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Control of Constrained Nonlinear Systems via Reference Management

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Abstract

For a broad class of nonlinear continuous-time systems this paper addresses the problem of satisfying input and/or state hard constraints. The approach consists of adding to a primal compensated nonlinear system a Reference Governor (RG). This is a predictive discrete-time device which, taking into account the current value of the state, filters the desired reference trajectory in such a way that a nonlinear primal compensated control system can operate in a stable way with satisfactory tracking performance and no constraint violation. The resulting hybrid system is proved to fulfill the constraints, as well as stability and tracking requirements, and the related computational burden turns out to be moderate and executable with current computing hardware.

1. Introduction

Recently, there has been a significant research interest in feedback control of dynamic systems with input and/or state constraints [1, 2]. Moving horizon optimal control [3, 4, 5] and model predictive control [6, 7] have been proved to be effective tools to deal with tracking problems with input/state constraints. These methods are based on the *receding horizon* philosophy: a sequence of future control actions is chosen according to a prediction of the future evolution of the system and applied to the plant until new measurements are available. Then, a new sequence is evaluated which replaces the previous one. Each sequence is evaluated by means of an optimization procedure which take into account two objectives: maximize the tracking performance, and protect the system from possible constraint violations. However, when applied to models described by nonlinear differential equations, this requires the on-line solution of high dimensional nonlinear optimization problems. In order to drastically reduce the related computational burden, we assume that a primal controller has already been designed to stabilize the system and provide nice tracking properties *in the absence of constraints*, and only consider the constraint fulfillment task. This is left to a *reference governor* (RG), a nonlinear device which is added to the primal compensated nonlinear system. Whenever necessary, the RG modifies the reference supplied to the primal control system so as to enforce the fulfillment of the constraints. The RG operates in accordance with the receding horizon strategy mentioned above, by selecting on-line optimal reference input sequences which are parameterized by a scalar quantity.

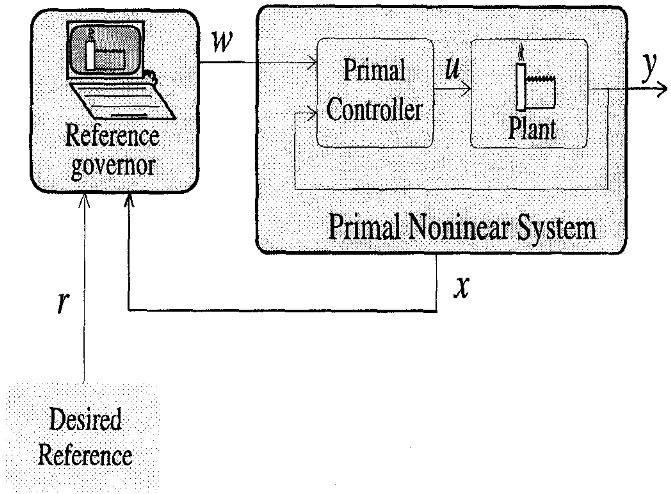

Figure 1: Control scheme with RG.

Previous studies along these lines have already appeared in [8, 9, 10], [11, 12] and [13, 14] for linear control systems.

2. Problem Formulation and Assumptions

Consider the following nonlinear system

$$\begin{cases} \dot{x}(t) &= \Phi(x(t), w(t)) \\ y(t) &= H(x(t), w(t)) \\ c(t) &= \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \end{cases} \tag{1}$$

representing in general a (nonlinear) plant under (nonlinear) feedback, where: $x(t) \in \mathbb{R}^n$ is the state vector, which collects both plant and controller states; $w(t) \in \mathbb{R}^p$ is the reference input, which in the absence of constraints would coincide with a desired reference $r(t) \in \mathbb{R}^p$; $y(t) \in \mathbb{R}^p$ is the output vector which shall track $r(t)$. Since input and/or state variables of the plant can be expressed as a function of $x(t)$ and $w(t)$, without loss of generality we define $c(t) \in \mathbb{R}^{n+p}$ as the vector to be constrained within a given set $\mathcal{C}$.

Assumption 1 *$\mathcal{C}$ is compact and has a nonempty interior.*

Compactness of $\mathcal{C}$ is non restrictive since in practice the desired references and state variables are bounded. Since we are interested in operating on vectors $[x' \; w']'$ in $\mathcal{C}$, we restrict the properties required by system (1) to the projections of $\mathcal{C}$ on the x-space

$$\mathcal{X} \triangleq \left\{ x \in \mathbb{R}^n : \; \exists w \in \mathbb{R}^p, \begin{bmatrix} x \\ w \end{bmatrix} \in \mathcal{C} \right\}$$

and the projection on the w-space $\mathcal{W}$, which is defined analogously. It is easy to show that compactness of $\mathcal{C}$ implies that both $\mathcal{X}$ and $\mathcal{W}$ are compact. System (1) is required to fulfill the following assumptions.

Assumption 2 *$\forall w \in \mathcal{W}$, there exists a unique equilibrium state $x_w \in \mathcal{X}$.*

We denote by

$$X(\cdot) : \mathbb{R}^p \mapsto \mathbb{R}^n \tag{2}$$

the function implicitly defined by $\Phi(X(\cdot), \cdot) = 0$, and define $x_w \triangleq X(w)$, $c_w \triangleq [x_w' \; w']'$. Notice that in general $w \in \mathcal{W} \not\Rightarrow c_w \in \mathcal{C}$.

Assumption 3 *The mapping $\Phi(x, w) : \mathcal{X} \times \mathcal{W} \mapsto \mathbb{R}^n$ is continuous in (x, w).*

Consider now an arbitrarily small scalar $\delta > 0$, and define the following set

$$\widehat{\mathcal{W}}_\delta \triangleq \{w \in \mathcal{W} : \; B(c_w, \delta) \subseteq \mathcal{C}\}. \tag{3}$$

where $B(c_w, \delta)$ denotes the closed ball $\{c \in \mathbb{R}^{n+p} : \|c - c_w\| \leq \delta\}$. We restrict the set of reference inputs w which can be supplied by assuming that

Assumption 4 *(Reference Input Conditioning) The class of reference inputs is restricted to a convex, nonempty, and compact set $\mathcal{W}_\delta \subseteq \widehat{\mathcal{W}}_\delta \subset \mathcal{W}$.*

Assumption 4 is needed to prevent that the border of $\mathcal{C}$ is approached in steady-state, and is required later to prove Theorem 2. The constraint $c \in \mathcal{C}$ and the reference input conditioning can be summarized as the unique constraint

$$c \in \mathcal{C}_\delta \triangleq \mathcal{C} \bigcap (\mathcal{X} \times \mathcal{W}_\delta) \tag{4}$$

where $\mathcal{C}_\delta$ is compact. We fix $\delta > 0$ such that $\mathcal{C}_\delta$ is nonempty. In order to derive the properties proved in Sect. 3., system (1) is supposed to satisfy some extra assumptions.

Assumption 5 *For all piece-wise constant reference input signals $w(t) \in \mathcal{W}_\delta$, $t \in [0, +\infty)$, and for all initial states $x(0) \in \mathcal{X}$, there exists a unique solution $x(t, x(0), w(t))$ of (1) defined $\forall t \in [0, +\infty)$.*

In the following we shall denote by $x(t, x(0), w)$ the solution corresponding to a constant reference $w(t) \equiv w$, $\forall t \in [0, +\infty)$.

Assumption 6 *(Converging Input Converging State Stability) Let $w(t) \to w \in \mathcal{W}_\delta$, and each component of vector $w(t)$ be monotonically convergent. Then, $\forall x(0) \in \mathcal{X}$, $\lim_{t\to\infty} x(t, x(0), w(t)) = x_w$.*

In particular, Assumption 6 ensures that x_w is an asymptotically stable solution of $\dot{x}(t) = \Phi(x(t), w)$.

Assumption 7 *(Uniform-in-$\mathcal{W}_\delta$ Stability) Let $w(t) \equiv w \in \mathcal{W}_\delta$. Then, $\forall \lambda > 0$ there exists $\alpha(\lambda) > 0$ such that $\|x(0) - x_w\| \leq \alpha(\lambda) \;\Rightarrow\; \|x(t, x(0), w) - x_w\| \leq \lambda$, $\forall t \geq 0$, $\forall w \in \mathcal{W}_\delta$.*

The aim of this paper is to design a *Reference Governor* (RG), a discrete-time device which, based on the current state $x(t)$ and desired reference $r(t)$, generates the reference input $w(t)$ so as to satisfy the constraint (4) and minimize the tracking error. The RG operates in discrete-time, in that it is applied every *RG period* T. The reference input $w(t)$ is generated on-line in a predictive manner: at time $t = kT$ a *virtual* reference input signal $\{w(t+\sigma)\}$, $\sigma \in (0, +\infty)$, is selected in such a way that the corresponding predicted evolution $c(t + \sigma, x(t), w(t + \sigma))$ lies within $\mathcal{C}_\delta$, $\forall \sigma > 0$. Then, according to a *receding horizon* strategy, the virtual signal is applied during the following interval $(t, t + T]$; at time $t + T$ a new selection is performed.

Consider the class of virtual constant reference input signals, introduced by [13], which are parameterized by the scalar β and defined by

$$\left\{ \begin{array}{rcl} w(kT + \sigma, \beta) & \equiv & r(kT) + \beta[w((k-1)T) - r(kT)] \\ & \triangleq & w_\beta, \; \forall \sigma > 0, \; k \in \mathbb{N}, \\ w(-T) & = & w_0 \end{array} \right. \tag{5}$$

where $\mathbb{N} = \{0, 1, \ldots\}$. At each time kT a parameter $\beta(kT) \in \mathbb{R}$, and the corresponding constant reference input $w_k \triangleq w_{\beta(kT)}$, are selected in accordance with the optimization criterion

$$\beta(kT) = \left\{ \begin{array}{l} \arg\min_{\beta \geq 0} \beta^2 \\ \text{subj. to } c(kT + \sigma, x(kT), w(kT + \sigma, \beta)) \in \mathcal{C}_\delta, \\ \forall \sigma \in (0, +\infty) \end{array} \right. \tag{6}$$

and

$$w(t) \equiv w_k,\ \forall t \in (kT, (k+1)T]$$

Notice that by minimizing β^2 one attempts to minimize $\|w - r\|^2$, and therefore $\|y - r\|^2$. A parameter β, or a constant reference w, satisfying the constraints in (6) will be referred to as *admissible.*

Assumption 8 (Feasible Initial Condition) *The initial state $x(0)$ is such that there exists at least one admissible virtual constant reference input $w_0 \in \mathcal{W}_\delta$.*

For instance Assumption 8 is satisfied for an equilibrium states $x(0) = x_{w_0}$ corresponding to $w_0 \in \mathcal{W}_\delta$.

3. Main Results

Proposition 1 *Suppose that $r(t) \equiv r$, $\forall t \geq 0$, and Assumptions 3-4 hold. Then there exists $\lim_{t\to\infty} w(t) \triangleq w_\infty \in \mathcal{W}_\delta$. In addition, each component of vector $w(t)$ is monotonically convergent.*

Proof. If $w_0 = r$, then $\beta(kT) = 0$ is admissible, $\forall k \in \mathbb{N}$. Therefore, $w(t) = r$, $\forall t > 0$, and $w_\infty = r$ (the RG behaves as an all-pass filter). Suppose $w_0 \neq r$. Since $\beta(kT) \geq 0$, $w_k = r + \frac{d_k}{\|w_0 - r\|}[w_0 - r]$, where $d_k \triangleq \|w_k - r\|$. By construction, at time $(k+1)T$, $\beta = 1$ is admissible, and hence $\beta((k+1)T) \leq 1$. Then, $0 \leq d_{k+1}^2 = \beta^2((k+1)T)d_k^2 \leq d_k^2$, $\forall k \in \mathbb{N}$, and hence there exists $d_\infty = \lim_{k\to\infty} d_k$. Consequently, $\lim_{t\to\infty} w(t) = w_\infty \triangleq r + \frac{d_\infty}{\|w_0 - r\|}[w_0 - r]$. By compactness of $\mathcal{W}_\delta$, $w_\infty \in \mathcal{W}_\delta$ follows. □

Lemma 1 *Suppose that Assumptions 1-5 and 7 hold. Consider two reference inputs $w_a, w_b \in \mathcal{W}_\delta$, $w_a \neq w_b$. Let $x(kT) = x_{w_a} + \Delta x \in \mathcal{X}$, and let η such that $B(c_{w_a}, \eta) \subseteq \mathcal{C}$. Then there exists a $\bar{\gamma} > 0$, dependent on w_a and η, such that reference input $w_a + \gamma(w_b - w_a)$ is admissible for all $\|\Delta x\| \leq \frac{1}{2}\alpha(\eta/2)$, and for all $0 \leq \gamma \leq \bar{\gamma}$.*

Proof. See [15]. □

Proposition 2 *Suppose that $r(t) \equiv r$, $\forall t \geq 0$, and Assumptions 1-8 hold. Then $\lim_{t\to\infty} w(t) = w_r \in \mathcal{W}_\delta$ with*

$$w_r = \arg\min_{\rho\in[0,1]} \begin{cases} \|w - r\| \\ \text{subject to } w = r + \rho[w_0 - r] \in \mathcal{W}_\delta \end{cases} \tag{7}$$

where $w_0 \in \mathcal{W}_\delta$ is an admissible reference input at time $t = 0$.

Proof. By Prop. 1 there exists $\lim_{t\to\infty} w(t) = w_\infty \in \mathcal{W}_\delta$, and the convergence is component-by-component monotonic. Suppose by contradiction $w_\infty \neq w_r$. By Assumption 6, there exists a time t_0 such that $\|x(t_0, x(0), w(t_0)) - x_{w_\infty}\| \leq \alpha\left(\frac{\delta}{2}\right)$. Hence, by Lemma 1, there exists a constant $\bar{\gamma} > 0$ such that $w_\gamma \triangleq w_\infty + \gamma(w_r - w_\infty)$ is admissible at time t_0, $\forall \gamma$ such that $0 < \gamma \leq \bar{\gamma}$. Then, $\|w(t) - r\| \leq \|w_\gamma - r\|$. Since r, $w(t)$, w_γ, w_∞ are collinear, it follows that $\|w(t) - w_\infty\| = \|w(t) - w_\gamma\| + \|w_\gamma - w_\infty\| \geq \gamma\|w_r - w_\infty\| > 0$, $\forall t \geq t_0$, which contradicts the hypothesis $\lim_{t\to\infty} w(t) = w_\infty$. □

Lemma 2 *(Finite Stopping Time) Under the hypotheses of Prop. 2, there exists a stopping time t_s such that $w(t) = w_r$ for all $t \geq t_s$.*

Proof. See [15]. □

Next Theorem 1 summarizes the previous results.

Theorem 1 *Suppose $r(t) \equiv r$, $\forall t \geq 0$, and Assumptions 1-8 hold. Then, after a finite time t_s the RG generates a constant reference input $w(t) \equiv w_r$, where w_r is given by (7). Consequently, system (1) is asymptotically driven from $x(0)$ to x_{w_r} with no constraint violation.*

Notice that, when $r \in \mathcal{W}_\delta$, the RG has no effect on the asymptotic behavior of $y(t)$, which instead depends on the tracking properties of the primal system (1).

The optimization criterion (6) requires that the constraint $c(kT + \sigma, x(kT), w_\beta) \in \mathcal{C}_\delta$ is checked for all $\sigma > 0$. In this section, we show that it suffices to verify this condition over a finite prediction horizon $(0, T_\infty]$.

Definition 1 *(Constraint Horizon) The constraint horizon T_∞ is defined as the shortest prediction horizon such that $c(t + \sigma, x(t), w) \in \mathcal{C}_\delta$, $\forall \sigma > 0 \Leftrightarrow c(t + \sigma, x(t), w) \in \mathcal{C}_\delta$, $\forall 0 < \sigma \leq T_\infty$, $\forall x(t) \in \mathcal{X}$, $\forall w \in \mathcal{W}_\delta$.*

When $w(t) \equiv w$, the following Theorem 2, whose proof is reported in [15], proves that, for a fixed scalar $\lambda > 0$, the state $x(t)$ converges to the ball $B(x_w, \lambda)$ in a finite time T which is not dependent of the initial state $x(0) \in \mathcal{X}$ and reference input $w \in \mathcal{W}_\delta$.

Theorem 2 *Let Assumptions 1, 3, and 5-7 be satisfied. Then for all $\lambda > 0$ there exists a finite time $T(\lambda)$ such that, $\forall c(0) = [x'(0)\ w']' \in \mathcal{C}_\delta$,*

$$\|x(t, x(0), w) - x_w\| \leq \lambda,\ \forall t \geq T(\lambda). \tag{8}$$

By (3) and Assumption 4, Theorem 2 proves that T_∞ exists and satisfies the inequality $T_\infty \leq T(\delta)$.

4. RG Numerical Implementation

The numerical implementation of the RG involves some approximations, which arise from two main reasons. The evolution $c(kT + \sigma, x(kT), w_\beta)$ is evaluated by numerical integration of (1) from initial state $x(kT)$, and consequently constraint fulfillment is only checked at integration steps. Moreover, the optimal $\beta(kT)$ defined in (6) is found by numerical optimization.

Numerical integration is performed by using a standard fourth order Runge-Kutta method with adaptive stepsize control, using the constants reported in Cash and

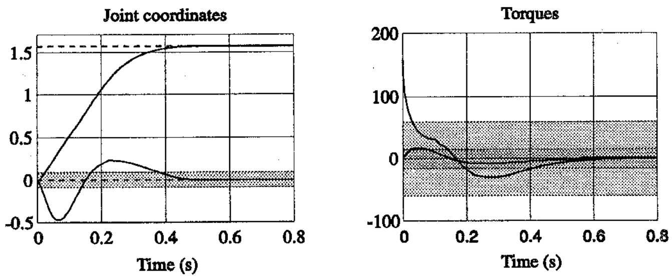

Figure 2: Response without RG.

Karp [16]. Constraint fulfillment is checked during integration.

On-line optimization is performed by using a bisection method over the interval $[0, 1]$. A finite number N of candidates is evaluated, where the number of trials N equals the number of numerical integrations. The selection of the optimal $\beta(kT)$ is performed according to the following algorithm:

Algorithm 1

1. If $\beta = 0$ is admissible, set $\beta(kT) := 0$ and STOP.
2. Set $\beta^- := 0$, $\beta^+ := 1$.
3. Set $\beta := \frac{1}{2}(\beta^+ + \beta^-)$.
4. If β is admissible, set $\beta^+ := \beta$. Otherwise, set $\beta^- = \beta$.
5. Execute steps (3), (4), and (5) $N-1$ times.
6. Set $\beta(kT) := \beta^+$ and STOP.

Notice that if constraint fulfilment is never achieved when Procedure A is executed, the algorithm yields $\beta(kT) = 1$, which, by construction, is always admissible.

For a given T, N is determined by both the desired integration accuracy and the constraint horizon T_∞. Since admissibility of $\beta = 0$ is always tried first, the optimal $\beta(kT)$ is evaluated with a worst case precision of $2^{-(N-1)}$. It is clear that if global minimization procedures were adopted in selecting $\beta(t)$, better tracking properties might be achieved, at the expense of an increased computational effort.

5. Simulation Results

The performance of the RG presented in the previous sections has been tested by computer simulations on a two link robot moving on a horizontal plane.

5.1. Nonlinear Model

Each joint is equipped with a motor for providing input torque, encoders and tachometers are used for measuring the joint positions θ_1, θ_2, and velocities $\dot{\theta}_1$, $\dot{\theta}_2$. By using Lagrangian equations, and by setting

$$x = [\theta_1\ \dot{\theta}_1\ \theta_2\ \dot{\theta}_2]',\ y = [\theta_1\ \theta_2]',\ \mathcal{T} = [\mathcal{T}_1\ \mathcal{T}_2]',\ w = [\theta_{1d}\ \theta_{2d}],$$

where θ_{1d}, θ_{2d} denote the desired values for joint positions and $\mathcal{T}_1$, $\mathcal{T}_2$ the motor torques, the dynamic model of the robot can be expressed as

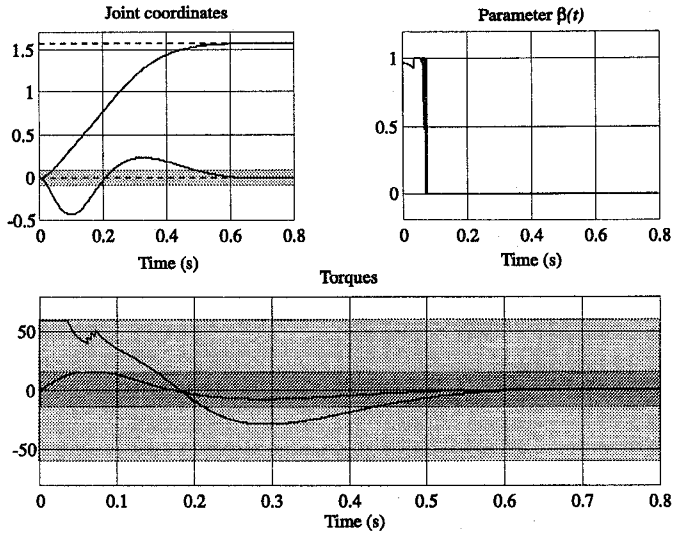

Figure 3: Response with RG ($T = 0.001$ s).

$$H(x)\begin{bmatrix}\dot{x}_2\\ \dot{x}_4\end{bmatrix} + C(x)\begin{bmatrix}x_2\\ x_4\end{bmatrix} = \mathcal{T} \tag{9}$$

where $H(x)$, $C(x)$ are reported in [15]. Individual joint PD controllers

$$\mathcal{T} = -\begin{bmatrix}k_{p1}(x_1 - w_1) + k_{d1}x_2\\ k_{p2}(x_3 - w_2) + k_{d2}x_4\end{bmatrix} \tag{10}$$

provide reference tracking. As a general rule to design controllers to be used in connection with a RG, in order to maximize the properties of tracking one should try to select a primal controller which provides a fast closed-loop response of system (1). Usually this corresponds to large violations of the constraints, which therefore can be enforced by inserting a RG. In [15], it is shown that system (9)–(10) fulfills the required assumptions.

Simulations have been carried out with the system parameters reported in [17]. Fig. 2 shows the closed loop system behavior for a constant desired reference $r_1(t) \equiv \frac{\pi}{2}$, $r_2(t) \equiv 0$, $t \in \mathbb{R}_+$, in the absence of the RG. In order to bound the input torques within the range

$$|\mathcal{T}_1| \leq 60 \text{ Nm},\ |\mathcal{T}_2| \leq 15 \text{ Nm}, \tag{11}$$

which has been represented by shadowed areas in Fig. 2, the RG is applied. The initial condition $\theta_1(0) = \theta_2(0) = 0$, $\dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$ and $w_0 = [0\ 0]'$ satisfy Assumption 8. A RG period $T = 0.001$ s, a constraint horizon $T_\infty = 0.4$ s, $N = 10$ admissibility evaluations per period, and $\delta \approx 0$ are selected as parameters of the RG. The set $\mathcal{C}$ is determined by (11) and by further limiting the state and reference input in such a manner that only constraints (11) become active. The resulting trajectories are depicted in Fig. 3. The further constraint

$$|\theta_2| \leq 0.2 \text{ rad}$$

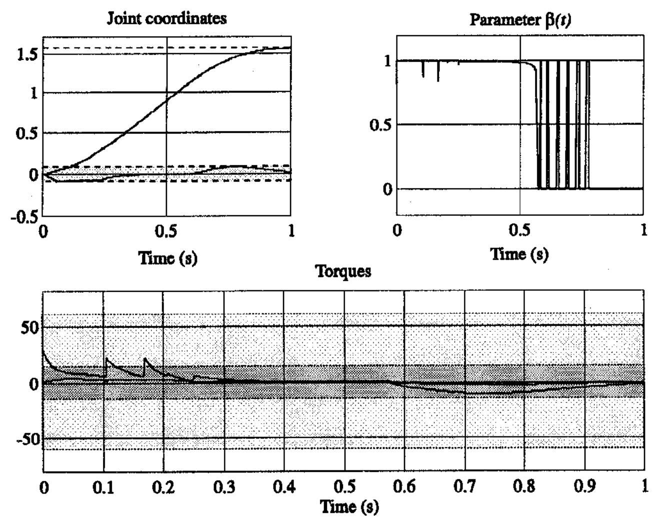

Figure 4: Response with RG, torque constraints, and the constraint $|\theta_1 - \theta_2| \leq 0.2$ rad. The generated reference input is depicted (thin line) together with the joint trajectories (thick lines)

is taken into account by the RG and the related simulated trajectories are depicted in Fig 4 ($T = 0.001$ s). The slight chatter on the β and torque trajectories is caused by the approximations involved in the optimization procedure described in Sect. 4.. Finally, the RG is applied in connection with a nonconstant desired reference trajectory, as shown in Fig. 5. The results described above were obtained on a 486 DX2/66 personal computer, using Matlab 4.2 and Simulink 1.3 with embedded C code. The CPU time required by the RG to select a single $\beta(t)$ ranged between 7 and 18 ms.

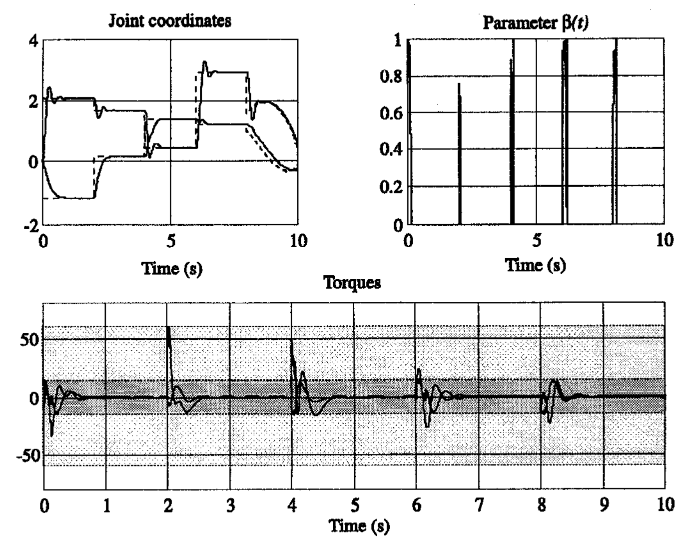

Figure 5: Response with RG for a nonconstant desired reference trajectory.

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