_{t\to\infty}[V(t) - V(t+1)] = 0$, and by (27)

$$\lim_{t\to\infty} \mu(t) = 0_p \quad (28)$$

$$\lim_{t\to\infty} \|y(t) - w(t)\|_{\Psi_y} = 0. \quad (29)$$

Lemma 2: Consider (1) controlled by the CG (24), (25). Assume that (A.1)–(A.3) are satisfied. Let $x(0)$ be admissible and $\mathcal{V}(x(t))$ closed and convex at each $t \in Z_+$. Let $r(t) \equiv r, \forall t \geq t^* \in Z_+$. Then

$$V(\infty) := \lim_{\tau\to\infty} V(\tau) \leq V(t+1) \leq V(t), \quad \forall t \geq t^* \quad (30)$$

(28) and (29) hold, and the CG output exhibits asymptotically vanishing variations

$$\lim_{t\to\infty}[w(t+1) - w(t)] = 0_p. \quad (31)$$

Further

$$\lim_{t\to\infty}[x(t) - x_{w(t)}] = 0_p \quad (32)$$

where

$$x_{w(t)} := (I - \Phi)^{-1} G w(t)$$

and

$$V(\infty) = \lim_{t\to\infty} \|w(t) - r\|^2_{\Psi_w}. \quad (33)$$

Proof: Equation (30) has already been proved under the stated assumptions. It follows by strict positivity of Ψ_μ and Ψ_w that the CG output sequence $g(\cdot) = \mu(\cdot) + w(\cdot)$ is bounded. Hence, by (28) and the stability of (1), the system state evolution $x(\cdot)$ remains bounded as well. Let $\bar{\theta}$ be defined in terms of θ as in (14). Then, along the trajectories of the system for each $t \geq t^*$

$$V(t) = J(x(t+1), r, \bar{\theta}(t)) + (1-\gamma^2)\|\mu(t)\|^2_{\psi_\mu} + \|y(t) - w(t)\|^2_{\psi_y}$$

and, by (8), (24), and convexity of J

$$V(t+1) = J(x(t+1), r, \theta(t+1)) \leq \underline{J}(t, \alpha) \leq J(x(t+1), r, \bar{\theta}(t)) \quad (34)$$

where $\alpha \in [0,1]$ and

$$\underline{J}(t,\alpha) := J(x(t+1), r, \bar{\theta}(t) + \alpha[\theta(t+1) - \bar{\theta}(t)]). \quad (35)$$

Now, taking into account (22), it is easy to see that

$$\underline{J}(t,\alpha) = \alpha^2\|\theta(t+1) - \bar{\theta}(t)\|^2_\Psi + \alpha c_1(t) + c_2(t) \quad (36)$$

with $\Psi = \Psi' > 0$, and $c_1(\cdot)$ and $c_2(\cdot)$ bounded real-valued sequences. Then, because $\lim_{t\to\infty} \underline{J}(t,1) = \lim_{t\to\infty} \underline{J}(t,0) = V(\infty)$, from (34) and (36) it follows that

$$\lim_{t\to\infty} \underline{J}(t,\alpha) = V(\infty), \quad \forall \alpha \in [0,1]. \quad (37)$$

Consequently, $\lim_{t\to\infty}[\theta(t+1) - \bar{\theta}(t)] = 0$, and hence (31) from (14). Equations (1), (25), (28), and (31) imply that $\lim_{t\to\infty}[x(t+1) - x(t)] = 0_p$. Hence, $\lim_{t\to\infty}[x(t) - \Phi x(t) + Gw(t)] = 0_p$ from which (32) follows. To show (33), consider that $y(k, x(t), \theta(t)) = y(k, x_{w(t)}, [0_p' \quad w'(t)]') + y(k, \tilde{x}(t), [\mu'(t) \quad 0_p']') = w(t) + y(k, \tilde{x}(t), [\mu'(t) \quad 0_p']')$, $\tilde{x}(t) := x(t) - x_{w(t)}$. Then, $\sum_{k=0}^\infty \|y(k, x(t), \theta(t)) - w(t)\|^2_{\Psi_y} = \|[\tilde{x}'(t) \quad \mu'(t)]'\|^2_Q$ for some symmetric nonnegative-definite matrix Q. Because of (28) and (32), the last quantity goes to zero as $t \to \infty$. This proves (33). □

We are now ready to prove that under the conditions stated after (23), the output of the system controlled by the CG converges to the best possible approximation to the reference.

Proposition 2: Under the same assumptions as in Lemma 2, the prescribed constraints are satisfied at every $t \in Z_+$, and

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} g(t) = w_r \quad (38)$$

$$w_r := \arg\min_{w\in\mathcal{W}_\delta} \|w - r\|^2_{\Psi_w}. \quad (39)$$

Proof: Because (33) implies that $V(\infty) \geq V_r := \min_{w\in\mathcal{W}_\delta}\|w - r\|^2_{\Psi_w}$, (38) is proven if we show that $V(\infty) = V_r$. Assume to the contrary that $V_r < V(\infty)$. Under this assumption, we show that, along the trajectory of the system controlled by the CG, for t large enough we can find a virtual command sequence $v(\cdot, \theta_\alpha(t))$ such that $J(x(t), r, \theta_\alpha(t)) < V(\infty)$. Because $V(t) \leq J(x(t), r, \theta_\alpha(t))$ and (30), the previous inequality contradicts the assumption. For $\alpha \in (0,1]$ let

$$\theta_\alpha(t) := \begin{bmatrix} 0_p \\ w_\alpha(t) \end{bmatrix}, \quad w_\alpha(t) := (1-\alpha)w(t) + \alpha w_r. \quad (40)$$

Because $x(t) - x_{w(t)} \to 0$, by Proposition 1 there is a time $t_1 \geq t^*$ and a positive real $\alpha_1 \in (0,1]$ such that $\forall t \geq t_1$, $\forall \alpha \in [0, \alpha_1]$, $(x(t), \theta_\alpha(t))$ is executable. Look next at $J(x_{w(t)}, r, \theta_\alpha(t))$. It can be found that

$$J(x_{w(t)}, r, \theta_\alpha(t)) = (1-\alpha)^2\|w(t) - w_r\|^2_{\Psi_w} + 2(1-\alpha)(w_r - r)'\Psi_w(w(t) - w_r) + \alpha^2\|w(t) - w_r\|^2_{\mathcal{K}} + \|r - w_r\|^2_{\Psi_w} \quad (41)$$

where $\mathcal{K} = \mathcal{K}' \geq 0$ is such that $\sum_{k=0}^\infty \|y(k, x_{w(t)}, \theta_\alpha(t)) - w_\alpha(t)\|^2_{\Psi_y} = \alpha^2\|w(t) - w_r\|^2_{\mathcal{K}}$. Such a matrix equals $\mathcal{K} = G'(I-\Phi)^{-T}\mathcal{L}(I-\Phi)^{-1}G$, where $X^{-T} := (X^{-1})'$ and $\mathcal{L} = \mathcal{L}' \geq 0$ is the observability Gramian of the pair $(\Phi, \Psi_y^{\frac{1}{2}}H)$. From (41), we find that

$$J(x_{w(t)}, r, \theta_\alpha(t)) < \|w(t) - r\|^2_{\Psi_w} \quad (42)$$

provided that

$$\frac{\alpha}{2}\|w(t) - w_r\|^2_{(\Psi_w + \mathcal{K})} < \|w(t) - w_r\|^2_{\Psi_w} + (w_r - r)' \times \Psi_w(w(t) - w_r). \quad (43)$$

By convexity of $\mathcal{W}_\delta$, the right-most term of (43) is nonnegative. Hence, for every $\alpha \in (0, \bar{\alpha}(t))$

$$\bar{\alpha}(t) := \frac{2\|w(t) - w_r\|^2_{\Psi_w}}{\|w(t) - w_r\|^2_{\Psi_w + \mathcal{K}}}$$

(43) is satisfied. Then, because $x(t) - x_{w(t)} \to 0$ and the continuity of J w.r.t. x, there is a time $t_2 \geq t_1$ such that for every $t \geq t_2$ and $\alpha \in (0, \underline{\alpha}_2)$, $0 < \underline{\alpha}_2 < \min\{\alpha_1, \bar{\alpha}(t)\}$, $(x(t), \theta_\alpha(t))$ is executable, and $J(x(t), r, \theta_\alpha(t)) < V(\infty)$. □

III. Solvability and Computability

It remains to find existence conditions for the minimizer (24). Further, even if solvability is guaranteed, (24) embodies an infinite number of constraints. For practical implementation, we must find out if and how these constraints can be reduced to a finite number of constraints whose time locations be determinable off-line. To this end, it is convenient to introduce some extra notation. We express the response of (1) from an event $(0, x)$ to the command sequence (11), (12) as follows:

$$\begin{cases} z(k+1) = Az(k), \text{ with } z(0) = \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} x \\ \mu \\ w \end{bmatrix} \in \mathbb{R}^n \times \Theta, \\ \quad c(k) := c(k, x, \theta) \\ \qquad = E_c z(k) \end{cases} \tag{44}$$

where

$$A = \begin{bmatrix} \Phi & G & G \\ 0_{p\times n} & \gamma I_p & 0_{p \times p} \\ 0_{p\times n} & 0_{p\times p} & I_p \end{bmatrix}, \quad E_c = [H_c \quad D \quad D]. \tag{45}$$

For $i \in Z_1 := \{1, 2, 3, ...\}$, consider the following sets:

$$\mathcal{Z} := \{z \in \mathbb{R}^n \times \Theta : q_j(E_c A^{k-1} z) \leq 0, j \in \underline{n}_q, k \in \underline{i}\} \tag{46}$$

$$\mathcal{Z} := \bigcap_{i=0}^{\infty} \mathcal{Z}_i. \tag{47}$$

$\mathcal{Z}_i$ is the set of initial states z with $w \in \mathcal{W}_\delta$ which gives rise to evolutions fulfilling the constraints over the first ith time steps $k = 0, 1, \cdots, i-1$, while $\mathcal{Z}$ is the set of all executable pairs (x, θ). $\mathcal{Z}_{i+1} \subset \mathcal{Z}_i$, $\forall i \in Z_1$, and under (A.2), all the $\mathcal{Z}_i$'s, and hence $\mathcal{Z}$, are closed and convex. Moreover, by the viability property of Proposition 1, $\mathcal{Z}$ is nonempty. The lemma that follows can be proved as in [16], taking into account (44)–(46).

Lemma 3:

$$\mathcal{Z}_i = \mathcal{Z}_{i+1} \Rightarrow \mathcal{Z}_i = \mathcal{Z}.$$

Consider next the "slice" of $\mathcal{Z}$ along x

$$\mathcal{V}(x) := \left\{ \begin{bmatrix} \mu \\ w \end{bmatrix} \in \Theta : \begin{bmatrix} x \\ \mu \\ w \end{bmatrix} \in \mathcal{Z} \right\}. \tag{48}$$

If x is admissible for some $\theta \in \Theta$, $\mathcal{V}(x)$ is nonempty. In addition, it is closed, being the intersection of two closed sets, $\mathcal{V}(x) = \mathcal{Z} \cap \{\{x\} \times \Theta\}$. $\mathcal{V}(x)$ is also convex because the "slicer" operator is convexity-preserving. Then, existence and uniqueness of the minimizer (24) follows, provided that the initial state of (1) is admissible. Practical implementation of the CG requires an effective way to solve the optimization problem (24). Notice in fact that there might be no algorithmic procedure capable of computing the exact minimizer, unless $\mathcal{Z}$ is finitely determined, viz. $\mathcal{Z} = \mathcal{Z}_i$ for some $i \in Z_+$. In what follows, we shall show that only a finite number of pointwise-in-time constraints suffices to determine $\mathcal{Z}$. To this end, let (A_o, E_{co}), with $A_o \in \mathbb{R}^{n_o \times n_o}, n_o \leq n + 2p$, be an observable subsystem obtained via the canonical observability decomposition of (A, E_c). Then

$$c(k) = E_{c_o} A_o^k z_o(0) \tag{49}$$

with $z_o = P_o z$, P_o defined by the observability decomposition. Consequently, define the following sets:

$$\mathcal{Z}_i^o := \{P_o z \in \mathbb{R}^{n_o} : z \in \mathcal{Z}_i\}, \quad \mathcal{Z}^o := \bigcap_{i=0}^{\infty} \mathcal{Z}_i^o. \tag{50}$$

It is easy to see that $\mathcal{Z}_i^o$ and $\mathcal{Z}^o$ own the same properties shown to hold for $\mathcal{Z}_i$ and, respectively, $\mathcal{Z}$. In particular, they are nonempty, closed, and convex. Moreover, the following result holds.

Proposition 3: Let (A.1)–(A.3) be fulfilled. Then, $\mathcal{Z}_i^o, \forall i \geq n_o$ is compact and convex. Moreover, there exists an integer $i_o \geq n_o$ such that $\mathcal{Z}_{i_o} = \mathcal{Z}$.

Proof: See the Appendix. □

It follows that $\mathcal{Z}^o$, and hence $\mathcal{Z}$ as well, is finitely determined; that is, it suffices to check the constraints over the initial i_oth time steps in order to ensure constraint fulfillment over Z_+. Consequently, problem (24) is equivalent to the following finite dimensional convex constrained optimization problem at each $t \in Z_+$:

$$\begin{aligned} \theta(t) = \begin{bmatrix} \mu(t) \\ w(t) \end{bmatrix} &:= \arg \min_{\mu \in \mathbb{R}^p, w \in \mathcal{W}_\delta} J(x(t), r(t), \theta) \\ &\quad \text{subject to } q_j(c(i-1, x(t), \theta)) \\ &\qquad \leq 0, j \in \underline{n}_q, i \in \underline{i}_o. \end{aligned} \tag{51}$$

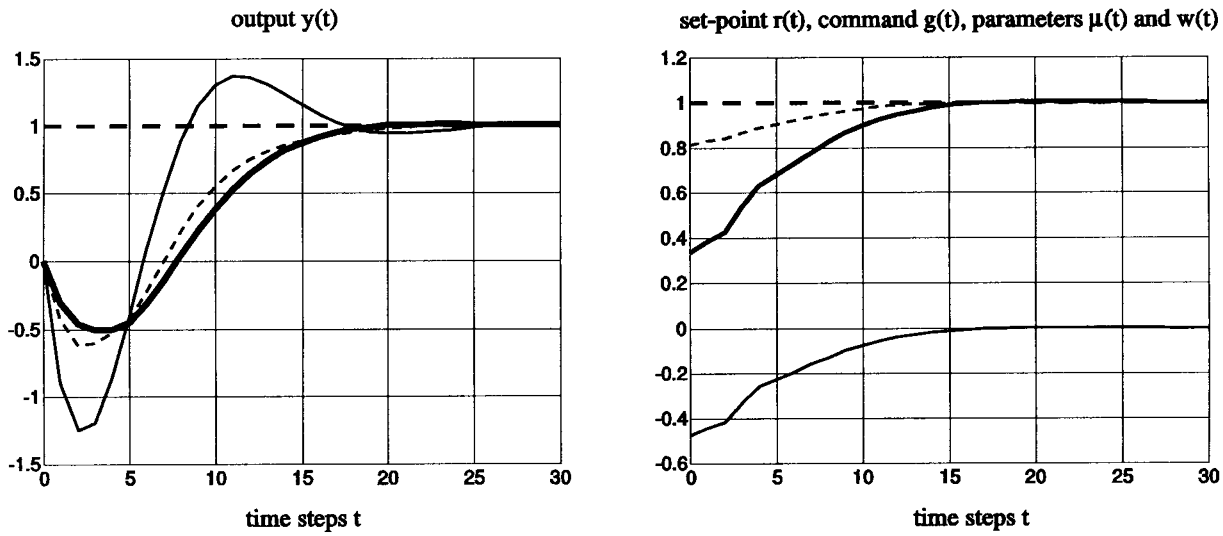
The Gilbert and Tan algorithm [16] can be adapted to the present case to find $i_o = \min_{i \geq n_o}\{i \mid \mathcal{Z}_i^o = \mathcal{Z}^o\}$. To this end, let

$$\begin{aligned} G_i(j) := \max_{x \in \mathbb{R}^n, \theta \in \Theta} &\{q_j(c(i, x, \theta))\}, \quad j \in \underline{n}_q, i = 1, 2, \cdots \\ &\text{subject to } q_j(c(k-1, x, \theta)) \leq 0, j \in \underline{n}_q, k \in \underline{i}. \end{aligned} \tag{52}$$

Then, i_o can be computed off-line via the following algorithm:

$$\left. \begin{array}{ll} 1. & i \leftarrow n_o; \\ 2. & \text{Solve } G_i(j), \quad \forall j \in \underline{n}_q; \\ 3. & \text{If } G_i(j) \leq 0, \quad \forall j \in \underline{n}_q, \text{ let } i_o = i \text{ and stop;} \\ 4. & \text{Otherwise } i \leftarrow i + 1, \text{ and go to 2.} \end{array} \right\}. \tag{53}$$

Notice that Step 2 in (53) is well posed because, according to Proposition 3, the implied maximization is carried out over a compact and convex set. In conclusion, we have found that our initial optimization problem, having an infinite number of constraints, is equivalent to a convex constrained optimization problem with a finite number of constraints.



(a) (b)

Fig. 1. Example 1: (a) Unit step response with no CG (thin line) and with CG ($\delta = 0.05$; $\gamma = 0.9$; $\Psi_\mu/\Psi_w = 0.1$; $\Psi_y/\Psi_w = 0.01$; thick line) for the nominal plant (54); response with CG for the perturbed plant (55) (dashed line). (b) Reference trajectory $r(t)$ (thick dashed line); generated command trajectory $g(t)$ (thick line); minimizing parameters $\mu(t)$ (thin solid line) and $w(t)$ (thin dashed line).

Theorem 1: Let (A.1)–(A.3) be fulfilled. Consider (1) with the CG (24), (25) and let $x(0)$ be admissible. Then we have the following.

1) The J-minimizer (24) uniquely exists at each $t \in Z_+$ and can be obtained by solving a convex constrained optimization problem with inequality constraints $q_j(c(i-1, x(t), \theta)) \leq 0, j \in \underline{n}_q$, limited to a finite number i_o of time-steps, viz. $i = 1, \cdots, i_o$.
2) The integer i_o can be computed off-line via (53).
3) The overall system satisfies the constraints and is asymptotically stable and offset free in that the conclusions of Proposition 2 hold.

IV. Simulation Studies

We investigate in some detail how to tune the free parameters of the CG, with direct reference to two different examples. The simulation results reported hereafter were obtained under Matlab 4.0 + Simulink 1.2 on a 486 DX2/66 personal computer, with no particular care of code optimization. The standard Matlab QP.M routine was used for quadratic optimization.

Example 1: Consider the following nonminimum-phase single-input/single-output system:

$$y(t) = \frac{-0.8935z + 1.0237}{z^2 - 1.5402z + 0.6703} g(t). \tag{54}$$

The unit step response of (54) is depicted in Fig. 1(a) (thin line). The task of the CG is to bound the output between -0.5 and 5. Accordingly, $c(t) = y(t)$ and $\mathcal{C} = [-0.5, 5]$. The CG has the following free parameters: δ; γ; Ψ_μ; Ψ_w; Ψ_y. They will be referred to as CG *design knobs*. The δ and γ knobs affect the resulting number of constraints involved in the on-line optimization. This number, which in general is given by the minimal constraint horizon i_o in (53) minus the delay between $c(t)$ and $g(t)$, equals here $i_o - 1$. Fig. 2 shows i_o as a function of δ and γ. For small values of δ, which are the ones of practical interest, i_o is larger at intermediate values of γ. Thus, in this respect, it is convenient to restrict γ close either to one or zero. Another item that can be affected by the choice of γ is the set of admissible states. For $\delta = 0.05$, this set is depicted in Fig. 3 for two candidate values of γ, viz. $\gamma = 0.1$ and 0.9. For intermediate values of γ, the set of admissible states is approximatively comprised within the two depicted sets. The conclusion is that here γ affects the size and the shape of the set only slightly. Before choosing either $\gamma = 0.9$ or 0.1, we focussed on the remaining tuning knobs. The choice $\Psi_\mu/\Psi_w \leq 0.1$ turned out to be an appropriate one in that, in practice, it entails no limitation on the values that $\mu(t)$ can take on. Choosing $\Psi_\mu/\Psi_w = 0.1$, we considered the constrained unit step response as a function of Ψ_y/Ψ_w for both the candidate values of γ. Globally, the shape of this response turned out to suggest the choice $\gamma = 0.9$. Fig. 4 depicts the constrained unit step response for $\delta = 0.05$, $\gamma = 0.9$, $\Psi_w = 10$, $\Psi_\mu = 1$, as a function of Ψ_y. As can be expected, a nonzero Ψ_y slows down the response of the overall system. Notice that for $\Psi_y = 0$, the dynamics of the system is unchanged when the constraints are inactive because the CG sets $g(t) = r(t)$. Taking into account the foregoing simulation analysis, the final selected tuned knobs were $\delta = 0.05$, $\gamma = 0.9$, $\Psi_\mu = 1$, $\Psi_w = 10$, $\Psi_y = 0.1$. Algorithm (53) was executed on the above machine and took 3.8 s to give $i_o = 14$. The related constrained unit step response is shown in Fig. 1(a) (thick line). This was computed in 0.15 s per time step. Fig. 1(b) depicts the generated command trajectory $g(t)$ (thick line), the reference trajectory $r(t)$ (thick dashed line), and minimizing parameters $\mu(t)$ (thin line) and $w(t)$ (thin dashed line).

In order to consider the effects of model uncertainties, the same CG as the one designed for the nominal plant (54) was

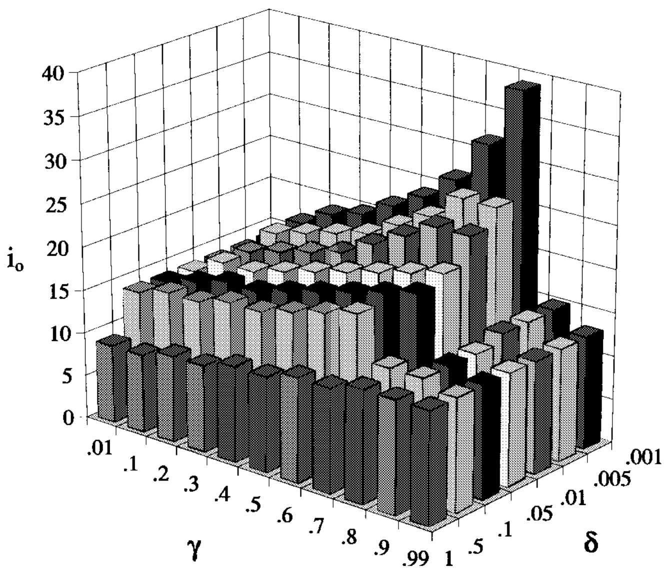


Fig. 2. Example 1. Minimal constraint horizon i_o computed via (53) as a function of γ and δ.

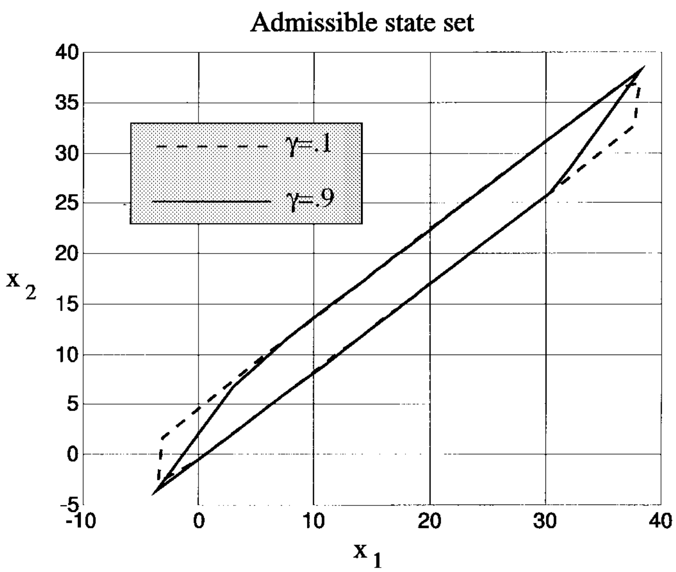


Fig. 3. Example 1. Set of admissible states x for different values of γ ($\delta = 0.05$).

used with the plant

$$y(t) = \frac{-1.2517z + 1.4352}{z^2 - 1.4657z + 0.6492} u(t). \tag{55}$$

Fig. 1(a) exhibits the related output response (thick dashed line). The prescribed lower bound is slightly violated.

Example 2: The CG is applied to the AFTI-16 aircraft modeled in continuous-time as in [15]. The elevator and the flaperon angles are the inputs u to the plant. They are subject to the physical constraints $|u_i| \leq 25°$, $i = 1, 2$. Then, $c = u$. The attack and the pitch angles are the outputs y. The task is to get zero offset for piecewise-constant references while avoiding input saturations. The continuous-time model in [15] is sampled every $T_s = .05$ s, and a zero-order hold is used at

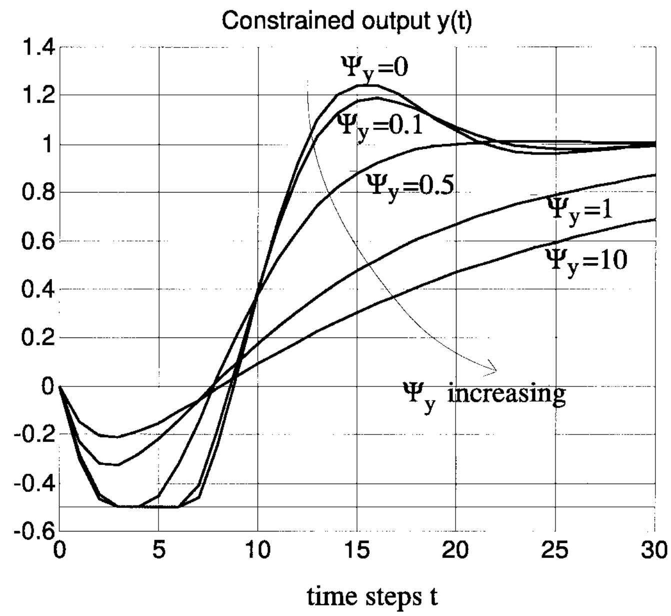


Fig. 4. Example 1. Constrained variable (output) response for different values of parameter Ψ_y.

the input. The following linear compensator:

$$u(t) = \begin{bmatrix} 0.000\,05 & 1.256\,01 & -0.178\,72 & 0.556\,20 \\ -0.000\,43 & 13.711\,01 & 4.069\,60 & -0.373\,50 \end{bmatrix} x(t) + \begin{bmatrix} 1.934\,76 & -0.556\,18 \\ -21.189\,23 & 0.373\,51 \end{bmatrix} g(t) \tag{56}$$

was designed, with no concern of the constraints, so as to obtain both adequate dynamic decoupling and fast transient response. Fig. 5 shows the response of the compensated linear system with no CG. Note that the constraints are violated. It can be shown that if the linear compensator outputs given by (56) are saturated so as to fulfill the constraints, the system becomes unstable. Fig. 6 depicts the trajectories resulting when the CG is activated so as to constrain the two plant inputs within the prescribed bounds. To this end, after some simulation analysis, we tuned the CG design knobs as follows: $\gamma = 0.9$; $\delta = 0.1$; $\Psi_\mu = 10I_2$; $\Psi_w = I_2$; $\Psi_y = 0$. The last choice was made in order to leave unchanged the dynamics with inactive constraints. Under these choices, (53) finds $i_o = 140$. Simulations were carried out with a computational time of 0.91 s per step. Heuristically, it was found that for the reference in Fig. 5, indistinguishable results can be obtained with a constraint horizon equal to 5 in 0.13 s per time step. Though these computational times exceed the sampling interval T_s, the simulation results indicate the performance achievable by using faster processors with software specifically optimized for the application at hand. Because of vector optimization, the reference is filtered both in modulus and direction. This explains the coupling between the two outputs. In order to let the direction of $g(t)$ be as close as possible to that of $r(t)$, Ψ_μ and Ψ_w were modified by penalizing at each time t the component of $g(t)$ orthogonal to $r(t)$. This is accomplished by adding to Ψ_μ and Ψ_w the weighting matrix

$$100\left(I - \frac{r(t)r'(t)}{r'(t)r(t)}\right) = \begin{bmatrix} 100 & 0 \\ 0 & 0 \end{bmatrix}. \tag{57}$$

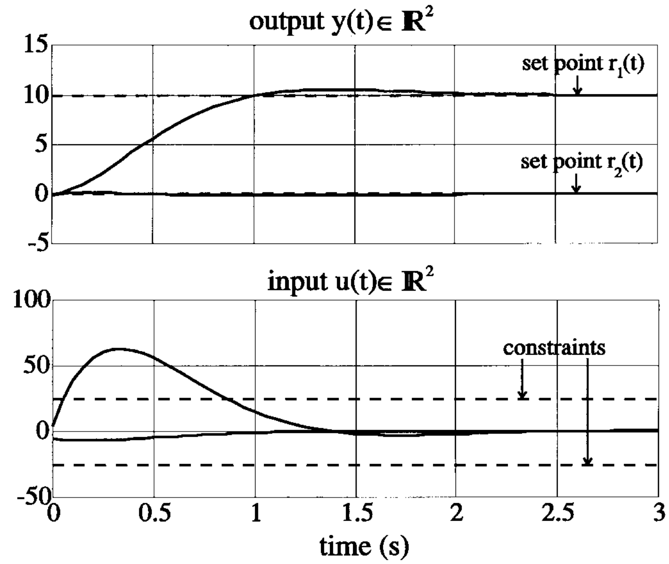


Fig. 5. Example 2. Compensated AFTI-16 response with no CG.

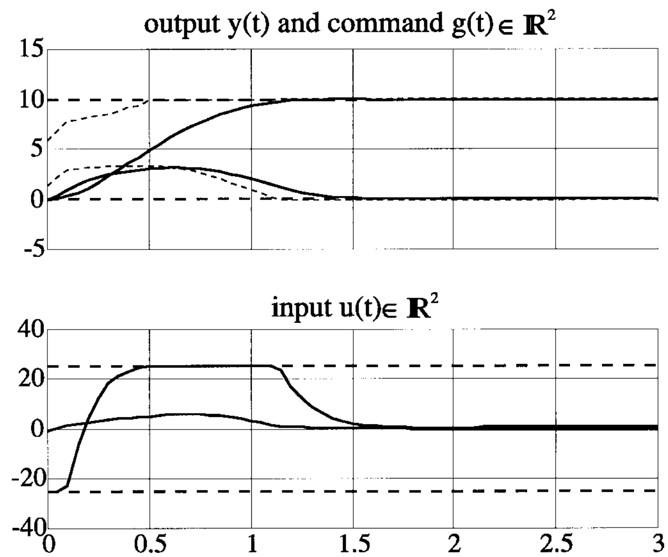


Fig. 6. Example 2. Response with the CG: output $y(t)$ (solid line), command $g(t)$ (thin dashed line), and reference $r(t)$ (thick dashed line).

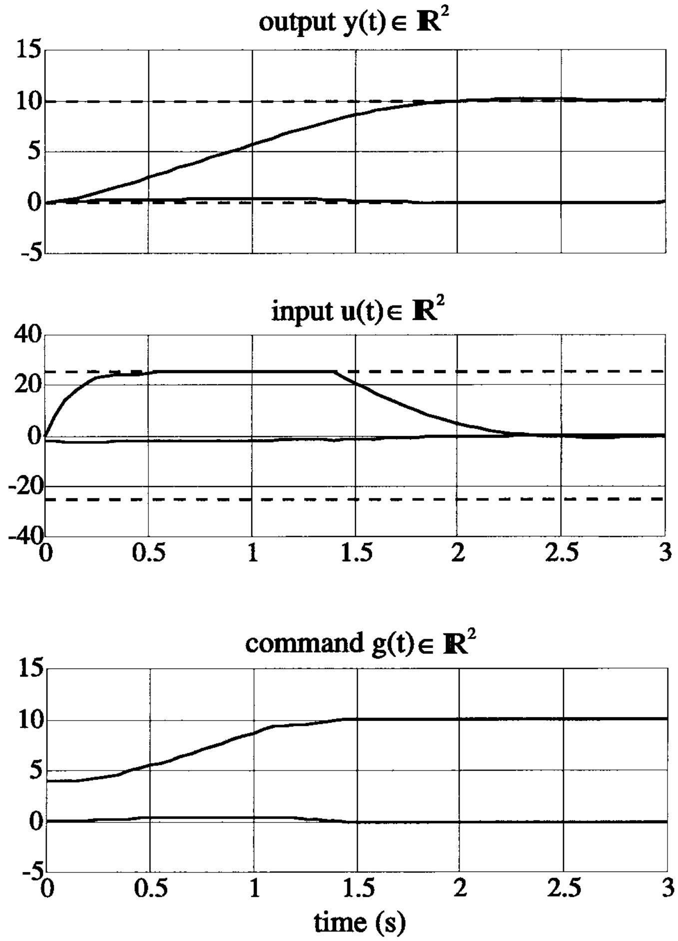


Fig. 7. Example 2. Response with the CG penalizing the component of μ and w orthogonal to $r(t)$.

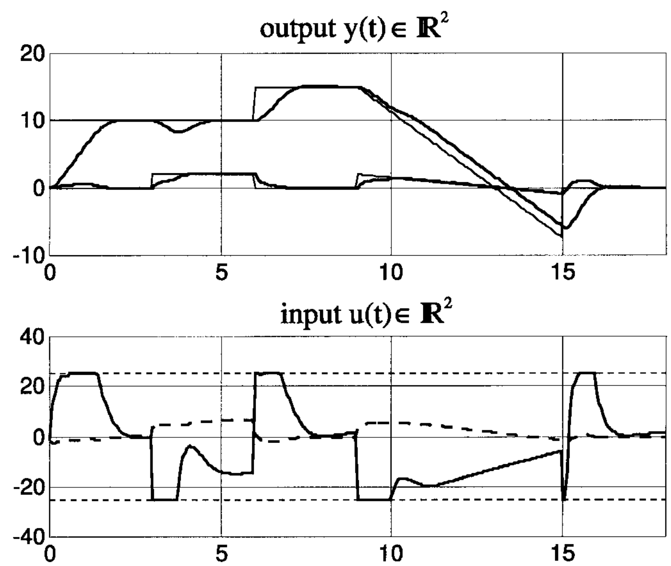


Fig. 8. Example 2. Output $y(t)$ (thick line) and reference $r(t)$ (thin line); input $u(t)$ (solid and dashed line).

This modification does not affect the analysis in Section II, where $r(t)$ is assumed to become constant. The trajectories related to the modification (57), as depicted in Fig. 7, exhibit a reduced crosscoupling at the cost of longer settling times. Fig. 8 shows the performance of the system with the same CG when the reference exhibits time-variations in such a way that transients take place also from nonequilibrium states, if by an equilibrium state we mean a vector x_w, $w \in \mathcal{W}_\delta$, as in (4).

Finally, the behavior of the command governor in the presence of an output zero-mean white Gaussian sensor noise with covariance $0.2 I_2$ was simulated under the same CG knob choice as in Fig. 7. This behavior is depicted in Fig. 9. In this case, the state $x(t)$ used by the CG is replaced by an estimate $\hat{x}(t)$ provided by the state Kalman observer. Notice that the constraints can be still fulfilled.

V. Conclusions

The CG problem, viz. the one of on-line designing a command input in such a way that a primal compensated

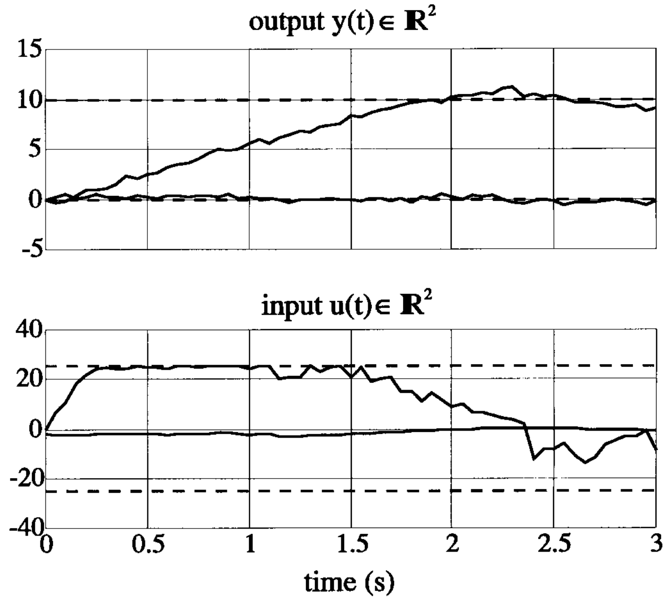


Fig. 9. Example 2. Response with the CG and output measurement noise.

control system can operate in a stable way with satisfactory tracking performance and no constraint violation, has been addressed by exploiting some ideas originating from predictive control. In this connection, the concept of a "virtual" command sequence is instrumental to synthesize a CG having the stated properties along with a moderate computational burden. This is achieved by: first, linearly parameterizing the virtual command sequences by a vector of twice the dimension of the reference and defining the functional form of the sequence so as to ease stability analysis; and second, choosing at each sampling time the free parameter vector as the one minimizing a constrained quadratic selection index.

It has been shown how to use off-line an iterative algorithm so as to restrict to a fixed finite integer the infinite number of time-instants over which the prescribed constraints must be checked in order to decide admissibility of virtual command sequences. A stability analysis based on a Lyapunov function argument shows that if the reference becomes constant, the system output asymptotically converges to the closest admissible approximation to the reference. Simulations have shown the effectiveness of the CG when applied to systems with input and/or state-related constraints. Future research on the subject should be directed to analyze effects of disturbances and modeling errors, as well as to compare the CG of this paper with other possible solutions (e.g., the ones in [15] and [19]) and direct use of constrained predictive control, in terms of performance, robustness, and computational burden.

APPENDIX

Proof of Proposition 3: Let $z_o \in \mathcal{Z}^o_{n_o}$. Because (A_o, E_{c_o}) is an observable pair, $\Theta'\Theta$ is a nonsingular matrix, where $\Theta := [E'_{c_o} | (E_{c_o} A_o)' | \cdots | (E_{c_o} A_o^{n_o - 1})']'$. It follows that, $z_o = (\Theta'\Theta)^{-1}\Theta' R$, with $R = [c'(0, x, \theta), \cdots, c'(n_o - 1, x, \theta)]'$. Then, being $\mathcal{C}$ bounded, $\mathcal{Z}^o_{n_o+l}$, $\forall l \in Z_+$, is bounded as well, because $\mathcal{Z}^o_{n_o+l} \subset \mathcal{Z}^o_{n_o}, \forall l \in Z_+$. In order to show that $\mathcal{Z}$ is finitely determined, note that $\lim_{i\to\infty} c(i, x, \theta) = c_w$. Now

$$\begin{aligned} c(i, x, \theta) - c_w &= E_c M^i (z - z_w) \\ &= E_{c_o} M_o^i (z_o - z_{wo}) \end{aligned}$$

where

$$M = \begin{bmatrix} \Phi & G & 0_{n\times p} \\ 0_{p\times n} & \gamma I_p & 0_{p\times p} \\ 0_{p\times n} & 0_{p\times p} & 0_{p\times p} \end{bmatrix}, \quad z_w = \begin{bmatrix} x_w \\ 0 \\ w \end{bmatrix}.$$

M_o is obtained from M in the same way as A_o from A, $z_o = P_o z$, and $z_{wo} = P_o z_w$. Then

$$\|c(i, x, \theta) - c_w\| \le \bar{\sigma}\left(E_{co} M_o^i\right)(\|z_o\| + \|z_{wo}\|).$$

Because $z_{wo} \in \mathcal{Z}^o$, $\|z_o\| + \|z_{wo}\|$ is bounded for all $z_o \in \mathcal{Z}^o$. Therefore, the existence of an integer i_o such that

$$i \ge i_o \Rightarrow \|c(i, x, \theta) - c_w\| \le \delta, \quad \forall z \in \mathcal{Z}$$

follows from asymptotic stability of M.

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