Constraint Fulfilment in Feedback Control via Predictive Reference Management*

A. Bemporad and E. Mosca†

Adaptive Systems Laboratory,
D.S.I., University of Florence,
Via S. Marta, 3 - 50139 Florence, Italy
phone: +39.55.4796258
fax: +39.55.4796363
E-mail: mosca@ingfi1.ing.unifi.it

Keywords: **Control with hard constraints; Predictive control; Intelligent control; Nonlinear systems.**

Abstract

The problem of satisfying input and state-dependent inequality constraints in feedback control systems is addressed. The proposed solution is based on predicting the evolution of the constrained vector and, accordingly, selecting on line the future reference based on both the current state and the desired set-point changes. The achievable performance is first investigated via simulations, and compared with the one obtained via a receding horizon controller which uses on line a mathematical programming solver. Finally, an analysis is carried out so as to estabilish stability and offset-free properties of the method.

1. Introduction

The problem of determining a feedback control law capable of stabilizing a given plant in the presence of input and state-related inequality constraints, is one of the fundamental issues in control applications. In this context even the conceptually simple case of a linear plant with saturated inputs gives rise to challenging stability problems [5]. For the discrete-time regulation problem, [6] showed that, under feasibility conditions, zero terminal state receding horizon control [8] with input and state-related constraints yields a stable feedback system. Under quite general conditions, [6] proved in fact this to hold true even if the plant to be regulated is nonlinear and time-varying. Extensions of similar results to the continuous-time regulation problem are tackled in [9] and discussed in [10]. Specific feedback regulation systems for linear plants with input saturations are treated in [7] and [3]. For a different approach see [4].

For 2-DOF (two degrees of freedom) control problems with hard constraints, in recent years a great deal of interest has been focussed on applying predictive control techniques [16], [14], via the on-line use of a mathematical programming solver [15].

The present paper tackles the control problem with constraints along the lines of predictive control but, unlike the previous contributions, sidesteps the need of using a mathematical programming solver by adopting a suitable on-line management of the reference to be tracked. It can be shown that, under some conditions, the proposed on-line *predictive reference management* (PRM) suffices for solving the constrained control problem, and that the associated computational load turns out to be much smaller than that of a mathematical programming solver. This is a very important feature in industrial control applications where the sampling time cannot be too large, and the computing power must be as small as possible.

The paper is organized as follows. Sect. 2 describes the on-line PRM for a given feedback control system. Via simulation experiments we show that the proposed on-line reference management fulfils input and state-dependent inequality constraints with results that can be made indistinguishable from those obtained by a mathematical programming solver. Sect. 4 analyses the feedback control system governed by the proposed on-line PRM. Some conclusive remarks are finally presented in Sect. 5.

2. On-line Predictive Reference Management

Consider the control system depicted in Fig. 1 where $y(t)$, $u(t)$, $\delta u(t)$ and $x(t)$ are respectively output, input, input increments and state of the controlled plant. Further, $c(t) = c(x(t), \delta u(t))$ is the constrained vector. The underlying controller can implement any stabilizing control law dependent

*Work supported in part by MURST and CNR.

†Author to whom all correspondence should be addressed.

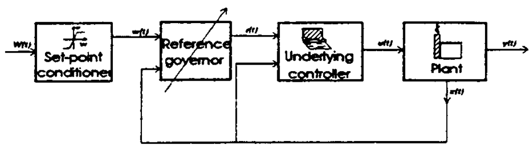

Fig. 1. Control system with PRM

on the reference sequence $r(\cdot)$. We assume also that there is an integral action in the feedback loop, and zero-offset for a constant reference results. We distinguish between the set-point trajectory $w(\cdot)$ to be tracked and the actual reference sequence $r(\cdot)$ applied to the controller. The latter is in fact selected on-line by the *reference governor* as explained in detail below. The *set-point conditioner* is needed to threshold the desired trajectory $W(t)$ in such a way that the output $w(t)$ of the conditioner is an *admissible* set-point. This means that when in steady state the plant output y reaches the level $w(t)$, c fulfils the constraints. See (22).

The procedure implemented by the reference governor aims at smoothing out transitions from the current value of the plant output to the desired set-point. Accordingly, the reference pattern selected by the reference governor at each step t is given as follows

$$r(t+i|t) = \lambda^{i+1}(t)y(t) + (1-\lambda^{i+1}(t))w(t) \qquad (1)$$

where t, $i \in \mathbb{Z}_+$ and $\lambda(t) \in [0,1)$. Let $c(\cdot|t) = \{c(t+i|t)\}_{i=0}^{\infty}$ be the hypothetical evolution of the constrained vector corresponding to the use of the reference pattern $r(\cdot|t) = \{r(t+i|t)\}_{i=0}^{\infty}$. $c(\cdot|t)$ will be called the *prediction* of the constrained variable for given $r(\cdot|t)$ and $x(t)$. Then, when a set-point change occurs, one can smooth out the dynamic range of $c(\cdot|t)$ by choosing the "time constant" $\lambda(t)$ so as to possibly keep $c(\cdot|t)$ admissible. In fact the closer $\lambda(t)$ to 1, the smoother $c(\cdot|t)$ will be and the larger the resulting settling-time. The idea is to choose at each t the time constant $\lambda(t)$ which gives the shortest settling time while keeping $c(\cdot|t)$ admissible. At each time step t the reference governor executes the following algorithm:

1. Construct the reference pattern $r(t+i|t) \equiv w(t)$, $\forall i \in \mathbb{Z}_+$ (i.e. $\lambda(t) = 0$);
2. Make a prediction $c_M(\cdot|t) := \{c(t+i|t)\}_{i=0}^{M-1}$ over an M-step horizon by iterating the plant model and the underlying controller fed by the selected reference pattern;
3. Does $c_M(\cdot|t)$ fulfil the constraints?
   (a) Yes: Use the current reference pattern as the actual reference, compute $u(t)$ to be given to the actuator, and go to 5;
   (b) No: Go to 4
4. Can other reference patterns corresponding to a larger value of $\lambda(t) < 1$ be constructed?
   (a) Yes: Construct the reference pattern by increasing the parameter $\lambda(t)$, and go to 2;
   (b) No: Set $r(t+i|t) = r(t+i|t-1)$. Go to 5;
5. Stop

In the deterministic case, step (3a) ensures that the constraints will be fulfilled at least for M steps. Ideally M should be infinite. In practice the prediction horizon M must be finite. In fact, the shorter it is, the lighter the computational burden. However, too small values of M can lead the system into a "blind alley", where no choice of future reference patterns will avoid violation of the constraints. A rule of thumb is to set MT_s (T_s=sampling period) equal to the settling time of $c(\cdot)$ in the presence of the prescribed constraints, given that, because of set-point conditioning, the constraints can be violated only during transients.

Another issue is how many sequences the governor can try before giving up and execute step (4b). Our solution is to set up a grid G made up of n_G values $0 = \lambda_0 < \lambda_1 < \ldots < \lambda_{n_G-1}$ suitably distributed on $[0,1)$: the higher n_G the better the performance, but the heavier the computational burden.

Although the strategy described so far can be applied to nonlinear MIMO plants and any underlying stabilizing control law, we investigate how to use it in order to control a discrete-time SISO linear time-invariant plant

$$\begin{cases} x(t+1) &= \Phi x(t) + G\delta u(t) \\ y(t) &= Hx(t) \end{cases} \qquad (2)$$

or, being d the unit delay operator,

$$\begin{cases} (1-d)A(d)y(t) = B(d)\delta u(t) \\ (1-d)A(d) \text{ and } B(d) \text{ are coprime} \end{cases} \qquad (3)$$

and

$$u(t) = u(t-1) + \delta u(t) \qquad (4)$$

$$c(t) = Cx(t) + D\delta u(t) \in \mathbb{R}^{n_c} \qquad (5)$$

The underlying control law we consider

$$R(d)\delta u(t) = -S(d)y(t) + \sum_{i=1}^{\infty} v_i r(t+i) \qquad (6)$$

is the one minimizing the quadratic performance index

$$J = \sum_{i=0}^{\infty}\{[(y(t+i) - r(t+i)]^2 + \rho[\delta u(t+i)]^2\} \quad (7)$$

$\rho > 0$, assuming that $\{r(t+i)\}_{i=0}^{\infty}$ is bounded and known. The resulting control law will be referred to as *LQ control with preview*. It consists of the 2-DOF control law (6) where $R(d)$, $S(d)$ are polynomials satisfying the Diophantine equation

$$(1-d)A(d)R(d) + B(d)S(d) = E(d)/E(0), \quad (8)$$

with $S(d)$ of minimum degree, and $E(d)$ is a strictly Hurwitz polynomial which solves the spectral factorization problem

$$\begin{aligned} E(d)E(d^{-1}) \;=\; & B(d)B(d^{-1}) + \\ & +\rho(1-d)A(d)A(d^{-1})(1-d^{-1}) \end{aligned} \quad (9)$$

The solution of the LQ control problem with preview in the form (6) is given in [12] and [11]. Since the transfer function from $r(t)$ to $y(t)$ for the system (3), (6), is given by $\frac{B(d)B(d^{-1})}{E(d)E(d^{-1})}$, we see that zero-offset results. With $r(\cdot)$ as in (1), (6) becomes

$$R(d)\delta u(t) = -[S(d) - V(\lambda(t))]y(t) + [V(1) - V(\lambda(t))]w(t) \quad (10)$$

where

$$V(d) = \sum_{i=1}^{\infty} v_i d^i = \frac{B(d)}{E(0)E(d)}. \quad (11)$$

Instead of LQ control with preview, any stabilizing control law with zero-offset could be chosen. E.g. the 2-DOF stabilizing input/output receding horizon control law (SIORHC) [13], or classical 1-DOF controllers like PID or lead-lag.

An advantage with LQ control with preview unlike other predictive controllers such as SIORHC is that its control law can be given in a compact explicit polynomial form as in (6)-(11). This makes the overall operation of the control system under consideration better understandable. We finally point out that, even if the plant and underlying control law are linear, the resulting controller is a nonlinear function of the current closed-loop state and the desired trajectory.

3. Simulation results

Example 1. Consider the linear discrete-time plant

$$(1-1.9517d+0.9517d^2)y(t) = (-0.0488d+0.0488d^2)u(t) \quad (12)$$

obtained by sampling every $T_s = 0.005$s and zero-order holding the input of continuous-time unstable plant

$$y(\tau) = \frac{1+10s}{(1+0.1s)(1-10s)}u(\tau) \quad (13)$$

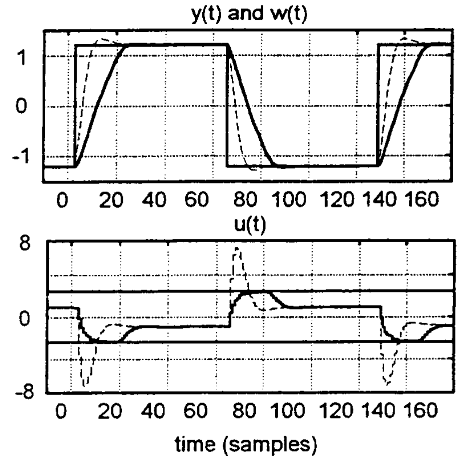

Fig. 2. 2-DOF LQ, no constraint (dashed line), and with PRM and constraint $|u(t)| \leq 3$.

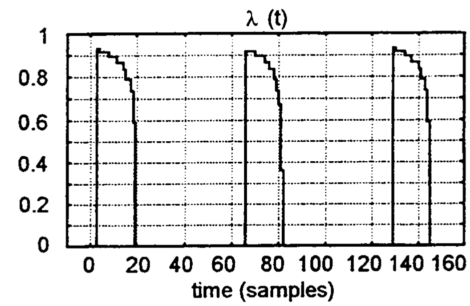

Fig. 3. $\lambda(t)$ for the example of Fig. 2.

for which a square-wave is chosen as desired trajectory $W(\cdot)$. Fig. 2 shows the behaviour of the 2-DOF LQ regulated system without constraints ($r(\cdot) \equiv w(\cdot) \equiv W(\cdot)$, dashed line) and how the control system with PRM behaves in the presence of the constraint

$$|u(t)| \leq 3 \quad (14)$$

(solid line). Because of open-loop instability the transfer function from $w(t)$ to $u(t)$ is non-minimum phase. This explains the large overshoots in the input. As reported in Fig. 3, the reference governor chooses a non-zero $\lambda(t)$, i.e. transforms $w(\cdot)$ into $r(\cdot|t)$, only during transients, when constraints would be violated. Moreover, in order to yield the shortest settling-time, it always selects the smallest value for $\lambda(t)$ compatible with the prescribed constraints.

Example 2. We show how the proposed reference governor generates controls practically indistinguishable from those obtained by a receding horizon technique which uses on-line mathematical programming. Consider the linear discrete-time plant

$$(1-1.7535d+0.8353d^2)y(t) = (0.5830d-0.5013d^2)u(t) \quad (15)$$

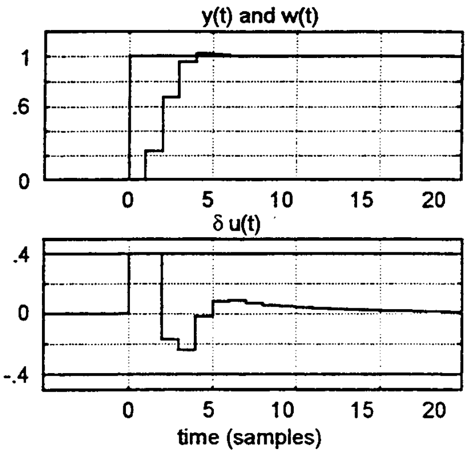

Fig. 4. Receding horizon controller (RHC) with constraint $|\delta u(t)| \leq 0.4$.

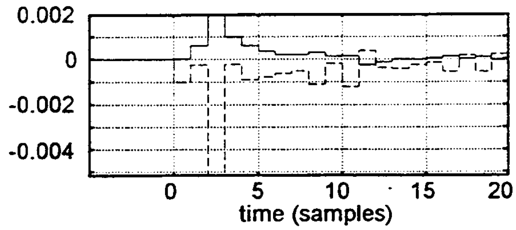

Fig. 5. Solid line: $y_{PRM}(t) - y_{RHC}(t)$. Dashed line: $\delta u_{PRM}(t) - \delta u_{RHC}(t)$.

obtained by sampling every $T_s = 0.03$s and zero-order holding the input of the second-order under-dumped continuous-time plant

$$y(\tau) = \frac{1 + 0.2s}{1 + \frac{2\zeta}{\omega_n} s + \frac{1}{\omega_n^2} s^2} u(\tau), \quad \left\{ \begin{array}{rcl} \zeta & = & 0.3 \\ \omega_n & = & 10\text{rad/s} \end{array} \right. \tag{16}$$

with a constraint on $|\delta u(t)|$. Fig. 4 shows the unit step-response of (15) regulated by a receding horizon control law. At each step t this chooses $\delta u(t) = \delta \hat{u}(t)$, where the finite sequence $\delta \hat{u}_{[t,t+N)}$ minimizes

$$J = \sum_{i=0}^{N} \{[y(t+i) - r(t+i)]^2 + \rho[\delta u(t)]^2\} \tag{17}$$

with the constraint $|\delta \hat{u}(t+i)| \leq 0.4$ $\forall i = 0, 1, N-1$ ($\rho = 0.1, N = 8$). Fig. 5 depicts the difference between the sequences $\delta u(\cdot)$, $y(\cdot)$ generated by the PRM strategy (same ρ, $M = 2$) and the one shown in Fig. 3. Fig. 6 also shows the values of $\lambda(t)$ chosen by the reference governor. For the PRM strategy a grid $G = \{0, 1-\mu, \cdot, 1-\mu^{n_G - 1}\}$, $\mu = .99$, $n_G = 646$ was chosen. Trying with $\mu = .7$, $n_G = 10$ we got qualitatively the same $y(\cdot)$ and $\delta u(\cdot)$, as depicted in

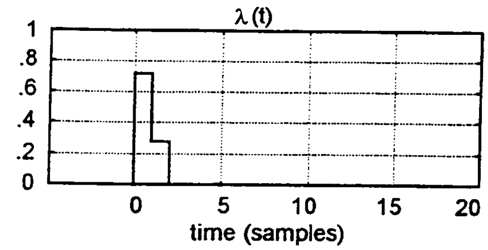

Fig. 6. $\lambda(t)$ for Example 2.

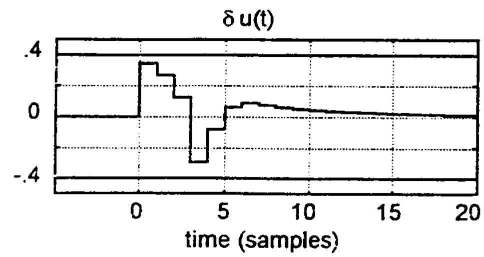

Fig. 7. PRM with a coarser grid G.

Fig. 7. A coarser grid lightens calculations but gives trajectories more distant from the optimal ones.

4. Analysis

The aim hereafter is to analyze the 2-DOF control system equipped with the reference governor, and described in the previous sections. To this end, it is convenient to represent the control law (6) in state-space form as follows

$$\delta u(t) = F x(t) + v(t) \tag{18}$$

with $v(t)$ the command input. System (2), (5) and (18) can be rewritten as

$$\left\{ \begin{array}{rcl} x(t+1) & = & \Phi_F x(t) + G v(t) \\ c(t) & = & C_F x(t) + D v(t) \in \mathcal{C} \end{array} \right. \tag{19}$$

$$v(t) = \sum_{i=1}^{\infty} v_i r(t+i|t) \tag{20}$$

where $\Phi_F := \Phi + GF$ is a stability matrix, $C_F := C + DF$, and $c(t) \in \mathbb{R}^{n_c}$ the vector to be constrained in the set $\mathcal{C} \subset \mathbb{R}^{n_c}$. If $\mathcal{C} = \mathbb{R}^{n_c}$, (19)-(20) enjoys the *zero-offset property*

$$\left\{ \begin{array}{rcl} y_w & := & H x_w = w \\ x_w & := & (I - \Phi_F)^{-1} G v_w \\ v_w & := & V(1) w \end{array} \right. \tag{21}$$

for every $w \in \mathbb{R}$ [$V(d)$ is defined in (11)]. This means that if $r(\cdot|t) = w$, $\forall t \in \mathbb{Z}_+$, the plant output coincides in steady-state with the constant set-point w. We define the *set W of admissible set-points*

$$W := \{w \in \mathbb{R} | c_w := C_F x_w + D v_w \in \mathcal{C}\} \tag{22}$$

Given a bounded reference sequence $r(\cdot|t) = r(\cdot|0) =: r(\cdot)$, $\forall t \in \mathbb{Z}_+$, we denote by $c(k, 0, x, r(\cdot))$, $k \in \mathbb{Z}_+$, the vector $c(k)$ produced by the application of $r(\cdot)$ to the system (19)-(20) from the state x at time zero. Then, we define the *$r(\cdot)$-admissible state set* $\mathcal{X}(r(\cdot))$

$$\mathcal{X}(r(\cdot)) := \{x \in \mathbb{R}^{n_x} \,|c(k, 0, x, r(\cdot)) \in \mathcal{C}, \forall k \in \mathbb{Z}_+\} \tag{23}$$

Because the maps $w \mapsto c_w$ and $x \mapsto \{c(k, 0, x, r(\cdot))\}_{k=0}^{\infty}$ are continuous, we have

$$\mathcal{C} \text{ open } \Rightarrow W \text{ and } \mathcal{X}(r(\cdot)) \text{ open} \tag{24}$$

Further,

$$\mathcal{C} \text{ convex } \Rightarrow W \text{ and } \mathcal{X}(r(\cdot)) \text{ convex} \tag{25}$$

$$\mathcal{C} \text{ symmetric } \Rightarrow W \text{ symmetric} \tag{26}$$

We introduce without proof the following fundamental lemma.

Lemma 1. Let $\mathcal{C}$ and W be bounded, with $\mathcal{C}$ open and convex. Then, given two admissible set-points $\bar{w}$ and $\hat{w}$, $\bar{w}$, $\hat{w} \in W$, there exist $\lambda \in [0, 1)$ such that the plant can be transferred from the equilibrium state $x_{\bar{w}}$ at time 0 to the equilibrium state $x_{\hat{w}}$ by driving the system (19)-(20) by the reference trajectory

$$r_\lambda(i) := \lambda^{i+1}\bar{w} + (1 - \lambda^{i+1})\hat{w}, \; i \in \mathbb{Z}_+ \tag{27}$$

or, equivalently, for some $\lambda \in [0, 1)$ and $\forall \bar{w}, \hat{w} \in W$

$$x_{\bar{w}} := (I - \Phi_F)^{-1}GH(1)\bar{w} \in \mathcal{X}(r_\lambda(\cdot)) \tag{28}$$

Lemma 2. Under the same assumptions as in Lemma 1, whenever $\bar{w}, \hat{w} \in W$, there exist $\lambda \in [0, 1)$ and $\epsilon > 0$ such that

$$x_{\bar{w}} + \tilde{x} \in \mathcal{X}(r_\lambda(\cdot)), \; \forall \tilde{x} \in \mathbb{R}^{n_x}, \; \|\tilde{x}\| \leq \epsilon \tag{29}$$

where $r_\lambda(\cdot)$ is as in (27).

Proof. By Lemma 1, $x_{\bar{w}} \in \mathcal{X}(r_\lambda(\cdot))$ for some $\lambda \in [0, 1)$. Since $\mathcal{C}$ open implies that $\mathcal{X}(r_\lambda(\cdot))$ open, the conclusion follows as once. □

It remains to specify how the "reference to go" $r(\cdot|t)$ is generated by the reference governor. While in this respect we refer the reader to the specific point discussed in Sect. 2, hereafter we shall adopt the following simplified criterion for selecting $\lambda(t)$. Let $w(t) \in W$ be the desired (conditioned) set-point at time t. For $t, i \in \mathbb{Z}_+$, set

$$\Lambda(t) := \{\lambda \in [0, 1) | x(t) \in \mathcal{X}(r_\lambda(\cdot))\} \tag{30}$$

where

$$r_\lambda(t + i|t) := \lambda^{i+1}r(t + i|t - 1) + (1 - \lambda^{i+1})w(t) \tag{31}$$

Then, take

$$\lambda(t) = \begin{cases} 1 & , \text{if } \Lambda(t) = \emptyset \\ 0 & , \text{if } 0 \in \Lambda(t) \\ \text{any } \lambda \in \Lambda(t) & , \text{otherwise} \end{cases} \tag{32}$$

and set

$$r(t + i|t) := r_{\lambda(t)}(t + i|t) \tag{33}$$

While this criterion for choosing $\lambda(t)$ must be further elaborated so as to improve the performance of the control system, nonetheless it suffices to prove next Theorem 1. Before proceding any further, notice that if

$$x(0) = x_w, \; w \in W, \text{ and } r(\cdot| - 1) \equiv w \tag{34}$$

it follows that, $\forall t \geq 1$,

$$x(0) \in \mathcal{X}(r_1(\cdot|0)) \text{ and } x(t) \in \mathcal{X}(r(\cdot|t)) \tag{35}$$

Theorem 1 (*Conditional stability and offset-free behaviour*). Assume that:

(i) $w(t) \in W$, $\forall t \in \mathbb{Z}_+$ (set point conditioning)
(ii) $x(0) \in \mathcal{X}(r(\cdot|0))$
(iii)$\exists \bar{t} \geq 0$ such that $w(t) = w \; \forall t \geq \bar{t}$.

Then, under the same assumptions as in Lemma 1, and provided that the reference governor action be defined as in (32), there exists a finite time $\hat{t}$, $\hat{t} \geq \bar{t}$, such that $\lambda(t) = 0$ or $r(\cdot|t) \equiv w$, $\forall t \geq \hat{t}$, and consequently

$$\lim_{t\to\infty} y(t) = w$$

Proof. Let $t \geq \bar{t}$. Consider that $\lim_{i\to\infty} r(t - 1 + i|t - 1) = \bar{w} \in \{w(0), w(1), \ldots, w(t - 1)\} \subset W$. Further, $\forall \tau \geq t$, $r(\tau + i|\tau) = \lambda^{i+1}(\tau)r(\tau + i|\tau - 1) + [1 - \lambda^{i+1}(\tau)]w$. Assume now that $\lambda(\tau) = 1 \; \forall \tau \geq t$. Consequently, $\lim_{\tau\to\infty} r(\tau + i|\tau) = \bar{w}$. In turn this implies $\lim_{\tau\to\infty} x(\tau) = x_{\bar{w}}$. Therefore, there exists a finite $\bar{\tau}$ such that $x(\bar{\tau}) = x_{\bar{w}} + \tilde{x}$ with $\|\tilde{x}\| \leq \epsilon$, for any $\epsilon > 0$. By Lemma 2, this contradicts the assumption that $\lambda(\tau) = 1$, $\forall \tau \geq t$. Therefore, there is a finite $\bar{\tau} \geq t$ such that $\lambda(\bar{\tau}) \in [0, 1)$. Then, $r(\bar{\tau} + i|\bar{\tau}) = \lambda^{i+1}(\bar{\tau})r(\bar{\tau} + i|\bar{\tau} - 1) + [1 - \lambda^{i+1}(\bar{\tau})]w$, $r(\bar{\tau} + 1 + i|\bar{\tau} + 1) = \lambda(\bar{\tau})[\lambda(\bar{\tau})\lambda(\bar{\tau} + 1)]^{i+1}r(\bar{\tau} + 1 + i|\bar{\tau} - 1) + \{1 - \lambda(\bar{\tau})[\lambda(\bar{\tau})\lambda(\bar{\tau} + 1)]^{i+1}\}w$ and so on. Hence, irrespective of $\lambda(\bar{\tau} + 1)$, $\lambda(\bar{\tau} + 2), \ldots$, $\lim_{t\to\infty} r(t + i|t) = w$. This implies that, as $t \to \infty$, $r(t + i|t) = w + \tilde{w}(t, i)$ and $x(t) = x_w + \tilde{x}(t)$. Consequently $c(t + i, t, x_w + \tilde{x}(t), w + \tilde{w}(t, i)) = c(t + i, t, x_w + \tilde{x}(t), w) + c(t + i, t, 0_x, \tilde{w}(t, i))$, where, being $\mathcal{C}$ open, the first term for a finite t belongs to $\mathcal{C}$ and the second goes to zero exponentially fast. The conclusion is therefore that there exists a finite time $\hat{t}$, $\hat{t} \geq t$, such that $\lambda(k) = 0$, $\forall k \geq \hat{t}$. □

In practice, instead of the selection rule (30)-(33), another possibility is, as indicated in Sect. 2, to construct the "reference to go" as follows. For $t, i \in \mathbb{Z}_+$, set

$$\Lambda(t) := \{\lambda \in [0, 1) | x(t) \in \mathcal{X}(r_\lambda(\cdot|t))\} \tag{36}$$

where

$$r_\lambda(t + 1|t) := \lambda^{i+1}y(t) + (1 - \lambda^{i+1})w(t) \tag{37}$$

Then, take

$$r(t+i|t) := \begin{cases} r(t+i|t-1) & , \text{if } \Lambda(t) = \emptyset \\ r_{\lambda(t)}(t+i|t) & , \text{otherwise} \end{cases} \quad (38)$$

where $\lambda(t) \in \Lambda(t)$. Also under this choice, provided that

$$x(0) \in \mathcal{X}(r(\cdot|0)),$$

$x(t) \in \mathcal{X}(r(\cdot|t))$. The analysis of the feedback system under the rule (36)–(38) turns out to be less direct and harder than the one in Theorem 1. For the sake of brevity we shall refrain from embarking ourselves into such an analysis. See [1] and [2] for details.

5. Conclusions.

Set-point conditioning and on-line reference managing schemes can be effective tools for solving feedback control problems in the presence of input and state-dependent constraints. These schemes can be embodied in any feedback control system, provided that model based predictions of the constrained vector can be carried out within two subsequent sampling times. The specific underlying control law considered in this paper consists of an optimal LQ controller with preview where constraints can be input saturations, input increment saturations, output over/undershoot limitations, etc.. Simulation experiments show that the reference governor can operate with no appreciable performance degradation, but substancial saving in computational load, w.r.t. a similar control system embodying a mathematical programming solver. Stability and offset-free properties are estabilished. For the 1-DOF control case, on-line PRM tecniques should be compared with the existing techniques [3] and [7].

References

[1] Bemporad, A. (1993). "Predictive control in the presence of constraints and reference managing," *M.S. Thesis (in Italian), Dipartimento Sistemi e Informatica, University of Florence*, Florence, Italy.

[2] Bemporad, A. and E. Mosca (1994). "Constraint fulfilment in control systems via predictive reference management," submitted, *33rd IEEE Control and Decision Conference.*

[3] Gilbert, E.G. and K. Tin Tan (1991). "Linear systems with state and control constraints: the theory and applications of maximal output admissible sets," *IEEE Trans. Automat. Control*, **AC-36**, 1008-1020.

[4] Graettinger, T. J. and B. H. Krogh (1992). "On the computation of reference signal constraints for guaranteed tracking performance," *Automatica*, **28**, 1125-1141.

[5] Gutman, P. O. and P. Hagander (1985). "A new design of constrained controllers for linear systems," *IEEE Trans. Automat. Control*, **AC-30**, 22-33.

[6] Keerthi, S. S. and E. G. Gilbert (1988). "Optimal infinite-horizon feedback control laws for a general class of constrained discrete-time systems: stability and moving-horizon approximations," *J. Optimization Theory and Applications*, **57**, 265-293.

[7] Kapasouris, P., Athans, M. and G. Stein (1988). "Design of feedback control systems for stable plants with saturating actuators," *Proc. 27th IEEE Conference on Decision and Control*, 469-479, Austin, Texas, U.S.A..

[8] Kwon, W. H. and A. E. Pearson (1978). "On feedback stabilization of time-varying discrete linear systems," *IEEE Trans. Automat. Control*, **AC-23**, 479-481.

[9] Mayne, D. Q. and H. Michalska (1990). "Receding horizon control of nonlinear systems," *IEEE Trans. Automat. Control*, **AC-35**, 814-824.

[10] Mayne, D. Q. and E. Polak (1993). "Optimization based design and control," *Preprints IFAC 12th World Congress*, **Vol. 3**, 129-138, Sydney, Australia.

[11] Mosca, E. (1994). *"Optimal, Predictive, and Adaptive Control"*. Prentice Hall, in print.

[12] Mosca, E. and G. Zappa (1989). "Matrix fraction solution to the discrete-time LQ stochastic tracking and servo problems," *IEEE Trans. Automat. Control*, **AC-34**, 240-242.

[13] Mosca, E. and J. Zhang (1992). "Stable redesign of predictive control,". *Automatica*, **28**, 1229-1233.

[14] Richalet, J. (1993). "Industrial applications of model based predictive control," *Automatica*, **29**, 1251-1274.

[15] Scokaert, P.O.H. and D. W. Clarke (1993). "Stability questions in predictive control. Part II: incorporation of hard inequality constraints," *Report no. 1983/93, University of Oxford, Dept. of Engineering Science*, Oxford, U.K..

[16] Tsang, T. T. C. and D. W. Clarke (1988). "Generalized predictive control with input constraints," *IEE Proc. Pt. D*, **135**, 451-460.