Energy-Aware Robust Model Predictive Control with Feedback from Multiple Noisy Wireless Sensors

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Abstract— **Wireless Sensor Networks (WSNs) are becoming fundamental components of modern control systems. Although WSNs provide tremendous advantages in versatility, their use poses new issues in the design of the control system, in particular the discharge of batteries of sensor nodes, which is mainly due to radio communications, must be taken into account. In a previous work, for the case of a single wireless measurement device and no measurement noise, we have provided a general transmission strategy for communication between controller and sensors and an energy-aware robust Model Predictive Control (MPC) algorithm that achieve a profitable trade-off between transmission rate (battery energy savings) and loss of closed-loop system performance. In this paper we extend the approach by taking into account unknown but bounded noise on state measurements, and by considering a local area network of multiple wireless sensors measuring the state vector for disturbance rejection.**

I. INTRODUCTION

Very recently wireless sensor networks (WSNs) are becoming fundamental components of modern control systems because of their low cost, flexibility, ease of deployment and reconfiguration. However, to obtain an expected lifetime of battery-operated sensor nodes in the order of months to years and therefore prevent frequent cumbersome and expensive replacements of batteries, energy budget is highly constrained. Energy consumption is therefore a crucial factor to take into account in the design of the nodes themselves and in their use in a control system.

In a wireless node the radio chip is the primary source of energy consumption. Hence, minimizing the radio utilization is necessary to achieve a satisfying network lifetime [1]. In recent years some interesting work was done addressing the energy problem from the point of view of both the communication network and the system architecture, i.e., by proposing consumption-efficient routing protocols [2], or dynamic power management techniques [3].

In this paper we consider the case of a WSN responsible for providing feedback to close the loop in a control system. Previous works related to this topic include [4], where a properly tuned control law is designed to achieve real-time regulation of the network transmission rate, and [5], where a predictive controller is proposed to optimize the trade-off between power transmission and system performance, as a function of the estimated wireless channel reliability.

Our framework considers a few key aspects to obtain effective energy savings. Firstly, we do consider energy costs of receiving packets, which are similar to the costs of transmitting packets [6], [7]. Secondly, it is necessary to avoid idle listening as much as possible, i.e., the radio chip should be completely turned off when no communication is taking place [1]. Finally, as common wireless protocols have a relatively large fixed cost due to communication overheads [6], although a single measurement acquisition can be stored in few bytes the transmission of small packets has a disproportionately high energy price. As a result, transmitting n measurements, with n sufficiently small, costs almost as transmitting a single measurement.

By taking into account the above aspects, in this paper we address the energy problem of the WSN together with the optimal control problem of the networked process. The main idea is to develop a network transmission strategy where measurements are transmitted to the controller *only when necessary*, and to design a control scheme aware of that transmission strategy, which allows a substantial reduction of radio utilization without excessively sacrificing closed-loop performance. Our previous work on energy-aware control based on wireless sensor feedback [8] is extended to the case of multiple noisy sensors. In particular we consider a network formed by a remote controller and a short range WSN of several nodes, which embed sensors measuring the same physical quantities. Redundancy is exploited here in order to mitigate measurement errors. Sensor nodes intercommunicate with low power transmissions, and determine the measurement to send to the remote controller via a wireless communication channel by means of an estimation algorithm. In the following, we restrict ourselves to the case of an ideal communication channel, i.e., no packet losses or delays are considered.

This paper is organized as follows. The proposed transmission strategy is presented in Section II, along with the description of the considered networked plant. A set-membership estimation algorithm, tailored for the WSN communication policy, is given in Section III. Then, an energy-aware robust control algorithm is formulated in Section IV. Results of closed-loop simulations are reported in Section V, and conclusions are drawn in Section VI.

II. WIRELESS TRANSMISSION STRATEGY

Consider a discrete-time linear system of the form

$$x(k+1) \quad = \quad Ax(k) + Bu(k) + w(k), \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state, $u(k) \in \mathbb{R}^{n_u}$ is the input, $w(k) \in \mathbf{W}$ is an additive disturbance, and k is the time index. $\mathbf{W} \subset \mathbb{R}^{n_x}$ is a given polytope containing the origin. State and input vectors are subject to the constraints $x \in \mathbf{X}$, $u \in \mathbf{U}$, where $\mathbf{X}$, $\mathbf{U}$ are polyhedra containing the origin in their interior. We assume that the measurements are given by a local WSN of m nodes, indexed by $i = 0, \dots, m-1$, each one measuring the state vector $x(k)$. The actual measurement given by the i-th node is

$$y^i(k) = x(k) + v^i(k), \tag{2}$$

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where $v^i \in \mathbf{V^i}$ is an unknown but bounded disturbance, with

$$\mathbf{V^i} = \{v \in \mathbb{R}^{n_x} : |v_j| \leq v^i_{j,max},\ j = 1, \ldots, n_x\}. \quad (3)$$

The vectors $v^i_{max} = [v^i_{1,max} \ \ldots \ v^i_{n_x,max}]$, $i = 0, \ldots, m-1$, are assumed to be known to every node. The optimal input values $u(k)$ are computed by a centralized remote controller, which communicates with the local WSN through a long-range wireless channel, and with the actuation device through a wired channel.

Sensor transmission strategy. At time step k let $h = \lfloor \frac{k}{n} \rfloor \bmod m$. We refer to node h as the *master* node. The positive integer parameter n represents the number of consecutive time-steps for which the master node does not change. Then, a short range transmission of $y^i(k)$ takes place from every node $i \neq h$, called *slave* nodes, to the master node. The master node performs a simple set-membership estimation by computing the box set

$$\mathcal{Y}(k) = \bigcap_{i=0}^{m-1} \mathcal{Y}^i(k) \quad (4a)$$

$$= \{x : b_{j,min} \leq x_j \leq b_{j,max},\ j = 1, \ldots, n_x\}, \quad (4b)$$

with

$$b_{j,min} = \max_{i=0,\ldots,m-1} y^i_j(k) - v^i_{j,max}, \quad (5a)$$

$$b_{j,max} = \min_{i=0,\ldots,m-1} y^i_j(k) + v^i_{j,max}, \quad (5b)$$

where

$$\mathcal{Y}^i(k) = \{x : |y^i(k) - x| \leq v^i_{max}\},\ i = 0, \ldots, m-1 \quad (6)$$

are all the feasible sets of states according to each node's measurements. Finally, the master node transmits $\mathcal{Y}(k)$ to the controller if and only if

$$\exists j \in \{1, \ldots, n_x\} \quad : \quad \begin{array}{l} [b_{j,min} - \hat{x}_j(k) < -\varepsilon_j] \\ \vee\ [b_{j,max} - \hat{x}_j(k) > \varepsilon_j], \end{array} \quad (7)$$

where $\hat{x}(k) \in \mathbb{R}^{n_x}$ is a prediction of the current state value $x(k)$ precalculated by the controller and transmitted beforehand to the actual master node [8]. Condition (7) is equivalent to

$$\mathcal{Y}(k) \supset \mathbf{E} \oplus \{\hat{x}(k)\}, \quad (8)$$

where $\mathbf{E} = \{x \ : \ |x_j| \leq \varepsilon_j,\ j = 1, \ldots, n_x\}$ is the box defined by the threshold vector $\varepsilon = [\varepsilon_1 \ \ldots \ \varepsilon_{n_x}]^T$, and $\oplus$ is the Minkowski sum operator. We represent the transmission condition (8) with $[\delta(k) = 1]$, where δ is a binary variable.

Then, if at time k it holds that $\delta(k) = 1$, the controller receives $\mathcal{Y}(k)$ from the master node, computes a set of M updated state predictions $\{\hat{x}(k+j)\}_{j=1}^M$, and transmits them back to the master node. Hence, there is a two-way communication between controller and master node when $\delta(k) = 1$. Moreover, if the controller does not receive any measurement for M time steps, i.e. $\delta(k) = \delta(k-1) = \ldots = \delta(k-M+1) = 0$, a one-way communication from controller to the master node takes place to send M updated predictions $\{\hat{x}(k+j)\}_{j=1}^M$. We refer to M as the *estimation horizon*, and to $\{\hat{x}(k+j)\}_{j=1}^M$ as the *prediction buffer*. We propose to compute these predictions using a set-membership estimation algorithm and a model predictive controller, as detailed in the following sections.

Note that the predictions $\hat{x}$ are used at every time step only by the actual master node. Let $h_k = \lfloor \frac{k}{n} \rfloor \bmod m$ and $h_{k-1} = \lfloor \frac{k-1}{n} \rfloor \bmod m$. At time k, if $h_k \neq h_{k-1}$, then the h_{k-1}-th node (the old master) is required to transmit, along with the measurement, also the prediction buffer to the h_k-th node (the new master).

The value of the estimation horizon M must be chosen according to a trade-off between energy consumption and predictions reliability. In fact, due to the presence of disturbances, the accuracy of the predicted state values decreases with the horizon length, i.e., if $\delta(k) = \delta(k+1) = \ldots = \delta(k+t) = 0$, the difference $|x_i(k+t) - \hat{x}_i(k+t|k)|$, in general, is likely to grow with $t \in \{0, 1, \ldots, M-1\}$. Hence, a too high value of M will lead to unnecessary transmission costs, due to predictions too far in the future which are useless because never evaluated by the master node.

Here we assume the energy cost for a short range transmission to be very small with respect to a long range transmission, so the communication activity between local sensor nodes is neglected. However, it is easy to extend this approach by using a threshold logic also for local transmissions, by having slave sensors send the measurements only when they are sufficiently far from a given value.

A notable consequence of the proposed transmission strategy is that idle listening is almost completely avoided, since the implicit synchronization between the master node and the remote controller allows the master node to wake up its radio chip only in the real presence of communications.

III. Set-membership State Estimation

It is easy to see that the transmission strategy (7) is such that the actual state $x(k)$ is always subject to a known set-membership relation, regardless of the master node's decision to transmit the data. In particular, it is possible to show that

$$x(k) \in \left\{ \begin{array}{ll} \mathcal{Y}(k) & \text{if } \delta(k) = 1, \\ \mathbf{E} \oplus \{\hat{x}(k)\} & \text{otherwise.} \end{array} \right. \quad (9)$$

Hence, in the absence of packet drops, information on the measured variables is gathered even when no feedback data are transmitted (i.e., when $\delta(k) = 0$). We propose to use a set-membership estimation algorithm [9] which takes into consideration this property to reduce the uncertainty on the state.

Let $\mathcal{X}(k|k)$ and $\mathcal{X}(k+1|k)$ be the sets of all the possible values of the state x at time k and $k+1$, respectively, given the feedback information sequence for time $t = 0, 1, \ldots, k$. The set $\mathcal{X}(k+1|k)$ is defined by the *prediction step*

$$\mathcal{X}(k+1|k) = A\mathcal{X}(k|k) \oplus B\{u(k)\} \oplus \mathbf{W}, \quad (10)$$

while the set $\mathcal{X}(k|k)$ is obtained through the *correction step*

$$\mathcal{X}(k|k) = \mathcal{X}(k|k-1) \cap \left\{ \begin{array}{ll} \mathcal{Y}(k) & \text{if } \delta(k) = 1, \\ \mathbf{E} \oplus \{\hat{x}(k)\} & \text{otherwise.} \end{array} \right. \quad (11)$$

The estimation $\bar{x}(k|k)$ of the actual state $x(k)$, given the feedback information at time k, is defined as the centroid of $\mathcal{X}(k|k)$, computed as

$$\bar{x}(k|k) = c\big(\mathcal{X}(k|k)\big) = \frac{1}{n_v} \sum_{i=1}^{n_v} v_i^{\mathcal{X}}, \quad (12)$$

where n_v is the number of vertices $v_1^{\mathcal{X}}, \ \ldots, \ v_{n_v}^{\mathcal{X}}$ of $\mathcal{X}(k|k)$.

This kind of set-membership algorithm can lead to very complex representations of $\mathcal{X}$ (i.e., with a high number of vertices). In order to lower the computational burden and preserve the implementability of the control scheme, in the simulations presented in Section V we use a sub-optimal estimation algorithm, where outbounding parallelotopes are used to approximate the actual state set $\mathcal{X}(k|k)$ (see [10] for further details).

IV. Energy-Aware Robust Model Predictive Control

Our goal is to design a control scheme for a regulation problem, where the set-point for the state x is the origin, and where the radio usage of the WSN is minimized in order to take explicitly into account the energy-constrained nature of the nodes. We have to deal with three sources of uncertainty: the additive disturbance w, the measurement noise v^i, $i = 0, 1, \ldots, m-1$, and the feedback error due to the wireless transmission strategy (8). Due to the presence of these persistent disturbances, the state cannot be directly steered to the origin. Hence, we setup a dual mode MPC [11], [12] where the *outer control mode* provides a (possibly time-varying) feedback control law $u = \kappa(x)$ which regulates the state to a given set $\mathbf{X_0}$ despite the sources of disturbance, and the *inner control mode* robustly keeps the state x in $\mathbf{X_0}$ by means of a time-invariant control law $u = Kx$. The target set $\mathbf{X_0}$ is assumed robust positively invariant with respect to additive disturbances, as from the following definition [13].

Definition 1: The set $\mathbf{X_0} \in \mathbb{R}^{n_x}$ is *robust positively invariant (RPI)* for a system of the form $x(k+1) = f(x(k), w(k))$ if and only if $\forall x(0) \in \mathbf{X_0}$ and $\forall w(k) \in \mathbf{W}$ the solution $x(k) \in \mathbf{X_0}$, $\forall k \in \mathbb{N}$.

See [13]–[15] for details about theory and design of RPI sets.

In the following, the proposed inner and outer control modes are presented. Their formulation is an extension of previous work to the case of feedback from multiple noisy sensors, see [8] for further details and explanations.

A. Inner Mode

Consider the switching feedback control law $u(k) = K\bar{x}(k|k)$, where $\bar{x}(k|k)$ is the current estimated value of the state obtained from the set-membership estimation algorithm presented in Section III. The closed-loop evolution of system (1) is

$$x(k+1) = Ax(k) + BK\bar{x}(k|k) + w(k). \tag{13}$$

Note that

$$x(k) - \bar{x}(k|k) \in \mathbf{F}(k) \doteq \begin{cases} \mathbf{V}_{\mathcal{I}} & \text{if } \delta(k) = 1, \\ \mathbf{E} & \text{otherwise,} \end{cases} \tag{14}$$

where $\mathbf{V}_{\mathcal{I}} = \bigcap_{i=0}^{m-1} \mathbf{V^i}$. Hence, (13) can be recast as

$$x(k+1) = (A + BK)x(k) + w(k) - BKf(k), \tag{15}$$

where $f(k) \in \mathbf{F}(k)$ is an unknown but bounded disturbance, and $\mathbf{F}(k)$ is a switching set in accordance with (14). Now, we can compute $\mathbf{X_0}$ as an RPI set for the piecewise affine (PWA) system (15) (see [15]–[17]). Introducing some conservativeness, $\mathbf{X_0}$ can be computed as an RPI set for the linear system obtained by (15) with $f(k) \in \mathbf{F}_{\mathcal{H}} \supseteq \mathbf{F}(k)$, $\forall k$, where

$$\mathbf{F}_{\mathcal{H}} = \text{hull}\{\mathbf{E}, \mathbf{V}_{\mathcal{I}}\}, \tag{16}$$

which is easily computed from the vertices of the box sets $\mathbf{E}$, $\mathbf{V}_{\mathcal{I}}$ (see [11], [13], [14] for RPI sets in the linear case).

B. Outer Mode

To design the outer controller, we propose an algorithm based on explicit Model Predictive Control (MPC).

MPC is widely spread in industry for control design of highly complex multivariable processes under constraints on input and state variables [18], [19]. The idea behind MPC is to solve at each sampling time an open-loop finite-horizon optimal control problem based on a given prediction model of the process, by taking the current state of the process as the initial state. Only the first sample of the sequence of future optimal control moves is applied to the process. At the next time step, the remaining moves are discarded and a new optimal control problem based on new measurements is solved over a shifted prediction horizon. An alternative approach to evaluate the MPC law was proposed in [20]: rather then solving the QP problem on line for the current state vector, by employing techniques of multiparametric QP the problem is solved off line for all state vectors within a given range, providing the *explicit* dependence of the control input on the state and reference, which is piecewise affine (PWA) and continuous. For a survey on explicit MPC the reader is referred to [21].

In our framework, an explicit formulation of MPC is a natural choice for many reasons: primarily, it can handle constraints and can be formulated to achieve robust control in presence of disturbances. Moreover, it allows the cheap computation of the prediction buffer $\{\hat{x}(k+j)\}_{j=1}^{M}$, by iterating the evaluation of a simple PWA function.

Let $\{w^{\ell}_{k+j|k}\}$ denote all the possible realizations of the disturbance w, indexed by $\ell \in \mathcal{L}$. Further, let $\{u^{\ell}_{k+j|k}\}$ denote a control sequence associated with the ℓ-th such realization, and $\{x^{\ell}_{k+j|k}\}$ the corresponding state value. By ignoring the wireless transmission strategy and assuming direct state-feedback, the min-max MPC problem is expressed as in [11]

$$\min_{\{u^{\ell}_{k+j|k}\}} \quad \max_{\ell \in \mathcal{L}} \sum_{j=0}^{N-1} L(x^{\ell}_{k+j|k}, u^{\ell}_{k+j|k}) \tag{17a}$$

$$\text{s.t.} \quad (1), \tag{17b}$$

$$x^{\ell}_{k+j|k} \in \mathbf{X}, u^{\ell}_{k+j|k} \in \mathbf{U}, x^{\ell}_{k+N|k} \in \mathbf{X_T}, \tag{17c}$$

$$x^{\ell_1}_{k+j|k} = x^{\ell_2}_{k+j|k} \Rightarrow u^{\ell_1}_{k+j|k} = u^{\ell_2}_{k+j|k}, \tag{17d}$$

$$j = 0, \ldots, N-1, \ \forall \ell, \ell_1, \ell_2 \in \mathcal{L},$$

where N is the control horizon, $x^{\ell_1}_{k+j|k} = x^{\ell_2}_{k+j|k} \Rightarrow u^{\ell_1}_{k+j|k} = u^{\ell_2}_{k+j|k}$ is the *causality constraint*, which enforces a single control input for each state, reducing the freedom on the control sequence and making the control law independent of the path taken to reach that state, and $x^{\ell}_{k+N|k} \in \mathbf{X_T}$ is the terminal set constraint.

Assumption 1: The stage cost L is convex over $\mathbf{X} \times \mathbf{U}$ and such that

$$L(x, Kx) \leq L(y, u), \quad \forall x \in \mathbf{X_T}, \forall y \notin \mathbf{X_T}, \forall u \in \mathbf{U}, \tag{18a}$$

$$L(x, u) \geq \alpha(d(x, \mathbf{X_T})), \quad \forall x \notin \mathbf{X_T}, \forall u \in \mathbf{U}, \tag{18b}$$

where α is a $\mathcal{K}$-function and $d(x, \mathbf{X_T})$ denotes a distance of x from the set $\mathbf{X_T}$.

Lemma 1: Let $\mathbf{X_0} = \{x : \ A_0 x \leq b_0\}$, $A_0 \in \mathbb{R}^{n_r \times n_x}$, $b_0 \in \mathbb{R}^{n_r}$. Let

$$\begin{aligned} c_0^i \quad &= \quad \min_{x,s} \quad s && \text{(19a)} \\ \text{s.t.} \quad & s \geq \|Q_x x\|_\infty, && \text{(19b)} \\ & A_0^i x = b_0^i, && \text{(19c)} \end{aligned}$$

for $i = 1, \dots, n_r$, with A_0^i the i-th row of A_0 and b_0^i the i-th element of b_0, where $\|Q_x x\|_\infty = \max_{i=1,\dots,n_x} |Q_x^i x|$, with Q_x^i the i-th row of Q_x, and define

$$c_0 = \min_{i=1,\dots,n_r} c_0^i \ . \tag{20}$$

Then the set

$$\mathbf{X_T} = \left\{ x : \begin{bmatrix} Q_x \\ -Q_x \end{bmatrix} x \leq c_0 \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \right\} \tag{21}$$

is such that $\mathbf{X_T} \subseteq \mathbf{X_0}$ and the stage cost

$$L(x, u) = \|Q_x x\|_\infty \tag{22}$$

satisfies Assumption 1.

Proof: By (19a), (20) and (21) it follows that the terminal set $\mathbf{X_T}$ is the largest level set of $\|Q_x x\|_\infty$ such that $\mathbf{X_T} \subseteq \mathbf{X_0}$. By construction, $\forall y \notin \mathbf{X_T}$, $\|Q_x y\|_\infty > c_0$, as $\exists i : (Q_x^i y > c_0) \vee (-Q_x^i y > c_0)$, which proves (18a). Define $d(y, \mathbf{X_T}) = \inf_{x \in \mathbf{X_T}} \|Q_x(y - x)\|_\infty$. As $0 \in \mathbf{X_T}$, by definition of inf $d(y, \mathbf{X_T}) \leq \|Q_x(y - 0)\|_\infty$. By letting $\alpha(\phi) = \phi$, (18b) follows. Hence, Assumption 1 is satisfied. ∎

The approach taken in this paper slightly differs from the one taken in [8], where the assumptions on L were not satisfied by the adopted stage cost.

Note that the input u is not taken into account in (22). This is not likely to have a major impact on the closed-loop behavior of the process, because usually in the outer mode the state x is relatively far from the origin, and u saturates irrespective to the choice of the stage cost L.

Note also that the stage cost (22) is only used in the outer mode: as soon as $x(k)$ enters $\mathbf{X_0}$, the inner mode $u(k+t) = Kx(k+t)$ is applied for all $t \geq 0$ (see next Theorem 1), where the gain K can be designed with arbitrary (yet stabilizing) performance criteria.

The optimal input resulting from the solution of Problem (17) ensures the convergence of the state x of (1) to the target set $\mathbf{X_T}$, in the absence of measurement noise (this can be proved similarly to the proof in [11]). By solving N mp-LPs as in [22], the solution for $u_{k|k}$ to Problem (17) is obtained in state-feedback piecewise affine form

$$u^*(x) = P_i x + q_i \quad \text{if } x \in \mathbf{X_i}, \tag{23}$$

where $P_i \in \mathbb{R}^{n_u \times n_x}$, $q_i \in \mathbb{R}^{n_u}$ and $\mathbf{X_i} = \{x \in \mathbb{R}^{n_x} : H_i x \leq k_i\}$. Now consider the feedback law derived from (23)

$$u(k) = P_j \bar{x}(k|k) + q_j, \tag{24}$$

where $\bar{x}(k|k) \in \mathbf{X_j}$. Let $x(k) \in \mathbf{X_i}$. Then, the difference between the optimal input (23) and the actual input (24) is $u(k) - u^*(x(k)) = P_j \bar{x}(k|k) + q_j - P_i x(k) - q_i$. Hence, the evolution of the system in closed-loop with (24) can be recast as

$$x(k+1) = (A + BP_i)x(k) + Bq_i + w(k) + e(k), \tag{25}$$

where

$$e(k) = B\big(P_j \bar{x}(k|k) + q_j - P_i x(k) - q_i\big) \tag{26}$$

is the error made with respect to the optimal input contribution due to the lack of information on the exact value of $x(k)$. We consider e as an additional unknown but bounded disturbance, with $e(k) \in \mathbf{Q} = \{e \in \mathbb{R}^{n_x} : \text{(26), (14) hold}\}$, and our goal is to find a control law which is robust with respect to this disturbance. To this aim, we introduce an iterative algorithm, which at every step h computes the updated set $\mathbf{Q^h}$, the gains $\{P_i\}_{i \in \mathcal{I}}^h$, $\{q_i\}_{i \in \mathcal{I}}^h$ and the partition $\{\mathbf{X_i}\}_{i \in \mathcal{I}}^h$ as a function of the previous set $\mathbf{Q^{h-1}}$. Let us consider the linear system

$$x(k+1) = Ax(k) + Bu(k) + w(k) + e(k) \tag{27}$$

and the associated min-max MPC problem

$$\begin{aligned} \min_{\{u_{k+j|k}^\ell\}} \quad & \max_{\ell \in \mathcal{L}} \sum_{j=0}^{N-1} L(x_{k+j|k}^\ell, u_{k+j|k}^\ell) \\ \text{s.t.} \quad & \text{(27), (17c), (17d),} \end{aligned} \tag{28}$$

together with its explicit solution in state feedback form. The proposed offline iterative procedure is defined by Algorithm 1.

Algorithm 1 Iterative explicit min-max MPC

1. Set $h = 0$, $Q^{-1} = \emptyset$, $Q^0 = \{0_{n_x}\}$;
2. while $Q^h \not\subseteq Q^{h-1}$,
  2.1. solve (28) with $e \in \mathbf{Q^h}$,
  get the explicit solution data $\{P_i,\ q_i,\ \mathbf{X_i}\}_{i \in \mathcal{I}}^h$;
  2.2. set $\mathbf{Q^{h+1}} = \ \text{hull}\{\mathbf{Q_{ij}^{h+1}}\}_{(i,j) \in \mathcal{I} \times \mathcal{I}}$, where[1]
  $\mathbf{Q_{ij}^{h+1}} = \{e \in \mathbb{R}^{n_x} : (26), x - \bar{x} \in \mathbf{F}_\mathcal{H}, x \in \mathbf{X_i^h}, \bar{x} \in \mathbf{X_j^h}\}$;
3. set $\{P_i,\ q_i,\ \mathbf{X_i}\}_{i \in \mathcal{I}} = \{P_i,\ q_i,\ \mathbf{X_i}\}_{i \in \mathcal{I}}^{h-1}$; **end**.

Remark 1: Since the actual value of the state $x(k)$ is not known, and only a set-membership relation $x(k) \in \mathcal{X}(k|k)$ is available, the controller is supposed to switch from outer to inner mode at time k if and only if

$$\mathcal{X}(k|k) \subseteq \mathbf{X_0}, \tag{29}$$

which ensures that $x(k) \in \mathbf{X_0}$.

We can finally define the structure of the Robust Energy-Aware MPC with Noisy measurements (REAN-MPC) in Algorithm 2. Its convergence properties are given in the following theorem.

Theorem 1: The state x of (1) controlled by the closed-loop scheme given by Algorithm 2 converges asymptotically to the target set $\mathbf{X_T} \subseteq \mathbf{X_0}$. If $\mathcal{X}(k|k) \subseteq \mathbf{X_0}$ at any time step k, then $x(k+t) \in \mathbf{X_0}$, $\forall t \in \mathbb{N}$.

Proof: The dual-mode explicit control law in Algorithm 2 is designed to be robust with respect to the additive disturbance w, the feedback error e induced by the threshold ε, and the measurement noise v^i, $i = 0, 1, \dots, m-1$. Moreover, $x \in \mathbf{X_T}$ if and only if $\|Q_x x\|_\infty \leq c_0$. Then, $\forall x(k) \notin \mathbf{X_T}$, $\forall \ell \in \mathcal{L}$, $\|Q_x x_{k|k}^\ell\|_\infty - \|Q_x x_{k+N|k}^\ell\|_\infty \geq 0$. Hence, the outer mode stage cost can be shown to be non-increasing in time, and the proof follows in a similar fashion of the proof of Theorem 1 in [11]. ∎

[1] We use $x - \bar{x} \in \mathbf{F}_\mathcal{H}$ instead of $x - \bar{x} \in \mathbf{F}$ in the definition of e to preserve linearity.

Remark 2: As observed earlier, when the controller switches to the inner mode, the state $x(k)$ lies in the terminal set $\mathbf{X_0}$ and will never exit it. Then, we can reduce conservativeness in the estimation algorithm by using the following prediction step

$$\mathcal{X}(k+1|k) = (A\mathcal{X}(k|k) \oplus B\{u(k)\} \oplus \mathbf{W}) \cap \mathbf{X_0} \quad (30)$$

instead of (10), if at time k the inner control mode is active.

Algorithm 2 Robust Energy-Aware MPC with Noisy measurements (REAN-MPC)

Offline:
run Algorithm 1 and get $\{P_i,\ q_i,\ \mathbf{X_i}\}_{i\in\mathcal{I}}$;
compute K, $\mathbf{X_0}$ and $\mathbf{X_T}$ as in Section IV.

At $k = 0$:
receive $\mathcal{Y}(0)$ from the master node;
set $\mathcal{X}(0|0) = \mathcal{Y}(0)$;
set $\bar{x}(0|0) = c(\mathcal{X}(0|0))$;
set $u(0) = B(P_i\bar{x}(0|0) + q_i),\ \bar{x}(0|0) \in \mathbf{X_i}$;
set $\mathcal{X}(1|0) = A\mathcal{X}(0|0) \oplus B\{u(0)\} \oplus \mathbf{W}$;
set $\hat{x}(0) = \bar{x}(0|0)$;
set $\hat{x}(j+1) = (A + BP_i)\hat{x}(j) + Bq_i$,
$\hat{x}(j) \in \mathbf{X_i},\ j = 0, \dots, M-1$;
transmit $\{\hat{x}(j)\}_{j=1}^{M}$ and ε to the master node.

For all $k > 0$:
if $\mathcal{Y}(k)$ is received from the master node
(because (8) is satisfied):
set $\delta(k) = 1$, otherwise $\delta(k) = 0$;
set $\mathcal{X}(k|k) = \mathcal{X}(k|k-1) \cap \begin{cases} \mathcal{Y}(k) & \text{if } \delta(k) = 1, \\ \mathbf{E} \oplus \{\hat{x}(k)\} & \text{otherwise;} \end{cases}$
set $\bar{x}(k|k) = c\big(\mathcal{X}(k|k)\big)$;
if $\mathcal{X}(k|k) \subseteq \mathbf{X_0}$:
set $u(k) = K\bar{x}(k|k)$;
set $\mathcal{X}(k+1|k) = (A\mathcal{X}(k|k) \oplus B\{u(k)\} \oplus \mathbf{W}) \cap \mathbf{X_0}$;
else
set $u(k) = B(P_i\bar{x}(k|k) + q_i),\ \bar{x}(k|k) \in \mathbf{X_i}$;
set $\mathcal{X}(k+1|k) = A\mathcal{X}(k|k) \oplus B\{u(k)\} \oplus \mathbf{W}$;
if $\delta(k) = 1$ **or** $\hat{x}(k+1)$ has not yet been computed:
set $\hat{x}(k) = \bar{x}(k|k)$;
if $\mathcal{X}(k|k) \subseteq \mathbf{X_0}$:
set $\hat{x}(k+j+1) = (A+BK)\hat{x}(k+j), j = 0, \dots, M-1$;
else
set $\hat{x}(k+j+1) = (A + BP_i)\hat{x}(k+j) + Bq_i$,
$\hat{x}(k+j) \in \mathbf{X_i},\ j = 0, \dots, M-1$;
transmit $\{\hat{x}(k+j)\}_{j=1}^{M}$ to the master node.

A nominal version of the REAN-MPC controller can be directly derived by solving (28) with $w(k) = e(k) = 0$, $\forall k$, $\mathbf{X_T} = \mathbb{R}^{n_x}$, $\mathbf{X_0} = \{\mathbf{0}_{n_x}\}$, and with a stage cost $L(x,u) = \|Q_x x\|_\infty + \|Q_u u\|_\infty$. This nominal controller, called Nominal Energy-Aware MPC with Noisy measurements (NEAN-MPC), can be in general less conservative, but no stability properties can be guaranteed in this case. The NEAN-MPC control scheme will be used for comparison purposes in Section V.

V. Simulation Results

To evaluate the performance of the proposed control scheme we consider the open-loop unstable second order discrete-time linear system (1), with $A = \left[\begin{smallmatrix} 0.02 & 1 \\ -0.49 & 0.02 \end{smallmatrix}\right]$, $B = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$, and threshold vector $\varepsilon = \left[\begin{smallmatrix} 0.1 \\ 0.1 \end{smallmatrix}\right]$. Limits on state, input, and disturbance variables are $|x_i| \le \bar{x}_i$, $|u| \le \bar{u}$, $|w_i| \le \bar{w}_i$, $i = 1, 2$, with $\bar{x} = \left[\begin{smallmatrix} 2 \\ 2 \end{smallmatrix}\right]$, $\bar{u} = 1$, $\bar{w} = \left[\begin{smallmatrix} 0.05 \\ 0.1 \end{smallmatrix}\right]$. We assume that $m = 3$ sensor nodes provide feedback to the controller. The bounds on measurement noise are $v^i_{max} = \left[\begin{smallmatrix} 0.03 \\ 0.03 \end{smallmatrix}\right]$, $i = 0, 1, 2$.

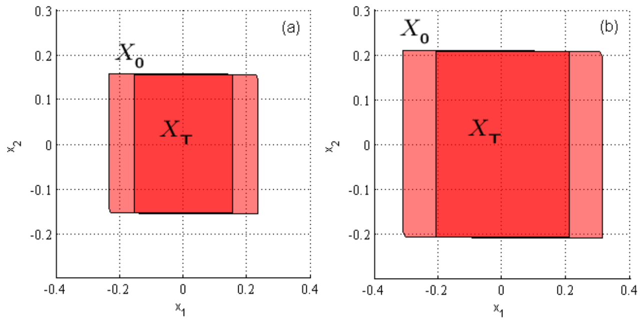

Fig. 1. Terminal sets for RNMPC (a) and REAN-MPC (b).

TABLE I
Energy-Aware MPC vs. Standard MPC: Simulation Results

Controller	J_{exp}	Tx. Rate
Standard Nominal MPC (NMPC)	0.1533	100.00%
Standard Robust MPC (RMPC)	0.1535	100.00%
Nominal Energy-Aware MPC (NEAN-MPC)	0.1549	56.42%
Robust Energy-Aware MPC (REAN-MPC)	0.1563	56.08%

The presented energy-aware control schemes are compared with standard nominal and robust MPCs, referred as NMPC and RMPC, respectively. These controllers are standard in the sense that the radio utilization minimization is not taken into account, i.e., no predictions are sent to the master node, and at every time step k the measurement set $\mathcal{Y}(k)$ is transmitted, regardless of the threshold logic. In other words, NMPC and RMPC control algorithms are a special case of NEAN-MPC and REAN-MPC, respectively, where $\varepsilon = \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]$.

The weight matrices are $Q_x = I_2$, $Q_u = 0.1$, the estimation horizon is $M = 10$, and the control horizon is $N = 3$. Different terminal sets $\mathbf{X_T}$ and RPI sets $\mathbf{X_0}$ are used for REAN-MPC and RMPC (see Figure 1). This is due to the fact that the REAN-MPC is more conservative, since it is robust with respect to the error e also. The constant gain used in the inner mode is $K = [0.4667\ \ -0.0280]$.

We run $N_s = 50$ simulations of $T_{sim} = 50$ time steps each, choosing the initial states $x(0)$ randomly in the feasible set of the REAN-MPC controller. The achieved average performance is evaluated as

$$J^i_{exp} = \frac{1}{T_{sim}} \sum_{k=1}^{T_{sim}} \left(\|Q_x x^i(k)\|_\infty + \|Q_u u^i(k)\|_\infty \right), \quad (31)$$

where i indexes the ith simulation. The average radio usage is computed assuming equal power consumption in transmitting and receiving packets, as usual for short range wireless nodes [7]. The average results are reported in Table I. Figures 2 and 3 show histograms of J_{exp} and radio utilization for NEAN-MPC and REAN-MPC controllers, respectively (data are given as percentage of analogue values obtained from standard MPCs).

Both nominal and robust energy-aware controllers achieve a good trade-off between performance and energy consumption: The NEAN-MPC grants a reduction in radio utilization by 43.58%, with a 1.04% loss in the experimental cost

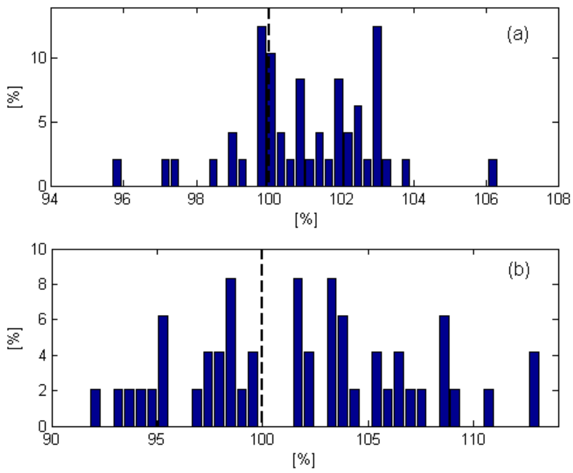

Fig. 2. Histogram of J^i_{exp} for NEAN-MPC (a) and REAN-MPC (b), normalized with respect to $J^{(NMPC)i}_{exp}$ and $J^{(RMPC)i}_{exp}$, respectively.

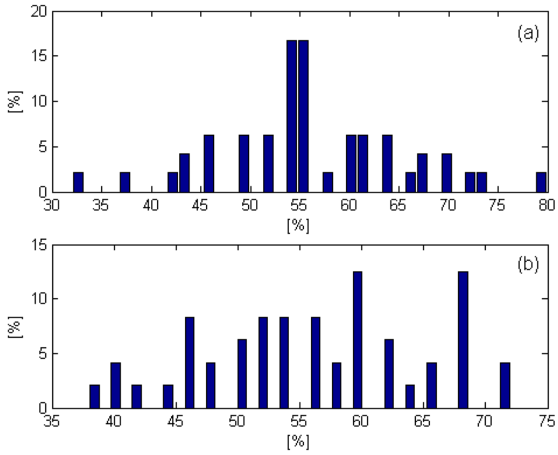

Fig. 3. Histogram of radio utilization for NEAN-MPC (a) and REAN-MPC (b).

function, with respect to the NMPC; the REAN-MPC obtains similar results, with a 43.92% saving in transmission rate, and a loss of 1.82% in performance, in comparison to the RMPC.

VI. Conclusions

This paper has extended the previous work on energy-aware MPC control [8] to the case of wireless feedback from multiple and noisy sensor nodes. A network with a short range WSN and a remote controller is investigated, where sensor nodes measure the same physical quantities for noise rejection. Sensor nodes intercommunicate with low power transmissions, and determine whether to send the measurement to the remote controller (and which value to send) by means of a simple estimation algorithm. A robust control scheme based on explicit MPC with guaranteed convergence properties was presented. Simulation results have shown that a substantial reduction in radio utilization ($\simeq 44\%$) can be achieved with little loss in closed-loop performance ($< 2\%$).

Future work on energy-aware MPC techniques will take into consideration some aspects of real WSNs such as packet loss, delays, and multi-hop routing protocols.

References

[1] V. Raghunathan, C. Schurgers, S. Park, and M. Srivastava, "Energy aware wireless microsensor networks," *IEEE Signal Processing Magazine*, vol. 19, pp. 40–50, 2002.

[2] C. E. Jones, K. M. Sivalingam, P. Agrawal, and J. C. Chen, "A Survey of Energy Efficient Network Protocols for Wireless Networks," *Wireless Networks*, vol. 7, no. 4, pp. 343–358, 2004.

[3] V. Raghunathan, C. L. Pereira, M. B. Srivastava, and R. K. Gupta, "Energy Aware Wireless Systems with Adaptive Power-Fidelity Tradeoffs," *IEEE Transactions On Very Large Scale Integration (VLSI) Systems*, vol. 13, no. 2, 2005.

[4] M. Walsh and M. Hayes, "A Robust Throughput Rate Control Mechanism for an 802.15.4 Wireless Sensor Network - An Anti-Windup Approach," *American Control Conference*, pp. 3071–3076, 2007.

[5] D. Quevedo, A. Ahlén, and G. Goodwin, "Predictive Power Control of Wireless Sensor Networks for Closed Loop Control," in *Proc. Int. Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control*, Pavia, Italy, 2008.

[6] L. M. Feeney and M. Nilsson, "Investigating the energy consumption of a wireless network interface in an ad hoc networking environment," *20th Annual Joint Conference of the IEEE Computer and Communications Societies*, vol. 3, pp. 1548–1557, 2001.

[7] M. Corporation, *Tmote Sky: Datasheet*, 2006.

[8] D. Bernardini and A. Bemporad, "Energy-Aware Robust Model Predictive Control Based On Wireless Sensor Feedback," in *IEEE Conference on Decision and Control*, Cancun, Mexico, 2008.

[9] M. Milanese and A. Vicino, "Optimal estimation theory for dynamic systems with set membership uncertainty: an overview," *Automatica*, vol. 27, no. 6, pp. 997–1009, 1991.

[10] L. Chisci, A. Garulli, and G. Zappa, "Recursive state bounding by parallelotopes," *Automatica*, vol. 32, pp. 1049–1055, 1996.

[11] P. O. M. Scokaert and D. Q. Mayne, "Min-max feedback model predictive control for constrained linear systems," *IEEE Transactions On Automatic Control*, vol. 43, pp. 1136–1142, 1998.

[12] E. C. Kerrigan and J. M. Maciejowski, "Robustly stable feedback min-max model predictive control," in *Proc. of the American Control Conference*, vol. 4, Denver, Colorado, USA, 2003, pp. 3490–3495.

[13] F. Blanchini, "Set invariance in control – a survey," *Automatica*, vol. 35, no. 11, pp. 1747–1768, 1999.

[14] I. Kolmanovsky and E. G. Gilbert, "Theory and computation of disturbance invariant sets for discrete-time linear systems," *Mathematical Problems in Engineering*, vol. 4, no. 4, pp. 317–367, 1998.

[15] E. C. Kerrigan, "Robust constraint satisfaction: Invariant sets and predictive control," Ph.D. dissertation, University of Cambridge, 2000.

[16] A. Alessio, M. Lazar, A. Bemporad, and W. P. M. H. Heemels, "Squaring the circle: An algorithm for generating polyhedral invariant sets from ellipsoidal ones," *Automatica*, vol. 43, pp. 2096–2103, 2007.

[17] S. V. Raković, P. Grieder, M. Kvasnica, D. Q. Mayne, and M. Morari, "Computation of invariant sets for piecewise affine discrete time systems subject to bounded disturbances," in *IEEE Conference on Decision and Control*, vol. 2, Atlantis, Paradise Island, Bahamas, 2004, pp. 1418–1423.

[18] E. F. Camacho and C. Bordons, *Model Predictive Control*, 2nd ed., ser. Advanced Textbooks in Control and Signal Processing. London: Springer-Verlag, 2004.

[19] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, 2003.

[20] A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos, "The Explicit Linear Quadratic Regulator for Constrained Systems," *Automatica*, vol. 38, no. 1, pp. 3–20, 2002.

[21] A. Alessio and A. Bemporad, "A survey on explicit model predictive control," in *Proc. Int. Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control*, Pavia, Italy, 2008.

[22] A. Bemporad, F. Borrelli, and M. Morari, "Min-max control of constrained uncertain discrete-time linear systems," *IEEE Transactions On Automatic Control*, vol. 48, 2003.