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Constrained Model Predictive Control of Spacecraft Attitude with Reaction Wheels Desaturation

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***Abstract*— In this paper we propose a Model Predictive Controller for spacecraft attitude tracking with reaction wheel actuators. The controller is designed for desaturation of the reaction wheels. In contrast with standard desaturation techniques, which rely on the activation of thrusters, the proposed strategy does not need to consume fuel as it exploits external torques derived from gravity gradients and/or the Earth magnetic field. The controller also guarantees that the spacecraft attitude is constrained within specified bounds during desaturation.**

I. Introduction

Model Predictive Control (MPC) is a popular technology for many industrial applications [1], [2]. It is suitable for multi-input multi-output systems, and generates a control action that minimizes a given performance index while satisfying actuator and state constraints. The computation of such an action requires that, at each sampling step, a Quadratic Programming (QP) problem is solved. This limited its scope to applications with slow systems and large computational resources.

Over the last several years, a significant progress in embedded optimization technologies has been made, coming from dedicated algorithms and hardware improvements. This paves the way for new research in MPC for aerospace applications. Recent developments include [3] where an explicit solution is derived from a linearized spacecraft model, [4] where MPC is applied to spacecraft attitude control using magnetic actuators, [5] which demonstrates a global in orientation attitude stabilization using a Lie group variational integrator-based model, and [6] that proposes MPC implementation suitable for a fixed-point processor. Moreover, it has been shown to be an effective approach for rendezvous problems, as in [7], [8], [9], [10].

Reaction wheels, a type of momentum-exchange devices, are a common way to control the attitude in a spacecraft, requiring only electrical power to operate [11], [12]. However, the presence of external disturbances can lead to a constant increase of the wheels rotational speeds, and ultimately to a saturation. Typically, this problem is solved by periodically activating mass-expulsion devices such as thrusters [13], [14], or by using magnetic coils [15], [16].

In this paper we investigate MPC formulations for spacecraft attitude control and reaction wheel desaturation, without the need of fuel-consuming thrusters. We propose two different approaches, exploiting either the gravity gradients effects or the Earth magnetic field, and demonstrate that both are viable solutions. A comparison with LQR control [17] is also presented, and used to emphasize that the constraint handling capability of MPC is a key enabler for better desaturation performance. In particular, MPC can satisfy the prescribed attitude pointing constraints during the desaturation of reaction wheels.

This paper is organized as follows. In Section II the control problem is introduced, along with the MPC formulation. Then, desaturation techniques based on gravity gradients and magnetic moments are detailed in Sections III and IV, respectively; for each of them background information is presented, followed by the nonlinear and control model formulations and simulation results. Finally, conclusions are drawn in Section V.

II. MPC for Spacecraft Attitude

Consider a spacecraft equipped with three reaction wheels, each one of them aligned with one of its body principal axes. The control objective is to desaturate the wheels by decreasing their rotational speeds below a target speed, while maintaining the spacecraft attitude constrained within specified bounds. This can be achieved by exploiting torques generated by gravity gradients (see Section III-A) or by means of additional magnetic actuators (see Section IV-A).

The control scheme is depicted in Figure 1. The setup is composed by two MPC controllers, which share the same formulation but with different tuning: one for faster wheel desaturation, one for responsive attitude tracking. The controllers are activated according to the speeds of the reaction wheels. The attitude tracking controller includes an external integral action scheme, which acts as a reference governor by feeding the controller with a set-point $r(t)$ such that

$$r(t) = \bar{r}(t) - \int_0^t \left(y(\tau) - \bar{r}(\tau)\right) \,\mathrm{d}\tau, \tag{1}$$

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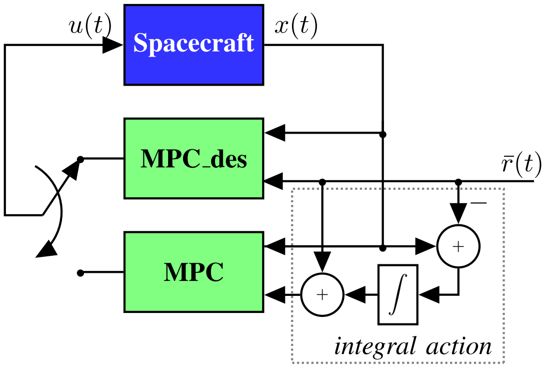

Fig. 1: Control scheme. $u(t)$: controlled variables; $x(t)$: measured variables; $\bar{r}(t)$: references; *MPC_des*: MPC controller tuned for fast reaction wheels desaturation; *MPC*: MPC controller tuned for orientation tracking.

where $\bar{r}(t)$ is the desired set-point and $y(\tau)$ is the measured system output. This allows one to achieve offset-free tracking without increasing the complexity of the controller (see [6] for details).

The constrained optimal control problem solved at each sampling step to compute the control action is

$$\begin{aligned} \min_{\Delta u, \tilde{x}} \quad & \sum_{k=0}^{N-1} (y_k - r(t))' Q (y_k - r(t)) \\ & + \Delta u_k' R \Delta u_k \qquad\qquad (2) \\ \text{subject to} \quad & x_{k+1} = Ax_k + Bu_k, \\ & y_k = Cx_k, \\ & u_k = u_{k-1} + \Delta u_k, \ k < N_c, \\ & u_k = u_{k-1}, \ k \geq N_c, \\ & u_{-1} = u(t-1), \ x_0 = x(t), \\ & (x_k, u_k) \in \mathcal{Z}, \end{aligned}$$

where $u(t-1)$ is the vector of the previous-step control inputs, Q and R are weight matrices of appropriate dimensions, and $\mathcal{Z}$ is the polytope associated with the state and input constraints. The formulations for the system states and inputs, as well as for the prediction model defined by the matrices (A, B, C), are specific for each of the two desaturation techniques and are detailed in Sections III-C and IV-C.

The solution of (2) is assigned to an algorithm specifically tailored for fixed-point, embedded implementations. This algorithm is introduced in [18]. A fixed-point extension is developed in [19], and its applicability to spacecraft attitude control is investigated in [6].

III. Desaturation by Gravity Gradients

A. Background

The *gravity gradients* are torques caused by the Earth gravitational field effects on the spacecraft body; since the gravity force decreases proportionally to the square of the distance, the spacecraft sections closer to the Earth receive a slightly larger pull.

By including the gravity gradients effects into the spacecraft and reaction wheel kinematic and dynamic equations, one is able to derive a *completely controllable* state-space model; this means that is possible, in principle, to steer both the spacecraft attitude and the reaction wheel speeds to specified set-points.

Thanks to the adoption of an MPC-based controller, desaturation of the wheels is achieved by optimally exploiting the torques induced by the gravity gradients, while maintaining the spacecraft attitude constrained in a specified set.

B. Nonlinear Model

The spacecraft rotational kinematics, assuming that the body fixed frame is the principal frame with the origin at the center of mass, are

$$\begin{bmatrix} \dot{\phi}(t) \\ \dot{\theta}(t) \\ \dot{\psi}(t) \end{bmatrix} = \frac{1}{c(\theta)} \begin{bmatrix} c(\theta) & s(\phi)s(\theta) & c(\phi)s(\theta) \\ 0 & c(\phi)c(\theta) & -s(\phi)c(\theta) \\ 0 & s(\phi) & c(\phi) \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \end{bmatrix} + \\ +n \begin{bmatrix} c(\theta)s(\psi) + c^{-1}(\theta)\left(\sigma_2 s(\phi)s(\theta) - \sigma_1 c(\phi)s(\theta)\right) \\ \sigma_2 c(\phi) + \sigma_1 s(\phi) \\ c^{-1}(\theta)\left(\sigma_2 s(\phi) - \sigma_1 c(\phi)\right) \end{bmatrix}, \tag{3}$$

where $\sigma_1 \triangleq c(\psi)s(\phi) - c(\phi)s(\psi)s(\theta)$, $\sigma_2 \triangleq c(\phi)c(\psi) + s(\phi)s(\psi)s(\theta)$, and $n \triangleq \sqrt{\frac{\mu}{R_0^3}}$, $c(\cdot) \triangleq \cos(\cdot)$, $s(\cdot) \triangleq \sin(\cdot)$; $\phi(t)$,$\theta(t)$, $\psi(t)$ (rad) are the roll, pitch and yaw angles, $\omega_1(t)$, $\omega_2(t)$, $\omega_3(t)$ (rad/s) are the spacecraft angular velocities; μ is the gravitational constant and R_0 is the nominal orbital radius.

The spacecraft rotational dynamics is

$$\begin{aligned} J_1\dot{\omega}_1 &= (J_2 - J_3)\left(\omega_2\omega_3 - 3n^2 s(\phi)c(\phi)c^2(\theta)\right) + \\ &\quad - J_\alpha\left(\ddot{\alpha}_1 + \dot{\omega}_1\right), \\ J_2\dot{\omega}_2 &= (J_3 - J_1)\left(\omega_1\omega_3 + 3n^2 c(\phi)c(\theta)s(\theta)\right) + \\ &\quad - J_\alpha\left(\ddot{\alpha}_2 + \dot{\omega}_2\right), \\ J_3\dot{\omega}_3 &= (J_1 - J_2)\left(\omega_1\omega_2 + 3n^2 s(\phi)s(\theta)c(\theta)\right) + \\ &\quad - J_\alpha\left(\ddot{\alpha}_3 + \dot{\omega}_3\right), \end{aligned} \tag{4}$$

where J_i (kgm^2), $i = (1, 2, 3)$, are the spacecraft principal moments of inertia, J_α is the wheel inertia, and $\ddot{\alpha}_i$ (rad/s^2) are the angular accelerations of the wheels.

Finally, the reaction wheels rotational dynamics is

$$\begin{aligned}\ddot{\alpha}_1 &= n\dot{\alpha}_3 + u_1,\\ \ddot{\alpha}_2 &= u_2,\\ \ddot{\alpha}_3 &= -n\dot{\alpha}_1 + u_3,\end{aligned} \tag{5}$$

where $u_i,\ i = (1, 2, 3)$, are the rotational accelerations induced on the wheels by the electric motors.

C. Control Model

Due to its nonlinear nature, the model described in Section III-B is not a good prediction model for an embedded MPC implementation. We therefore obtain a simpler state-space control model by linearizing (3)-(4) around the nominal conditions

$$\bar{\phi} = 0,\ \bar{\theta} = 0,\ \bar{\psi} = 0,\ \bar{\omega}_1 = 0,\ \bar{\omega}_2 = -n,\ \bar{\omega}_3 = 0, \tag{6}$$

and choosing the vector of system states

$$x \ = \ [\Delta\phi\ \Delta\theta\ \Delta\psi\ \Delta\omega_1\ \Delta\omega_2\ \Delta\omega_3\ \dot{\alpha}_1\ \dot{\alpha}_1\ \dot{\alpha}_1]', \tag{7}$$

where Δ denotes a variation from the nominal condition.

The resulting model is

$$\dot{x}(t) = A_C x(t) + B_C \left[u_1(t) \quad u_2(t) \quad u_3(t)\right]',$$

$$A_C \triangleq \left[\begin{array}{ccc|ccc|ccc} 0 & 0 & n & 1 & 0 & 0 & & & \\ 0 & 0 & 0 & 0 & 1 & 0 & & & \\ -n & 0 & 0 & 0 & 0 & 1 & & \mathbf{0}^{6\times 3} & \\ c_1 & 0 & 0 & 0 & 0 & c_3 & & & \\ 0 & c_2 & 0 & 0 & 0 & 0 & & & \\ 0 & 0 & 0 & c_4 & 0 & 0 & & & \\ \hline & & & & & & 0 & 0 & n \\ & \mathbf{0}^{3\times 3} & & & \mathbf{0}^{3\times 3} & & 0 & 0 & 0 \\ & & & & & & -n & 0 & 0 \end{array}\right],$$

$$B_C \triangleq \left[\begin{array}{ccc} & \mathbf{0}^{3\times 3} & \\ \hline -\sigma_1 & 0 & 0 \\ 0 & -\sigma_2 & 0 \\ 0 & 0 & -\sigma_3 \\ \hline & \mathbf{I}^3 & \end{array}\right], \tag{8}$$

where $\sigma_i \triangleq J_\alpha \left(J_i + J_\alpha\right)^{-1}$, $c_1 \triangleq -3\sigma_1 n^2(J_2 - J_3)$, $c_2 \triangleq 3\sigma_2 n^2(J_3 - J_1)$, $c_3 \triangleq -\sigma_1 n(J_2 - J_3)$, and $c_4 \triangleq \sigma_3 n(J_1 - J_2)$.

The controllability matrix of the pair (A_C, B_C) is full rank; this property does not hold if the gravity gradient effects are neglected.

D. Results

The wheel desaturation process exploiting gravity gradients is shown in Figure 2. The simulation covers a period of 22 hours. The controller is set to regulate spacecraft attitude and reaction wheel speeds, operating at a sampling time of 0.5 s and using a discretized version of (8) as prediction model over a 10 steps horizon. The control horizon is 2 steps. The resulting QP has 6 decision variables and 72 constraints. The system in initialized with the spacecraft body frame aligned with the *LVLH* (Local Vertical/Local Horizontal) frame, and all the 3 reaction wheels spinning at 10 rad/s. Roll, pitch and yaw angles are constrained in the set $[-0.5,\ 0.5]$ rad. The values of the chosen spacecraft and wheels moments of inertia are $J_1 = 1400$, $J_2 = 1700$, $J_3 = 1000$, and $J_\alpha = 50\ kgm^2$.

Simulation results show how the controller drives the spacecraft to perform oscillations along the roll and yaw angles, while maintaining an offset in the pitch angle. As a result, the wheels are desaturated while maintaining the spacecraft attitude within the specified constraints. The whole process takes about 8 hours to halve wheel speeds, and 16 hours to bring them close to zero rad/s. We note that similar results are obtained for different initial speeds, and it is not necessary to bring the wheel speed all the way to zero during practical desaturation maneuvers.

E. Comparison with LQR

It is interesting to compare the behavior and performance of MPC with respect to a standard *LQR* controller. The latter does not allow imposing constraints on the spacecraft attitude through the desaturation process. Instead, one is forced to tune properly weights in the cost function for the spacecraft orientation and wheel speeds. A fast desaturation is obtained by increasing the wheel weights; however, this subjects the spacecraft to large oscillations (see dashed lines in Figure 3). This behavior is not desirable, as the controller is based on a model linearized for small angles (and the model mismatch may become intolerable). The other option is to increase the weights on spacecraft attitude (see solid lines in Figure 3): now its oscillations are smaller, but the wheels desaturation performance is significantly degraded.

IV. Desaturation by Magnetic Moments

A. Background

The desaturation by magnetic moments is possible when the spacecraft is equipped with magnetic actuators (usually three, aligned with the body frame axes). These devices are composed by a magnetic core and a coil; when current flows through the latter, a magnetic dipole is generated which interacts with the Earth magnetic field, resulting in a control torque on the spacecraft.

The magnetic moments are stronger than the moments due to gravity gradients, allowing for faster wheel desaturation. However, they require additional equipment on the spacecraft and they are based on the interaction with the Earth magnetic field, which varies along the orbit. Moreover, the spacecraft/wheels system is not

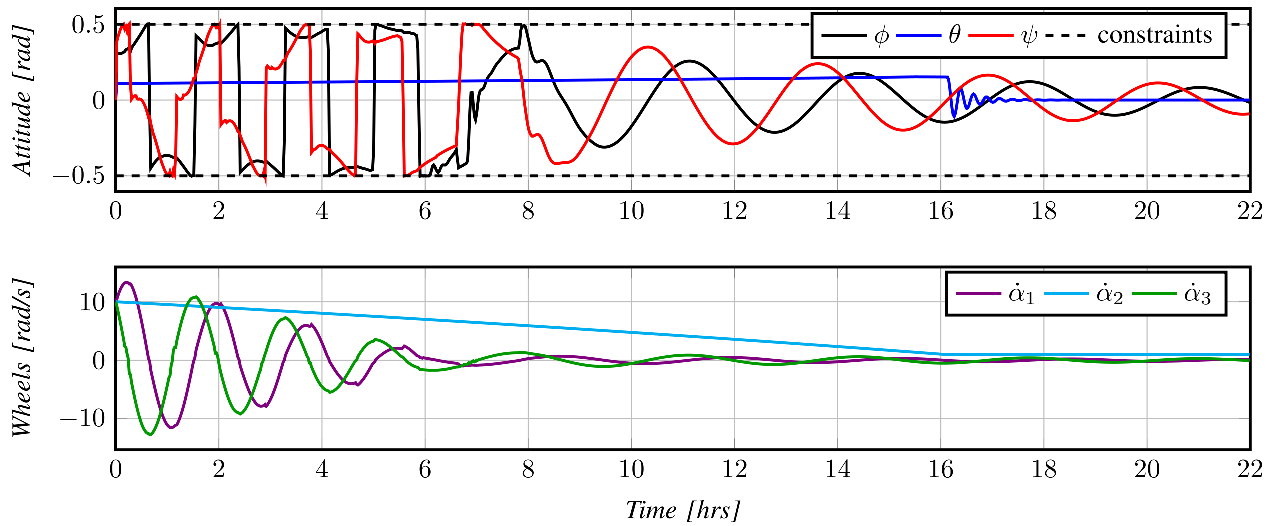

Fig. 2: Closed-loop simulation with MPC for reaction wheels desaturation using the gravity gradients, starting from $10\ rad/s$. Top: spacecraft roll, pitch and yaw angles. Bottom: reaction wheels speed.

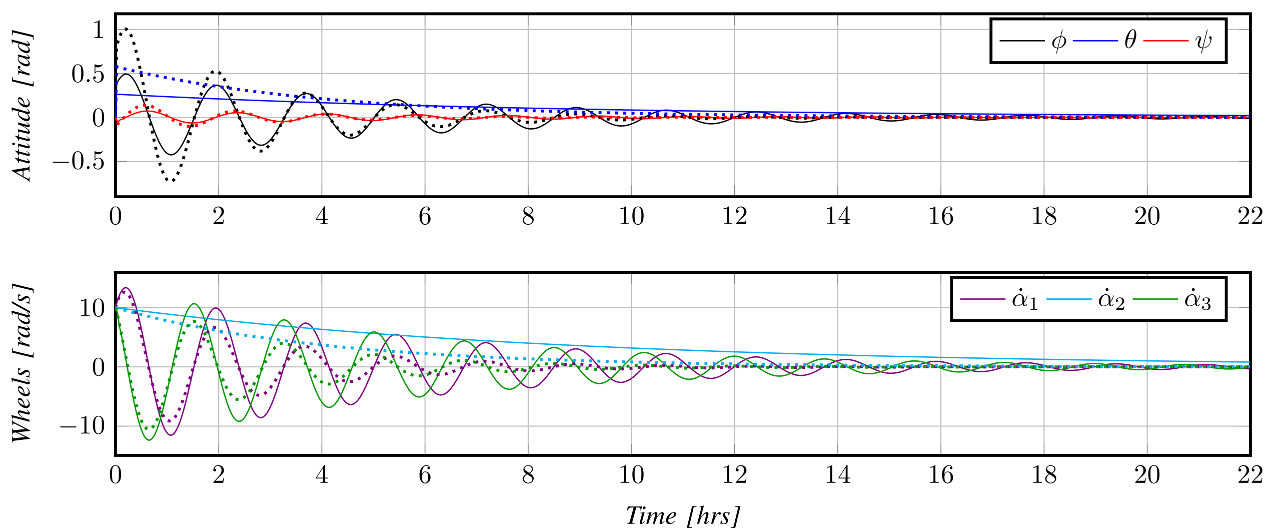

Fig. 3: Closed-loop simulation using the gravity gradients with two LQR controllers, one tuned for fast desaturation (dotted lines), one for slow desaturation (solid lines). Top: spacecraft roll, pitch and yaw angles. Bottom: reaction wheels speed.

completely controllable, since it is not possible to generate magnetic torques along the Earth magnetic field direction. However, since this direction varies as the spacecraft moves along its orbit, an MPC controller can still achieve wheel desaturation.

B. Nonlinear Model

The spacecraft rotational kinematics and the wheel dynamics are as in (3) and (5) with $n = 0$.

The spacecraft dynamics is

$$\begin{aligned} J_1\dot{\omega}_1 &= (J_2 - J_3)\,\omega_2\omega_3 - J_\alpha\,(\ddot{\alpha}_1 + \dot{\omega}_1) + M_1^B, \\ J_2\dot{\omega}_2 &= (J_3 - J_1)\,\omega_1\omega_3 - J_\alpha\,(\ddot{\alpha}_2 + \dot{\omega}_2) + M_2^B, \\ J_3\dot{\omega}_3 &= (J_1 - J_2)\,\omega_1\omega_2 - J_\alpha\,(\ddot{\alpha}_3 + \dot{\omega}_3) + M_3^B, \end{aligned} \tag{9}$$

where M_i^B denote the torques generated by the magnetic actuators. Those can be computed as

$$M \triangleq \begin{bmatrix} M_1^B & M_2^B & M_3^B \end{bmatrix}' = m \times B, \tag{10}$$

$$m \triangleq \begin{bmatrix} N_x A_x i_x \\ N_y A_y i_y \\ N_z A_z i_z \end{bmatrix} = \begin{bmatrix} u_1^B \\ u_2^B \\ u_3^B \end{bmatrix}, \tag{11}$$

where B is the Earth magnetic field vector, expressed in the spacecraft body fixed frame; $N_{(\cdot)}$, $A_{(\cdot)}$, $i_{(\cdot)}$ are, respectively, the number of coil turns, their areas, and the currents flowing through them, for each of the three magnetic actuators mounted along the x, y and z axes of the spacecraft body frame.

C. Control Model

The control model is obtained by linearizing the nonlinear model of Section IV-B around the origin. The state vector is the same as in (7), while the input vector takes the form $u = \begin{bmatrix} u_1 & u_2 & u_3 & u_1^B & u_2^B & u_3^B \end{bmatrix}'$.

The resulting model is

$$\begin{aligned} \dot{x}(t) &= A_C x(t) + B_C(t) u(t), \\ A_C &\triangleq \left[\begin{array}{c|c|c} \mathbf{0}^{3\times3} & \mathbf{I}^3 & \mathbf{0}^{3\times3} \\ \hline \mathbf{0}^{6\times3} & \mathbf{0}^{3\times3} & \mathbf{0}^{3\times3} \end{array}\right], \\ B_C(t) &\triangleq \left[\begin{array}{ccc|ccc} & \mathbf{0}^{3\times3} & & & \mathbf{0}^{3\times3} & \\ \hline -\sigma_1 & 0 & 0 & 0 & -\eta_1 & \eta_2 \\ 0 & -\sigma_2 & 0 & \eta_1 & 0 & -\eta_3 \\ 0 & 0 & -\sigma_3 & -\eta_2 & \eta_3 & 0 \\ \hline & \mathbf{I}^3 & & & \mathbf{0}^{3\times3} & \end{array}\right], \\ \sigma_i &\triangleq J_\alpha\,(J_i + J_\alpha)^{-1}, \\ \eta_1 &\triangleq B^{long}\theta - B^{lat}\phi - B^v, \\ \eta_2 &\triangleq B^{lat}\psi - B^v\theta - B^{long}, \\ \eta_3 &\triangleq B^v\phi - B^{long}\psi - B^{lat}, \end{aligned} \tag{12}$$

where B^{long}, B^{lat}, B^v are, respectively, the longitudinal, latitudinal and vertical components of the Earth magnetic field.

Model (12) is linear time-varying (LTV); therefore, an additional computational effort is required to form the quadratic programming problem at each sampling step.

D. Results

To test the desaturation by magnetic moments we simulated a low-earth test orbit at an altitude of $420\ km$ and with an orbital period of $1.55\ hrs$. The QP solved to compute the control action has 12 decision variables and 84 constraints. The system is initialized with reaction wheels spinning at $100\ rad/s$. The goal is to lower their speed below $30\ rad/s$.

The Earth magnetic field is generated with data from the World Magnetic Model [20].

Figure 4 shows the results of the simulation obtained with MPC based on the LTV model (12). Within 2.5 hours (less than two orbit revolutions) the desaturation process is completed, with all the reaction wheel speeds below the target of $30\ rad/s$. Then, in 1.5 additional hours, the spacecraft attitude is driven to the rest position, with the controller waiting for favorable Earth magnetic field directions to steer the roll, pitch and yaw angles. Note that, for the whole process, the attitude has been constrained within a small box of side $0.1\ rad$.

V. Conclusions

In this paper we presented MPC formulations for spacecraft attitude tracking that ensure reaction wheel desaturation capabilities, without relying on thrusters actuation. This is achieved by either exploiting the gravity gradient effect, i.e., the torque generated by an uneven distribution of the gravity force, or the Earth magnetic field. By means of simulations we showed that both are viable solutions, with the latter achieving faster response at the cost of increased complexity due to the need of additional magnetic actuators and a time-varying prediction model.

Model Predictive Control technology is still in an early stage of development in the spacecraft application domain, but its adoption enables advanced control strategies with tangible benefits, as we demonstrated for the reaction wheel desaturation problem in this paper.

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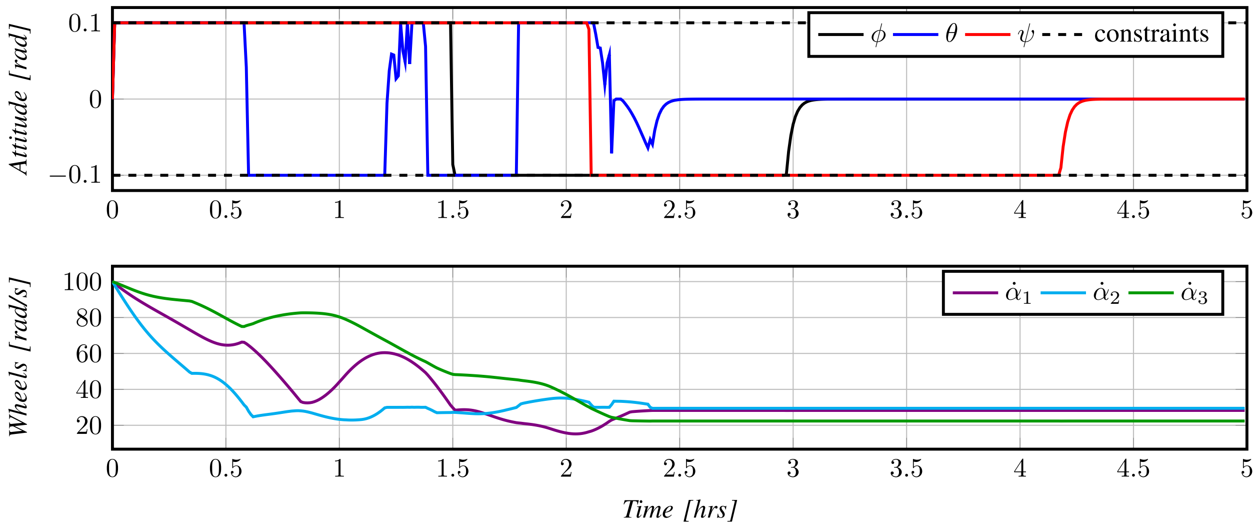

Fig. 4: Closed-loop simulation with LTV-MPC for reaction wheels desaturation using the Earth magnetic field. The goal is to lower wheel speed below 30 rad/s, starting from 100 rad/s. Top: spacecraft roll, pitch and yaw angles. Bottom: reaction wheels speed.

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