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Robust Nonlinear Reference Filtering for Constrained Linear Systems with Uncertain Impulse/Step Responses

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Abstract

A method based on conceptual tools of predictive control is described for solving tracking problems wherein pointwise-in-time input and/or state inequality constraints and model uncertainties are present. It consists of adding to a primal compensated system a nonlinear device called Predictive Reference Filter (PRF) which manipulates the desired trajectory in order to fulfill the prescribed constraints. Provided that an admissibility condition on the initial state is satisfied, the control scheme is proved to fulfill the constraints and be asymptotically stable for all the systems whose impulse-response and step-response descriptions lie within given uncertainty ranges.

1 Introduction

In recent years there have been substantial theoretical advancements in the field of feedback control of dynamic systems with input and/or state-related constraints. The main goal of the present paper is to address this issue by laying down guidelines for synthesizing *reference filters* based on predictive control ideas [5]. A reference filter is a nonlinear device which is added to a primal compensated control system. The latter, is designed so as to perform satisfactorily in the absence of constraints. Whenever necessary, the filter modifies the input to the primal control system so as to avoid violation of the constraints. Hence, the action is finalized to let the primal control system operate linearly within a wider dynamic range. Preliminary studies along these lines have already appeared in [1], [2]. For approaches arising form different perspectives see [3] and [4]. This paper extends the results obtained in [2] by considering system uncertainties. These are modeled in terms of both impulse-response and step-response interval ranges. The filtering action is operated on-line by performing a scalar quadratic optimization for the worst-case system. A simulation example is presented so as to exhibit the results achievable by the method.

2 Problem Formulation

Consider the following asymptotically stable linear system [1] $\Sigma \in \mathcal{S}$

$$\left\{\begin{array}{rcl} x(\tau+1) & = & \Phi x(\tau) + Gg(\tau) \\ y(\tau) & = & Hx(\tau) + Dg(\tau) \\ c(\tau) & = & H_c x(\tau) + D_c g(\tau) \end{array}\right. \tag{1}$$

along with a desired output reference $r(\tau) \in \mathbb{R}^p$. System (1) can possibly represent a plant under robustly stabilizing feedback, where $x \in \mathbb{R}^n$ collects both plant and controller states, $g \in \mathbb{R}^p$ is the reference input, y the output, and $c \in \mathbb{R}^q$ the vector to be constrained within a given set $\mathcal{C}$. By the set-membership inclusion $\Sigma \in \mathcal{S}$, we mean that system (1) has parametric uncertainties. The following

$$\left\{\begin{array}{rcl} \hat{x}(\tau+1) & = & \hat{\Phi}\hat{x}(\tau) + \hat{G}g(\tau) \\ c(\tau) & = & \hat{H}_c\hat{x}(\tau) + \hat{D}_c g(\tau) \end{array}\right. \tag{2}$$

will be referred to as the *nominal* system $\hat{\Sigma} \in \mathcal{S}$. As discussed later, the uncertainty is modeled in terms of interval ranges of the step-response and impulse-response coefficients. We wish to design a *predictive reference filter* (PRF), a device which transforms the desired reference r in an actual reference g so as to have possibly constraints fulfilment for all possible plants inside the family $\mathcal{S}$. Filtering is operated on-line in a predictive manner: at time τ a *virtual* reference sequence $g(\tau+t|\tau)$ is selected in such a way that the corresponding predicted evolution $c(t+\tau|\tau,\Sigma,g(\cdot))$ lies within $\mathcal{C}$, for all $t \geq 0$, and for all systems $\Sigma \in \mathcal{S}$. Then, accordingly to a *receding horizon* strategy, the first sample of the virtual sequence is applied at time τ, $g(\tau) = g(\tau|\tau)$, a new selection being carried out at time $\tau+1$.

Assumption 1 *The set $\mathcal{C}$ is a convex polytope.*

W. l. o. g. we assume that $\mathcal{C}$ has the form

$$\mathcal{C} = \{c \in \mathbb{R}^q : \ c \leq B_c\} \tag{3}$$

[1] The results presented in this paper can be extended to time-varying linear systems

In fact, in general one has a set of constraints $A_c c \leq B_c$. These can be rewritten in the form (3) by defining a new vector $c^* = A_c c$ and, accordingly, new matrices $\hat{H}_c^* = A_c \hat{H}_c$, $\hat{D}_c^* = A_c \hat{D}_c$.

Several criteria [1]-[3] can be adopted in order to select the class of virtual references. For reasons that will be clearer soon, we restrict upon the following scalarly-parameterized structure

$$g(t+\tau|\tau) = g(\tau-1) + \beta[r(\tau) - g(\tau-1)],\ \forall t \geq 0 \quad (4)$$

where the free parameter $\beta \in \mathbb{R}$ is selected along with the optimization criterion

$$\beta = \begin{cases} \arg\min \|g(\tau|\tau) - r(\tau)\|^2 \\ \text{subject to } c(t+\tau|\tau, \Sigma, g(\tau|\tau)) \in \mathcal{C},\ \forall t \geq 0, \forall \Sigma \in \mathcal{S} \end{cases} \quad (5)$$

A parameter β or equivalently a virtual sequence $\{g(t+\tau|\tau)\}_{t=0}^{\infty}$ satisfying (4)-(5) will be referred to as *admissible*.

Assumption 2 (Feasible Initial Condition) *At time $\tau = 0$ there exists an admissible virtual reference sequence $\{g(t|0)\}_{t=0}^{\infty} \equiv g(0)$.*

As an example, Assumption 2 is satisfied for an equilibrium initial state $x(0) = (I - \Phi)^{-1} G g_0$.

2.1 Models of Uncertainty

Model uncertainty can be described in various ways. In our case, frequency domain descriptions are not convenient because of the time-domain strategy on which the PRF design is based. State space realization uncertainties could be adopted. However, if this is the case, the effect of matrix perturbations on the predicted evolution become cumbersome to compute. E. g., a free response of the form $(\hat{\Phi} + \tilde{\Phi})^k x(0)$ gives rise to prediction perturbations which are nonlinear in the uncertain parameter $\tilde{\Phi}$. On the contrary, uncertainties on the step-response or impulse-response coefficients provide a practical description in many applications and turn out to be reasonably simple to compute predictions. In this paper, both step-responses and impulse-responses perturbations will be jointly used.

The impulse response H_t from g to c can be expressed as sum of a nominal impulse response

$$\hat{H}_t = \begin{cases} \hat{H}_c \hat{\Phi}^t \hat{G} & \text{if } t > 0 \\ \hat{D}_c & \text{if } t = 0 \end{cases}$$

and an uncertainty $\tilde{H}_t$, with range intervals $\tilde{H}_t \in [H_t^-, H_t^+]$, $t = 0, 1, \ldots, N-1$. We suppose that the model set $\mathcal{S}$ satisfies the following asymptotic stability assumption.

Assumption 3 *There exist a matrix $M \in \mathbb{R}^{q \times p}$ and a scalar λ, $0 \leq \lambda < 1$, such that for all plants $\Sigma \in \mathcal{S}$*

$$|H_t^{ij}| \leq M^{ij} \lambda^t \quad (6)$$

holds for all $t \geq N$

The positive real $\lambda \in [0, 1)$ is related to an upperbound for the dominant poles of all plants $\Sigma \in \mathcal{S}$. Similarly, we describe impulse-response uncertainties for $t \geq N$ as

$$|\tilde{H}_t^{ij}| \leq E^{ij} \lambda^t,\ E \in \mathbb{R}^{q \times p} \quad (7)$$

for each component ij, $i = 1, \ldots, q$, $j = 1, \ldots, p$.

In the same way, the step-response from g to c can be expressed by means of a nominal response $\hat{W}_t = \sum_{k=0}^{t-1} \hat{H}_c \hat{\Phi}^k \hat{G} + \hat{D}_c$ along with range intervals

$$\tilde{W}_t \in [W_t^-, W_t^+] \quad (8)$$

for $t = 0, 1, \ldots, N-1$. For $t \geq N$, the same specifications (7) can be expressed as

$$|\tilde{W}_t^{ij} - \tilde{W}_{t-1}^{ij}| \leq E^{ij} \lambda^t \quad (9)$$

$i = 1, \ldots, q$, $j = 1, \ldots, p$. Despite these two formulations are seemingly equivalent in that

$$H_t = \begin{cases} W_t & \text{if } t = 0 \\ W_t - W_{t-1} & \text{if } t > 0 \end{cases}$$

it turns out that they can be conveniently considered in a cooperative fashion.

3 From Infinite to Finite Number of Constraints

The optimization criterion formulated in (5) involves an infinite number of constraints, arising from the semi-infinite horizon prediction. In order to compute (5), we need to reduce the constraints to a finite number. This can be made possible by introducing some additional hypothesis.

Assumption 4 (Set-Point Conditioning) *For all $\tau \geq 0$ the reference vector $r(\tau)$ belongs to a bounded and convex set $\mathcal{R}$.*

Assumption 4 amounts to assuming that either the class of references to be tracked is bounded, or a clamping device is inserted in the PRF mechanism. In practice, this is not a restriction since bounds on the reference will be dictated by the physical context of the application.

Assumption 5 *For all $\tau > 0$, $g(-\tau) \in \mathcal{R}$.*

Lemma 1 *Provided that Assumptions 4 and 5 are satisfied, $g(\tau) \in \mathcal{R}$, $\forall \tau \geq 0$.*

Proof: As shown in next Sect. 6, the domain $D_\beta = \{\beta : \beta^- \leq \beta \leq \beta^+\}$ (possibly $\beta^- = -\infty$, $\beta^+ = +\infty$) given by (28) is convex. Since $\beta = 0$ is admissible by construction, then the optimal $\beta(\tau)$ will be always comprised within 0 and the unconstrained minimizer 1. By this, $g(\tau)$ lies on the segment whose vertices are $g(\tau-1)$, $r(\tau)$. By convexness of $\mathcal{R}$, the result follows by induction. □