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Decentralized Model Predictive Control of Constrained Linear Systems

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Abstract— **This paper proposes a novel decentralized model predictive control (MPC) design approach for open-loop asymptotically stable processes whose dynamics are not necessarily decoupled. A set of partially decoupled approximate prediction models are defined and used by different MPC controllers. Rather than looking for a-priori conditions for asymptotic stability of the overall closed-loop system, we present a sufficient criterion for analyzing a posteriori the asymptotic stability of the process model in closed loop with the set of decentralized MPC controllers. The degree of decoupling among submodels represents a tuning knob of the approach: the less coupled are the submodels, the lighter the computational burden and the load for transmission of information among the decentralized MPC controllers, but the less performing is the control system and the less likely the proposed stability test succeeds. The designer can therefore trade off between simplicity of computations/limited transmitted information and performance/stability.**

I. INTRODUCTION

In *decentralized control* of multivariable systems the achievement of a global control task is obtained by the cooperation of many controllers, each one computing a subset of control commands individually under a possibly limited exchange of information with the other controllers.

Compared to centralized schemes, while decentralized control has the disadvantage of inevitably leading to a loss of performance, it has a twofold technological advantage: (i) no need for a high-performance central processing unit performing complex global control algorithms that take into account the overall system dynamics, replaced by several simpler (and therefore cheaper) units; (ii) all process measurements [command variables] do not need to be conveyed to [issued from] a single unit, therefore limiting the exchange of information between spatially distributed components of the process.

The interest in decentralized control schemes dates back to several decades ago [1], [2], [3], [4]. More recently the attention was revived [5], [6], [7], [8], [9], [10], [11], [12] by the interest in increasingly complex distributed sensing and actuation systems interacting with the environment that arise in several application domains, including civil engineering (control of large structures), and military (unmanned air vehicles). The control of a multitude of sensing and actuation devices is naturally tackled by decentralized control designs, conceived to ensure both local self-properties and global coordination. From a control theoretical viewpoint, a centralized approach with complete knowledge of the overall system has the potential of providing significant properties like stability and optimality [5]; a similar advantage arises in optimization while comparing a global solution to a collection of locally optimal solutions.

Several approaches to decentralized control design have been proposed in the literature. They differ from each other in the assumptions made on the kind of interactions between different components of the overall system, the model of information exchange between subsystems, and the control design technique used for each subsystem. A very promising design approach to decentralized control was proposed in the context of model predictive control (MPC) [13], [14], [15], [16], [17]. Motivated by describing problems of formation control of unmanned air vehicles, of networks of independently actuated systems, and on optimal strategies for multi-agents in a game theoretical setting, most contributions focused the attention on dynamically decoupled systems, i.e. dynamical systems decomposed into distinct subsystems that can be independently actuated. For each subsystem a distinct MPC controller computes the local control action based on the measurements (and predictions) of the states of its corresponding subsystem and of its *neighbors*. Prediction is based on a linear-discrete time model of each subsystem. Typically the interaction among neighbors is represented by an "interaction graph", where each node represents a single subsystem and an arc between two nodes denotes a coupling term in the goal and/or in the constraints associated to the nodes [17], [13], [16]. The set of neighbors associated to a single subsystem (node) is represented by the set of nodes directly connected to that node.

The main issues in decentralized MPC are *feasibility* and *stability*. These two properties are, in general, not guaranteed, unless additional (usually conservative) constraints are included, for instance imposing the robust fulfillment of constraints ensuring feasibility [14]. This is due to the fact that predictions of trajectories of the neighbors are often wrong, since a subsystem may have the knowledge of its neighbors' state but neither of the state of the neighbors' neighbors, nor of the neighbor's moves. In [18] min-max optimization is used to handle the uncertainties in the disturbances from neighbor controllers. In [16], [14], [15] stability analysis of decentralized MPC of decoupled systems was investigated. In [16] a cooperative behavior among the controllers is assumed, as the cost function of the

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optimization problem solved by each subsystem measures the entire system performance. In [14] a sufficient condition for the stability of each individual subsystem is given, while in [15] closed-loop stability is achieved through the inclusion of a contractive constraint in the optimization problem.

This work proposes an approach to decentralized MPC design for a *process model* that is not necessarily dynamically decoupled. The decoupling assumption only appears in the *prediction models* used by different MPC controllers. The degree of chosen decoupling represents a tuning knob of the approach. Rather than looking for a-priori conditions for asymptotic stability of the overall closed-loop, we present a sufficient criterion for analyzing a posteriori the asymptotic stability of the process model in closed loop with the set of decentralized MPC controllers. If this condition is not verified, then the degree of decentralization can be modified by augmenting the level of coupling of the dynamics of the prediction models, increasing consequently the number of exchanged information about state measurements among MPC controllers.

II. Centralized MPC

Consider the standard MPC problem based on the linear discrete-time prediction model

$$x(t+1) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the vector of command variables at time step t, and the following finite-time optimal control problem

$$V(x(t)) = \min_{U} \quad x_N^\top P x_N + \sum_{k=0}^{N-1} x_k^\top Q x_k + u_k^\top R u_k \tag{2a}$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k,\ k = 0, \ldots, N-1 \tag{2b}$$

$$x_0 = x(t) \tag{2c}$$

$$u_{\min} \le u_k \le u_{\max},\ k = 0, \ldots, N_u - 1 \tag{2d}$$

$$u_k = 0,\ k = N_u, \ldots, N-1 \tag{2e}$$

where N is the prediction horizon, $N_u \le N$ is the input horizon, and $u_{\min} < 0 < u_{\max} \in \mathbb{R}^m$ define saturation constraints on input variables, and "$\le$" denotes component-wise inequalities.

Problem (2) can be recast as a Quadratic Programming (QP) problem (see e.g. [19], [20]), whose solution

$$U^*(x(t)) \triangleq [u_0^{*\top}(x(t)) \ \ldots \ u_{N-1}^{*\top}(x(t))]^\top \in \mathbb{R}^{Nm}$$

is a sequence of optimal control inputs. In (2) we assume that $Q = Q^\top \ge 0$, $R = R^\top > 0$ are square weight matrices defining the performance index, and that the terminal weight $P = P^\top \ge 0$ is a square matrix satisfies the Lyapunov equation

$$A^\top P A - P = -Q \tag{3}$$

so that the cost (2a) is equal to $\sum_{k=0}^{\infty} x_k^\top Q x_k + u_k^\top R u_k$. The existence of matrix P is ensured by the following assumption

Assumption 1: Matrix A is strictly Hurwitz[1].

Assumption 1 restricts the strategy and stability results of this paper to processes that are open-loop asymptotically stable, leaving to the controller the mere role of optimizing the performance of the closed-loop system. Another restriction taken in this paper is that problem (2) only tackles input constraints (2d), which makes problem (2) feasible for any value of the state vector $x(t) \in \mathbb{R}^n$

At each sampling time t, problem (2) is solved for the given measured (or estimated) current state $x(t)$. Only the first optimal move $u_0^*(x(t))$ of the optimal sequence $U^*(x(t))$ is applied to the process,

$$u(t) = u_0^*(x(t)), \tag{4}$$

the remaining optimal moves are discarded and the optimization is repeated at time $t+1$.

Theorem 1 ([21]): Under Assumption 1, system (1) in closed-loop with the MPC algorithm (2), (4) is asymptotically stable.

III. Decentralized MPC

The centralized MPC algorithm described in Section II requires that all the n components of the state vector $x(t)$ are transmitted to a (possibly remote) central unit, that needs to solve a QP with mN_u decision variables and $2mN_u$ inequality constraints, and transmit back m signals to different actuators. To avoid the need for such a centralized computing power and a star-like network topology, in this section we proposed a decentralized scheme where simpler QP problems can be solved in a spatially distributed way.

A. Decentralized Prediction Models

Let the system to be controlled be described again by the process model (1). Matrices A, B will have a certain number of zero components corresponding to partially dynamically decoupled subsystems, or even be block diagonal in case of total dynamical decoupling (this is the case for instance of independent moving agents each one having its own dynamics).

Let M be the number of decentralized control actions that we want to design, for example $M = m$ in case each individual actuator is governed by its own controller. For all $i = 1, \ldots, M$, we define $x^i \in \mathbb{R}^{n_i}$ as the vector collecting a subset $I_{xi} \subseteq \{1, \ldots, n\}$ of the state components,

$$x^i = W_i^\top x = \begin{bmatrix} x^{i1} \\ \vdots \\ x^{in_i} \end{bmatrix} \in \mathbb{R}^{n_i}$$

where $W_i \in \mathbb{R}^{n \times n_i}$ collects the n_i columns of the identity matrix of order n corresponding to the indices in I_{xi}, and, similarly,

$$u^i = Z_i^\top u = \begin{bmatrix} u^{i1} \\ \vdots \\ u^{im_i} \end{bmatrix} \in \mathbb{R}^{m_i}$$

[1] While usually a matrix A is called *Hurwitz* if all its eigenvalues have strictly negative real part, in this paper we adapt the notion to discrete-time systems, saying that a matrix A is called *Hurwitz* if all the eigenvalues λ_i of A are such that $|\lambda_i| < 1$.

as the vector of input signals tackled by the i-th controller, where $Z_i \in \mathbb{R}^{m \times m_i}$ collects m_i columns of the identity matrix of order m corresponding to the set of indices $I_{ui} \subseteq \{1, \ldots, m\}$. Note that

$$W_i^\top W_i = I_{n_i},\ Z_i^\top Z_i = I_{m_i},\ \forall i = 1, \ldots, M. \tag{5}$$

By left-multiplying (1) by $W_i^\top$, we obtain

$$x^i(t+1) = W_i^\top x(t+1) = W_i^\top Ax(t) + W_i^\top Bu(t). \tag{6}$$

An *approximation* of (1) is obtained by changing $W_i^\top A$ into $W_i^\top AW_iW_i^\top$ and $W_i^\top B$ into $W_i^\top BZ_iZ_i^\top$, therefore getting the new prediction model of reduced order

$$x^i(t+1) = A_ix^i(t) + B_iu^i(t) \tag{7}$$

where matrices $A_i = W_i^\top AW_i \in \mathbb{R}^{n_i \times n_i}$ and $B_i = W_i^\top BZ_i \in \mathbb{R}^{m_i \times m_i}$ are submatrices of the original A and B matrices, respectively, describing in a possibly approximate way the evolution of the states of subsystem #i.

Assumption 2: Matrix A_i is strictly Hurwitz, for all $i = 1, \ldots, M$.

Model (7) has a smaller size than the original process model (1). The choice of the dimensions n_i,m_i and of matrices W_i,Z_i are a tuning knob of the proposed decentralized procedure and should be inspired by the inspection of zero or negligible entries in A,B (or in other words by physical insight on the process dynamics) and by taking into account the requirement stated in Assumption 2.

We want to design a controller for each set of moves u^i according to the prediction model (7) and based on feedback on x^i, for all $i = 1, \ldots, M$. Note that in general different states x^i, x^j and different u^i, u^j may share common components. In particular, to avoid ambiguities on the control action to be provided to the process, we impose that only a subset $I_{ui}^\# \subseteq I_{ui}$ of input signals computed by controller #i is actually applied to the process, with the following conditions

$$\bigcup_{i=1}^{M} I_{ui}^\# = \{1, \ldots, m\} \tag{8}$$

$$I_{ui}^\# \cap I_{uj}^\# = \emptyset,\ \forall i, j = 1, \ldots, M,\ i \neq j. \tag{9}$$

Condition (8) ensures that all actuators are commanded, Condition (9) that each actuator is commanded by only one controller. For the sake on simplicity of notation, since now on we will assume that $M = m$ and that $I_{ui}^\# = i$, $i = 1, \ldots, m$, i.e., that each controller #i only controls the ith input signal. As observed earlier, in general $I_{xi} \cap I_{xj} \neq \emptyset$, meaning that controller #i may partially share the same feedback information with controller #j, and $I_{ui} \cap I_{uj} \neq \emptyset$, meaning that controller #i may take into account the effect of control actions that are actually decided by another controller #j, $i \neq j$, $i, j = 1, \ldots, M$.

B. Decentralized Optimal Control Problems

For all $i = 1, \ldots, M$ consider the following infinite-time constrained optimal control problem

$$V_i(x(t)) = \min_{u_0^i} \sum_{k=0}^{\infty} x_k^{i\top} W_i^\top QW_ix_k^i + u_k^{i\top} Z_i^\top RZ_iu_k^i = \tag{10a}$$

$$= \min_{u_0^i} x_1^{i\top} P_ix_1^i + x^{i\top}(t)W_i^\top QW_ix^i(t) + u_0^{i\top} Z_i^\top RZ_iu_0^i \tag{10b}$$

$$\text{s.t. } x_1^i = A_ix^i(t) + B_iu_0^i, \tag{10c}$$

$$x_0^i = W_i^\top x(t) = x^i(t) \tag{10d}$$

$$u_{\min} \leq u_0^i \leq u_{\max}, \tag{10e}$$

$$u_k^i = 0,\ \forall k \geq 1 \tag{10f}$$

where $P_i = P_i^\top \geq 0$ is the solution of the Lyapunov equation

$$A_i^\top P_iA_i - P_i = -W_i^\top QW_i, \tag{11}$$

that exists by virtue of Assumption 2. Problem (10) corresponds to a finite horizon problem with control horizon $N_u = 1$.

At time t, each controller MPC #i measures (or estimates) the state $x^i(t)$ (usually corresponding to local and neighboring states), solves problem (10), and obtains the optimizer $u_0^{*i} = [u_0^{*i1}, \ldots, u_0^{*ii}, \ldots, u_0^{*im_i}]^\top \in \mathbb{R}^{m_i}$. In the simplified case $M = m$ and $I_{ui}^\# = i$, only the i-th sample of u_0^{*i}

$$u_i(t) = u_0^{*ii} \tag{12}$$

will determine the i-th component $u_i(t)$ of the input vector actually implemented to the process at time t. The inputs u_0^{*ij}, $j \neq i$, $j \in I_{ui}$ to the neighbors are discarded, their only role is to provide a better prediction of the state trajectories x_k^i, and therefore a possibly better performance of the overall system.

The collection of the optimal inputs of all the M MPC controllers,

$$u(t) = [u_0^{*11}\ \ldots\ u_0^{*ii}\ \ldots\ u_0^{*mm}]^\top \tag{13}$$

is the actual input commanded to process (1). The optimizations (10) are repeated at time $t+1$, based on the new states $x^i(t+1) = W_i^\top x(t+1)$, according to the usual receding horizon control paradigm.

C. Stability of Decentralized MPC

As mentioned in the introduction, one of the major issues in decentralized RHC is to ensure stability of the overall closed-loop system. The non-triviality of this issue is due to the fact that the prediction of the state trajectory made by MPC #i about state $x^i(t)$ is often wrong, because of partial state information and of the mismatch between u^{*ij} (desired by controller MPC #i) and u^{*jj} (computed and applied to the process by controller MPC #j).

In order to ensure stability of the decentralized control scheme, an approach was proposed in [14] based on the computation of decoupled terminal regions which ensure closed-loop stability of the entire system and constraint fulfillment. The approach computes a hyper-rectangular inner approximation between the feasible space defined by the interconnection constraints and a controlled invariant set of the i-th subsystem. The authors give a sufficient condition for the asymptotically stability of the closed-loop system.

The authors in [16] propose a "feasible cooperation-based MPC" where they modify the objective functions of the subsystems' MPCs by replacing the single i-th objective function with an objective that measures the entire system performance. Nominal closed-loop stability and optimality are established by using an algorithm that at every time step solves M QP problems iteratively until it does reach a certain level of agreement between subsystems, and then applies the computed inputs. A similar approach was taken in [9] where, contrarily to our approach, the authors consider situations where it is possible for the agents to exchange information several times while they are solving their local optimization problems at each control instant.

In this paper we provide "a posteriori" verifiable stability conditions for decentralized MPC. Coupling constraints are not additional constraints in the optimization problem. Dynamical coupling between different parts of the model are captured by matrices (A_i, B_i). Once the structure of the controllers for the subsystems to be actuated is chosen, the stability test presented in the next section ensures stability of the entire system (1) in closed-loop with the M decentralized MPC controllers. If the test fails, it is always possible to find larger matrices A_i, B_i, i.e., a larger degree of coupling between subsystems, and consequently a different set of MPC controllers that stabilizes the entire system (in the worst case, $A_i = A$, $B_i = B$, i.e., all MPC controllers solve the global MPC problem (2), therefore obtaining consistent solutions $u_0^{*ii} = u_0^{*ji}$, $\forall i, j = 1, \ldots, M$).

Let $\Delta u^i(t) \triangleq u(t) - Z_i u_0^{*i}(t)$, $\Delta x^i(t) \triangleq (I - W_i W_i^\top)x(t)$, $\Delta A^i \triangleq (I - W_i W_i^\top)A$, $\Delta B^i \triangleq B - W_i W_i^\top B Z_i Z_i^\top$. Since $x^i(t) = W_i^\top x(t)$ and by exploiting (11), at time t the optimal cost $V_i(x(t))$ can be rewritten as

$$\begin{aligned} V_i(x(t)) &= (W_i^\top x(t))^\top (W_i^\top Q W_i) W_i^\top x(t) + \\ &(A_i W_i^\top x(t) + B_i u_0^{*i}(t))^\top P_i (A_i W_i^\top x(t) + B_i u_0^{*i}(t)) + \\ &u_0^{*i\top}(t) Z_i R Z_i u_0^{*i}(t). \end{aligned} \tag{14}$$

As the input $u_0^i = 0$ satisfies the constraints $u_{\min} \le u_0^i \le u_{\max}$, by (11) the optimal cost at time $t+1$ satisfies the following inequality

$$\begin{aligned} V_i(x(t+1)) &\le (W_i^\top x(t+1))^\top (W_i^\top Q W_i) W_i^\top x(t+1) + \\ &(A_i W_i^\top x(t+1))^\top P_i A_i W_i^\top x(t+1) = \\ &= (W_i^\top x(t+1))^\top (A_i^\top P_i A_i + W_i^\top Q W_i) W_i^\top x(t+1) = \\ &= x(t+1)^\top W_i P_i W_i^\top x(t+1). \end{aligned} \tag{15}$$

By rewriting $x(t+1) = Ax(t) + Bu(t) = (A \pm W_i W_i^\top A)(x(t) \pm W_i W_i^\top x(t)) + (B \pm W_i W_i^\top B Z_i Z_i^\top)(u(t) \pm Z_i u_0^{*i}(t)) = W_i(A_i W_i^\top x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t))$, where $\Delta Y^i(x(t)) \triangleq W_i W_i^\top (A \Delta x^i(t) + B Z_i Z_i^\top \Delta u^i(t)) + \Delta A^i x(t) + \Delta B^i u(t)$, from (15) and recalling (5) we obtain

$$\begin{aligned} V_i(x(t+1)) &\le (W_i(A_i W_i^\top x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t)))^\top \cdot \\ &\cdot W_i P_i W_i^\top (W_i(A_i W_i^\top x(t) + B_i u_0^{*i}(t)) + \Delta Y^i(x(t))) = \\ &= (A_i W_i^\top x(t) + B_i u_0^{*i}(t))^\top P_i (A_i W_i^\top x(t) + B_i u_0^{*i}(t)) + \\ &+ \Delta S^i(x(t)), \end{aligned}$$

where $\Delta S^i(x(t)) \triangleq 2(A_i W_i^\top x(t) + B_i u_0^{*i}(t))^\top P_i W_i^\top \Delta Y^i(x(t)) + \Delta Y^{i\top}(x(t)) W_i P_i W_i^\top \Delta Y^i(x(t))$. By (14), we obtain

$$\begin{aligned} V_i(x(t+1)) &\le V_i(x(t)) - x^\top(t) W_i W_i^\top Q W_i W_i^\top x(t) + \\ &- u_0^{*i\top}(t) Z_i^\top R Z_i u_0^{*i}(t) + \Delta S^i(x(t)). \end{aligned} \tag{16}$$

We are now ready to state the following theorem:

Theorem 2: If the condition

$$(i)\ x^\top \left(\sum_{i=1}^{M} W_i W_i^\top Q W_i W_i^\top \right) x - \sum_{i=1}^{M} \Delta S^i(x) \ge 0,\ \forall x \in \mathbb{R}^n, \tag{17}$$

is satisfied, or the condition

$$\begin{aligned} (ii)\ x^\top &\left(\sum_{i=1}^{M} W_i W_i^\top Q W_i W_i^\top \right) x - \alpha x^\top x - \sum_{i=1}^{M} \Delta S^i(x) + \\ &\sum_{i=1}^{M} u_0^{*i\top}(x) Z_i^\top R Z_i u_0^{*i}(x) \ge 0,\ \forall x \in \mathbb{R}^n \end{aligned} \tag{18}$$

is satisfied for some scalar $\alpha > 0$, then the decentralized MPC scheme defined in (10)–(13) in closed loop with (1) is globally asymptotically stable.

Proof: Either because of (16), (17), and positive definiteness of R and hence of his principal minor $Z_i^\top R Z_i$ (case i) or (16) and (18) (case ii), the function

$$V(x(t)) \triangleq \sum_{i=1}^{M} V_i(W_i^\top x(t)) \tag{19}$$

is non-increasing. Since $V(x(t)) \ge 0$, $\forall t \ge 0$, it follows that there exists $\lim_{t\to\infty} V(x(t)) = \lim_{t\to\infty} V(x(t+1))$. Hence, by (16) it also follows that $\lim_{t\to\infty} x^\top(t) \left(\sum_{i=1}^{M} W_i W_i^\top Q W_i W_i^\top \right) x(t) - \sum_{i=1}^{M} \Delta S^i(x(t)) + \sum_{i=1}^{M} u_0^{*i\top}(x(t)) Z_i^\top R Z_i u_0^{*i}(x(t)) = 0$. (i) Because of (17), it follows that $\lim_{t\to\infty} \sum_{i=1}^{M} u_0^{*i\top}(x(t)) Z_i^\top R Z_i u_0^{*i}(x(t)) = 0$, and by positive definiteness of $Z_i^\top R Z_i$, that $\lim_{t\to\infty} u_0^{*i}(x(t)) = 0$, and hence that $\lim_{t\to\infty} u_0^{*ii}(x(t)) = 0$, $\forall i = 1, \ldots, M$, which in turn implies $\lim_{t\to\infty} u(t) = 0$. As by Assumption 1 the open-loop process (1) is linear and asymptotically stable, and therefore input-to-state stable, it also follows that $\lim_{t\to\infty} x(t) = 0$. (ii) Because of (18), it follows that $\lim_{t\to\infty} \alpha x^\top(t) x(t) = 0$, which in turn implies that $\lim_{t\to\infty} x(t) = 0$. □

D. Stability Tests

By using the explicit MPC results of [20], each optimizer function $u_0^{*i}: \mathbb{R}^n \mapsto \mathbb{R}^{m_i}$, $i = 1, \ldots, M$, can be expressed as a piecewise affine function of x

$$u_0^{*i}(x) = F_{ij}x + G_{ij} \text{ if } H_{ij}x \leq K_{ij},\ j = 1, \ldots, n_{ri} \quad (20)$$

Hence, both condition (17) and condition (18) are a composition of quadratic and piecewise affine functions, so that *global stability* can be tested through linear matrix inequality relaxations [22] (cf. the approach of [23]).

As $u_{\min} < 0 < u_{\max}$, there exists a ball around the origin $x = 0$ contained in one of the regions, say $\{x \in \mathbb{R}^n : H_{i1}x \leq K_{i1}\}$, such that $G_{i1} = 0$. Therefore, around the origin both (17) and (18) become a quadratic form of x, and hence *local stability* of (10)–(13) in closed loop with (1) can be simply tested by checking positive semidefiniteness of a square $n \times n$ matrix.

IV. Experimental Results

Consider the following asymptotically stable linear time-invariant discrete-time system

$$\left\{\begin{array}{rcl} x(t+1) & = & \begin{bmatrix} 0.9429 & -0.02798 & -0.2611 \\ 0.02224 & 0.9798 & -0.02135 \\ 0.2616 & 0.01452 & 0.943 \end{bmatrix} x(t)+ \\ & + & \begin{bmatrix} .009384 & .005471 & -.00072 \\ -.001563 & .00931 & -.00055 \\ -.002088 & -.00147 & .005401 \end{bmatrix} u(t) \end{array}\right. \quad (21)$$

We choose here to design three decentralized controllers, one for each input, i.e., the i-th MPC controller decides the value of the i-th input to be commanded. The decoupled prediction models are chosen according to the following set of state/input indices:

$$I_{x1} = I_{u1} = \{1, 3\}; I_{x2} = I_{u2} = \{1, 2\}; I_{x3} = I_{u3} = \{1, 3\}.$$

Accordingly, the pairs of matrices (A_i, B_i) are defined as

$$A_1 = \begin{bmatrix} 0.9429 & -0.2611 \\ 0.2616 & 0.943 \end{bmatrix}$$
$$B_1 \begin{bmatrix} .009384 & -.00072 \\ -.002088 & .005401 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0.9429 & -0.02798 \\ 0.02224 & 0.9798 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} .009384 & .005471 \\ -.001563 & .00931 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0.9429 & -0.2611 \\ 0.2616 & 0.943 \end{bmatrix}$$
$$B_3 = \begin{bmatrix} .009384 & -.00072 \\ -.002088 & .005401 \end{bmatrix}$$

corresponding to the choice of matrices (W_i, Z_i)

$$\begin{array}{ll} W_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; & Z_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \\ W_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}; & Z_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \\ W_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; & Z_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}. \end{array} \quad (22)$$

The controllers are designed using the decentralized MPC scheme (10)–(13) based on the following weights

$$\begin{array}{l} Q = \begin{bmatrix} 4 & -0.2 & -0.3 \\ -0.1 & 5 & -0.4 \\ -0.6 & -0.1 & 5 \end{bmatrix} \\ R = 10^{-6} \cdot I_3. \end{array} \quad (23)$$

By computing the unconstrained solution to the MPC problems (corresponding to the critical region of the explicit version of the MPC controller related to no constraint active at optimality), we obtain the matrix gains F_i, $i = 1, 2, 3$

$$\begin{array}{l} F_1 = \begin{bmatrix} -107.7638 & 14.7213 \\ -89.7068 & -168.7838 \end{bmatrix} \\ F_2 = \begin{bmatrix} -96.9938 & 58.1031 \\ -19.2757 & -95.4731 \end{bmatrix} \\ F_3 = \begin{bmatrix} -107.7638 & 14.7213 \\ -89.7068 & -168.7838 \end{bmatrix}. \end{array} \quad (24)$$

Note that different gains computed for the same command input by different MPC controllers look similar (or are even equal), meaning that the guess u^{*ij} made by MPC #i on the move u^{*jj} planned by MPC #j will not be very inaccurate.

In a region around the origin where no constraint is active, the decentralized MPC control law is

$$u(t) = \begin{bmatrix} -107.7638 & 0 & 14.7213 \\ -19.2757 & -95.4731 & 0 \\ -89.7068 & 0 & -168.7838 \end{bmatrix} x(t) \quad (25)$$

while the centralized MPC control gain is given by

$$u(t) = \begin{bmatrix} -93.7776 & 58.3261 & 17.5837 \\ -23.5555 & -95.7687 & -4.8290 \\ -91.0992 & -6.3035 & -169.0643 \end{bmatrix} x(t).$$

System (21) in closed-loop with the decentralized MPC controller (25) is asymptotically stable since condition (17) is verified. In fact, condition (17) under the feedback gains (24) leads to the following condition

$$x^\top \begin{bmatrix} 12.0169 & -0.1988 & -0.6000 \\ -0.0987 & 5.0001 & 0 \\ -1.1997 & 0 & 9.3572 \end{bmatrix} x \geq 0 \quad (26)$$

that is verified since the eigenvalues of the matrix associated with the quadratic form (26) are 12.3287, 9.6911, 4.9972. Figures 1–3 show the state and input trajectories, starting from the initial state $x_0 = [-0.01\ 0.1\ 0.09]^\top$. It also shows that a completely decoupled approach $I_{xi} = I_{ui} = i$ to the design of decentralized controller would lead to instability.

V. Conclusions

In this paper we have proposed a decentralized MPC scheme and two different associated sufficient stability criteria for testing asymptotic stability of the overall closed-loop system. The degree of decoupling among submodels is a tuning knob of the approach: more coupling leads to generally better performance and to stability guarantees, at the price of a heavier computational burden of the decentralized MPC controllers and of the need for transmitting a larger amount of information about state values. In this paper we have assumed that the open-loop process is asymptotically stable. Extensions of the approach to open-loop unstable systems by using Riccati terminal weights instead of Lyapunov weights

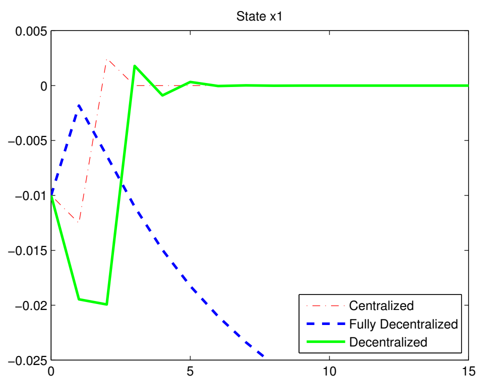

Fig. 1. Centralized MPC controller, decentralized MPC controller, and fully decentralized MPC controller in closed loop with the process model (21)

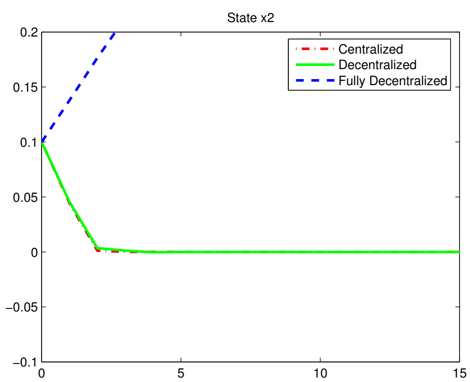

Fig. 2. Centralized MPC controller, decentralized MPC controller, and fully decentralized MPC controller in closed loop with the process model (21)

are currently under investigation. Also, the use of state-observers for estimating the effects of neglected states that are detectable from local state information is a current research item.

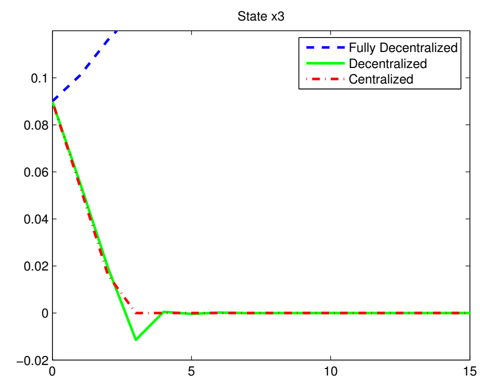

Fig. 3. Centralized MPC controller, decentralized MPC controller, and fully decentralized MPC controller in closed loop with the process model (21)

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