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Anti-windup synthesis via sampled-data piecewise affine optimal control☆

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Abstract

Discrete-time receding horizon optimal control is employed in model-based anti-windup augmentation. The optimal control formulation enables designs that minimize the mismatch between the unconstrained closed-loop response with a given controller and the constrained closed-loop response with anti-windup augmentation. Recently developed techniques for off-line computation of the constrained linear regulator's solution, which is piecewise affine, facilitate implementation. The resulting sampled-data, anti-windup closed-loop system's properties are established and its performance is demonstrated on a simulation example.

Keywords: Anti-windup design; Input saturation; Sampled-data systems; Piecewise affine systems

1. Introduction

1.1. Background

In this paper we bring the receding horizon optimal control (RHOC) to bear on the anti-windup synthesis problem. The anti-windup problem is a particular control design task for linear systems with input constraints. The problem specifies a linear controller that produces a desirable small-signal closed-loop behavior but an unsatisfactory large-signal closed-loop behavior. The objective is to augment the controller to produce an acceptable large-signal closed-loop response without changing the small-signal closed-loop response. In particular, the closed-loop should not be modified until the saturation occurs.

The RHOC is a general nonlinear control strategy that has been used successfully over the years in many different applications, especially for discrete-time linear systems with input constraints (see the excellent survey paper (Mayne, Rawlings, Rao, & Scokaert, 2000)). In RHOC, often referred to as "model predictive control", an open-loop optimal control problem is solved over a finite horizon, the first element of the optimizing sequence is applied as a feedback control, and the process is repeated at the next iteration. In general, the RHOC requires a relatively formidable on-line computational effort which has limited its applicability to relatively slow processes. Recently, Bemporad, Morari, Dua, and Pistikopoulos (2002) showed that the RHOC control law for linear systems with constraints is a piecewise affine function of the state, and that it can be computed off-line by employing the techniques of multiparametric quadratic programming (Bemporad et al., 2002; Tøndel, Johansen, & Bemporad, 2003). Thus, the on-line complexity of RHOC reduces to the evaluation of such a piecewise affine map.

The RHOC algorithms are very effective at stabilizing linear systems with saturation. However, in their standard

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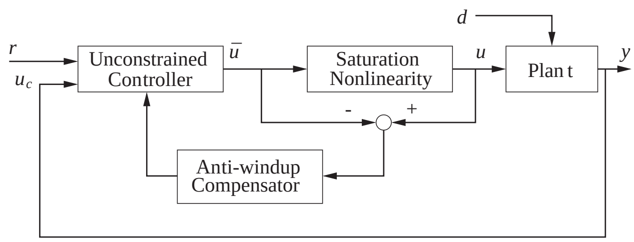

Fig. 1. The direct approach to anti-windup synthesis.

formulation (especially those that are computationally feasible), it is not possible to make them perform like an arbitrary, given, continuous-time controller for small signals. This is one of the main requirements of anti-windup synthesis. The main way in which optimal control has been brought to bear on the anti-windup problem previously is through the so-called "reference governor" which appeared in Kapasouris, Athans, and Stein (1988) and which has been extended and/or developed in discrete-time in Gilbert and Tan (1991), Gilbert, Kolmanovsky, and Tan (1995), Gilbert and Kolmanovsky (1999), Bemporad, Casavola, and Mosca (1997) and Shamma (2000), where the "measurement governor" is introduced. In the reference governor approach to anti-windup, the reference signal input to the prespecified controller is modified, using an optimization-based nonlinear and dynamic filter, in an attempt to keep the output of the prespecified controller from activating the saturation nonlinearity in the closed loop. The structure of most of the reference governor schemes is such that when the reference is identically zero, like in many vibration attenuation problems for example, essentially no modifications are made to the closed loop. Moreover, most reference governor schemes are not well suited for arbitrarily large exogenous disturbances acting on the closed loop. Finally, reference governors typically try to anticipate saturation and so may modify the closed loop in situations where the future values of the reference would not actually lead to saturation.

A more common approach to anti-windup uses the architecture shown in Fig. 1. In this architecture, the difference between the saturated and unsaturated control signal passes into an unbiased dynamical system and the output of this dynamical system augments the prespecified linear controller. Early designs using this architecture were mostly ad hoc, and often limited to static linear gains. (See, e.g., Hanus, 1988; Kothare, Campo, Morari, & Nett, 1994 for surveys of early anti-windup techniques.) The systematic design algorithms admitting *linear*, *dynamic* anti-windup compensators have emerged more recently. (See, e.g., Bemporad et al., 1997; Edwards & Postlethwaite, 1999; Gilbert et al., 1995; Grimm, Hatfield, Poslethwaite, Teel, Turner, & Zaccarian, 2003; Miyamoto & Vinnicombe, 1996; Mulder, Kothare, & Morari, 2001; Shamma, 2000; Teel & Kapoor, 1997; Zheng, Kothare, & Morari, 1994.) To the best of our knowledge, the only guidelines for the synthesis of *nonlinear*, *dynamic* anti-windup in the architecture of Fig. 1 have been given in Teel and Kapoor (1997), with the follow-up work appearing in Barbu, Reginatto, Teel, and Zaccarian (2001), Morabito, Teel, and Zaccarian (2002), Teel (1999a) and Zaccarian and Teel (2001).

1.2. Contribution

In this paper, we will combine the model-based anti-windup synthesis ideas from Teel and Kapoor (1997) with the results from RHOC to produce a nonlinear, dynamic, sampled-data anti-windup compensator that fits the architecture of Fig. 1, for all intents and purposes. (Actually, it fits a slightly different architecture where the anti-windup compensator has an extra input corresponding to the reference r. However, it will still have the property that if the saturation is never reached and the anti-windup compensator dynamics are initialized to zero then the outputs of the anti-windup compensator are identically zero.) We will show how this approach enables a design aimed at keeping small[1] the $\mathscr{L}_2$ error between the constrained and unconstrained closed-loop responses, which is the definition of successful anti-windup proposed in Teel and Kapoor (1997). We will focus on achieving the successful anti-windup for reference signals that converge to (feasible) constants and for disturbances that converge to zero, both in an $\mathscr{L}_2$ sense. Because of this, the results do not apply to a class of references and disturbances that is as broad as that considered in Teel and Kapoor (1997). However, it is for this subclass of references and disturbances that RHOC's constrained linear regulator solution is most powerful in the anti-windup setting.[2]

Being more precise about our proposed synthesis, we will pick the *anti-windup compensator* block in Fig. 1 to have the form

$$\dot{x}_{aw} = Ax_{aw} + B\kappa(\xi_{aw}, \rho) + B[u - \bar{u}],$$

$$v_1 = \kappa(\xi_{aw}, \rho),$$

$$v_2 = -C_y x_{aw} - D_y \kappa(\xi_{aw}, \rho) - D_y[u - \bar{u}], \tag{1}$$

where

(1) (A, B, C_y, D_y) represents the plant from input to measured output,
(2) ξ_{aw} represents a sampled and held version of the state x_{aw}, and ρ represents an averaged, sampled and held version of the reference input; the sampling period T

[1] What we will provide is a suboptimal solution to the $\mathscr{L}_2$ minimization problem. This solution cannot be optimal in the continuous time sense because of the limitations due to the sample data nature of the solution and to the finite horizon effects.

[2] It is possible to use RHOC in the anti-windup problem for the more general class of references and disturbances considered in Teel and Kapoor (1997). However, in the most straightforward application, many of the advantages of optimal control are lost because of having to account for the worst case disturbance and reference profiles.

is determined by the computational capabilities of the available hardware, and

(3) $\kappa(\cdot,\cdot)$ is synthesized using the tools of constrained discrete-time receding horizon control for the discrete-time linear system $\xi^+ = A_d\xi + B_d v$ (i.e., $\xi(k+1) = A_d\xi(k) + B_d v(k)$), where
   (a) the constraints on v are dictated by the characteristics of sat$(\cdot)$ (see the following Assumption 2) and the reference to the prespecified controller,
   (b) A_d and B_d are given by

$$A_d := e^{AT}, \quad B_d := e^{AT}\left(\int_0^T e^{-A\tau}\,d\tau\right)B, \tag{2}$$

   (c) the stage cost in the receding horizon problem is used to tune anti-windup performance; in particular, to minimize the mismatch between the constrained and unconstrained closed-loop responses.

The main technical challenges in proving that the control laws from Bemporad et al. (2002) provide a suboptimal solution to the nonlinear $\mathscr{L}_2$ anti-windup problem revolve around combining the discrete-time RHOC algorithm of Bemporad et al. (2002) with the continuous-time controller/plant interconnection that is prone to windup in the presence of input saturation. When using a discrete-time anti-windup algorithm, the dynamics that characterize the mismatch between the unconstrained closed-loop behavior and the saturated closed-loop behavior is a sampled-data nonlinear system whose $\mathscr{L}_2$ stability needs to be established. Results on $\mathscr{L}_2$ (in fact, $\mathscr{L}_p$) stability for *linear* sampled-data systems can be found in Chen and Francis (1991) for the time-invariant case, in Iglesias (1995) for the time-varying case and in the recent paper (Zaccarian, Teel, & Nešić, 2003) for a class of nonlinear systems. Here, we will use the main result from Zaccarian et al. (2003) to establish the $\mathscr{L}_2$ stability of the sampled-data system that arises in our anti-windup problem.

The rest of our paper is organized as follows. In Section 2 we define the windup problem and clarify the goals of our anti-windup construction. In Section 3.1 we describe the sampled-data anti-windup scheme and in Section 3.2 we provide a specific sampled-data anti-windup construction based on the explicit RHOC techniques. In Section 4 we illustrate the proposed solution on a simulation example and in Section 5, we provide a proof for the main theorem. Finally, in the appendix, we report proofs of some technical results.

1.3. Notation

We use $\mathbb{Z}_{\geqslant k}$ (respectively, $\mathbb{R}_{\geqslant\alpha}$) to denote the set of integers (respectively, reals) greater than or equal to the integer k (respectively, to the real α). Given a signal $x(t)$, $t\in\mathbb{R}_{\geqslant 0}$, we denote by $\dot{x}(t) = dx(t)/dt$ its derivative with respect to time (often we will use the shortcut $\dot{x}$ in place of $\dot{x}(t)$). Given a sequence $\xi(h)$, $h\in\mathbb{Z}_{\geqslant 0}$, we denote by $\xi^+(h) = \xi(h+1)$ its one-step translation with respect to time (often we will use the shortcut ξ^+ in place of $\xi^+(h)$). For a function $v:\mathbb{R}_{\geqslant 0}\to\mathbb{R}^m$, we define $\|v(\cdot)\|_{\mathscr{L}_2} := \left(\int_0^\infty |v(\tau)|^2\,d\tau\right)^{1/2}$ and for a function $\xi:\mathbb{Z}_{\geqslant 0}\to\mathbb{R}^n$, we define $\|\xi(\cdot)\|_{\ell_2} := \left(\sum_{k\geqslant 0}|\xi(k)|^2\right)^{1/2}$. When $\|v(\cdot)\|_{\mathscr{L}_2}<\infty$, respectively $\|\xi(\cdot)\|_{\ell_2}<\infty$, we say that $v(\cdot)\in\mathscr{L}_2$, respectively, $\xi(\cdot)\in\ell_2$. Given a matrix Q, Q' denotes the transpose of Q and, if Q is square, $Q>0$ means that Q is positive definite, while $\lambda_{\min}(Q)$, $\lambda_{\max}(Q)$ denote the eigenvalues of Q whose modulus is minimum and maximum, respectively. Given two sets $\mathscr{A},\mathscr{B}$, $\mathscr{A}\subset\mathscr{B}$ means that $\mathscr{A}$ is contained or equal to the set $\mathscr{B}$.

With a slight abuse of notation, given two vectors $a = [a_1,\dots,a_n]'$, and $b = [b_1,\dots,b_n]'$, $a\leqslant b$ means that $a_i\leqslant b_i$ for all $i = 1,\dots,n$. A matrix is called Hurwitz if all its eigenvalues have negative real part and it is called discrete-time Hurwitz if all its eigenvalues have modulus smaller than 1.

2. Problem definition

2.1. Unconstrained closed-loop and saturation

For the anti-windup synthesis problem, a linear continuous-time plant is given. We represent it in state–space form as

$$\begin{aligned}
\dot{x} &= Ax + Bu + B_d d + \psi_x,\\
z &= C_z x + D_z u + D_{dz} d + \psi_z,\\
y &= C_y x + D_y u + D_{dy} d + \psi_y,
\end{aligned} \tag{3}$$

where $x\in\mathbb{R}^n$ is the state, $y\in\mathbb{R}^{n_y}$ is the measured output, $z\in\mathbb{R}^{n_z}$ is the performance output, $u\in\mathbb{R}^{n_u}$ is the control input, d is a disturbance input and ψ_x,ψ_z,ψ_y are the outputs of a linear dynamic system representing uncertain dynamics, which can be written as

$$\Psi := \begin{bmatrix}\psi_x\\ \psi_z\\ \psi_y\end{bmatrix} = \Delta(s)\begin{bmatrix}x\\ u\\ d\end{bmatrix} = \begin{bmatrix}\Delta_x(s)\\ \Delta_z(s)\\ \Delta_y(s)\end{bmatrix}\begin{bmatrix}x\\ u\\ d\end{bmatrix}, \tag{4}$$

where $\Delta(s)$ is a linear time-invariant asymptotically stable system with $\mathscr{L}_2$ gain smaller than $\gamma_\Delta>0$.

A linear controller[3] is also given in the anti-windup problem. Its state–space representation is

$$\begin{aligned}
\dot{x}_c &= A_c x_c + B_{cu} u_c + B_{cr} r,\\
y_c &= C_c x_c + D_{cu} u_c + D_{cr} r,
\end{aligned} \tag{5}$$

where $x_c\in\mathbb{R}^{n_c}$ is the controller state, $u_c\in\mathbb{R}^{n_y}$ and $y_c\in\mathbb{R}^{n_u}$ are the controller input and output, respectively, and $r\in\mathbb{R}^{n_z}$ is the reference input. The dynamical system (5) will be denoted as the *unconstrained controller* henceforth, and

[3] In general, controller (5) does not need to be linear for the anti-windup construction to be applicable. However, in this paper we will assume it to be linear, to keep the discussion simple.

corresponds to the block in the upper left-hand corner of Fig. 1. We assume that the unconstrained controller has been designed so that the feedback connection of the linear plant (3) with the unconstrained controller (5) via the unconstrained interconnection equations

$$u = y_c, \quad u_c = y \tag{6}$$

(which will be referred to as the *unconstrained closed-loop system*), satisfies the following assumption:

Assumption 1. The linear unconstrained closed-loop system (3), (5), (6) with $\Psi \equiv 0$ is well-posed and internally stable.

Lemma 1. *Under Assumption* 1, *if the* $\mathscr{L}_2$ *gain* γ_Δ *of* $\Delta(s)$ *is sufficiently small, the (perturbed) unconstrained closed-loop* (3)–(6) *is well-posed and internally stable.*

Proof. Observe first that well-posedness of the unperturbed closed-loop implies that $(I - D_y D_{cu})$ is invertible. On the other hand, for the perturbed closed-loop system to be well-posed it is necessary that $(I - D_y D_{cu} - \Delta_{uy})$ is invertible, where $\Delta_{uy} := \Delta_y(\infty)[0\ D_c'\ 0]'$ and $\Delta_y(\infty)$ denotes the input–output direct link of the dynamical system $\Delta_y(s)$. Then, since Δ_{uy} can be made arbitrarily small by making γ_Δ sufficiently small, well-posedness follows. Moreover, the fact that internal stability is preserved for the perturbed system can be shown by a small gain argument and selecting once again γ_Δ sufficiently small. □

2.2. Input saturation and anti-windup goals

If saturation is present at the input u of the linear plant (3), the unconstrained interconnection (6) is replaced by the *saturated interconnection*

$$u = \mathrm{sat}(y_c), \quad u_c = y \tag{7}$$

and the linearity of the closed-loop system is lost. For simplicity, we will consider the decentralized saturation function, as detailed in the following assumption:

Assumption 2. The input nonlinearity $\mathrm{sat}(\cdot): \mathbb{R}^{n_u} \to \mathbb{R}^{n_u}$ is the standard decentralized saturation function, namely $\mathrm{sat}(u) := [\mathrm{sat}_1(u_1), \ldots, \mathrm{sat}_{n_u}(u_{n_u})]'$, where $\mathrm{sat}_i(u_i) := \max\{u_{mi}, \min\{u_{Mi}, u_i\}\}$ and where $u_{mi} < u_{Mi}$ for all $i = 1, \ldots, n_u$.

The closed-loop system (3), (5), (7), which we will call the *saturated closed-loop system* henceforth, often exhibits unpredictable behavior and, typically, performance loss and instability. This phenomenon is often referred to in the literature as "windup". External anti-windup augmentation, which we consider in this paper, consists in modifying the unconstrained interconnection equations (6) as follows:

$$u = \mathrm{sat}(y_c + v_1), \quad u_c = y + v_2, \tag{8}$$

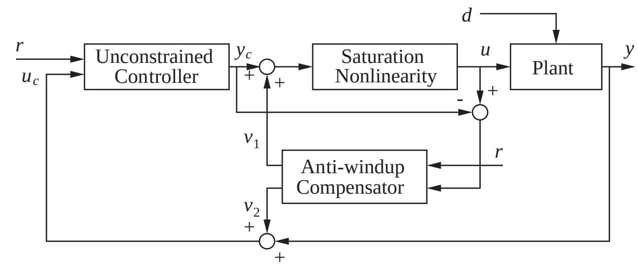

Fig. 2. The $\mathscr{L}_2$ (external) anti-windup scheme.

where v_1 and v_2 are the outputs of a suitable (dynamical, in general) system

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \Sigma_{aw}(x_{aw}(0), y_c, y), \tag{9}$$

which is introduced in the closed-loop as shown in Fig. 2. The complete nonlinear closed-loop system of Fig. 2, described by Eqs. (3), (5), (8), (9) will be called the *anti-windup closed-loop system* henceforth.

Generally speaking, filter (9) should be designed to guarantee the following qualitative goals for any selection of the external inputs (r, d) and of the initial conditions:

(1) if the actuators never saturate, then the unconstrained response is retained (namely, $v_1(\cdot) \equiv 0$ and $v_2(\cdot) \equiv 0$);
(2) if the actuators saturate, then the performance output $z(\cdot)$ is as close as possible to the performance output $z_\ell(\cdot)$ of the (ideal) unconstrained response starting from the same plant/controller initial conditions.

To suitably formalize the second requirement above, we will measure the deviation between $z(\cdot)$ and $z_\ell(\cdot)$ in terms of the $\mathscr{L}_2$ norm of the signal $(z - z_\ell)(\cdot)$. This is a useful measure of anti-windup quality because when $\|(z - z_\ell)(\cdot)\|_{\mathscr{L}_2}$ is finite, then under minor conditions on $z(\cdot)$ and $z_\ell(\cdot)$ (Teel, 1999b) (indirectly, on the external inputs $r(\cdot)$ and $d(\cdot)$), this implies that $z(\cdot)$ converges asymptotically to $z_\ell(\cdot)$, thus guaranteeing asymptotic recovery of the unconstrained performance output. When formally characterizing the two qualitative anti-windup goals mentioned above, it helps to restrict the attention to a limited set of references. To this aim, we formalize in the following the concept of *feasible* reference, which, as stated in Lemma 1, is well defined provided the $\mathscr{L}_2$ gain γ_Δ of the unmodeled dynamics is sufficiently small.

Definition 1. A constant reference r_o is said to be *feasible* if the response of the perturbed unconstrained closed-loop (3)–(6) to the input $(r, d) = (r_o, 0)$ is such that the steady-state value $y_{c,\infty}(r_o)$ of the unconstrained controller output satisfies

$$y_{c,\infty}(r_o) = \mathrm{sat}(y_{c,\infty}(r_o)). \tag{10}$$

Based on the above definition, we are now ready to introduce a formal property capturing the goals of our sampled-data anti-windup design.

Property 1. *For each reference-disturbance pair* (r, d) *and initial conditions, let* z_ℓ *represent the performance output and* u_ℓ *represent the plant input for the perturbed unconstrained closed-loop system* (3)–(6); *let z represent the performance response of the anti-windup closed-loop system* (3)–(5), (8), (9). *The following holds*:

(1) *if the anti-windup compensator* (9) *has zero initial conditions and* $u_\ell(\cdot) \equiv \mathrm{sat}(u_\ell(\cdot))$, *then* $z(\cdot) \equiv z_\ell(\cdot)$;
(2) *if the* $\mathscr{L}_2$ *gain* γ_Δ *of the unmodeled dynamics* (4) *is sufficiently small, then for each feasible reference* r_o, *there is a global finite* $\mathscr{L}_2$ *gain from* $(d(\cdot), r(\cdot) - r_o)$ *to* $(z_\ell - z)(\cdot)$.

Property 1 formalizes the two particular goals of anti-windup designs, formulated above from an intuitive viewpoint. Item (1) imposes that whenever the initial conditions and external inputs are such that the arising trajectory would not exceed the saturation limits, then the anti-windup compensator must not enforce any modification to the linear closed-loop transfer function. Furthermore, item (2) formalizes the requirement that, whenever saturation is activated (thus making the desired linear trajectory unfeasible for the saturated plant), the performance output must at least converge (in an $\mathscr{L}_2$ sense) to the unconstrained performance output. Evidently, the smaller the deviation $z_\ell - z$ between the unconstrained performance and the actual one, the better the anti-windup goal has been accomplished. For this reason, among all of the anti-windup compensators that solve this problem, we are interested in ones that are effective at making $\|(z_\ell - z)(\cdot)\|_{\mathscr{L}_2}$ small.

3. Sampled-data $\mathscr{L}_2$ anti-windup

3.1. Anti-windup augmentation scheme

Consider the case where plant (3) with $\Psi \equiv 0$ is asymptotically stable. Then, based on Teel and Kapoor (1997), we can select the anti-windup compensator (9) as the following filter:

$$\dot{x}_{\mathrm{aw}} = Ax_{\mathrm{aw}} + B(\mathrm{sat}(y_c + v_1) - y_c),$$

$$v_2 = -C_y x_{\mathrm{aw}} - D_y(\mathrm{sat}(y_c + v_1) - y_c), \quad (11)$$

which consists of a model of the plant transfer function from u to y and where the signal v_1 is a free design parameter (to be defined later). Notice that this agrees with Eq. (1) in the Introduction by adding and subtracting Bv_1 to each equation and selecting $\bar{u} = y_c + v_1$.

The closed-loop system represented in Fig. 2 corresponds to a particular embodiment of the general scheme of Fig. 1. The sampled-data nature of our approach stands in the selection of the signal v_1 in (8) and (11), which will be defined by assigning the anti-windup compensator block of Fig. 2 as detailed in Fig. 3. This sampled-data anti-windup compensator contains two principal blocks: a continuous-time one, reproducing the dynamics of the plant and corresponding to Eq. (11), and a discrete-time one, driven by sampled versions of the state of the previous block and of an averaged version of the reference input.

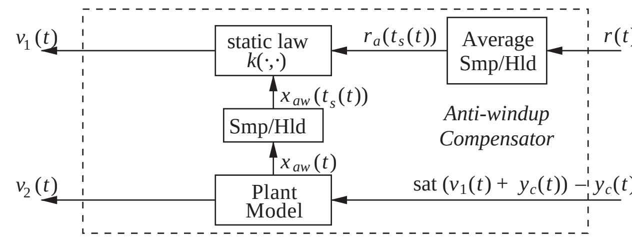

Fig. 3. The sampled-data anti-windup compensator.

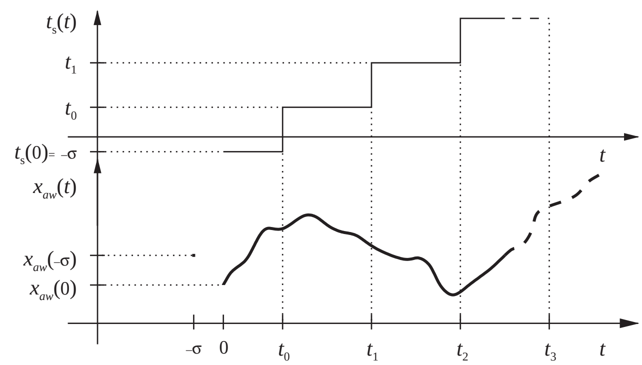

Fig. 4. The state–space response of the sampled-data compensator (11), (13) with indications of its initial conditions.

Note that, in Fig. 3, two time scales are represented: the continuous time t and the sampled time $t_s(t)$, which, by way of the two sample and hold devices of the figure, are related as depicted in Fig. 4. Note also that, for the sake of generality, $t = 0$ is not necessarily a sampling instant. To suitably represent this sampled-data nature of the anti-windup compensator, the sampling instants $t_s(t)$ are related to the continuous time t by

$$t_s(t) = \lfloor t + \sigma \rfloor_T - \sigma, \quad \sigma \in [0, T), \quad (12a)$$

where T denotes the sampling period, $\lfloor s \rfloor_T := T\lfloor s/T \rfloor$, and $\lfloor q \rfloor := \max\{\chi \in \mathbb{Z}, \chi \leqslant q\}$, for all $q \in \mathbb{R}$. The value $\sigma := -t_s(0) = -t_{-1}$ is the time elapsed between the initial time $t = 0$ and the most recent sampling instant, denoted by t_{-1}. The values t_k representing the sampling instants associated with the sampled-data block can also be defined as follows

$$t_k := t_s((k+1)T) = (k+1)T + t_s(0). \quad (12b)$$

According to the notation above for the discrete and continuous time scales, the discrete-time static law in Fig. 3 corresponds to

$$v_1(t) = \kappa(x_{\mathrm{aw}}(t_s(t)), r_a(t_s(t))), \quad (13)$$

where $r_a(t_s(\cdot))$ is a piecewise constant average of the reference signal r and is defined for all $k \geqslant 0$ as

$$r_a(t_s(t)) := \frac{1}{T}\int_{t_{k-1}}^{t_k} r(\tau)\,d\tau, \quad \forall t \in [t_k, t_{k+1}). \tag{14}$$

As shown in Fig. 4, the information $x_{aw}(0)$, $x_{aw}(t_s(0))$, $r_a(t_s(0))$ (where $t_s(0) = -\sigma$), $r(t)$, $\forall t \in [-\sigma, 0)$ is a minimal representation of the initial conditions of the anti-windup compensators (11)–(13). In the simulations reported in Section 4, we will use the initial conditions $(x_{aw}(0), x_{aw}(t_s(0)), r_a(t_s(0))) = (0,0,0)$ and $r(t) = r(0)$, $\forall t \in [-\sigma, 0)$.

In this work, the synthesis of the static function $\kappa(\cdot,\cdot)$ in Fig. 3 will be based on the discrete-time model

$$\xi^+ = A_d\xi + B_d v, \tag{15}$$

where the input v is subjected to suitable constraints corresponding to the continuous-time input nonlinearity of (11) and where the matrices A_d and B_d are defined in (2). The sampled-data feedback will have the form (13), namely $v = \kappa(\xi,\rho)$, with $(\xi,\rho) = (x_{aw}(t_s(t)), r_a(t_s(t)))$ at all (continuous) times t, and (in the next section) we will design (high performance) selections for the function $\kappa(\cdot,\cdot)$ that satisfy the following property whenever plant (3) is asymptotically stable:

Property 2. *The function $\kappa(\cdot,\cdot)$ is such that*

(1) *it is globally Lipschitz,*
(2) $\text{sat}(y_{c,\infty}(r_o) + \kappa(\xi, r_o)) = y_{c,\infty}(r_o) + \kappa(\xi, r_o)$, $\forall \xi \in \mathbb{R}^n$, r_o feasible,
(3) *the origin is a globally exponentially stable equilibrium point of the system $\xi^+ = A_d\xi + B_d\kappa(\xi, r_o)$.*

We can now state the following main result:

Theorem 1. *Under Assumptions* 1 *and* 2, *if the function $\kappa(\cdot,\cdot)$ satisfies Property* 2, *the anti-windup closed-loop system* (3)–(5), (8), (11), (13) *satisfies the anti-windup Property* 1.

Proof. See Section 5. □

Remark 1. We emphasize that Theorem 1 establishes a result for system (11), (13) where the second argument of $\kappa(\cdot,\cdot)$ is time-varying, yet assumes properties for $\kappa(\cdot,\cdot)$ only when its second argument is a constant, feasible reference.

3.2. Synthesis via explicit RHOC techniques

In this section, we will introduce an RHOC-based design strategy for the synthesis of a feedback function $\kappa(\cdot,\cdot)$ that satisfies Property 2 (thus solving the $\mathscr{L}_2$ anti-windup problem by way of Theorem 1), while keeping small the ℓ_2 norm of the discrete time signal

$$\zeta_{aw} := C_z\xi + D_z[\text{sat}(y_{c,\infty}(r_o) + \kappa(\xi, r_o)) - y_{c,\infty}(r_o)], \tag{16}$$

where ξ is the trajectory of (15). Note that since (15) is the exact discretization of the continuous-time anti-windup compensator dynamics (11), in the case where $y_c = y_{c,\infty}(r_o)$, Eq. (16) corresponds to the sampled version of the output

$$z_{aw} := C_z x_{aw} + D_z(\text{sat}(y_c + v_1) - y_c), \tag{17}$$

which, according to the results in Teel and Kapoor (1997), provides a measure of the mismatch $z_\ell - z$ between the unconstrained and the actual output responses (in the case where $\Psi \equiv 0$ it can be actually shown that $z_{aw} = z - z_\ell$). Hence, minimizing the ℓ_2 norm of the signal ζ_{aw} is especially effective at providing a good solution to the $\mathscr{L}_2$ anti-windup problem when the sampling period is small and $y_c(\cdot)$ is similar to an impulse function. When this is the case, $y_c(\cdot)$ can be thought of as inducing an initial condition $\xi(0) = x_{aw}(t_0)$ and thereafter satisfying $y_c(t) \approx y_{c,\infty}(r_o)$ so that the real problem is very close to the problem for which the RHOC strategy was designed.

According to Assumption 2, define $u_m := [u_{m1} \cdots u_{mn_u}]'$ and $u_M := [u_{M1} \cdots u_{Mn_u}]'$. In the following, we will synthesize a globally Lipschitz static control law

$$v = \kappa(\xi, r_o) =: \bar{\kappa}(\xi, y_{c,\infty}(r_o)) \tag{18}$$

with the constraint

$$u_m \leqslant y_{c,\infty}(r_o) + v \leqslant u_M \tag{19}$$

(where the above inequalities should hold componentwise) that exponentially stabilizes the origin of

$$\xi^+ = A_d\xi + B_d v,$$
$$\zeta_{aw} = C_z\xi + D_z v \tag{20}$$

(i.e., it satisfies Property 2) and aims to minimize the ℓ_2 norm of its output ζ_{aw}.

For simplicity, we will use the notation $y_{c,\infty}$ to denote $y_{c,\infty}(r_o)$ hereafter. The design of $\bar{\kappa}(\cdot,\cdot)$ is based on the result of a finite horizon optimization problem

$$\bar{\kappa}(\xi, y_{c,\infty}) := v_0^*, \tag{21}$$

where, given a finite number of steps N, v_0^* is the first element of the minimizer $\mathscr{V}^*$ of the following optimization problem:

$$J^*(\xi, y_{c,\infty}) := \min_{\mathscr{V}} \left\{ J(\mathscr{V}, \xi, y_{c,\infty}) = \eta_N' P \eta_N + \sum_{i=0}^{N} (|\zeta_i|^2 + v_i' R v_i) \right\} \tag{22a}$$

$$\text{s.t.} \begin{cases} \eta_0 = \xi, \\ u_m \leqslant v_i + y_{c,\infty} \leqslant u_M, \; i = 0, \dots, N-1, \\ \eta_{i+1} = A_d \eta_i + B_d v_i, \; i = 0, \dots, N-1, \\ \zeta_i = C_z \eta_i + D_z v_i, \; i = 0, \dots, N-1, \end{cases} \tag{22b}$$

where $R = R' > 0$, P is the solution to the Lyapunov equation $P = A_d' P A_d + C_z' C_z \geqslant 0$, and $|\cdot|$ denotes the standard Euclidean norm. Moreover, η_0 is the current state and $\eta_1, \dots, \eta_N$ are the predicted states for the future N sampling instants; $\mathscr{V} := \{v_0, v_1, \dots, v_{N-1}\}$ denotes the set of free moves and $\mathscr{V}^* := \{v_0^*, v_1^*, \dots, v_{N-1}^*\}$ is the minimizer (the dependence on ξ and $y_{c,\infty}$ is omitted for simplicity).

One great advantage in selecting (21)–(22) for the sampled-data feedback (18) resides in the fact that, based on the results of Bemporad et al. (2002) and Tøndel et al. (2003), (21)–(22) can be computed analytically as a globally Lipschitz and piecewise affine control law

$$\bar{\kappa}(\xi, y_{c,\infty}) = F^i \xi + G^i y_{c,\infty} + a^i \quad \text{if } H^i \xi + K^i y_{c,\infty} \leqslant b^i, \; i = 1, \dots, n_r, \tag{23}$$

where F^i, G^i, a^i, H^i, K^i and b^i, $i = 1, \dots, n_r$, are matrices of suitable dimensions, whose values can be determined explicitly by following the construction in Bemporad et al. (2002) and Tøndel et al. (2003). This result is formalized in the following lemma:

Lemma 2. *The control law* (21)–(22) *has the form* (23) *and is globally Lipschitz.*

Proof. See Appendix A. □

The suitability of the proposed control law as a candidate for the solution of the anti-windup problem described in the previous section is formalized in the following lemma:

Lemma 3. *Let A_d be a discrete-time Hurwitz matrix and P be the solution to $P = A_d' P A_d + C_z' C_z$. Then, given any $N \geqslant 1$, the RHOC law defined by* (21)–(22) *globally exponentially stabilizes* (20) *while fulfilling constraint* (19) *at all sampling steps.*

Proof. See Appendix A. □

The proof of the anti-windup result is finally completed by noticing that whenever A is Hurwitz, by (2), A_d is discrete-time Hurwitz, hence Lemma 3 ensures that the items 2 and 3 of Property 2 hold. Moreover, since by linearity the map $r_o \mapsto y_{c,\infty}(r_o)$ is globally Lipschitz, Lemma 2 ensures that the item 1 of Property 2 holds too. Finally, once Property 2 is satisfied, Theorem 1 guarantees desirable properties of the anti-windup closed-loop system.

Theorem 2. *Under Assumptions* 1 *and* 2, *if A is Hurwitz, the anti-windup closed-loop system* (3), (5), (8), (11), (13) *with selection* (23) *satisfies the anti-windup Property* 1.

Remark 2. Like in Rawlings and Muske (1993), we have chosen P in (22a) as the solution to a Lyapunov equation to guarantee global stability. While this choice does not automatically address ℓ_2-optimality, this aim can be addressed by increasing the number N of free control moves in the optimization problem. The price paid for this typically is an increase in the number n_r of cells in the polyhedral partition of the piecewise affine control law (23). An alternative choice, as suggested in Bemporad et al. (2002), Chmielewski and Manousiouthakis (1996), Scokaert and Rawlings (1998), and Sznaier and Damborg (1987), is to choose P as the solution to the Riccati equation $P = (A_d + B_d K)' P (A_d + B_d K) + K' R K + C_z' C_z$, where $K = -(R + B_d' P B_d)^{-1} B - B_d' P A_d$ is the LQR gain. These references show that a semiglobal stability result is guaranteed by choosing a sufficiently large finite number N of free moves, which can be computed for any compact set as suggested in Bemporad et al. (2002) and Chmielewski and Manousiouthakis (1996). This choice for P is more directly aimed at ℓ_2-optimality.

Remark 3. From a purely theoretical viewpoint the result in Theorem 2 guarantees that, when the plant model is exact, the sampled-data scheme satisfies the anti-windup specifications of Property 1 regardless of the selection of T. Obviously, from a practical implementation viewpoint, T should be chosen properly. On one hand, the smaller T is, the better the anti-windup is expected to perform, as the continuous-time behavior is better mimicked by the discretized dynamics. On the other hand, for a given number of prediction steps N in problem (22a), if T is very small then the prediction horizon TN is very short, so that the anti-windup behavior may deviate considerably from the $\mathscr{L}_2$-optimal solution. Moreover, as it usually happens with discrete-time controllers, fast sampling, besides imposing more stringent hardware requirements, may lead to numerical problems, as the discrete-time model (2) used in (22b) tends to become just a pool of discrete-time integrators.

Regarding the complexity of the piecewise affine control law $\bar{\kappa}(\cdot,\cdot)$ in (23), this is related to two factors: (i) the dimensions n of the state vector ξ and n_u of $y_{c,\infty}$, and (ii) the number n_r of polyhedral cells. Thanks to the robustness property with respect to the unmodeled dynamics addressed and proven in our result (which guarantees that approximate models can be implemented in the anti-windup filter), the former source of complexity can be attacked by reducing the order of the model. On the contrary, the number n_r is mainly related to the horizon length N and to the constraints in (22a), rather than to $n + n_u$ (Bemporad et al., 2002), and it usually increases exponentially with N. According to the authors' experience, problems with $n = 2$–10 states, $n_u = 1$–3 inputs, and $N = 1$–5 horizon lengths are tractable in practice on average. The complexity of the implementation also depends on how the control law (23) is evaluated: while for small n_r (namely, $n_r \leqslant 100$) the piecewise affine map can be easily stored in memory and the current cell searched sequentially on line, for larger partitions more efficient approaches both in terms of memory requirements and computation complexity have been proposed in Borrelli, Baotic, Bemporad, and Morari (2001) and Tøndel et al. (2003). For the reasons above, as a rule of thumb, the largest T and the smallest N that ensure an acceptable anti-windup performance should be chosen.

4. Simulation example

To illustrate the performance of the anti-windup construction proposed in Section 3, we consider the high pressure boiler first studied in Chien, Ergin, Ling, and Lee (1958) and summarized in Davison (1990) (this model was also used in Grasselli, Menini, & Valigi, 2002). The boiler model is a 9th order plant with three control inputs u_1, u_2, u_3, two accessible outputs y_1, y_2, and a scalar disturbance input that we neglect here. The model corresponds to a linearization of the nonlinear boiler dynamics around the operating point of 44% full load, whose values are listed in Table 1.

In order to guarantee robust asymptotic tracking of constant references, we construct a standard (unconstrained) linear dynamic output-feedback controller by first augmenting the system with output integrators, and then, for the resulting extended system, by designing an observer (via pole placement) and a stabilizing LQR controller. The overall size of the controller state is $n_c = 2 + (9 + 2) = 13$, where the first term accounts for the output integrators, and the last term is the dimension of the observer.

Selecting the reference signal as

$$r(t) = \begin{cases} \begin{bmatrix} 60 \\ 0 \end{bmatrix} \begin{matrix} \text{bar} \\ \text{m} \end{matrix}, & 0 \leqslant t \leqslant 150 \text{ s}, \\ \begin{bmatrix} 60 \\ 0.03 \end{bmatrix} \begin{matrix} \text{bar} \\ \text{m} \end{matrix}, & t > 150 \text{ s}, \end{cases} \quad (24)$$

the unconstrained response corresponds to the dashed curves in Figs. 5 and 6.

According to the steady-state values listed in Table 1, we assume that the control inputs of the (linearized) plant are saturated at the values $u_{M1} = -u_{m1} = 0.6$ kg/s, $u_{M2} = -u_{m2} = 9$ kg/s, $u_{M3} = -u_{m3} = 30$°C, respectively, which correspond to about 50%, 60%, and 17% of the corresponding input operating values. The response of the system when the linear controller is saturated is represented by the dotted curves in Figs. 5 and 6. Note the persistent oscillations of the output signals and the loss of stability.

We first construct an RHOC-based anti-windup compensator for this system by using a full order prediction model, a sample time $T = 0.5$ s, a horizon length $N = 1$ and by selecting $z = [y_1 \;\; y_2]'$ and $R = 10^{-6} I_3$, where I_3 is the identity matrix of order 3. The arising piecewise affine discrete-time law is defined over 27 regions in the state and input spaces $\mathbb{R}^9 \times \mathbb{R}^3$. The results of the corresponding simulation are reported in Figs. 5–7, which confirm the desirable unconstrained response recovery proven in Theorem 2. From Fig. 7, it is evident that the anti-windup compensator makes large use of all three inputs. This behavior can be modulated

Table 1
Physical meaning of the boiler inputs and outputs and their values at the operating point of 44% full load

Name	Physical quantity	Values	Unit
u_1	Input fuel flow	1.08	kg/s
u_2	Input water flow	14.80	kg/s
u_3	Input water temperature	171.1	°C
y_1	Pressure	84.04	bar
y_2	Water level in boiler	44%	m
d	Output steam flow	14.80	kg/s

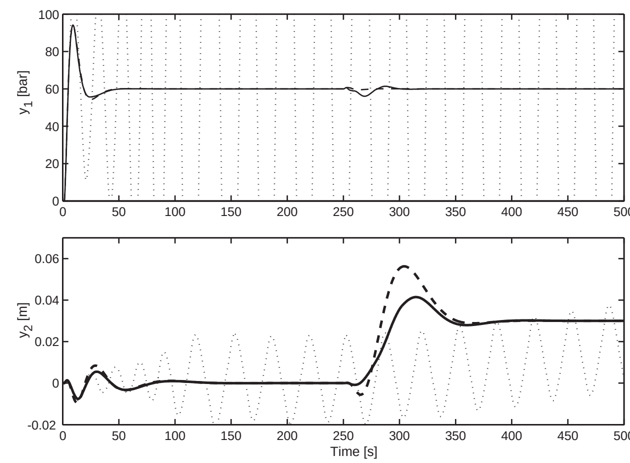

Fig. 5. Output responses to the reference signals in (24). Unconstrained response (dashed), anti-windup response (solid), saturated response without anti-windup (dotted).

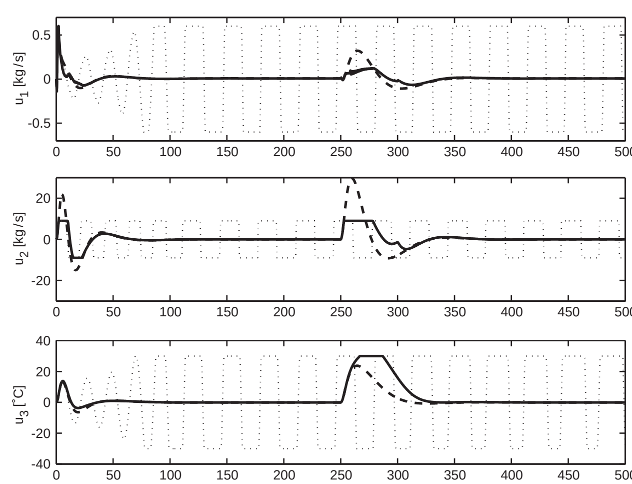

Fig. 6. Input responses to the reference signals in (24). Unconstrained response (dashed), anti-windup response (solid), saturated response without anti-windup (dotted).

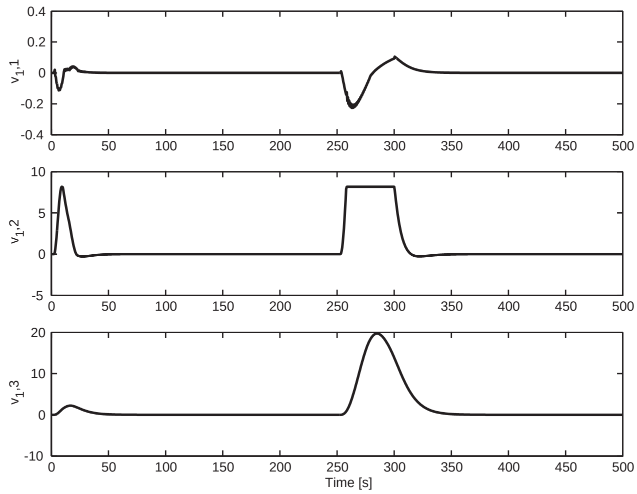

Fig. 7. Variable v_1 generated by the anti-windup closed-loop system in the response to the reference signals in (24).

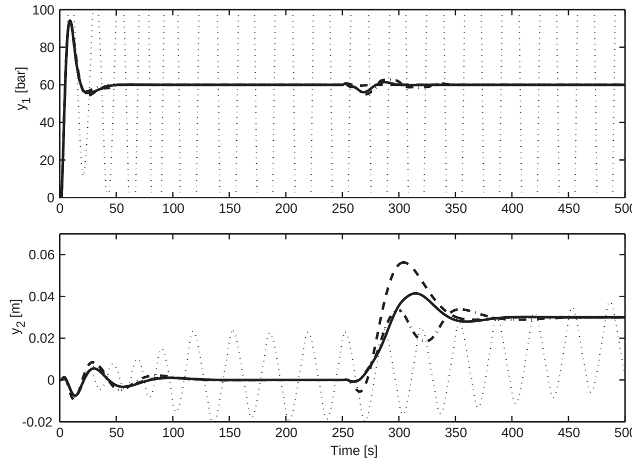

Fig. 9. Reduced order anti-windup response (dash–dotted) compared to the full order anti-windup response (solid), to the unconstrained response (dashed) and to the saturated response without anti-windup (dotted).

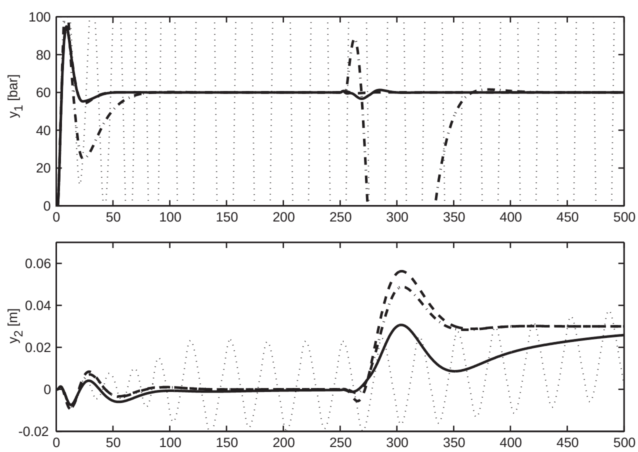

Fig. 8. Anti-windup responses with different selections of the performance output: anti-windup response with $z \approx y_1$ ($z = [y_1 \;\; 0.05y_2]'$, solid); anti-windup response with $z \approx y_2$ ($z = [10^{-5}y_1 \;\; y_2]'$, dash–dotted); unconstrained response (dashed); saturated response without anti-windup (dotted).

by selecting different diagonal entries in matrix R for each input.

Output tracking performances can be tuned by changing the selection of the performance output z. Fig. 8 shows the closed-loop output responses when selecting first $z \approx y_1$ ($z = [y_1 \;\; 0.05y_2]'$, solid curves) and then $z \approx y_2$ ($z = [10^{-5}y_1 \;\; y_2]'$, dash–dotted curves). Once again, the unconstrained system response corresponds to the dashed curves and the saturated system response without anti-windup corresponds to the dotted curve. Note that both the anti-windup responses show an improved behavior on the performance output (the one most considered in the performance figure) and, respectively, a degradation of the other output (which is almost disregarded in the optimization).

Finally, to investigate the robustness properties of the sampled-data anti-windup scheme, we compare the anti-windup responses when the associated discrete-time optimal control problem is based on a reduced-order model of the plant, instead of the full-order model considered so far. The reduced-order model is obtained by removing the four least significant states from a balanced realization of the full-order one. The arising piecewise affine discrete-time control law is defined over 27 regions in the state and input spaces $\mathbb{R}^5 \times \mathbb{R}^3$. Because of the reduced space dimension, the corresponding controller matrices require approximately 30% less space than those of the full-order controller to be stored in the controller memory. Simulation comparisons are shown in Fig. 9, where $z = [y_1 \;\; y_2]'$ is used both for the response of the full-order anti-windup (solid curves) and for the response of the reduced-order anti-windup (dash–dotted curves). Note that due to the model reduction, the anti-windup closed-loop system response exhibits a worse tracking of the unconstrained response. However, the overall performance is still more than satisfactory and the closed-loop stability is retained, thus showing the robustness properties of the proposed anti-windup scheme.

5. Proof of Theorem 1

Proof of Theorem 1. Define $(x_\ell, x_{c\ell})$ as the response of the perturbed unconstrained closed-loop system (3)–(6). Also denote by $u_\ell = y_{c\ell}$ the corresponding controller output, and by z_ℓ the corresponding performance output. Then, introduce the following variables:

$$\varepsilon := \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} := \begin{bmatrix} x - x_\ell - x_{aw} \\ x_c - x_{c\ell} \end{bmatrix}. \qquad (25)$$

After some computations, the following behavior results for the dynamics of the ε variables:[4]

$$\dot{\varepsilon} = A_{\rm cl}\varepsilon + \begin{bmatrix} \Delta_x + BD_{\rm cu}(I - D_y D_{\rm cu})^{-1}\Delta_y \\ B_{\rm c}(I - D_y D_{\rm cu})^{-1}\Delta_y \end{bmatrix} \begin{bmatrix} x - x_\ell \\ u - u_\ell \\ 0 \end{bmatrix} \quad (26)$$

where $A_{\rm cl}$ is the closed-loop matrix of the unconstrained closed-loop system (3), (5), (6), with $\Psi \equiv 0$, which is Hurwitz by Assumption 1. First note that, based on (25) and on (17), the following relations hold:

$$y_{\rm c} - y_{{\rm c}\ell} = (I - D_{\rm cu}D_y)^{-1}(C_{\rm c}\varepsilon_2 + D_{\rm cu}C_y\varepsilon_1) + D_{\rm cu}(I - D_y D_{\rm cu})^{-1}\Delta_y \begin{bmatrix} x - x_\ell \\ u - u_\ell \\ 0 \end{bmatrix}, \quad (27)$$

$$x - x_\ell = \varepsilon_1 + x_{\rm aw}, \quad (28)$$

$$z - z_\ell = z_{\rm aw} + C_z\varepsilon_1 + D_z(y_{\rm c} - y_{{\rm c}\ell}) + \Delta_z \begin{bmatrix} x - x_\ell \\ u - u_\ell \\ 0 \end{bmatrix}. \quad (29)$$

To show item 1 of Property 1, note that by item 3 of Property 2, $\kappa(0,\cdot) \equiv 0$. Hence, since $u_\ell(t) = {\rm sat}(u_\ell(t))$ for all $t \geqslant 0$, it is easy to verify that $(x(t), x_{\rm c}(t), x_{\rm aw}(t)) = (x_\ell(t), x_{{\rm c}\ell}(t), 0)$ is a solution. Since the saturation function is globally Lipschitz and the unmodeled dynamics (4) are LTI, then the right-hand side of the overall system is globally Lipschitz and solutions are unique and defined for all times. Finally, by uniqueness, the one above is the only solution, which by (25) implies $(x(t), x_{\rm c}(t)) = (x_\ell(t), x_{{\rm c}\ell}(t))$ at all times and item 1 of Property 1 follows from Eqs. (29) and (17).

In the rest of the proof we will show item 2 of Property 1, based on a small gain argument where the $\mathscr{L}_2$ gain γ_Δ of (4) is restricted to be sufficiently small. Given any feasible reference $r_{\rm o}$, by item 2 of Property 2, we have

$$\begin{aligned} &{\rm sat}(\kappa(x_{\rm aw}, r_{\rm o}) + y_{{\rm c},\infty}) - y_{{\rm c},\infty} \\ &\quad = \kappa(x_{\rm aw}, r_{\rm o}) =: \kappa_{\rm o}(x_{\rm aw}), \quad \forall x_{\rm aw} \in \mathbb{R}^n. \end{aligned} \quad (30)$$

We rewrite the continuous time anti-windup compensator dynamics as

$$\dot{x}_{\rm aw}(t) = Ax_{\rm aw}(t) + B\kappa_{\rm o}(x_{\rm aw}(t_{\rm s}(t))) + Bw(t), \quad (31a)$$

$$z_{\rm aw}(t) = C_z x_{\rm aw}(t) + D_z\kappa_{\rm o}(x_{\rm aw}(t_{\rm s}(t))) + D_z w(t), \quad (31b)$$

where

$$\begin{aligned} w(t) := {} & {\rm sat}(\kappa(x_{\rm aw}(t_{\rm s}(t)), r_{\rm a}(t_{\rm s}(t))) + y_{\rm c}(t)) \\ & - y_{\rm c}(t) - \kappa_{\rm o}(x_{\rm aw}(t_{\rm s}(t))) \end{aligned} \quad (31c)$$

and augment the continuous-time dynamics with the following sampled-data components:

$$\begin{aligned} & t_{\rm s}(t) = \lfloor t + \sigma \rfloor_T - \sigma, \quad \sigma \in [0, T), \\ & t_k = t_{\rm s}((k+1)T), \\ & \xi_{\rm aw}(k) = x_{\rm aw}(t_k). \end{aligned} \quad (31d)$$

To complete the proof, it will be useful to establish the global, finite gain $(\mathscr{L}_2, \ell_2)$ stability from w to $(x_{\rm aw}, \xi_{\rm aw})$ for the sampled-data system (31), by which we mean that there exists $c > 0$ such that, for each $\sigma \in [0, T)$, $x_{\rm aw}(0) \in \mathbb{R}^n$, $x_{\rm aw}(t_{\rm s}(0)) \in \mathbb{R}^n$, $w \in \mathscr{L}_2$,

$$\begin{aligned} & \max\{\|x_{\rm aw}(\cdot)\|_{\mathscr{L}_2}, \|\xi_{\rm aw}(\cdot)\|_{\ell_2}\} \\ & \quad \leqslant c(|x_{\rm aw}(0)| + |x_{\rm aw}(t_{\rm s}(0))| + \|w(\cdot)\|_{\mathscr{L}_2}). \end{aligned} \quad (32)$$

To prove (32), we apply Theorem 1 of Zaccarian et al. (2003) to the sampled-data system (31), based on the global Lipschitz property of the right-hand side of (31a), (31b) and on items 1 and 3 of Property 2, which imply that $\kappa_{\rm o}(\cdot)$ is globally Lipschitz and that the discrete-time system corresponding to (31) with $t_{\rm s}(0) = 0$ and $w \equiv 0$, which is given by

$$\xi_{\rm aw}^+ = {\rm e}^{AT}\xi_{\rm aw} + \left(\int_0^T {\rm e}^{A(T-\tau)}B\,{\rm d}\tau\right)\kappa_{\rm o}(\xi_{\rm aw}) \quad (33)$$

has a globally exponentially stable equilibrium point at the origin.

For each $t \geqslant 0$, define $v_1(t) = \kappa(x_{\rm aw}(t_{\rm s}(t)), r_{\rm a}(t_{\rm s}(t)))$. Then, based on Eq. (31c) and by the Lipschitz properties of $\kappa_{\rm o}(\cdot)$, there exists $L_\kappa > 0$ such that $|{\rm sat}(v_1(t) + y_{\rm c}(t)) - y_{\rm c}(t)| = |w(t) + \kappa_{\rm o}(x_{\rm aw}(t_{\rm s}(t)))| \leqslant |w(t)| + L_\kappa|x_{\rm aw}(t_{\rm s}(t))|$, which can be integrated on both sides to get[5]

$$\|{\rm sat}(v_1 + y_{\rm c}) - y_{\rm c}\|_{\mathscr{L}_2} \leqslant \|w\|_{\mathscr{L}_2} + TL_\kappa\|\xi_{\rm aw}\|_{\ell_2}. \quad (34)$$

Moreover, by Eq. (27) and the $\mathscr{L}_2$ gain assumptions on the unmodeled dynamics (4), there exist positive numbers L_ε and L_Δ, such that[6]

$$\|y_{\rm c} - y_{{\rm c}\ell}\|_{\mathscr{L}_2} \leqslant L_\varepsilon\|\varepsilon\|_{\mathscr{L}_2} + \gamma_\Delta L_\Delta \left\|\begin{matrix} x - x_\ell \\ u - u_\ell \end{matrix}\right\|_{\mathscr{L}_2} \quad (35)$$

[4] With a slight abuse of notation, we will use in the following Δ_x, Δ_y and Δ_z as a shortcut for the output Ψ of a state–space realization of the unmodeled dynamics (4).

[5] For simplicity, we drop the terms that would arise due to the deviation of the initial condition from the steady-state condition. These terms would appear in an additive manner with a linear gain.

[6] In the following, whenever confusion may not arise, we will sometimes denote signals and functions omitting the time dependence or the symbol "$(\cdot)$."

and by the Lipschitz property of the saturation function and (34), we also get

$$\begin{aligned}\|u-u_\ell\|_{\ell_2} &= \|\mathrm{sat}(y_{\rm c}+v_1)-y_{{\rm c}\ell}\|_{\mathscr{L}_2}\\ &\leqslant \|\mathrm{sat}(y_{\rm c}+v_1)-y_{\rm c}\|_{\mathscr{L}_2}+\|y_{\rm c}-y_{{\rm c}\ell}\|_{\mathscr{L}_2}\\ &\leqslant \|w\|_{\mathscr{L}_2}+TL_\kappa\|\xi_{\rm aw}\|_{\ell_2}+\|y_{\rm c}-y_{{\rm c}\ell}\|_{\mathscr{L}_2}. \quad (36)\end{aligned}$$

Consider now items 1 and 2 of Property 2, and the globally Lipschitz property of the saturation function defined in Assumption 2. Then, substituting (30) into (31c), there exists L_κ such that

$$\begin{aligned}|w(t)| &\leqslant 2|y_{\rm c}(t)-y_{{\rm c},\infty}|+L_\kappa|r_{\rm a}(t_{\rm s}(t))-r_{\rm o}|\\ &\leqslant 2|y_{{\rm c}\ell}(t)-y_{{\rm c}\infty}|+2|y_{\rm c}(t)-y_{{\rm c}\ell}(t)|\\ &\quad +L_\kappa|r_{\rm a}(t_{\rm s}(t))-r_{\rm o}|. \quad (37)\end{aligned}$$

Recalling that $t_{\rm s}(t)=\lfloor t+\sigma\rfloor_T-\sigma$ and $t_k=t_{\rm s}((k+1)T)$ and using Holder's inequality to go from (38a) to (38b) below, we also have by the definition in (14):

$$\begin{aligned}\|r_{\rm a}(t_{\rm s}(\cdot))-r_{\rm o}\|^2_{\mathscr{L}_2} &= \int_0^\infty |r_{\rm a}(t_{\rm s}(t))-r_{\rm o}|^2\,{\rm d}t\\ &\leqslant T\sum_{k\in\mathbb{Z}_{\geqslant -1}}|r_{\rm a}(t_k)-r_{\rm o}|^2\\ &= \frac{1}{T}\sum_{k\in\mathbb{Z}_{\geqslant 0}}\left|\int_{t_{k-1}}^{t_k}(r(t)-r_{\rm o})\,{\rm d}t\right|^2\\ &\quad +T|r_{\rm a}(t_{-1})-r_{\rm o}|^2 \quad (38a)\end{aligned}$$

$$\leqslant \sum_{k\in\mathbb{Z}_{\geqslant 0}}\int_{t_{k-1}}^{t_k}|r(t)-r_{\rm o}|^2\,{\rm d}t+T|r_{\rm a}(t_{-1})-r_{\rm o}|^2 \quad (38b)$$

$$\leqslant \|r(\cdot)-r_{\rm o}\|^2_{\mathscr{L}_2}+\int_{t_{-1}}^0|r(t)-r_{\rm o}|^2\,{\rm d}t+T|r_{\rm a}(t_{-1})-r_{\rm o}|^2.$$

The proof will be completed based on a small gain argument. For simplicity, with a slight abuse of notation, we will use the symbol c to denote any positive constant (so that, for example, $c+c=c$). First note that Eqs. (29), (32) and (34) can be combined, also based on the definition in (17) to get

$$\|z-z_\ell\|_{\mathscr{L}_2}\leqslant c\left(\|w\|_{\mathscr{L}_2}+\|\varepsilon\|_{\mathscr{L}_2}+\|y_{\rm c}-y_{{\rm c}\ell}\|_{\mathscr{L}_2}+\left\|\begin{matrix}x-x_\ell\\ u-u_\ell\end{matrix}\right\|_{\mathscr{L}_2}\right). \quad (39)$$

So in the rest of the proof, we will show via a small gain argument that there is a finite $\mathscr{L}_2$ gain from $(r-r_{\rm o},d)(\cdot)$ to all the terms on the right-hand side of (39). Based on bounds (37) and (38), we have

$$\begin{aligned}\|w(\cdot)\|_{\mathscr{L}_2} &\leqslant 2\|y_{{\rm c}\ell}(\cdot)-y_{{\rm c}\infty}\|_{\mathscr{L}_2}\\ &\quad +2\|(y_{\rm c}-y_{{\rm c}\ell})(\cdot)\|_{\mathscr{L}_2}+L_\kappa\|r(\cdot)-r_{\rm o}\|_{\mathscr{L}_2}, \quad (40)\end{aligned}$$

which, together with (32), can be used in (28) and (36) to get

$$\left\|\begin{matrix}x-x_\ell\\ u-u_\ell\end{matrix}\right\|_{\mathscr{L}_2}\leqslant c\left(\|\varepsilon\|_{\mathscr{L}_2}+\|y_{\rm c}-y_{{\rm c}\ell}\|_{\mathscr{L}_2}+\|y_{{\rm c}\ell}-y_{{\rm c}\infty}\|_{\mathscr{L}_2}+\|r-r_{\rm o}\|_{\mathscr{L}_2}+\gamma_\Delta\left\|\begin{matrix}x-x_\ell\\ u-u_\ell\end{matrix}\right\|_{\mathscr{L}_2}\right). \quad (41)$$

Hence, by the small gain theorem applied to Eqs. (35) and (41), if γ_Δ is sufficiently small, then

$$\left\|\begin{matrix}x-x_\ell\\ u-u_\ell\\ y_{\rm c}-y_{{\rm c}\ell}\end{matrix}\right\|_{\mathscr{L}_2}\leqslant c(\|\varepsilon\|_{\mathscr{L}_2}+\|y_{{\rm c}\ell}-y_{{\rm c}\infty}\|_{\mathscr{L}_2}+\|r-r_{\rm o}\|_{\mathscr{L}_2}). \quad (42)$$

Since $A_{\rm cl}$ is Hurwitz and $\Delta(s)$ is asymptotically stable, by Eq. (26) and the $\mathscr{L}_2$ gain assumption on (4), there is a finite gain, which is proportional to γ_Δ from $(x-x_\ell,u-u_\ell)(\cdot)$ to $\varepsilon(\cdot)$, namely

$$\|\varepsilon(\cdot)\|_{\mathscr{L}_2}\leqslant c\gamma_\Delta\left\|\begin{matrix}(x-x_\ell)(\cdot)\\ (u-u_\ell)(\cdot)\end{matrix}\right\|_{\mathscr{L}_2}, \quad (43)$$

and applying once again the small gain theorem to (42) and to (43), if γ_Δ is sufficiently small, then

$$\left\|\begin{matrix}(x-x_\ell)(\cdot)\\ (u-u_\ell)(\cdot)\\ (y_{\rm c}-y_{{\rm c}\ell})(\cdot)\\ \varepsilon(\cdot)\end{matrix}\right\|_{\mathscr{L}_2}\leqslant c(\|y_{{\rm c}\ell}(\cdot)-y_{{\rm c}\infty}\|_{\mathscr{L}_2}+\|r(\cdot)-r_{\rm o}\|_{\mathscr{L}_2}). \quad (44)$$

Also note that, by the asymptotic stability of the linear unconstrained closed-loop system (3), (5), (6) stated in Assumption 1 and by Definition 1, there exists a small enough bound γ_Δ on the $\mathscr{L}_2$ gain of the unmodeled dynamics such that for any feasible reference $r_{\rm o}$ there is a finite $\mathscr{L}_2$ gain from $(r-r_{\rm o},d)(\cdot)$ to $(y_{{\rm c}\ell}-y_{{\rm c}\infty}(r_{\rm o}))(\cdot)$. Hence, combining this result with (39), (40) and (44), the proof follows. □

6. Conclusions

In this paper, we employed receding horizon optimal control (RHOC) in a model-based anti-windup architecture.

The anti-windup augmentation scheme consists of a continuous-time filter whose dynamics are based on an approximate model of the plant and a discrete-time control law which is designed based on a RHOC algorithm. By way of suitable modification signals injected at the input and at the output of the unconstrained controller, desirable stability properties of the augmented closed-loop can be asserted for generic linear control systems for asymptotically stable linear plants. In addition, the optimality properties of the receding horizon controller achieve high-performance anti-windup compensation. The recent results on explicit solutions to the constrained RHOC problem can be employed to allow the implementation of these techniques in high-speed control systems. To correctly represent the continuous- and discrete-time nature of the augmentation scheme, the system has been analyzed as a sampled-data linear system. The performance of the resulting construction has been studied on a simulation example.

Future work will include establishing bounds on the level of performance in terms of the input–output gain of the scheme with anti-windup augmentation. To this aim, a useful tool may be represented by the RHOC construction for systems with disturbance inputs, where the disturbance is accessible for measurement (see, e.g., (Mayne et al., 2000)), as this is the kind of setup that the anti-windup design reduces to when applying the sampled-data solution proposed here.

Appendix A. Proof of Lemmas 2 and 3

Before proving Lemma 2, we need the following well-known fact which can be proved, for example, by appealing to Corollary 3.7 in Clarke, Stern, and Wolenski (1993) which characterizes the global Lipschitz property in terms of the (local) lower Dini derivative.

Lemma A.1. *Let $f:\mathbb{R}^n \mapsto \mathbb{R}$ be a continuous and piecewise affine function, defined over a finite partition $\mathscr{P}$ of $\mathbb{R}^n$. Then f is globally Lipschitz.*

Proof of Lemma 2. Define $\theta := [\xi', y'_{c,\infty}]' \in \mathbb{R}^{n+n_u}$, and express problem (22) as the multiparametric quadratic program (mp-QP) Bemporad et al., 2002

$$\phi^*(\theta) = \frac{1}{2}\theta' Y\theta + \min_{\mathscr{V}} \frac{1}{2}\mathscr{V}' H\mathscr{V} + \theta' F\mathscr{V}$$
$$\text{s.t. } G\mathscr{V} \leqslant W + S\theta, \tag{A.1}$$

where $\phi^*:\mathbb{R}^{n_u+n} \mapsto \mathbb{R}$, the Hessian matrix $H = H' > 0$, and H, F, Y, G, W, S are easily obtained from (22).

Solving the mp-QP (A.1) amounts to determining the optimizer $\mathscr{V}^*$ and the value function ϕ^* as a function of the parameter $\theta \in \Theta$, where $\Theta \subseteq \mathbb{R}^{n+n_u}$ is a given set. We recall here from Bemporad et al. (2002) the main properties enjoyed by the solution to multiparametric quadratic programs.

Theorem A.1 (Bemporad et al., 2002). *Consider the multiparametric quadratic program (A.1) with $H > 0$ and Θ convex. Then the set of feasible parameters $\Theta_f \subseteq \Theta$ is convex, the optimizer $\mathscr{V}^*:\Theta_f \mapsto \mathbb{R}^{n_u N}$ is continuous and piecewise affine, and the optimal value $\phi^*:\Theta_f \mapsto \mathbb{R}$ is continuous, convex and piecewise quadratic.*

Remark A.1. Two different algorithms for solving mp-QP problems are described in Bemporad et al. (2002) and Tøndel et al. (2003), respectively. While the algorithm in Bemporad et al. (2002) assumes Θ to be a compact set, the approach of Tøndel et al. (2003), besides being computationally more efficient, allows $\Theta = \mathbb{R}^{n+n_u}$, and will be therefore used in this paper to characterize the optimizer $\mathscr{V}^*(\theta)$ globally.

By Theorem A.1, $\mathscr{V}^*(\theta)$ is piecewise affine and continuous, and therefore the same properties hold for its first component $\bar{\kappa}(\xi, y_{c,\infty})$. As a sequence $\mathscr{V}$ which is feasible for the constraints in (22) exists for all $\xi \in \mathbb{R}^n$ and for all $y_{c,\infty} \in \mathbb{R}^{n_u}$, problem (22) always admits an optimal solution, which implies that $\kappa(\cdot,\cdot)$ is defined over a partition of the whole space $\mathbb{R}^{n_u+n}$. Moreover, since the number n_r of affine functions constituting the multiparametric solution $\mathscr{V}^*(\theta)$ is smaller or equal than the number 2^{2Nn_u} of possible combinations of active constraints at optimality, then this number is finite. Hence, by Lemma 4, function (23) is globally Lipschitz. □

Proof of Lemma 3. Given a value for $y_{c,\infty}$ that satisfies Eq. (10), consider the optimization problem (22), and note that it is well defined for all $\xi \in \mathbb{R}^n$, because the sequence $\mathscr{V} = [0,\dots,0]$ always satisfies the constraints (since $y_{c,\infty}$ is fixed, in the following the dependence on $y_{c,\infty}$ will be omitted for simplicity of notation). Since constraint (19) is included in (22b), the solution to (always feasible) problem (22) satisfies (19).

In the rest of the proof we will show the exponential stability property. If $\mathscr{V}^* := [v_0^*,\dots,v_{N-1}^*]$ denotes the optimizer of (22), by defining $G(\xi) := A_d\xi + B_d v_0^*(\xi)$, and recalling that by Definition 1, $u_m \leqslant y_{c,\infty} \leqslant u_M$, then it follows that the shifted sequence $\mathscr{V}_1 := [v_1^*,\dots,v_{N-1}^*, 0]$ is a feasible (not optimal in general) solution to the optimization problem (22) applied to $G(\xi)$. Namely, $J^*(G(\xi)) \leqslant J(\mathscr{V}_1, G(\xi))$ and by standard direct methods (see, e.g., Mayne et al., 2000), the following holds:

$$J^*(G(\xi)) - J^*(\xi) \leqslant -|\zeta_0|^2 - \lambda_{\min}(R)|v_0^*(\xi)|^2. \tag{A.2}$$

Moreover, by selecting the feasible sequence $\mathscr{V} = [0,\dots,0]$, it is easily seen by (22) that there exists a large enough number M_J such that

$$J^*(\xi) \leqslant (A_d^N \xi)' P A_d^N \xi + \sum_{i=0}^{N-1} |C_z A_d^i \xi|^2 \leqslant M_J |\xi|^2. \tag{A.3}$$

Then, introducing the function $V : \mathbb{R}^n \to \mathbb{R}_{\geqslant 0}$ defined as $V(\xi) := \varepsilon \xi' Q \xi + J^*(\xi)$, (where $Q > 0$ satisfies the Lyapunov equation $A_d' Q A_d - Q = -I$), we will show in the rest of the proof that there exists a small enough $\varepsilon > 0$ such that

$$\varepsilon \lambda_{\min}(Q)|\xi|^2 \leqslant V(\xi) \leqslant (\varepsilon \lambda_{\max}(Q) + M_J)|\xi|^2,$$
$$V(G(\xi)) \leqslant (1 - \varepsilon/2) V(\xi), \tag{A.4}$$

Eqs. (A.4) are sufficient to prove the theorem because $\xi^+ = G(\xi) = A_d \xi + B_d v_0^*(\xi)$ corresponds to dynamics (20) in closed-loop with the RHOC control law (21), (22). Hence, the exponential decreasing property given by the second equation in (A.4) combined with the bounds in the first equation in (A.4) implies the exponential stability of (20)–(22).

The first equation of (A.4) follows directly from (A.3) and from the definition of $V(\cdot)$. To prove the second equation of (A.4), consider the following inequalities, which are easily derived from $A_d' Q A_d - Q = -I$ and from equation (A.2):

$$\begin{aligned} V(G(\xi)) - V(\xi) \leqslant\; & (A_d \xi + B_d v_0^*(\xi))' \varepsilon Q (A_d \xi + B_d v_0^*(\xi)) \\ & - \xi' \varepsilon Q \xi - \lambda_{\min}(R)|v_0^*(\xi)|^2 \leqslant -\varepsilon|\xi|^2 \\ & - \lambda_{\min}(R)|v_0^*(\xi)|^2 + 2\varepsilon(B_d v_0^*(\xi))' Q A_d \xi \\ & + \varepsilon(B_d v_0^*(\xi))' Q B_d v_0^*(\xi), \end{aligned}$$

which, by completing squares in the last two terms, and defining $c_v := 2(|B_d||QA_d|)^2 + |B_d||QB_d|$ (where the norms of matrices are induced norms), can be bounded as $V(G(\xi)) - V(\xi) \leqslant -\varepsilon/2|\xi|^2 - (\lambda_{\min}(R) - \varepsilon c_v)|v_0^*(\xi)|^2$. Then, it is sufficient to pick $\varepsilon < \lambda_{\min}(R)/c_v$ to prove the second equation of (A.4). □

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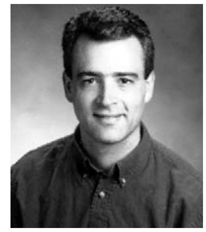

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