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Moving Horizon Estimation for Hybrid Systems and Fault Detection

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Abstract

A new approach for fault detection and state estimation of hybrid systems is presented. The method relies on the modeling framework for hybrid systems introduced by [4]. This framework considers interacting propositional logic, automata, continuous dynamics and constraints. The proposed approach is illustrated by considering the fault detection problem of the three-tank benchmark system.

1 Introduction

Many practical control, estimation and fault detection problems involve hybrid systems, here losely defined as systems involving both continuous and discrete variables. Various approaches have been proposed for modeling hybrid systems [10, 6]. Often engineering systems include "logic" components (eg. if-then-else rules, finite state machine, etc.) which are conveniently described via propositional logic. Moreover, in addition to a quantitative system description there might be some available qualitative information about the behavior of the system, for instance in terms of heuristic knowledge.

Recently it was shown [4, 15] that expressing logical propositions in the form of linear constraints on integer variables leads to a powerful modeling framework, the so called *mixed logical dynamical* (MLD) form. It allows to describe a broad number of important classes of systems, like piecewise linear systems, systems with mixed discrete/continuous inputs and states, and many others more [4]. The framework permits to include and prioritize constraints, and incorporate heuristic rules in the description of the model.

In this work we first show that the MLD form can be used as a new tool to model systems with faults. Next we define the moving horizon estimation problem, which can be considered dual to (receding horizon) model predictive control [9]. At each time step we solve a least squares estimation problem over a finite horizon backwards from the current time. The resulting optimization problem is a mixed-integer quadratic program (MIQP), for which efficient solvers exist. We propose to use this moving horizon estimation formulation to estimate states and faults of hybrid systems in the MLD form.

To demonstrate the effectiveness of the modeling approach and the fault detection scheme, we apply the method to a well studied problem in the fault detection literature, namely the three tank benchmark system [11].

In the form presented in this paper the moving horizon estimator does not address all relevant issues in state estimation and fault detection. For example, it does not explicitly take into account any stochastic aspects. Nevertheless, it is a new and promising method to deal with state estimation and fault detection for the broad class of hybrid systems.

The paper is organized as follows. In Section 2 we recall the general MLD form. In Section 3 we present a first principles model of the three tank system and we derive its MLD form. Since this derivation is tedious and involves the application of a set of fixed rules, a compiler was developed for the translation into the MLD form. In Section 4 we present the compilable definition of the three tank system. In Section 5 we extend the ideas of [4] to the fault detection and the state estimation problem. The ideas are illustrated with simulations of the three tank system, which are presented in Sections 6 and 7. A few remarks about the computational aspects are given in Section 8. A more detailed version of this paper can be found in [1].

2 The Mixed Logic Dynamic Form

The mixed logical dynamic (MLD) form has been introduced in [4]. The general MLD form is:

$$x(t+1) = Ax(t) + B_1u(t) + B_2\delta(t) + B_3z(t) \quad (1a)$$

$$y(t) = Cx(t) + D_1u(t) + D_2\delta(t) + D_3z(t) \quad (1b)$$

$$E_2\delta(t) + E_3z(t) \leq E_1u(t) + E_4x(t) + E_5 \quad (1c)$$

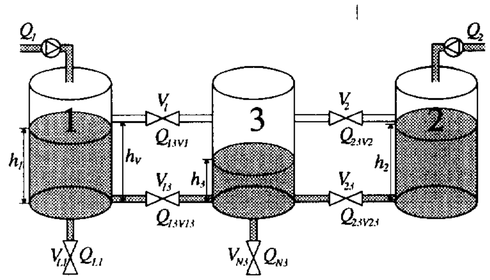

Figure 1: COSY Three-Tank Benchmark.

x are the continuous and binary states, u are the continuous and binary inputs, δ and z represent binary and continuous auxiliary variables. The latter are introduced when translating logic propositions into linear inequalities. All constraints are summarized in the inequality (1c). The description (1) only appears to be linear; the variables δ are constrained to be binary.

3 Model of Three Tank System

The three tank system represented in Fig. 1 has been adopted recently as a standard benchmark problem for fault detection and reconfigurable control [11, 5]. Here we report a simplified physical description of the system (more details can be found in [7]). From the conservation of mass in the tanks we obtain the differential equations

$$\dot{h}_1 = \frac{1}{A}(Q_1 - Q_{13V1} - Q_{13V13} - Q_{L1}) \quad (2)$$

$$\dot{h}_2 = \frac{1}{A}(Q_2 - Q_{23V2} - Q_{23V23}) \quad (3)$$

$$\dot{h}_3 = \frac{1}{A}(Q_{13V1} + Q_{13V13} + Q_{23V2} + Q_{23V23} - Q_N) \quad (4)$$

where the Q's denote flows and A is the cross-sectional area of each of the tanks [11]. By Torricelli's law, the flow through a lower valve V_{i3} $(i = 1,2)$ is

$$Q_{i3Vi3} = V_{i3} a_z S_{i3} \,\mathrm{sign}(h_i - h_3)\sqrt{|2g(h_i - h_3)|} \quad (5)$$

The flow through the valve V_{L1} (V_{N3}) is obtained by setting h_1 (h_3) in place of $(h_i - h_3)$ in (5), and through the upper valves V_i by setting $\max\{h_v, h_j\}$ in place of h_j, $(j = 1,2,3)$. In order to express the physical model (3)–(5) in the MLD form (1), we approximate the nonlinearity in (5) with a straight line, as follows:

$$Q_{i3Vi3} \approx k_{i3} V_{i3}(h_i - h_3), \; k_{i3} \triangleq a_z S_{i3}\sqrt{\frac{2g}{h_{\max}}} \quad (6)$$

Note that more accurate approximations of the square root could be used. According to [4] we have that:

$$[f(x) \le 0] \leftrightarrow [\delta = 1] \text{ is true iff } \begin{cases} f(x) \le M(1-\delta) \\ f(x) \ge \epsilon + (m - \epsilon)\delta \end{cases} \quad (7)$$

where M and m are upper and lower bounds on $f(x)$. Another fact we take from [4] is :

$$z = \delta f(x) \text{ is equivalent to } \begin{cases} z \le M\delta \\ z \ge m\delta \\ z \le f(x) - m(1-\delta) \\ z \ge f(x) - M(1-\delta) \end{cases} \quad (8)$$

(7) and (8) are used to obtain the MLD form. By introducing the auxiliary variables $z_{i3} = V_{i3}(h_i - h_3)$ $(i = 1,2)$, and (8), Eq. (6) can be expressed through mixed-integer linear inequalities. In order to take into account the flows through the upper valves V_1, V_2, define for $i = 1,2,3$ the auxiliary binary variables

$$[\delta_{0i}(t) = 1] \leftrightarrow [h_i(t) \ge h_v] \quad (9)$$

and continuous variables

$$z_{0i} \triangleq \max\{h_v, h_i\} - h_v = \delta_{0i}(h_i - h_v). \quad (10)$$

Then, for $i = 1,2$, and $z_i \triangleq V_i(z_{0i} - z_{03})$

$$Q_{i3Vi} \approx k_i z_i, \qquad k_i \triangleq a_z S_i \sqrt{\frac{2g}{h_{\max} - h_v}}$$

Similarly, one has $Q_{L1} \approx k_{L1} z_{L1}$ and $Q_{N3} \approx k_{N3} z_{N3}$, where k_{L1}, k_{N3} depend on S_{L1}, S_{N3} respectively and are defined as in (6), where $z_{L1} \triangleq V_{L1} h_1$ and $z_{N3} \triangleq V_{N3} h_3$.

In addition, h_i, Q_j must fulfill the operating constraints

$$0 \le h_i \le h_{\max}, \; (i = 1,2,3) \quad 0 \le Q_j \le Q_{\max}, \; (j = 1,2).$$

Finally, the differential equations (3) are discretized by replacing $\dot{h}_i(t)$ by $\frac{h_i(t+1) - h_i(t)}{T_s}$, where T_s is the sample time. Defining

$$\begin{aligned} x &\triangleq [h_1 \; h_2 \; h_3]' \\ u &\triangleq [Q_1 \; Q_2 \; V_{13} \; V_{23} \; V_1 \; V_2 \; V_{L1} \; V_{N3}\,]' \\ \delta &\triangleq [\delta_{01} \; \delta_{02} \; \delta_{03}\,]' \\ z &\triangleq [z_{13} \; z_{23} \; z_{01} \; z_{02} \; z_{03} \; z_1 \; z_2 \; z_{L1} \; z_{N3}]' \end{aligned}$$

one obtains the form (1). Tank 2 is used only for reconfiguration purposes.

4 System Description in HYSDEL

The transformation of first principles hybrid system descriptions, like (2) to (4) into the MLD form requires the application of a set of given rules. It is therefore a task that is preferably automated. To avoid the tedious procedure of deriving the MLD form by hand, a compiler is currently under development that generates the matrices A, B_i, C, D_i and E_i in (1). The problem specification language to the compiler is HYSDEL (HYbrid System DEscription Language). In Fig. 2 we report the description of the three-tank system developed in the previous section in HYSDEL.

```
% Description of variables and constants
state h1,h2,h3;                                  % Tank levels
input Q1,Q2;                                     % Input flows
input V1,V2,V13,V32,VL1,VN3;                     % Valves
const  A,Ts, k1, k2, kl, hv,hmax,Qmax,e;  % Constants

% Variable types
real  h1,h2, h3,z13,z32,z01,z02,z03, z1,z2,zl1,zn,Q1,Q2;
logic V1,V2,V13,V23,VL1,VN3,d01,d02,d03;

% Relations
d01 = {h1-hv >= 0, M1, m1, e};
d02 = {h2-hv >= 0, M1, m1, e};
d03 = {h3-hv >= 0, M1, m1, e};
z13 = V13*(h1-h3)  {hmax, -hmax, e};
z23 = V23*(h2-h3)  {hmax, -hmax, e};
z01 = d01*(h1-hv)  {hmax-hv, 0, e};
z02 = d02*(h2-hv)  {hmax-hv, 0, e};
z03 = d03*(h3-hv)  {hmax-hv, 0, e};
z1  = V1*(z01-z03) {hmax-hv, hv-hmax, e};
z2  = V2*(z02-z03) {hmax-hv, hv-hmax, e};
zl1 = VL1*h1       {hmax, 0, e};
zn  = VN3*h3       {hmax, 0, e};

% Other constraints
must h1 <= hmax, h2 <= hmax, h3 <= hmax;
must h1 >= 0,    h2 >= 0,    h3 >= 0;
must Q1 <= Qmax, Q2 <= Qmax;
must Q1 >= 0,    Q2 >= 0;

% Update
update h1 = h1+Ts/A*(Q1-k2*z1-k1*z13-kl*zl1);
update h2 = h2+Ts/A*(k2*z1+k1*z13+k2*z2+k1*z32-k1*zn);
update h3 = h3+Ts/A*(Q2-k1*z32-k2*z2);
```

Figure 2: HYSDEL description of the three-tank system.

5 Moving Horizon Estimation for MLD Systems

We mentioned above that the MLD framework (1) can be used for the synthesis of model predictive controllers [4]. The method requires the solution of an MIQP at each sample time. The dual problem, i.e. the moving horizon estimation problem [14, 13] can also be formulated in terms of the iterative solution of MIQPs. The goals of such an estimation can be state estimation, fault detection, disturbance estimation. The common feature in all these problems is the minimization of a quadratic cost function involving the quantities to be estimated. Contrary to the control problem, the estimation horizon extends backwards in time, allowing at time t to estimate the quantities of interest at times prior to t.

In the following we consider moving horizon estimation for the purpose of fault detection and state estimation in some detail. The three-tank system presented in Section 3 will be used as a benchmark example for the illustration of the method.

Consider an MLD system, where the occurrence of f faults can be modeled with unmeasured binary disturbances. We assume that the dynamics of the system in the presence of each fault is known. Therefore we extend the MLD model (1) by including three unmeasured variables:

- Fault, i.e. binary disturbance $\phi(t) \in \{0,1\}^f$
- Input disturbance $\xi(t) \in R^n$
- Output disturbance $\zeta(t) \in R^p$

We define the mixed logic dynamic fault (MLDF) form:

$$x(t+1) = Ax(t)+B_1u(t) + B_2\delta(t) + B_3z(t) + B_6\phi(t) + \xi(t) \tag{11a}$$

$$y(t) = Cx(t)+D_1u(t) + D_2\delta(t) + D_3z(t) + D_6\phi(t) + \zeta(t) \tag{11b}$$

$$E_2\delta(t) + E_3z(t) \leq E_1u(t) + E_4x(t) + E_5 + E_6\phi(t) \tag{11c}$$

A moving horizon estimator for (11) can be formulated as follows. At time t we know the last T input and output data $U(t)$ and $Y(t)$:

$$\begin{aligned} U(t) &= [u(t-T), u(t-T+1), \ldots, u(t-1), u(t)] \\ Y(t) &= [y(t-T), y(t-T+1), \ldots, y(t-1), y(t)] \end{aligned}$$

and the estimates $\hat{Z}(t-1)$, $\hat{\Delta}(t-1)$, $\hat{\Phi}(t-1)$ and $\hat{X}(t-1)$ from the estimation at time $t-1$:

$$\begin{aligned} \hat{Z}(t-1) &= [\hat{z}(t-T|t-1), \hat{z}(t-T+1|t-1), \ldots, \hat{z}(t-2|t-1)] \\ \hat{\Delta}(t-1) &= [\hat{\delta}(t-T|t-1), \hat{\delta}(t-T+1|t-1), \ldots, \hat{\delta}(t-2|t-1)] \\ \hat{\Phi}(t-1) &= [\hat{\phi}(t-T|t-1), \hat{\phi}(t-T+1|t-1), \ldots, \hat{\phi}(t-2|t-1)] \\ \hat{X}(t-1) &= [\hat{x}(t-T|t-1), \hat{x}(t-T+1|t-1), \ldots, \hat{x}(t-1|t-1)] \end{aligned}$$

At time t we can consider the following estimate evolution:

$$\left\{ \begin{aligned} \hat{x}(t-T|t) &\triangleq \hat{x}(t-T|t-1) + \Delta x(t) \\ \hat{x}(t+k+1|t) &= A\hat{x}(t+k|t) + B_1u(t+k) + B_2\hat{\delta}(t+k|t) + \\ &\quad B_3\hat{z}(t+k|t) + B_6\hat{\phi}(t+k|t) + \xi(t+k|t) \\ \hat{y}(t+k|t) &= C\hat{x}(t+k|t) + D_1u(t+k) + D_2\hat{\delta}(t+k|t) + \\ &\quad D_3\hat{z}(t+k|t) + D_6\hat{\phi}(t+k|t) + \zeta(t+k|t) \\ E_2\hat{\delta}(t+k|t)+E_3\hat{z}(t+k|t) &\leq E_4\hat{x}(t+k|t)+E_1u(t+k)+E_5+E_6\hat{\phi}(t+k|t) \end{aligned} \right. \tag{12}$$

for $k = -T, \ldots, -1$. Let us define the optimization variable at time t as:

$$\begin{aligned} \chi_t = [&\Delta x(t), \hat{\delta}(t-T+1|t), \ldots, \hat{\delta}(t-1|t), \hat{z}(t-T+1|t), \ldots, \\ &\hat{z}(t-1|t), \hat{\phi}(t-T+1|t), \ldots, \hat{\phi}(t-1|t), \\ &\xi(t-T+1|t), \ldots, \xi(t-1|t), \zeta(t-T+1|t), \ldots, \zeta(t|t)] \end{aligned}$$

and the cost function at time t as:

$$\begin{aligned} J(\chi_t) = &\|\Delta x(t)\|^2_{Q_9} + \textstyle\sum_{k=-T+1}^{0} \Big(\|\hat{y}(t+k|t) - y(t+k)\|^2_{Q_5} + \\ &\|\zeta(t+k|t)\|^2_{Q_8} + \|\hat{\phi}(t+k|t)\|_{Q_{10}}\Big) + \textstyle\sum_{k=-T+1}^{-1} \Big(\|\hat{x}(t+k|t) - \\ &\hat{x}(t+k|t-1)\|^2_{Q_4} + \|\xi(t+k|t)\|^2_{Q_7}\Big) + \textstyle\sum_{k=-T+1}^{-2} \Big(\|\hat{\delta}(t+k|t) - \\ &\hat{\delta}(t+k|t-1)\|^2_{Q_2} + \|\hat{z}(t+k|t) - \hat{z}(t+k|t-1)\|^2_{Q_3} + \\ &\|\hat{\phi}(t+k|t) - \hat{\phi}(t+k|t-1)\|^2_{Q_6}\Big) \end{aligned} \tag{13}$$

where the matrices Q_i are symmetric, positive semidefinite and have appropriate dimensions. The estimates at time t are obtained by solving the optimization problem:

$$\min_{\chi_t} J(\chi_t) \quad \text{subject to (12)}$$

With the estimates χ_t and with the estimate evolution (12), we can reconstruct the state estimate $\hat{X}(t)$:

$$\hat{X}(t) = [\hat{x}(t-T+1|t), \ldots \hat{x}(t-1|t), \ldots, \hat{x}(t|t)] \quad (14)$$

Note that the optimization problem is an MIQP. Setting Q_2, Q_3, Q_4 and Q_6 to zero in (13) reduces the estimator to an FIR filter with input $U(.)$ and $Y(.)$ and the estimates as output. This is one way to guarantee the stability of the estimator.

6 Simulations of the Three Tank System

The method described in Section 5 was used for the fault detection problem of the tank system. We consider the closed loop system with a PI controller controlling h_1 by manipulating Q_1 and a switching controller controlling h_3 by manipulating V_1. The control aim is to keep level $h_1 = 0.5$ and $h_3 = 0.1$, which cannot be met exactly because of the hysteresis of the switching controller for V_1 [11]. The following 3 types of faults are considered:

- ϕ_1 Leak in tank 1
- ϕ_2 Valve V_1 blocked closed
- ϕ_3 Valve V_1 blocked open

Fault ϕ_1 has already been considered in the modeling of Section 3 as binary input u_2. To model ϕ_2 and ϕ_3 we can "filter" the control signal u_5 to valve V_1 with a processing unit, that introduces the potential faults, see Fig. 3. The actual input to the valve V_1 is a new auxiliary variable $\bar{\delta}$. The

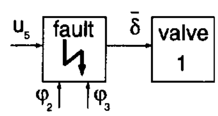

Figure 3: The faults ϕ_2 and ϕ_3 can override the binary control signal u_5 to valve V_1

defining relations for the actuator signal $\bar{\delta}$ can be formulated with logical connectives and then rewritten in conjunctive normal form (CNF). However, the translation of CNF into linear inequalities requires the introduction of additional auxiliary Boolean variables. It is preferable [2] to first build up the truth table for the relations between the involved variables and to find the inequalities delimiting the convex hull of the points corresponding to the rows of the truth table. The truth table is given in table 1. The interpretation of table 1 is that all combinations of $[u_5, \phi_2, \phi_3, \bar{\delta}] \in \{0,1\}^4$ not appearing as a row in table 1 cannot occur and are "invalid". The linear inequalities in table 1 exclude all invalid combinations and are fulfilled by the rows of the truth table. They can be found as described in [2]. The results in Fig. 4 show a simulation, where the leak

Table 1: Truth table for the relations between the control signal to V_1 and the faults and the corresponding linear inequalities

u_5	ϕ_2	ϕ_3	$\bar{\delta}$
0	0	0	0
0	0	1	1
0	1	0	0
1	0	0	1
1	0	1	1
1	1	0	0

$$\Rightarrow \begin{cases} -u_5 & & -\phi_3 & +\bar{\delta} & \leq 0 \\ u_5 & +\phi_2 & -\phi_3 & +\bar{\delta} & \leq 2 \\ & & \phi_3 & -\bar{\delta} & \leq 0 \\ u_5 & -\phi_2 & & -\bar{\delta} & \leq 0 \end{cases}$$

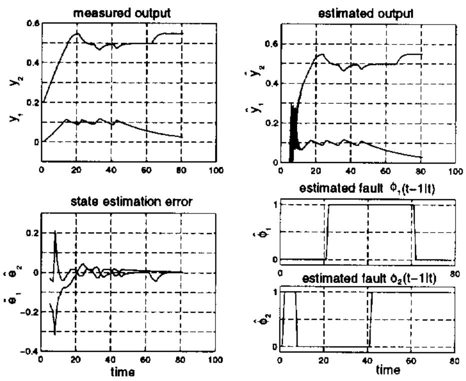

Figure 4: Simulation of a leak ϕ_1 from time $t = 20$ until $t = 60$, and a blocking valve ϕ_2 from time $t = 40$ until $t = 80$.

ϕ_1 and the blocking ϕ_2 occurs at different times. Here we have chosen a horizon of $T = 3$ steps. For $t = 0$ up to $t = 20$ no faults are simulated. From $t = 20$ to $t = 60$ there is a leak in tank 1, whereas from $t = 40$ to $t = 80$ the switching valve blocks. Both faults are detected correctly with a few time steps of delay. Note however that during the startup there are a few false alarms of fault ϕ_2, i.e. blocking of valve V_1. These wrongly detected faults are due to the fact, that the level in tank 1 has not yet reached the height of valve V_1. Therefore no liquid can pass through V_1, which is indistinguishable from a blocked valve V_1. To avoid this problem it is very natural to formulate the clause $[h_1 \leq h_v] \Rightarrow \phi_2 = 0$. According to [4] this additional specification can be translated into the linear inequality $0 \leq x_1 - h_v - m + \phi_2(m - \epsilon)$, which can be added to the MLD constraints

The fault estimates are free of any errors with this correction, as can be seen in Fig. 5. Figure 6 shows a simulation, where the leak is simulated from $t = 20$ until $t = 40$, valve V_1 is stuck closed from $t = 40$ until $t = 60$ and stuck open from $t = 60$ until $t = 80$. The faults are correctly identified with only a few time steps of delay.

7 State Estimation Problem

As we saw in the set up presented in Section 5, it is possible to compute state estimates of a system in

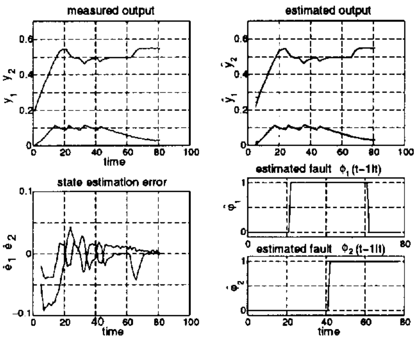

Figure 5: The same simulation as in Fig. 4, with the requirement $[h_1 \leq h_v] \Rightarrow \phi_2 = 0$

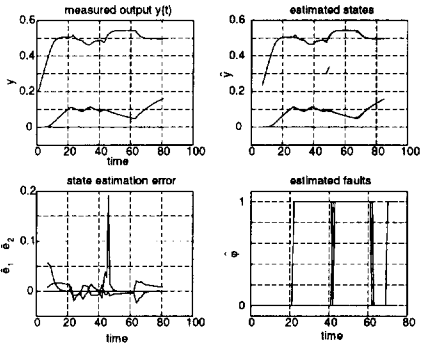

Figure 6: Simulation of the three types of faults in the tank system

MLD form, by solving an MIQP at each time step. To illustrate this, we assume that in the tank system only level h_1 is measured. In Fig. 7 we estimate level h_3 with this measurement.

8 Computational Aspects

The biggest handicap is the computational complexity of the MIQPs, that is exponentially increasing with the number of binary optimization variables δ and ϕ. This complexity is, however, inherent in the problem and not a particular disadvantage of the proposed method. On average, branch and bound algorithms are an efficient way to solve MIQPs [12, 8]. Problem specific knowledge can be incorporated in the node selection strategy and in the branching rule to speed up the computations dramatically [3].

9 Conclusions

We have extended the ideas of receding horizon control to the state estimation and fault detection problem. The three tank system benchmark demonstrated the usefulness of the new methodology. The mixed logic dynamic framework proves to be a convenient modeling tool for hybrid systems, which allows one to solve estimation and fault detection problems effectively.

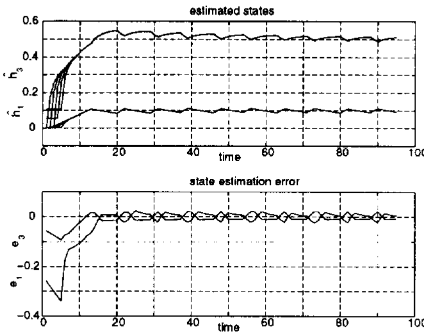

Figure 7: State estimation of h_3 with measurement of level h_1

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