Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Energy-Aware Robust Model Predictive Control Based On Wireless Sensor Feedback

D. Bernardini†, A. Bemporad†

Abstract— **Flexibility, ease of deployment and of spatial reconfiguration, and low cost make Wireless Sensor Networks (WSNs) fundamental component of modern networked control systems. However, due to the energy-constrained nature of WSNs, the transmission rate of the sensor nodes is a critical aspect to take into account in control design. Two are the main contributions of this paper. First, a general transmission strategy for communication between controller and sensors is proposed. Then, a scenario with a controller and a wireless node providing measures is investigated, and two energy-aware control schemes based on explicit Model Predictive Control (MPC) are presented. We consider both nominal and robust control in the presence of disturbances, and convergence properties are given for the latter. The proposed control schemes are tested and compared to traditional MPC techniques. The results show the effectiveness of the proposed energy-aware approach, which achieves a profitable trade-off between energy consumption of wireless sensors and loss in system performance.**

I. Introduction

Wireless sensor networks (WSNs) is an emerging technology that allows the deployment of a large number of cheap small sensors of low energy consumption and collect amounts of measurement data that were previously cost prohibitive. Some of the driving objectives for wireless sensing in automation are to reduce the cost of cablings, and avoid associated failures due to wear and tear. Another advantage is the possibility to rapidly reconfigure the communications infrastructure in case of failures or addition of system components. The first major applications of WSNs have been in goods and inventory tracking, and in environmental monitoring [1].

However, compared to standard wired sensors, WSNs pose new challenges, as the control design must take care of issues such as energy consumption and channel reliability. While some interesting work has been done for the latter, such as modeling packet dropouts and addressing time delays (see [16], [4], [3]), energy-aware control is still a rather open problem. The limited amount of energy available and the cumbersome replacement of batteries motivate the urge to develop new methods of control design that, aware of communication and power consumption aspects of the network, ensure an optimized controller-sensor operation.

As pointed out in [18], [20], the communication system is often the dominant power hog in a wireless device. Hence, it is desirable to turn the radio off in the absence of traffic, and activate it only when needed. For this reason, the reduction of the data transmission rate is a crucial point when aiming at limiting power consumption. Previous works addressing the trade-off between transmission rate and performance of control systems propose to reduce the amount of transmitted data using signal quantization and receding horizon control [12]. In our framework this approach is not viable, since we aim at reducing the amount of time in which the radio chip is turned on, in order to quit the battery use. We propose a network transmission strategy which implies a coordinated action between controller and sensor, so that the number of WSN transmissions (and not strictly their size) can be reduced, thus leading to effective power savings. Then, we design a control system which explicitly integrates the knowledge about the transmission strategy, taking into consideration both nominal and robust control in the presence of disturbances.

A related methodological approach has been adopted in [23], addressing communication between controller and actuators under nominal conditions. The authors propose a simple network transmission strategy, model the networked plant as a mixed logical dynamical (MLD) system [7], and formulate a nominal control problem based on mixed-integer programming. With respect to these works, we wish to avoid the need of an MLD system in order to reduce computation complexity and to improve system performance by exploiting two-way channel communication.

The paper is organized as follows. The proposed transmission strategy is introduced in Section II. Nominal and robust energy-aware control schemes for the networked plant are presented in Section III and Section IV, respectively. Proposed controllers are tested in comparison with traditional MPC techniques, and results are reported in Section V. Finally, conclusions are drawn in Section VI.

II. Wireless Transmission Strategy

Consider a control system which receives feedback by an ideal wireless sensor. Considerations on practical aspects of real sensor networks such as measurement noise, packet loss, channel delay, multi-hop, etc., are beyond the scope of this paper and will be addressed in future works.

Sensor transmission strategy. At time step k, a wireless node transmits the measurement of the state vector $x(k) \in \mathbb{R}^{n_x}$ of the controlled process if and only if

$$\exists i \in \{1, 2, \ldots, n_x\} : |x_i(k) - \hat{x}_i(k)| > \varepsilon_i \qquad (1)$$

where $\varepsilon = [\varepsilon_1 \ \varepsilon_2 \ \ldots \ \varepsilon_{n_x}]^T$ is the vector that collects threshold values ε_i for every component x_i of the state x. More compactly, condition (1) can be expressed as $x(k) - \hat{x}(k) \notin \mathbf{E}$, where $\mathbf{E} = \{x \ : \ |x_i| \leq \varepsilon_i, \ i = 1, 2, \ldots, n_x\}$

† Department of Information Engineering, University of Siena, Italy. Email: {bernardini,bemporad}@dii.unisi.it.

978-1-4244-3124-3/08/$25.00 ©2008 IEEE

is the box defined by threshold vector ε. Vector $\hat{x}(k)$ is a prediction of the measured value $x(k)$ precalculated by the controller and transmitted beforehand to the wireless node. We represent the transmission condition (1) as $[\delta(k) = 1]$, where δ is a binary variable.

The communication protocol between controller and node is defined by two types of communications. If $\delta(k) = 1$ at time step k, the controller receives the measurement $x(k)$, computes a set of M updated predictions $\{\hat{x}(k+j)\}_{j=1}^{M} = \{\hat{x}(k+1), \hat{x}(k+2), \dots, \hat{x}(k+M)\}$, and transmits them to the wireless node. Hence, there is a two-way communication between controller and node when $\delta(k) = 1$. Moreover, if the controller does not receive any measurement for M time steps, i.e., $\delta(k) = \delta(k-1) = \cdots = \delta(k-M+1) = 0$, a one-way communication from controller to node takes place to send M updated predictions $\{\hat{x}(k+j)\}_{j=1}^{M}$, computed using $\hat{x}(k)$ as an estimation of the current state $x(k)$. We refer to M as the *estimation horizon*.

Since values of $\hat{x}$ can be calculated with any application-dependent estimation technique, the proposed transmission strategy is very general and can be implemented in a wide set of frameworks. Note that the threshold logic (1) allows one to gather information on measured variables even when the measurement is not received: if $\delta(k) = 0$, it means that $x(k) \in \mathbf{E} \oplus \{\hat{x}(k)\}$, where $\oplus$ is the Minkowski sum operator. In other words, from the controller's point of view a non-measurement is a set-measurement and with an opportune choice of the threshold ε, this can be usefully exploited in set-membership estimation algorithms.

At this stage of work we restrict ourselves to consider the case where all the sensors providing feedback are embedded in one single wireless node. This simple scenario is the basis for the more practical case of a WSN composed by several nodes, which all measure the same variables and transmit data in accordance to an opportune estimation algorithm. In this case we can treat $x(k)$ as the result of the estimation, rather than a direct measurement from a single sensor.

III. Energy-Aware Control: The Nominal Case

Consider the linear time-invariant discrete-time system

$$x(k+1) \;=\; Ax(k) + Bu(k) + w(k) \tag{2}$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$ are the state and input vectors at time k, A and B are the state transition and input distribution matrices, with (A, B) controllable. Polyhedral constraints $x \in \mathbf{X}$, $u \in \mathbf{U}$ on the state and input are also given. We assume that $w(k) \in \mathbf{W}$ is an unknown but bounded disturbance, where $\mathbf{W} \subset \mathbb{R}^{n_x}$ is a given polytope containing the origin.

We aim at designing a nominal controller for (2) that regulates the state x to the origin and that is suitable for the proposed transmission strategy. To this end, we propose an algorithm based on Model Predictive Control (MPC).

MPC is widely spread in industry for control design of highly complex multivariable processes under constraints on input and state variables [10], [17]. The idea behind MPC is to solve at each sampling time an open-loop finite-horizon optimal control problem based on a given prediction model of the process, by taking the current state of the process as the initial state. Only the first sample of the sequence of future optimal control moves is applied to the process. At the next time step, the remaining moves are discarded and a new optimal control problem based on new measurements is solved over a shifted prediction horizon. An alternative approach to evaluate the MPC law was proposed in [8]: rather then solving the QP problem on line for the current state vector, by employing techniques of multiparametric QP the problem is solved off line for all state vectors within a given range, providing the *explicit* dependence of the control input on the state and reference, which is piecewise affine (PWA) and continuous.

In our framework, explicit formulation of MPC is a natural choice for many reasons: primarily, it can handle constraints and can be formulated to achieve both nominal and robust control in presence of disturbances (see Section IV). Moreover, it allows the cheap computation of future closed-loop values $\hat{x}(k+j)$, $j = 1, \dots, M$ by iterating the evaluation of a simple PWA function.

Note that in nominal conditions (i.e. $w(k) = 0$, $\forall k$) there is no prediction error, so $\hat{x}(k) = x(k)$ and $\delta(k) = 0$, $\forall k$. Hence, the wireless node never sends data and the forward transmission rate is $1/M$. Another notable consequence is that the network transmission strategy and the nominal controller are completely uncoupled: the WSN protocol does not have to be taken into account when designing the control system. Thus, the implementation of the proposed transmission strategy has a low impact on the behavior of the controlled system, as confirmed by numerical results in Section V.

At time k the optimization problem solved by MPC is

$$\begin{aligned}
\min_{u} \quad & \sum_{j=0}^{N-1} \left(\|Q_x x(k+j|k)\|_p + \|Q_u u(k+j|k)\|_p \right) + \\
& + \|Q_N x(k+N|k)\|_p \\
\text{s.t.} \quad & x(k|k) = x_k, \\
& x(k+j+1|k) = Ax(k+j|k) + Bu(k+j|k), \\
& x(k+j|k) \in \mathbf{X} \subseteq \mathbb{R}^{n_x}, \\
& u(k+j|k) \in \mathbf{U} \subseteq \mathbb{R}^{n_u}, \\
& j = 0, 1, \dots, N,
\end{aligned} \tag{3}$$

where N is the control horizon, $Q_x, Q_u, Q_N \succeq 0$ are weight matrices, and $\|Qx\|_2 = x^T Q x$, $\|Qx\|_\infty = \max_{i=1,\dots,n_x} |(Qx)_i|$, $\|Qx\|_1 = \sum_{i=1}^{n_x} |x_i|$. The initial state x_k is chosen in accordance with the transmission strategy:

$$x_k = \begin{cases} x(k) & \text{if } \delta(k) = 1, \\ \hat{x}(k) & \text{otherwise.} \end{cases} \tag{4}$$

Problem (3) is a linear programming (LP) problem if $p = 1/\infty$, and a QP if $p = 2$. The MPC formulation (3) can be rewritten as a multiparametric program (see [8], [5] for details), allowing one to express the MPC control law as an explicit function of the state:

$$u^*(x) = P_i x + q_i \qquad \text{if } x \in \mathbf{X_i}, \tag{5}$$

where $P_i \in \mathbb{R}^{n_u \times n_x}$, $q_i \in \mathbb{R}^{n_u}$ and the polyhedral sets $\mathbf{X_i} = \{x \in \mathbb{R}^{n_x} : H_i x \leq k_i\}$ are a partition of $\mathbf{X}$, such that $\mathbf{X} = \bigcup_{i \in \mathcal{I}} \mathbf{X_i}$.

Algorithm 1 summarizes the proposed Nominal Energy-Aware MPC (NEA-MPC) algorithm.

Algorithm 1 Energy-Aware Nominal MPC (NEA-MPC)

Offline:
solve (3) explicitly and get $\{P_i,\ q_i,\ \mathbf{X_i}\}_{i \in \mathcal{I}}$,

At $k = 0$**:**
receive $x(0)$ from the wireless node,
set $\hat{x}(0) = x(0)$,
set $\hat{x}(j+1) = (A + BP_i)\hat{x}(j) + Bq_i$,
$\hat{x}(j) \in \mathbf{X_i},\ j = 0, \ldots, M-1$,
transmit $\{\hat{x}(j)\}_{j=1}^{M}$ to the wireless node.

For all $k > 0$**:**
if $x(k)$ is received (because (1) is satisfied)
set $\delta(k) = 1$, otherwise $\delta(k) = 0$.
if $\delta(k) = 1$,
set $u(k) = B(P_i x(k) + q_i),\ x(k) \in \mathbf{X_i}$,
set $\hat{x}(k) = x(k)$,
set $\hat{x}(k+j+1) = (A + BP_i)\hat{x}(k+j) + Bq_i$,
$\hat{x}(k+j) \in \mathbf{X_i},\ j = 0, \ldots, M-1$,
transmit $\{\hat{x}(k+j)\}_{j=1}^{M}$ to the wireless node.
else
set $u(k) = B(P_i \hat{x}(k) + q_i),\ \hat{x}(k) \in \mathbf{X_i}$,
if $\hat{x}(k+1)$ has not yet been computed,
set $\hat{x}(k+j+1) = (A + BP_i)\hat{x}(k+j) + Bq_i$,
$\hat{x}(k+j) \in \mathbf{X_i},\ j = 0, \ldots, M-1$,
transmit $\{\hat{x}(k+j)\}_{j=1}^{M}$ to the wireless node.

IV. ENERGY-AWARE CONTROL: THE ROBUST CASE

Algorithm 1 does not take into account the presence of the additive disturbance w. To do that the WSN communication protocol cannot be anymore decoupled from the controller and has to be integrated into the optimization. Thus, we can recast (2) to include sensor logic (1), obtaining the piecewise linear (PWL) system[1]

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) \\ \hat{x}(k+1) &= \begin{cases} Ax(k) + Bu(k) & \text{if } \delta(k) = 1 \\ A\hat{x}(k) + Bu(k) & \text{otherwise} \end{cases} \end{aligned} \tag{6a}$$

$$\delta(k) = 1 \leftrightarrow [x(k) - \hat{x}(k) \notin \mathbf{E}] \tag{6b}$$

Looking for a trade-off between closed-loop performance and battery use (that is mainly due to wireless network transmission rate), we want to design a robust control algorithm for (6), which guarantees convergence properties of the state x despite the state disturbance and network feedback. Due to the presence of the threshold ε and of the persistent unknown disturbance w, the state cannot be directly regulated to the origin. Therefore, following the idea of dual mode MPC [21], [14], our goal is (i) to obtain a (possibly time-variant) feedback control law $u = \kappa(x)$ which steers the state to a target set $\mathbf{X_0}$ despite the disturbance w and the eventual lack of feedback on x due to ε, while satisfying the state and input constraints (the *outer control mode*); and (ii) to find a time-invariant feedback control law $u = Kx$ which robustly keeps the state in $\mathbf{X_0}$ (the *inner control mode*). The set $\mathbf{X_0}$ is designed to be robust positively invariant, according to the following definition [9].

Definition 1: The set $\mathbf{X_0}$ is *robust positively invariant (RPI)* for a system of the form $x(k+1) = f(x(k), w(k))$ if and only if $\forall x(0) \in \mathbf{X_0}$ and $\forall w(k) \in W$ the solution $x(k) \in \mathbf{X_0}$, $\forall k \in \mathbb{N}$.

For further details about theory and construction of RPI sets, see [9], [13], [15]. We remark that in this work robustness is intended with respect to additive disturbance only. Parametric uncertainty is not taken into account. Inner and outer control modes are defined below.

A. Inner Control Mode

Since the actual measurement $x(k)$ is not always available, we consider the switching feedback control law

$$u(k) = \begin{cases} Kx(k) & \text{if } \delta(k) = 1, \\ K\hat{x}(k) & \text{otherwise,} \end{cases} \tag{7}$$

to maintain x in $\mathbf{X_0}$. Combining the system behavior described by (6) with (7) we obtain

$$\begin{bmatrix} x(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{cases} \begin{bmatrix} (A+BK)x(k) + w(k) \\ (A+BK)x(k) \end{bmatrix} & \text{if } \delta(k) = 1, \\ \begin{bmatrix} Ax(k) + BK\hat{x}(k) + w(k) \\ (A+BK)\hat{x}(k) \end{bmatrix} & \text{otherwise.} \end{cases} \tag{8}$$

Note that $Ax(k) + BK\hat{x}(k) = (A+BK)x(k) - BK(x(k) - \hat{x}(k))$, where $-BK(x(k) - \hat{x}(k))$ can be seen as an unknown, but bounded disturbance $v \in \mathbf{V} = -BK\mathbf{E}$. Then, (8) can be recast as:

$$\begin{bmatrix} x(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{cases} \begin{bmatrix} (A+BK)x(k) + w(k) \\ (A+BK)x(k) \end{bmatrix} & \text{if } \delta(k) = 1, \\ \begin{bmatrix} (A+BK)x(k) + w(k) + v(k) \\ (A+BK)\hat{x}(k) \end{bmatrix} & \text{otherwise,} \end{cases} \tag{9}$$

which is nominally stable for all appropriate designs of K. Now, we can use known methods for the computation of RPI sets for PWL systems [2], [19], [13]. As a simpler alternative, one can define $\mathbf{X_0}$ as an RPI set for the linear system

$$x(k+1) = (A+BK)x(k) + w(k) + v(k) \tag{10}$$

(see [9], [15], [21] for methods to obtain RPI sets in the linear case). This is an RPI set also for (9), provided that $\mathbf{X_0} \subseteq \mathbf{X}$ and $K\mathbf{X_0} \subseteq \mathbf{U}$.

B. Outer Control Mode

For the outer controller we propose a scheme derived from min-max MPC [6], [21], [14], where the goal is to steer the state to the target set $\mathbf{X_0}$ while minimizing a given index performance. The basic idea is to include the knowledge about the transmission logic in the optimization problem, so that the evolution of the prediction $\hat{x}$ can be coherently modeled into the optimizer. In the following, we show how the common implicit formulation of min-max MPC proposed

[1] The set of states $[x(k)\ \hat{x}(k)]^T$ such that $\delta(k) = 1$ is not convex. This description of (6) is kept for ease of notation without loss of generality, as it is straightforward to build an equivalent PWA system with 2 modes and a partition of $2n_x + 1$ polyhedral sets.

in [21], [14] is not viable in our framework, and how to resort to an efficient explicit min-max [6].

Let $\{w^{\ell}_{k+j|k}\}$ denote all the possible realizations of the disturbance, indexed by $\ell \in \mathcal{L}$. Further, let $\{u^{\ell}_{k+j|k}\}$ denote a control sequence associated with the ℓ-th such realization, and $\{x^{\ell}_{k+j|k}\}$ the corresponding state value. In principle, the min-max MPC problem can be expressed as:

$$\min_{\{u^{\ell}_{k+j|k}\}} \quad \max_{\ell \in \mathcal{L}} \sum_{j=0}^{N-1} L(x^{\ell}_{k+j|k}, u^{\ell}_{k+j|k}) \tag{11a}$$

$$\text{s.t. } (6), \tag{11b}$$

$$x^{\ell}_{k+j|k} \in \mathbf{X}, u^{\ell}_{k+j|k} \in \mathbf{U}, x^{\ell}_{k+N|k} \in \mathbf{X_0}, \tag{11c}$$

$$x^{\ell_1}_{k+j|k} = x^{\ell_2}_{k+j|k} \Rightarrow u^{\ell_1}_{k+j|k} = u^{\ell_2}_{k+j|k}, \quad j = 0, \ldots, N-1, \ \forall \ell, \ell_1, \ell_2 \in \mathcal{L}, \tag{11d}$$

where N is the prediction horizon, $x^{\ell_1}_{k+j|k} = x^{\ell_2}_{k+j|k} \Rightarrow u^{\ell_1}_{k+j|k} = u^{\ell_2}_{k+j|k}$ is the *causality constraint*, which enforces a single control input for each state, reducing the freedom on the control sequence and making the control law independent of the path taken to reach that state, and $x^{\ell}_{k+N|k} \in \mathbf{X_0}$ is the terminal set constraint [21].

Assumption 1: The stage cost $L(x, u)$ satisfies the following conditions [21]: $L(\cdot) = 0$ if $x \in \mathbf{X_0}$, $L(\cdot)$ is convex over $\mathbf{X} \times \mathbf{U}$, and such that $L(x, u) \geq \alpha(d(x, \mathbf{X_0}))$ for all $(x, u) \in (\mathbf{X} \backslash \mathbf{X_0}) \times \mathbf{U}$, where α is a $\mathcal{K}$-function.

For linear systems, problem (11) can be solved using the extreme disturbance realizations. Due to possible loss of convexity, this method cannot be directly applied in PWL systems: the influence of the disturbance on the state is dependent on the mode switching sequence, and therefore it cannot be predicted independently of the system trajectory, as for linear systems. An usual strategy to overcome this problem is to restrict the admissible control sequences to only those which guarantee that the mode of the system is unique for every value of the disturbance w, at each time step k [22]. In our framework this approach is not viable: for instance, if $\mathbf{E} \subset \mathbf{W}$ then the restricted admissible input sequences set, when not empty, would necessarily generate a switching sequence $\delta(k) = 1, \forall k$, because the condition $x(k) - \hat{x}(k) \in \mathbf{E}$ could not be guaranteed at any time step. In general, transmission rate would grow proportionally to the size of the disturbance set $\mathbf{W}$. This behavior is not admissible, since our global aim is to obtain a good trade-off between transmission rate and system performance. So, we propose to look for an approximated (conservative) solution which exploits the linearity of the process model (2), thus lowering the computational demand and still ensuring convergence to the target set $\mathbf{X_0}$. This strategy relies on the explicit solution of min-max MPC, obtained via multiparametric programming [6].

Let us consider the formulation of the min-max MPC problem given by (11a) subject to (2) instead of (6). Note that the optimal input resulting from the solution of this problem ensures the convergence of the state x of (2) to the target set $\mathbf{X_0}$ (see proof in [21]). By using $p = \infty$ and solving N mp-LPs as in [6], this solution is obtained in state-feedback piecewise affine form

$$u^*(x) = P_i x + q_i \quad \text{if } x \in \mathbf{X_i} \tag{12}$$

where $P_i \in \mathbb{R}^{n_u \times n_x}$, $q_i \in \mathbb{R}^{n_u}$ and $\mathbf{X_i} = \{x \in \mathbb{R}^{n_x} : H_i x \leq k_i\}$. Now, let us suppose to apply the feedback law derived from (12) to the PWL system (6):

$$u(k) = \begin{cases} P_i x(k) + q_i & \text{if } \delta(k) = 1, \\ P_j \hat{x}(k) + q_j & \text{otherwise,} \end{cases} \tag{13}$$

where $x(k) \in \mathbf{X_i}$ and $\hat{x}(k) \in \mathbf{X_j}$. It is clear that, when $\delta(k) = 1$, $\forall k$, the input (13) is safe for system (6), since $u(k) = u^*$. Else, when $\delta(k) = 0$ and the exact value of $x(k)$ is not known, there is a difference between the optimal input u^* and the input actually applied to the system: $u(k) = u^* + P_j \hat{x}(k) + q_j - P_i x(k) - q_i$. So, the evolution of (6) in closed loop with (13) can be recast as

$$\begin{bmatrix} x(k+1) \\ \hat{x}(k+1) \end{bmatrix} = \begin{cases} \begin{bmatrix} (A+BP_i)x(k)+Bq_i+w(k) \\ (A+BP_i)x(k)+Bq_i \end{bmatrix} & \text{if } \delta(k)=1, \\ \begin{bmatrix} (A+BP_i)x(k)+Bq_i+w(k)+e(k) \\ (A+BP_j)\hat{x}(k)+Bq_j \end{bmatrix} & \text{otherwise,} \end{cases} \tag{14}$$

where

$$e(k) = B(P_j \hat{x}(k) + q_j - P_i x(k) - q_i) \tag{15}$$

is the error made with respect to the safe trajectory due to the lack of information on the exact value of $x(k)$. The basic idea of our strategy is to consider e as an additional unknown but bounded disturbance, with $e \in \mathbf{Q} = \{e \in \mathbb{R}^{n_x} : (15),\ x - \hat{x} \in \mathbf{E}\}$, and to find a control law which is robust with respect to this disturbance. We cannot directly set up a multiparametric optimization problem including e, since the polytope $\mathbf{Q}$ is dependent on $\{P_i\}_{i \in \mathcal{I}}, \{q_i\}_{i \in \mathcal{I}}$, which are nonlinear functions of x. To overcome this issue, we propose to design an iterative algorithm, which at every step h computes the updated set $\mathbf{Q^h}$, the gains $\{P_i\}^h_{i \in \mathcal{I}}, \{q_i\}^h_{i \in \mathcal{I}}$ and the partition $\{\mathbf{X_i}\}^h_{i \in \mathcal{I}}$ as a function of the previous set $\mathbf{Q^{h-1}}$. Let us consider the linear system

$$x(k+1) = Ax(k) + Bu(k) + w(k) + e(k) \tag{16}$$

and the associated min-max MPC problem

$$\min_{\{u^{\ell}_{k+j|k}\}} \quad \max_{\ell \in \mathcal{L}} \sum_{j=0}^{N-1} (\|Q_x x^{\ell}_{k+j|k}\|_p + \|Q_u u^{\ell}_{k+j|k}\|_p) \quad \text{s.t.} \quad (16),\ (11c),\ (11d). \tag{17}$$

together with its explicit solution in state feedback form. The structure of the proposed offline iterative algorithm can now be defined as in Algorithm 2.

Algorithm 2 Iterative explicit min-max MPC

1. set $h = 0$, $Q^{-1} = \emptyset$, $Q^0 = \{0_{n_x}\}$.
2. while $Q^h \not\subseteq Q^{h-1}$
  2.1. solve (17) with $e \in \mathbf{Q^h}$,
  get the explicit solution data $\{P_i,\ q_i,\ \mathbf{X_i}\}^h_{i \in \mathcal{I}}$.
  2.2. set $\mathbf{Q^{h+1}} = hull\{\mathbf{Q^{h+1}_{ij}}\}_{(i,j) \in \mathcal{I} \times \mathcal{I}}$, where[2]
  $\mathbf{Q^{h+1}_{ij}} = \{e \in \mathbb{R}^{n_x} :\ x - \hat{x} \in \mathbf{E}, x \in \mathbf{X^h_i},\ \hat{x} \in \mathbf{X^h_j},$
  $e = B(P^h_j \hat{x} + q^h_j - P^h_i x - q^h_i)\}$.
3. set $\{P_i,\ q_i,\ \mathbf{X_i}\}_{i \in \mathcal{I}} = \{P_i,\ q_i,\ \mathbf{X_i}\}^{h-1}_{i \in \mathcal{I}}$,

The transmission strategy has to be slightly modified in order to deal with the dual mode MPC coherently. In addition to (1), the wireless node is required to transmit the measurement also when the state $x(k)$ and its prediction $\hat{x}(k)$ lie in different control mode sets:

$$\begin{aligned}[\delta(k)=0] \quad \leftrightarrow \quad & [x(k)-\hat{x}(k) \in \mathbf{E}] \\ & \wedge [[x(k) \in \mathbf{X_0},\ \hat{x}(k) \in \mathbf{X_0}] \\ & \vee [x(k) \notin \mathbf{X_0},\ \hat{x}(k) \notin \mathbf{X_0}]].\end{aligned} \qquad (18)$$

We can finally define the structure of the Robust Energy-Aware MPC in Algorithm 3 and state its convergence property in the following theorem.

Algorithm 3 Robust Energy-Aware MPC (REA-MPC)

Offline:
run Algorithm 2 and get $\{P_i,\ q_i,\ \mathbf{X_i}\}_{i\in\mathcal{I}}$,
compute K and $\mathbf{X_0}$ as in Section IV-A.

At $k=0$**:**
receive $x(0)$ from the wireless node,
set $\hat{x}(0)=x(0)$,
set $\hat{x}(j+1)=(A+BP_i)\hat{x}(j)+Bq_i$,
$\hat{x}(j) \in \mathbf{X_i},\ j=0,\ldots,M-1$,
transmit $\{\hat{x}(j)\}_{j=1}^{M}$ and $\mathbf{X_0}$ to the wireless node.

For all $k>0$**:**
if $x(k)$ is received (because (18) is satisfied)
set $\delta(k)=1$, otherwise $\delta(k)=0$.
if $\delta(k)=1$,
if $x(k) \in \mathbf{X_0}$,
set $u(k)=Kx(k)$.
else
set $u(k)=B(P_i x(k)+q_i),\ x(k) \in \mathbf{X_i}$,
set $\hat{x}(k)=x(k)$,
set $\hat{x}(k+j+1)=(A+BP_i)\hat{x}(k+j)+Bq_i$,
$\hat{x}(k+j) \in \mathbf{X_i},\ j=0,\ldots,M-1$,
transmit $\{\hat{x}(k+j)\}_{j=1}^{M}$ to the wireless node.
else
if $\hat{x}(k) \in \mathbf{X_0}$,
set $u(k)=K\hat{x}(k)$.
else
set $u(k)=B(P_i\hat{x}(k)+q_i),\ \hat{x}(k) \in \mathbf{X_i}$,
if $\hat{x}(k+1)$ has not yet been computed,
set $\hat{x}(k+j+1)=(A+BP_i)\hat{x}(k+j-1)+Bq_i$,
$\hat{x}(k+j) \in \mathbf{X_i},\ j=0,\ldots,M-1$,
transmit $\{\hat{x}(k+j)\}_{j=1}^{M}$ to the wireless node.

Theorem 1: The state of (2), which receives feedback according to (18) and is controlled with Algorithm 3, converges asymptotically to the terminal set $\mathbf{X_0}$.

Proof: The proof follows from Theorem 1 in [21], noting that the explicit feedback control law given by Algorithm 2 is designed to be robust with respect to both disturbance w and feedback error e induced by the network transmission strategy (18). ■

V. Simulation Results

To illustrate the performance of the proposed algorithms consider the second order discrete-time linear system with sample time $T_s = 0.1s$, $A = \left[\begin{smallmatrix} 0.9988 & 0.25 \\ -0.01 & 0.9988\end{smallmatrix}\right]$, $B = \left[\begin{array}{cc} 0 & 1 \end{array}\right]^T$ and threshold vector $\varepsilon = \left[\begin{array}{cc} 0.1 & 0.1 \end{array}\right]^T$. Limits on state, input and disturbance variables are $|x_i| \le \bar{x}_i$, $|u| \le \bar{u}$, $|w| \le \bar{w}$, with $\bar{x} = \left[\begin{array}{cc} 5 & 5 \end{array}\right]^T$, $\bar{u}=5$, $\bar{w} = \left[\begin{array}{cc} 0.05 & 0.1 \end{array}\right]^T$, $i=1,2$.

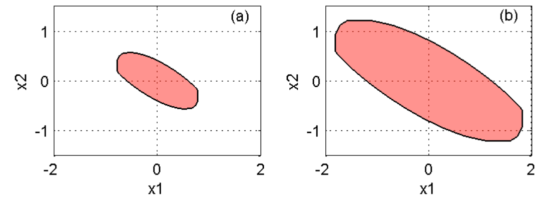

Fig. 1. Terminal set for RMPC (a) and REA-MPC (b).

The proposed energy-aware techniques are tested in comparison with standard deterministic and robust MPC control schemes, where the wireless node simply transmits the measurement at every time step. Prediction horizons used in nominal and robust control are $N_n = 10$ and $N_r = 3$, respectively, and the estimation horizon is $M = 10$. The weight matrices are $Q_x = Q_N = I_2$, $Q_u = 1$, and $p=\infty$. The constant gain $K = \left[\begin{array}{cc} -0.2339 & -0.4542 \end{array}\right]$ is used for the inner mode both in the standard robust MPC (RMPC) and in the REA-MPC algorithms. The terminal sets $\mathbf{X_0}$ for the two controllers are depicted in Figure 1. Since the energy-aware approach is more conservative, a larger terminal set is used in order to preserve feasibility of the outer mode controller (the use of the same set in Figure 1b for both algorithms would penalize the performance of RMPC).

As a benchmark to evaluate the performance of the proposed algorithms two quantities are considered: the transmission rate of the wireless node, calculated considering both transmitted and received packets[3], and the cumulated cost function J^i_{exp} defined as:

$$J^i_{exp} = \sum_{k=1}^{T} (\|Q_x x^i(k)\|_\infty + \|Q_u u^i(k)\|_\infty), \qquad (19)$$

where i denotes the i-th disturbance realization and $T = 100$ is the number of simulation time steps. $J^{(N)i}_{exp}$ and $J^{(R)i}_{exp}$ are the cumulated costs for a standard nominal and robust MPC algorithm, respectively, and are defined as in (19) on the corresponding closed-loop trajectories. In simulation, the initial states $x(0)$ are chosen randomly among the vertices of the feasible set of the REA-MPC controller.

Table I reports simulation results averaged over 100 realizations, while Figures 2 and 3 show histograms of J_{exp} and transmission rate for Energy-Aware Nominal and Robust Energy-Aware controllers (data are given as percentage of analogue values from standard MPCs).

NEA-MPC achieves a good trade-off, with a reduction in transmission rate of 55.4% and a loss in experimental cost function of 1.58%, on average, with respect to a standard nominal MPC. REA-MPC grants similar savings in radio utilization (−50.22%), with a higher J_{exp} average value in

[2] In the computation of $\mathbf{Q^h}$ the convex hull operator is used instead of the union operator to ease computability. Whether this introduces conservativeness is an open issue.

[3] We assume equal power consumption in transmitting and receiving packets, as is usual for short range wireless nodes (e.g. see [11]).

TABLE I

ENERGY-AWARE MPC VS. TRADITIONAL MPC: SIMULATION RESULTS

Controller	J_{exp}	Tx. Rate
Standard Nominal MPC	30.39	100.00%
Standard Robust MPC	31.39	100.00%
Nominal Energy-Aware MPC	30.87	44.66%
Robust Energy-Aware MPC	34.23	49.78%

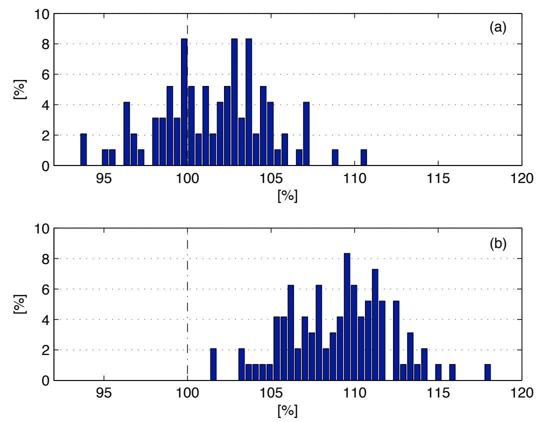

Fig. 2. Histogram of cost function J^i_{exp} for NEA-MPC (a) and REA-MPC (b), normalized with respect to $J^{(N)i}_{exp}$ and $J^{(R)i}_{exp}$, respectively.

comparison to standard RMPC ($+9.05\%$), owing to its more conservative nature.

VI. CONCLUSIONS AND FUTURE WORK

This paper has investigated an energy-aware approach in control design for systems with wireless sensor feedback, based on a novel WSN transmission strategy and a properly tuned MPC control algorithm. A nominal controller and a robust controller with guaranteed convergency properties are proposed. Energy-aware algorithms are tested in simulation in comparison with more traditional MPC techniques with continuous transmission rate. Results show a good trade-off between system closed-loop behavior and transmission rate: both the nominal and the robust control schemes provide an average transmission rate of about $45\% - 50\%$ (which roughly corresponds to doubling the life of the wireless sensor), with a narrow loss in system performance ($< 2\%$ in the nominal case, around 9% in the robust case).

Ongoing research on energy-aware MPC control includes aspects of real WSNs such as multiple sensing nodes, measurement errors, packet losses, and multi-hop protocols.

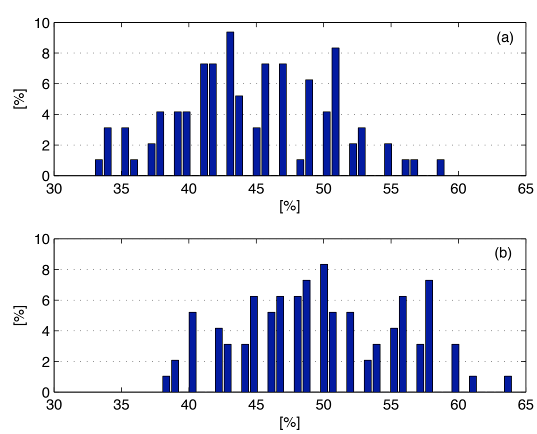

Fig. 3. Histogram of transmission rate for NEA-MPC (a) and REA-MPC (b).

REFERENCES

[1] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci. A survey on sensor networks. *IEEE Communications Magazine*, 40:102–114, 2002.

[2] A. Alessio, M. Lazar, A. Bemporad, and W. P. M. H. Heemels. Squaring the circle: An algorithm for generating polyhedral invariant sets from ellipsoidal ones. *Automatica*, 43:2096–2103, 2007.

[3] P. Antsakli and J. Baillieul. Special Issue on Networked Control Systems. *IEEE Transactions On Automatic Control*, 49(9), 2004.

[4] A. Bemporad. Predictive control of teleoperated constrained systems withunbounded communication delays. In *Proceedings of the 37th IEEE Conference on Decision and Control*, volume 2, pages 2133–2138, Tampa, FL, USA, 1998.

[5] A. Bemporad, F. Borrelli, and M. Morari. Model Predictive Control Based on Linear Programming The Explicit Solution. *IEEE Transactions On Automatic Control*, 47(12):1974–1985, 2002.

[6] A. Bemporad, F. Borrelli, and M. Morari. Min-max control of constrained uncertain discrete-time linear systems. *IEEE Transactions On Automatic Control*, 48, 2003.

[7] A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3):407–427, 1999.

[8] A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos. The Explicit Linear Quadratic Regulator for Constrained Systems. *Automatica*, 38(1):3–20, 2002.

[9] F. Blanchini. Set invariance in control – a survey. *Automatica*, 35(11):1747–1768, 1999.

[10] E. F. Camacho and C. Bordons. *Model Predictive Control*. Advanced Textbooks in Control and Signal Processing. Springer-Verlag, London, 2^{nd} edition, 2004.

[11] Moteiv Corporation. *Tmote Sky: Datasheet*, 2006.

[12] G. Goodwin, H. Haimovich, D. Quevedo, and J. Welsh. A moving horizon approach to networked control system design. *IEEE Transactions On Automatic Control*, 49:1427–1445, 2004.

[13] E. C. Kerrigan. *Robust Constraint Satisfaction: Invariant Sets and Predictive Control*. PhD thesis, University of Cambridge, 2000.

[14] E. C. Kerrigan and J. M. Maciejowski. Robustly stable feedback min-max model predictive control. In *Proceedings of the American Control Conference*, volume 4, pages 3490–3495, Denver, Colorado, USA, 2003.

[15] I. Kolmanovsky and E. G. Gilbert. Theory and computation of disturbance invariant sets for discrete-time linear systems. *Mathematical Problems in Engineering*, 4(4):317–367, 1998.

[16] P. R. Kumar. New technological vistas for system and control: The example of wireless networks. *IEEE Control Systems Magazine*, 21:24–37, 2001.

[17] S. J. Qin and T. A. Badgwell. A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7):733–764, 2003.

[18] V. Raghunathan, C. Schurgers, S. Park, and M. Srivastava. Energy aware wireless microsensor networks. *IEEE Signal Processing Magazine*, 19:40–50, 2002.

[19] S. V. Raković, P. Grieder, M. Kvasnica, D. Q. Mayne, and M. Morari. Computation of invariant sets for piecewise affine discrete time systems subject to bounded disturbances. *IEEE Conference on Decision and Control*, 2:1418–1423, 2004.

[20] C. Schurgers. *Energy-aware Wireless Communications*. PhD thesis, University of California, Los Angeles, 2002.

[21] P. O. M. Scokaert and D. Q. Mayne. Min-max feedback model predictive control for constrained linear systems. *IEEE Transactions On Automatic Control*, 43:1136–1142, 1998.

[22] M. P. Silva, A. Bemporad, M. A. Botto, and J. S. da Costa. Optimal control of uncertain piecewise affine/mixed logical dynamical systems. In *Proceedings of the European Control Conference*, 2003.

[23] G. Zhang, X. Chen, and T. Chen. A model predictive control approach to networked systems. In *Proceedings of the 46th IEEE Conference on Decision and Control*, New Orleans, Louisiana, USA, 2007.