Hybrid Model Predictive Control of Direct Injection Stratified Charge Engines

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***Abstract*—This paper illustrates the application of hybrid modeling and model predictive control techniques to the management of air-to-fuel ratio and torque in advanced technology gasoline direct-injection stratified-charge (DISC) engines. A DISC engine is an example of a constrained hybrid dynamical system, because it can operate in two distinct modes (stratified and homogeneous) and because the mode-dependent constraints on the air-to-fuel ratio and on the spark timing need to be enforced during its operation to avoid misfire, knock, and high combustion variability. In this paper, we approximate the DISC engine dynamics as a two-mode discrete-time switched affine system. Using this approximation, we tune a hybrid model predictive controller with integral action based on online mixed-integer quadratic optimization, and show the effectiveness of the approach through simulations. Then, using an offline multiparametric optimization procedure, we convert the controller into an equivalent explicit piecewise affine form that is easily implementable in an automotive microcontroller through a lookup table of linear gains.**

***Index Terms*—Automotive applications, direct-injection engines, model predictive control (MPC), powertrain control.**

I. Introduction

THE advanced technology direct-injection stratified-charge (DISC) engines can operate in either *stratified* or *homogeneous* combustion modes. Mode switches are performed by changing the fuel injection timing from late (for stratified combustion) to early (for homogeneous combustion). If the fuel injection is late and occurs in the compression stroke, the time available for fuel to mix with air is short and a non-homogeneous (stratified) air–fuel mixture forms in the cylinder. The air-to-fuel ratio distribution of this mixture is suitable for combustion to be initiated and proceed in the region near the spark plug. However, away from that region, the in-cylinder air-to-fuel ratio can be very lean (i.e., have significant excess air). If the fuel injection is early and occurs in the intake stroke, the fuel has then sufficient time to mix with air to form a homogeneous mixture in the cylinder with uniform air-to-fuel ratio distribution, as in conventional port-fuel-injected (PFI) engines.

Since the engine operation in the stratified mode occurs with excess air and, therefore, higher intake manifold pressure, engine pumping losses are reduced and fuel economy is improved. There are benefits to using the direct injection even in the homogeneous mode, including volumetric efficiency and peak power improvements, which make the direct injection synergistic with turbocharging and engine downsizing technologies.

The stratified operation can only be sustained in a restricted portion of the engine operating range, namely at low to medium engine speeds and loads. For higher engine speeds and loads, the transition to homogeneous combustion mode becomes necessary. In addition, periodic transitions from the stratified mode to the homogeneous mode are required even at lower speeds and loads to purge the lean NO_x trap (LNT), which is a catalyst specifically formulated to store oxides of nitrogen during lean operation and convert them during slightly rich operation in the homogeneous mode. Other requirements exist, including desulfation of the LNT, which also temporarily impede the stratified operation and necessitate transition to the homogeneous operation.

The control system for the DISC engine must accurately deliver the requested engine torque and air-to-fuel ratio in each of the modes through optimal coordination of throttle, spark timing, fueling, and the selection of the combustion mode. In addition, mode-dependent state and control constraints on the air-to-fuel ratio and spark timing need to be enforced. The control system must also seamlessly perform stratified to homogeneous mode transitions to avoid disturbance to the vehicle customers.

The existing approaches to this control problem rely on logic-based switching applied to a family of low-level controllers [1], [2]. The construction of the switching logic and low-level controllers in these references is, to a large extent, DISC engine-specific. From the standpoint of reducing the development time, more *systematic* control design procedures that can be effortlessly applied to various (DISC and non-DISC) engine and powertrain configurations with multiple operating modes and constraints are of significant interest.

The hybrid modeling and the model predictive control (MPC) framework discussed in this paper provide, in principle, a systematic control design procedure of this kind. With this approach, the MPC controller is first designed and tuned in simulations on a mixed logic dynamical (MLD) characterization of the hybrid dynamics. The tuning process involves adjusting the horizon and the weights in the cost function (which act as knobs with a very direct influence on shaping the closed-loop response) until the desired performance is attained. Then, the equivalent explicit piecewise linear form of the MPC controller is computed offline by using a multiparametric solver. If the explicit

Manuscript received October 29, 2005; revised June 16, 2006. Recommended by Guest Editor K. Jezernik.

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Digital Object Identifier 10.1109/TMECH.2006.882979

form of the receding horizon controller has a sufficiently small number of regions, it may be suitable for implementation in the automotive microcontroller, which, as compared to regular desktop computers, has only a limited computing power.

In this paper, we illustrate this control framework by considering a nonlinear DISC engine model, which enables us to numerically extract two linearized models representative of engine behavior in each of the two modes at a nominal engine speed. At every sampling time instant, the reference commands for intake manifold pressure, mass flow rate of air through the throttle, and mass flow rate of fuel that are consistent with the current engine speed and match in steady state the requested torque and air–fuel ratio commands, are calculated, thereby providing a feed-forward control. The feedback control, which is key to accomplishing our goals of transient response shaping and constraint enforcement, is then augmented in the form of a hybrid MPC controller designed based on the two-mode discrete-time linearized model. The MPC controller is tuned in simulations, in which a mixed-integer quadratic optimization problem is solved at each time step. Integrators are added to compensate for the mismatch between the nonlinear model and the linearized models used for the design. This approach, based on combining the feed-forward reference generation mechanism with the feedback hybrid MPC controller based on a simplified two-mode linearized model extracted at a nominal engine speed, achieves good torque and air-to-fuel ratio tracking. At the same time, it enforces pointwise-in-time constraints on the air-to-fuel ratio and spark timing even when the engine speed deviates from the nominal value. The transient response is shaped by changing the weights in the MPC cost function.

Once the MPC controller is tuned in simulation, its explicit version can be computed via offline multiparametric optimization procedures, obtaining a reasonably simple piecewise affine controller form that can be implemented in the automotive microcontroller.

The paper is organized as follows. A nonlinear model of the DISC engine is presented in Section II. The feedforward control is covered in Section III. The hybrid modeling and the MPC strategy are discussed in Sections IV and V, respectively. In Section VI, we report the simulation results. The implementation of the control law in the explicit piecewise affine form is discussed in Section VII. Finally, concluding remarks are made in Section VIII.

Preliminary results about the problem considered in this paper have appeared in our conference papers [3]–[5]. The main difference is that in this paper we utilize a more elaborate and complex simulation model of the DISC engine.

Other examples of powertrain systems with multiple operation modes include variable displacement engines (VDEs) that can vary the number of running cylinders while keeping others deactivated; variable compression ratio (VCR) engines in which the compression ratio can vary between high and low values; homogeneous charge compression ignition (HCCI) engines in which the combustion can occur by autoignition or be initiated by spark; and hybrid electric vehicles (HEVs) that can switch between different power transfer modes. The basic issues of controlling torque and air-to-fuel ratio, seamlessly handling mode transitions, and satisfying pointwise-in-time state and actuator constraints are similar in these applications to the DISC engine cade. The control development methodology proposed in this paper, based on the hybrid MPC paradigm, can be beneficially applied to other powertrain systems with multiple operating modes and constraints.

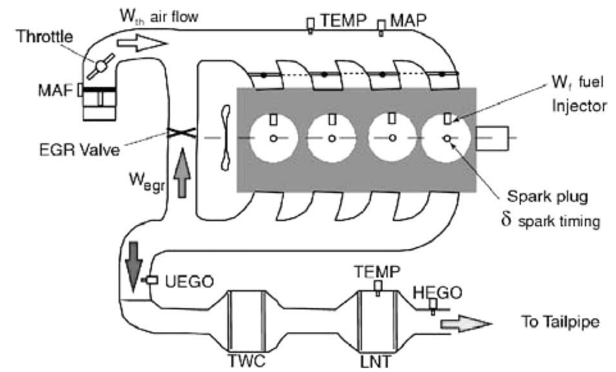

Fig. 1. DISC engine.

II. Nonlinear Model

Our developments are based on a nonlinear engine model extracted from a model [6] developed by Ford Research and Advanced Engineering for a DISC engine depicted in Fig. 1. As our main interest is in torque and air-to-fuel ratio control as well as in mode transitions, the exhaust gas recirculation (EGR) valve is considered closed (or, equivalently, the engine is assumed not to have external EGR). The intake manifold pressure and mass flow rates into the intake manifold are related by the equation

$$\dot{p}_{\mathrm{m}} = c_{\mathrm{m}}(W_{\mathrm{th}} - W_{\mathrm{cyl}})$$
$$W_{\mathrm{cyl}} = k_{\mathrm{cyl},0} + k_{\mathrm{cyl},1} p_{\mathrm{m}} \omega \tag{1}$$

where ω (r/min) is the engine speed; p_{m} [kPa] is the intake manifold pressure; $c_{\mathrm{m}} = \frac{RT_{\mathrm{m}}}{V_{\mathrm{m}}}$, where T_{m} [K] is the intake temperature, R is the difference of specific heats for air, and V_{m} is the intake manifold volume; W_{th} [g/s] is the air mass flow rate through the electronic throttle, which is a nonlinear function of p_{m} described later in (5); W_{cyl} is the mass flow rate of air into the engine cylinders, and $k_{\mathrm{cyl},0}, k_{\mathrm{cyl},1}$ are the engine pumping coefficients that depend on the intake temperature and engine speed. Equation (1) is obtained from the differentiated ideal gas law under isothermal conditions for the manifold filling dynamics.

The in-cylinder air-to-fuel ratio is defined as

$$\lambda = \frac{W_{\mathrm{cyl}}}{W_{\mathrm{f}}} \tag{2}$$

where W_{f} [g/s] is the mass flow rate of fuel into the engine cylinders.

The engine torque is a summation of three terms,

$$\tau = \tau_{\mathrm{mfr}} + \tau_{\mathrm{pump}} + \tau_{\mathrm{ind}} \tag{3}$$

where τ_{mfr}[N·m] and τ_{pump}[N·m] are the mechanical friction torque and the pumping torque, respectively, and are modeled by affine functions of p_{m} that also depend on the engine speed;

τ_{ind}[N·m] is the indicated torque

$$\tau_{\text{ind}} = (\theta_a + \theta_b(\delta - \delta_{\text{mbt}})^2)W_{\text{f}} \tag{4}$$

where θ_a, θ_b, and δ_{mbt} are functions of λ that depend on the spark timing δ, the engine speed ω, and the combustion mode ρ. In particular, $\rho = 0$ corresponds to the stratified mode, while $\rho = 1$ corresponds to the homogeneous mode. Such a binary nature of ρ is the main source of "hybridness" in the DISC engine model. We also note that δ_{mbt} is referred to as the maximum brake torque (MBT) spark timing because the engine brake torque is maximized when $\delta = \delta_{\text{mbt}}$.

Equations (1)–(3) can also be used in the control strategy to provide online estimates of the in-cylinder air-to-fuel ratio and of the torque that the engine generates. These estimates can be used by a controller, such as the MPC controller that we consider in the remainder of the paper, to shape transient response of the engine and satisfy the pointwise-in-time constraints. The parameters of (1)–(3) can be determined during engine calibration and may be adapted online from available measurements in the vehicle, such as engine speed and exhaust air-to-fuel ratio measurement, using various online estimation algorithms.

To represent the throttle flow in simulations, a standard orifice flow equation [7] was introduced

$$W_{\text{th}} = \frac{A_{\text{th}} P_{\text{amb}}}{\sqrt{T_{\text{amb}}}} \phi\left(\frac{p_{\text{m}}}{P_{\text{amb}}}\right) \tag{5}$$

where P_{amb} [kPa] and T_{amb} [K] are the ambient pressure and temperature, respectively, that we consider as constant values, and $A_{\text{th}}[\text{m}^2]$ is the effective throttle flow area (scaled by $\sqrt{R}$). The function ϕ represents the effects of the pressure ratio across the throttle

$$\phi(x) = \begin{cases} \gamma^{1/2}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/2(\gamma-1)} & \text{if } x \leq \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} \\ x^{1/\gamma}\left(\frac{2\gamma}{\gamma-1}\left(1 - x^{\frac{\gamma-1}{\gamma}}\right)\right)^{1/2} & \text{if } x > \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} \end{cases} . \tag{6}$$

where $\gamma = 1.4$ is the ratio of specific heats for air and is considered constant. The hybrid model used for MPC predictions and developed in Section III neglects throttle flow nonlinearity (5) and (6), and throttle flow is treated as one of the variables that is directly being manipulated. The throttle nonlinearity in (5) and the actuator dynamics are instead taken into account in the simulation model as follows. Given the request for the throttle air flow W_{th} prescribed by the controller, as well as the current intake manifold pressure p_{m}, the ambient temperature T_{amb}, and the ambient pressure P_{amb}, the throttle equation (5) is first inverted to compute the required A_{th}. Then, the required A_{th} is filtered through a first-order low-pass filter, emulating throttle body dynamics and passed through a saturation block in order to restrict it to a physical range. Then (5) and (6) are used to compute the actual throttle flow entering (1).

III. Reference Generation

The objective of this paper is to design a control law that generates the inputs $W_{\text{th}}, W_{\text{f}}, \delta$, and ρ as a function of the measurements (or estimates) of $p_{\text{m}}, \tau, \omega$, and λ so that the requested reference commands for $\tau, \lambda, \Delta\delta = \delta_{\text{mbt}} - \delta$ and ρ ($\tau_{\text{ref}}, \lambda_{\text{ref}}, \Delta\delta_{\text{ref}}$ and ρ_{ref}, respectively) are accurately tracked and several constraints (see Section IV) are satisfied to ensure that the engine operation is maintained within a feasible operating window. The references τ_{ref}, $\lambda_{\text{ref}}, \Delta\delta_{\text{ref}}$, and ρ_{ref} are generated by a higher level portion of the control strategy, based on vehicle operating conditions, including driver pedal position and the state of LNT. Once these *independent* reference commands have been prescribed, the *dependent* reference commands for $W_{\text{th}}, W_{\text{f}}$, and p_{m}, i.e., $W_{\text{th,ref}}, W_{\text{f,ref}}$, and $p_{\text{m,ref}}$, can be obtained as equilibrium values of the throttle air flow, fueling rate, and intake manifold pressure, respectively, yielding in the steady state the engine torque τ_{ref}, air-to-fuel ratio λ_{ref}, and spark retard $\Delta\delta_{\text{ref}}$ in the given combustion mode ρ and at a given engine speed ω. The calculation of $W_{\text{th,ref}}, W_{\text{f,ref}}$, and $p_{\text{m,ref}}$ can either be performed through an online numerical search or precomputed offline and embedded into lookup tables or appropriate regression models.

IV. Hybrid Model for Control

In view of the presence of the binary input ρ, we solve the feedback control problem within a hybrid systems framework. Hybrid systems provide a unified framework for describing processes evolving according to continuous dynamics, discrete dynamics, and logic rules [8]–[11]. The interest in hybrid systems is mainly motivated by the large variety of practical situations where physical processes interact with digital controllers as, for instance, in embedded systems. Several modeling formalisms have been developed to describe hybrid systems [12], among them the class of MLD systems [13]. Examples of real-world applications that can be naturally modeled within the MLD framework have been reported in [14]–[16]. The hybrid systems description language (HYSDEL) was developed in [17] to obtain MLD models from a high-level textual description of the hybrid dynamics, and it will be used in this paper.

The model described in Section II is approximated by a discrete-time hybrid model through the following steps.

1) *Linearization and time-discretization*. For the engine speed $\omega = 2000$ r/min, we define two operating points for each mode: $\tau^{\text{d}}(0) = 40$ N·m, $\lambda^{\text{d}}(0) = 30, \delta^{\text{d}}(0) = 16^\circ$ for the stratified mode and $\tau^{\text{d}}(1) = 40$ N·m, $\lambda^{\text{d}}(1) = 14, \delta^{\text{d}}(1) = 16^\circ$ for the homogeneous mode. Then, for each of the two modes $\rho = 1$ and $\rho = 0$, a linear model is obtained through standard numerical linearization routines in MATLAB and discretized in time with sampling period of $T = 10$ ms. In particular, τ and λ are approximated as mode-dependent affine functions: $\lambda = \ell_1^\lambda(\rho)p_{\text{m}} + \ell_2^\lambda(\rho)W_{\text{th}} + \ell_3^\lambda(\rho)W_{\text{f}} + \ell_4^\lambda(\rho)\delta + \ell_5^\lambda(\rho), \tau = \ell_1^\tau(\rho)p_{\text{m}} + \ell_2^\tau(\rho)W_{\text{th}} + \ell_3^\tau(\rho)W_{\text{f}} + \ell_4^\tau(\rho)\delta + \ell_5^\tau(\rho)$.
2) *Integrators*. The model is augmented by two integrators to obtain zero offsets in the steady state

$$\epsilon_\tau(t+1) = \epsilon_\tau(t) + T(\tau_{\text{ref}} - \tau) \tag{7a}$$

$$\epsilon_\lambda(t+1) = \epsilon_\lambda(t) + T(\lambda_{\text{ref}} - \lambda) \tag{7b}$$

where t represents the sampling step and the subscript "ref" represents the reference value. In particular, this augmentation of the integrators ensures zero offsets in τ and

λ from $\tau_{\rm ref}$ and $\lambda_{\rm ref}$ despite the model mismatch between the nonlinear simulation model and the linearized hybrid design model. In addition, we consider the spark timing deviation from the MBT value, $\Delta\delta = \delta_{\rm mbt} - \delta$, as an extra output for which a reference value is also prescribed.

3) *Constraints*. Constraints are added to guarantee the correct operation of the engine.

a) A constraint on the air-to-fuel ratio is imposed to prevent excessive engine roughness and misfiring at air-to-fuel ratios that are too lean, and the increase in hydrocarbon and smoke emissions at air-to-fuel ratios that are too rich. The constraint takes the form

$$\lambda_{\min}(\rho) \le \lambda \le \lambda_{\max}(\rho). \tag{8}$$

Note that the limits $\lambda_{\min}(\rho)$ and $\lambda_{\max}(\rho)$ depend on the combustion mode ρ.

b) A constraint on the mass flow rate through the electronic throttle, $0 \le W_{\rm th} \le K$, where K is the function of the intake manifold pressure and represents the physical limit of the throttle.

c) A constraint on the spark timing δ to avoid engine knock and maintain combustion stability

$$0 \le \delta \le \delta_{\rm mbt}(\rho, p_{\rm m}, W_{\rm th}, W_{\rm f}, \delta) \tag{9}$$

where $\delta_{\rm mbt}(\rho, p_{\rm m}, W_{\rm th}, W_{\rm f}, \delta) = \ell_1^\delta(\rho) p_{\rm m} + \ell_2^\delta(\rho) W_{\rm th} + \ell_3^\delta(\rho) W_{\rm f} + \ell_4^\delta(\rho)\delta + \ell_5^\delta \rho$ is also modeled as a switched affine function.

d) A bound on the derivative of the mass flow rate $\dot W_{\rm th}$ generated by the controller

$$\dot W_{\rm th}^{\min} \le \frac{W_{\rm th}(t) - W_{\rm th}(t-1)}{T} \le \dot W_{\rm th}^{\max} \tag{10}$$

where $\dot W_{\rm th}^{\min}$ and $\dot W_{\rm th}^{\max}$ are suitable constants.

The above dynamic equations and constraints have been modeled in the modeling language HYSDEL [17]. [1]

The HYSDEL compiler translates the description into the MLD form

$$x(t+1) = Ax(t) + B_1 u(t) + B_2\gamma(t) + B_3 z(t) \tag{11a}$$

$$y(t) = Cx(t) + D_1 u(t) + D_2\gamma(t) + D_3 z(t) \tag{11b}$$

$$E_2\gamma(t) + E_3 z(t) \le E_1 u(t) + E_4 x(t) + E_5. \tag{11c}$$

In our case, $x = [p_{\rm m}\ \epsilon_\tau\ \epsilon_\lambda\ W_{\rm th}(t-1)\ \tau_{\rm ref}\ \lambda_{\rm ref}] \in \mathbb{R}^6, y = [\tau - \tau_{\rm ref}\ \lambda - \lambda_{\rm ref}\ \delta_{\rm mbt} - \delta]' \in \mathbb{R}^3, u = [W_{\rm th}\ W_{\rm f}\ \delta\ \rho]' \in \mathbb{R}^3 \times \{0,1\}$, where $W_{\rm th}, W_{\rm f}, \delta$, and ρ are the manipulated variables, the reference commands $\tau_{\rm ref}$ and $\lambda_{\rm ref}$ are treated as (constant) states to be able to predict (and integrate) future tracking errors, and γ and z are auxiliary variables introduced for the translation of the constraints and dynamics into (11). In general, γ and z are, respectively, a binary and a real auxiliary vector whose values are determined uniquely by the inequalities (11c) once $x(t)$ and $u(t)$ are fixed [13]. In our case, the binary vector γ is empty, as no additional Boolean variables are needed to describe the hybrid dynamics of the DISC engine, and $z \in \mathbb{R}^5$.

[1]The corresponding description is available at http://www.dii.unisi.it/hybrid/automotive/disc.

V. MPC-Based Online Optimization

MPC has found many industrial applications and it has been successfully applied to hybrid dynamical systems [14]–[16]. In Section V, we show how we can derive an MPC controller for the DISC engine. In the MPC approach, at each sampling instant, a finite horizon open-loop optimization problem is solved. This is done by assuming the current state as the initial condition for the problem. The optimization provides an optimal control sequence, only the first element of which is applied to the hybrid system. This process is iteratively repeated at each subsequent time instant, thereby providing a feedback mechanism for disturbance rejection and reference tracking. The optimal control problem is defined as

$$\begin{aligned}\min_{\xi} J(\xi, x(t)) \triangleq \sum_{k=0}^{N-1} &(u_k - u_{\rm ref})^T R (u_k - u_{\rm ref}) \\ &+ (y_k - y_{\rm ref})^T Q (y_k - y_{\rm ref}) \\ &+ \sum_{k=1}^{N} (x_k - x_{\rm ref}) S (x_k - x_{\rm ref})\end{aligned} \tag{12a}$$

subject to

$$\begin{cases} x_0 = x(t) \\ x_{k+1} = Ax_k + B_1 u_k + B_2\gamma_k + B_3 z_k \\ y_k = Cx_k + D_1 u_k + D_2\gamma_k + D_3 z_k \\ E_2\gamma_k + E_3 z_k \le E_1 u_k + E_4 x_k + E_5 \end{cases} \tag{12b}$$

where N is the control horizon, $x(t)$ is the state of the MLD system at sampling time t, $\xi \triangleq [u_0', \gamma_0', z_0', \ldots, u_{N-1}', \gamma_{N-1}', z_{N-1}']'$ is the optimization vector, and Q, R, and S are weight matrices. In (12), we let

$$y_{\rm ref} \triangleq [0\ 0\ \Delta\delta_{\rm ref}]' \tag{13a}$$

$$u_{\rm ref} \triangleq [W_{\rm th,ref}\ W_{\rm f,ref}\ \delta_{\rm ref}\ \rho_{\rm ref}]' \tag{13b}$$

$$x_{\rm ref} \triangleq [p_{\rm m,ref}\ 0\ 0\ 0\ 0\ 0]'$$

where $\Delta\delta_{\rm ref}$ is the reference on $\delta_{\rm mbt} - \delta$, and

$$Q = \begin{bmatrix} q_\tau & 0 & 0 \\ 0 & q_\lambda & 0 \\ 0 & 0 & q_{\Delta\delta}\end{bmatrix}, \quad R = \begin{bmatrix} r_{W_{\rm th}} & 0 & 0 & 0 \\ 0 & r_{W_{\rm f}} & 0 & 0 \\ 0 & 0 & r_\delta & 0 \\ 0 & 0 & 0 & r_\rho \end{bmatrix},$$

$$S = \begin{bmatrix} s_{p_m} & 0 & 0 & 0 & 0 & 0 \\ 0 & s_{\epsilon_\tau} & 0 & 0 & 0 & 0 \\ 0 & 0 & s_{\epsilon_\lambda} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{14}$$

In (12) we assume that the possible physical and/or logical constraints on the variables of the hybrid system are already included in the mixed-integer linear constraints of the MLD model, as they can be conveniently modeled through the language HYSDEL. Problem (12) can be translated into a mixed integer quadratic program (MIQP), i.e., into the minimization of a quadratic cost function subject to linear constraints, where some of the variables are constrained to be binary (in our case $\rho(0), \ldots, \rho(N-1) \in \{0,1\}$) [18].

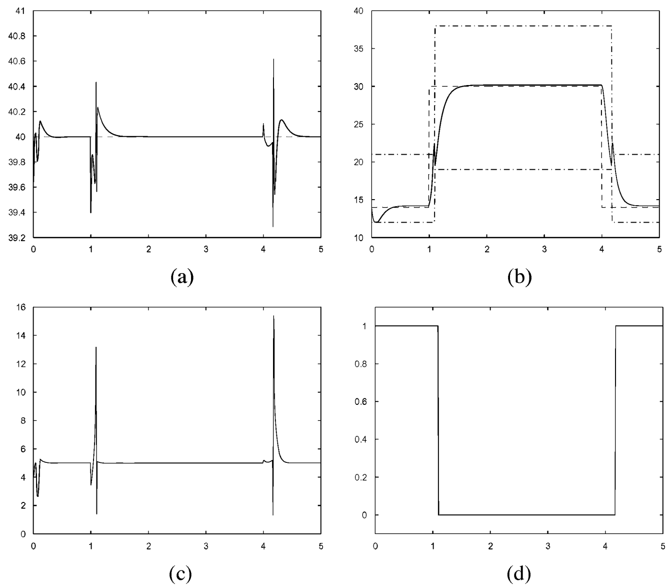

Fig. 2. Closed-loop response (nonlinear model + MPC controller) at nominal engine speed. (a) Engine torque $\tau(t)$ (*dashed line*: desired value, *solid line*: response of the nonlinear model). (b) Air-to-fuel ratio $\lambda(t)$ (*dashed line*: desired value, *solid line*: response of the nonlinear model, *dash-dot line*: A/F bounds). (c) Spark retard from MBT, $\delta_{\rm mbt}(t) - \delta(t)$. (d) Mode of combustion $\rho(t)$.

VI. Simulation Results

The closed-loop behavior of the DISC engine under MPC control has been evaluated in simulations by using the nonlinear model described in Section II. Our control design is based on a two-step strategy. First, given current engine speed and desired torque, air–fuel ratio, and spark retard references, the remaining dependent references are generated at each sampling time instant. Second, these references are passed to the hybrid MPC controller described in Section V.

As MPC design parameters in (12), we choose $N = 1$ and the weights

$$\begin{array}{llll} q_\tau = 1, & q_\lambda = 10^{-3}, & q_{\Delta\delta} = 0.01, & \\ r_{W_{\rm th}} = 0.01, & r_{W_{\rm f}} = 10^{-3}, & r_\delta = 0, & r_\rho = 1 \\ s_{p_{\rm m}} = 0.04, & s_{\epsilon_\tau} = 1.5 \cdot 10^3, & s_{\epsilon_\lambda} = 0.01. & \end{array}$$

Note that we select q_τ much greater than q_λ and set s_{ϵ_τ} to a large value to emphasize torque tracking as the primary objective. The weight r_ρ is set to a sufficiently small value to leave enough freedom to choose the best mode at each time instant, yet not too small in order to prevent engine mode chattering. We have assumed the following mode-dependent ranges on the air-to-fuel ratio (8)

$$\rho = 0: \quad \lambda_{\max} = 38, \quad \lambda_{\min} = 19$$

$$\rho = 1: \quad \lambda_{\max} = 21, \quad \lambda_{\min} = 12.$$

We consider two different scenarios. The first scenario (Fig. 2) is a cruise at constant nominal engine speed (same engine speed at which the engine dynamics were linearized) and constant engine torque request. At time $t = 1$, a mode transition from homogeneous mode to stratified mode is requested, and at time $t = 4$, a reverse mode transition is requested. Often during this type of constant speed/torque cruise, the mode transitions can be most easily noticed by vehicle customers and be perceived as a disturbance. Our controller successfully coordinates engine throttle, fueling, spark timing, and combustion mode selection to keep engine torque fluctations unnoticeable to the vehicle customers (torque deviation from the requested torque is less than 1 N·m) during the transitions, while closely the constraints (9) on $\delta_{\rm mbt} - \delta$ and (8) on λ.

The second scenario is aimed at testing the approach in the presence of the engine speed changes and is based on a 20-s segment (from 965 to 985 s) of the European drive cycle (NEDC). Based on the vehicle speed profile prescribed by the driving cycle, trajectories for $\tau_{\rm ref}$ and ω have been generated assuming particular vehicle and transmission shift schedule choices. The stratified mode $(\rho = 0)$ was enabled for $\omega \leq 2000$ r/min and $\tau_{\rm ref} \leq 50$ N·m. The target air-to-fuel ratio, $\lambda_{\rm ref}$, in the stratified mode was 40 if achievable, or as high as possible if not achievable. The target air-to-fuel ratio $\lambda_{\rm ref}$ in the homogeneous mode $(\rho = 1)$ was prescribed as 14.64. The reference $\Delta\delta_{\rm ref}$ for $\delta_{\rm mbt} - \delta$ was prescribed as 5°. The resulting closed-loop responses are shown in Figs. 3 and 4.

The simulation starts in the stratified combustion mode, the requested torque is 21 N·m, and the engine speed is under 2000 r/min. A step in the torque command occurs at $t = 5$, in response to which the controller changes the combustion mode from stratified to homogeneous synergistically with the adjustment of throttle, spark, and fuel rate, in order to track the changed references of τ and λ. From $t = 5$ s to $t = 16$s the requested torque and the engine speed progressively increase, except for some small variations in τ, until the maximum values of 70 N·m and 2450 r/min are reached at $t = 15$ s. At $t = 16$ s, $\tau_{\rm ref}$ decreases to 40 N·m, while the engine speed remains above 2000 r/min. In response to this, the controller does not change the operating mode, but reduces the mass flow rates of fuel and air to track the torque. The slight violation of the bound on air-to-fuel ratio around $t = 16$ is caused by the discrepancy between the hybrid prediction model (linearized at 2000 r/min) and the nonlinear simulation model (running at around 2400 r/min). Later, the requested torque and the engine speed drop below 50 N·m and 2000 r/min, respectively, thereby the controller can switch back to the stratified operation.

It took approximately 2.3 s to compute the dependent set points (offline), and another 9.4 s to simulate the closed-loop system on a PC Intel Xeon 2.8 GHz running the Hybrid Toolbox for Matlab [18] and the MIQP solver of CPLEX [19], of which 6.1 s are spent by CPLEX, that is an average of approximately 3 ms per time step. Because of the excessive CPU requirements for online optimization and because of the complexity of the software for solving the mixed-integer programs, the MPC controller cannot be directly implemented in a typical production automotive microcontroller. In Section VII, we compute an *explicit* version of the MPC controller that does not require online mixed-integer optimization, in order to circumvent such implementation problems.

VII. Explicit Hybrid MPC Controller

Since the MPC controller based on the optimal control problem (12) cannot be directly implemented in a standard

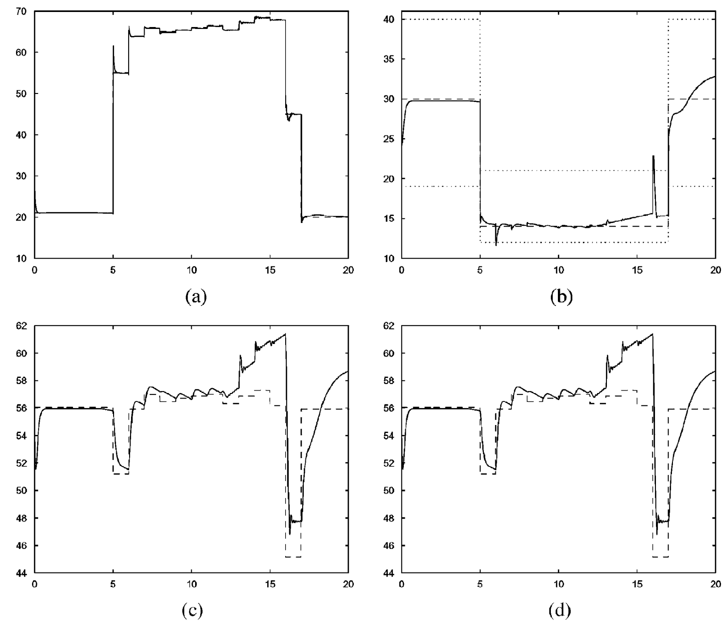

Fig. 3. Closed-loop response (nonlinear model + MPC controller) at variable engine speed. (a) Engine torque $\tau(t)$ (*dashed line*: desired value, *solid line*: response of the nonlinear model). (b) Air-to-fuel ratio $\lambda(t)$ (*dashed line*: desired value, *solid line*: response of the nonlinear model, *dotted line*: A/F bounds). (c) Air mass flow rate $W_{\mathrm{th}}(t)$ (*dashed line*: desired value, *solid line*: response of the controller + throttle dynamics). (d) Intake manifold pressure $p_{\mathrm{m}}(t)$ (*dashed line*: desired value, *solid line*: response of the nonlinear model).

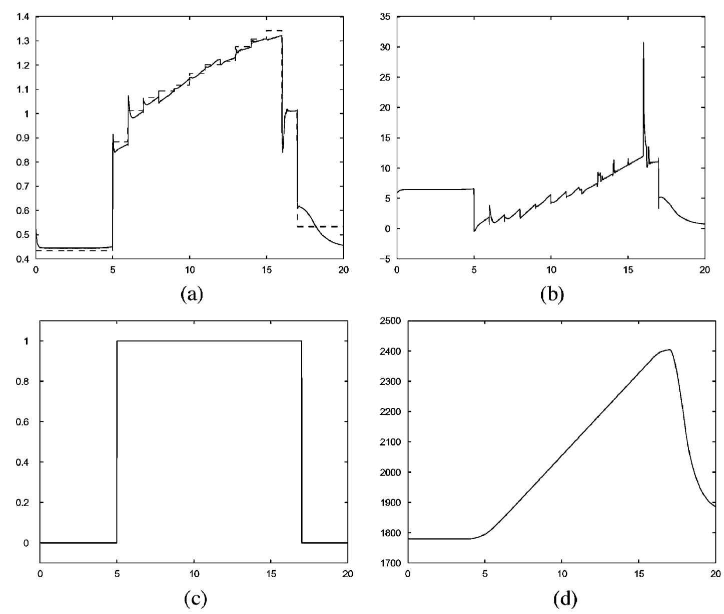

Fig. 4. Closed-loop response (nonlinear model + MPC controller) at variable engine speed. (a) Mass flow rate of fuel $W_{\mathrm{f}}(t)$ (*dashed line*: desired value, *solid line*: response of the controller). (b) Spark retard from MBT, $\delta_{\mathrm{MBT}}(t) - \delta(t)$. (c) Mode of combustion $\rho(t)$. (d) Engine speed ω.

automotive microcontroller, as it would require an MIQP to be solved online, the design of the controller is performed in two steps. First, the MPC controller is tuned in simulation using MIQP solvers, until the desired performance is achieved. Then, for implementation purposes, the explicit piecewise affine form of the MPC law is computed offline by using a combination of multiparametric quadratic programming [20] and dynamic programming, as described in [21] and implemented in the Hybrid Toolbox [18]. The value of the resulting piecewise affine control function is identical to the one that would be calculated by the MPC controller designed in the first phase, but the online complexity is reduced to the simple function evaluation instead of online optimization.

As shown in [22], the explicit representation $u(t) = f(\theta(t))$ of the MPC law (12), with $u = [W_{\mathrm{th}}\ \ W_{\mathrm{f}}\ \ \delta\,\rho]'$, is represented as a collection of affine gains over (possibly overlapping) polyhedral partitions of the set of parameters $\theta = [p_{\mathrm{m}}\ \epsilon_\tau\ \epsilon_\lambda\ W_{\mathrm{f}}\ \tau_{\mathrm{ref}}\ \lambda_{\mathrm{ref}}\ p_{\mathrm{m,ref}}\ W_{\mathrm{th,ref}}\ W_{\mathrm{f,ref}}\ \delta_{\mathrm{ref}}]'$ (the reference $\Delta\delta_{\mathrm{ref}}$ on $\delta_{\mathrm{mbt}} - \delta$ was fixed as 5° in our case).

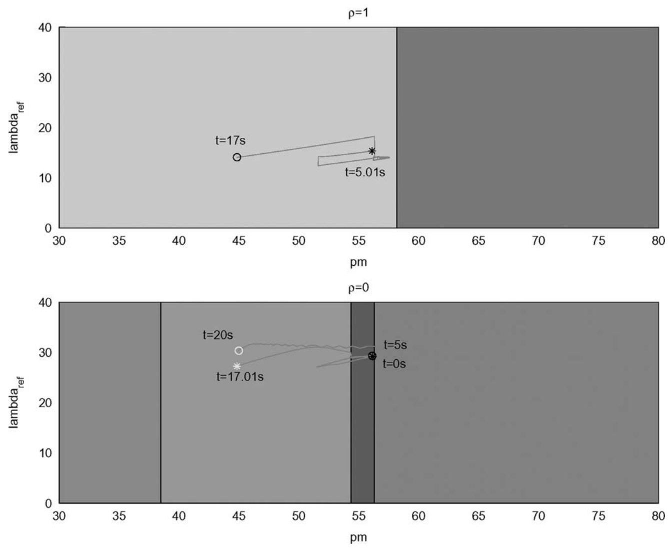

Fig. 5. Cross section by p_{m}–λ_{ref} plane and the simulated closed-loop trajectory with time stamps. Upper subplot: $\rho = 1$, lower subplot: $\rho = 0$.

For the control horizon $N = 1$, we obtain a piecewise affine control law defined over 96 polyhedral regions. This number is lower than the number obtained by using ∞-norm cost functions (as in [3], [4]).

The number of regions reduces to 42 when constraint (10) is removed. The number of regions may be further reduced by postprocessing them and eliminating those that tend to be inactive during realistic driving scenarios.

We note that, even if the engine is working in a wide speed range, the controller does not have the actual speed as an input in this design, and the dependence on speed is implicitly included through the dynamical references generation.

In Fig. 5, we report the cross section of the explicit controller regions by the p_{m}–λ_{ref} plane assuming that $\epsilon_\tau = 0, \epsilon_\lambda = 0, W_{\mathrm{f}}(t-1) = 0, \tau_{\mathrm{ref}} = 60$ N·m, $W_{\mathrm{f,ref}} = 1, p_{\mathrm{m,ref}} = 55$ kPa, $W_{\mathrm{th,ref}} = 20$, and $\delta_{\mathrm{ref}} = 16$.

Note that as ρ changes, the trajectory migrates from the lower plot to the upper plot and back at $t = 5$ s and $t = 17$ s, respectively.

While the online solution of the MPC optimal control problem and its explicit offline solution provide the same result, the explicit controller requires a lower computational effort.

In fact, the total simulation time reduces to 3.9 s on the same computer platform, of which the time to evaluate the explicit MPC control law is approximately 8 μs per time step. Note that the sampling period is $T = 10$ ms. The Hybrid Toolbox for Matlab [18] provides an option for automatically generating the C-code of the explicit solution that can then be used for the embedded implementation in the production microcontroller. For an earlier version of the controller reported in [5], which had a similar complexity of the explicit MPC control law, the requirements for the production microcontroller implementation were 43 kB of ROM (this is the total size of the code and the constants) and execution time of 3 ms to calculate the command input vector.

If we consider a control horizon $N = 2$, the number of regions increases to 3435 [or to 747 if constraint (10) is not taken into account). In Fig. 6, we report the simulations for this control horizon. The closed-loop behavior with $N = 2$ is very similar to the case $N = 1$. Simulations performed with increasing prediction horizons N also show very similar closed-loop trajectories, which seems to suggest that the control horizon $N = 1$ is the most adequate choice.

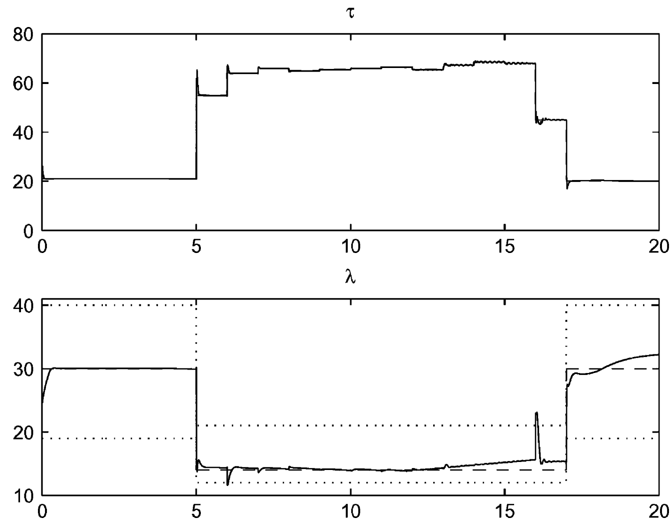

Fig. 6. Closed-loop response with control horizon $N = 2$.

VIII. Conclusion

In this paper, we presented a systematic approach for developing a hybrid model predictive controller for the DISC engine. The overall control strategy combines two parts. The first part computes the steady-state reference values for several engine internal states and inputs consistent with the current engine speed, while the second part is based on a hybrid MPC mechanism that, in turn, is based on a hybrid model at a nominal engine speed.

The controller simultaneously manipulates discrete and continuous control inputs of the engine to effect torque and air-to-fuel ratio tracking and to enforce pointwise-in-time state and control constraints on the air-to-fuel ratio and spark timing. The explicit implementation of the MPC controller, in the form of a piecewise affine control law computed offline, obviates the need for online optimization and makes the overall approach suitable for implementation in memory and chronometrics-constrained automotive microcontrollers.

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