

# A Stochastic Model Predictive Control Approach for Series Hybrid Electric Vehicle Power Management

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**Abstract**—This paper illustrates the use of stochastic model predictive control (SMPC) for power management in vehicles equipped with advanced hybrid powertrains. Hybrid vehicles use two or more distinct power sources for propulsion, and their complex powertrain architecture requires the coordination of all the subsystems to achieve target performances in terms of fuel consumption, driveability, component life-time, exhaust emissions. Many control strategies have been presented and successfully applied, mainly based on heuristics or rules and tuned on certain reference drive cycles. To take into account that cycles are not exactly known a priori in driving routine, this paper proposes a stochastic approach for the power management problem. We focus on a series hybrid electric vehicle (HEV), which combines an internal combustion engine and an electric motor. The power demand from the driver is modeled as a Markov chain estimated on several driving cycles and used to generate scenarios in the SMPC law. Simulation results over a standard driving cycle are presented to demonstrate the effectiveness of the proposed stochastic approach and compared with other deterministic approaches.

## I. INTRODUCTION

Increasing fuel economy and reducing greenhouse gas pollution have become a clear target of national policies, as announced in 2009 by the President of the United States. Achieving such a target has set an urgent need for advanced powertrain systems and for clean power sources, and a significant increase in electrification in vehicles is expected. Pure electric vehicles, developed for zero emissions, have limited capabilities, mainly due to their short driving range. Instead, Hybrid Electric Vehicles (HEVs) are a viable and alternative choice in the near term due to their improved fuel efficiency and lower emissions, while ensuring vehicle performance and driving requirements. However, contrarily to vehicles with a single power source, HEVs require new high-level control strategies to optimally use two or more power sources, dealing with complex configurations and operating modes. The performance of HEVs is tightly dependent on the power management strategies used to control the power flow between the different subsystems. This paper considers the problem of optimally splitting the power demand of the driver among the electric power source and the internal combustion engine (ICE) in a series HEV [1]–[4] schematized in Figure 1. Recent research efforts mostly focus on power split optimization for fuel economy, while satisfying

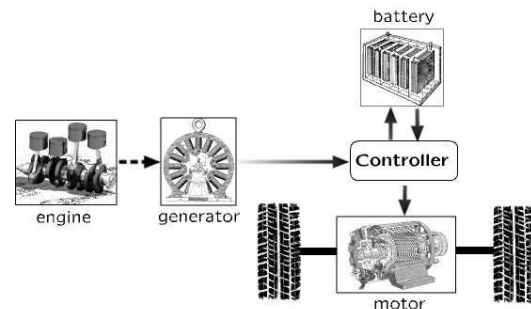


Fig. 1. Series hybrid configuration

constraints such as driveability, charge sustainability, and component reliability. Most of the existing techniques rely on knowing the future power demand to set up deterministic dynamic programming (DDP) problems [1], or rule-based (RB) algorithms [2]. Even though these techniques have been already tested in real vehicles with good results, they suffer a few drawbacks. Both RB and DDP strongly depend on the specific driving cycle used for their tuning, and might be neither optimal nor charge-sustaining under other cycles. Other approaches do not rely on the specific power demand profile. Stochastic dynamic programming (SDP) exploits a probabilistic distribution of the power demand obtained from many driving cycles [5]–[7]. However due to the large computation time needed to compute the control law, it can never be updated to accommodate changes in the power demand probability distribution. Instead of optimizing the entire driving cycle, which is not assumed to be known in advance, hybrid Model Predictive Control (MPC) strategies repeatedly optimize the decision on-line over short and receding future time horizons to coordinate powertrain subsystems and enforce pointwise-in-time state and control constraints [8].

This paper extends the last approach by developing a stochastic model predictive control (SMPC) algorithm for power management, with the goal of optimizing the way a HEV splits its overall power demand among its power sources, while fulfilling bounds on the state of charge of the battery and on the power availability. The underlying assumption of this approach is that the power requested from the driver is represented by a Markov model. Instead of optimizing over driving cycle known a priori, the SMPC strategy optimizes over a distribution of future requested power demand, given the current one, at each sample time. SDP solves instead a single infinite-horizon optimization

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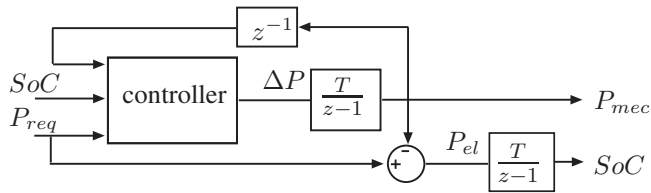


Fig. 2. Power management scheme

problem over a family of random driving cycles in an average sense. The advantage of using SMPC with respect to SDP is that SMPC optimization is feasible in real time [9], since it is based on quadratic programming (QP). As a consequence, the stochastic description of the system through a Markov model can be constantly updated on line by measuring driver's power requests, using for instance the algorithms presented in [10]. In this way the probability distribution will relate more to the particular driver and to the real daily use of the vehicle.

The paper is organized as follows: Section II describes the series HEV configuration, and proposes a mathematical model for optimization purposes. Section III illustrates the stochastic modeling of the power demand. The stochastic model predictive control approach is introduced in Section IV, followed in Section V by the *frozen-time* (no information about future power demand), and *prescient* (full information about future power demand) MPC approaches. Section VI presents simulation results comparing the SMPC power management with frozen-time (FTMPC) and prescient (PMPC) approaches. Conclusions are drawn in Section VII.

## II. POWERTRAIN MODEL

### A. Series hybrid vehicle

The hybrid vehicle model used in this work derives from a simplified series hybrid vehicle model, which is part of the QSS toolbox [11]. The topology of the powertrain under study is the one represented in Figure 1. In the series configuration the internal combustion engine (ICE) powers an electric generator, which converts the mechanical power into electrical power. The generator feeds an electrical bus, where also the battery and the electric motor that drives the wheels are connected. The electrical bus allows the power flow between engine, motor and battery. Compared to the *powersplit* configuration, where the power flow coupling involves mechanical powers and is obtained by a planetary gear set, the electrical bus of the series configuration has a higher efficiency [5]. On the other hand the mechanical power must be always converted to electrical power, with consequent power losses. Nonetheless, the series HEV configuration is an interesting candidate for implementation, and in particular is well suited for plug-in and fuel cells hybrid vehicles [7].

### B. Prediction model

As a prediction model for the MPC control problem, we consider the simplified powertrain scheme represented in Figure 2,

where  $P_{req}(k)$  is the total requested power that must be generated by the powertrain at the current sample step  $k$ ,  $P_{mec}(k)$ ,  $P_{el}(k)$  are the mechanical and the electric powers supplied by the ICE and the battery, respectively, and  $\Delta P(k) = P_{mec}(k) - P_{mec}(k-1)$  is the step-to-step variation of the mechanical power. The state of charge  $SoC(k)$  of the battery is normalized with respect to the battery capacity ( $SoC(k) = 1$  fully charged,  $SoC(k) = 0$  fully discharged), and it is modeled as the integrator

$$SoC(k+1) = SoC(k) - KTP_{el}(k), \quad (1)$$

where  $T = 1$  s is the sampling time and  $K > 0$  is a scalar parameter identified for a generic battery for hybrid cars. In this model,  $\Delta P$  is the only controlled variable, while  $P_{req}$  is an exogenous input given by the driver (i.e., a measured disturbance). The control strategy based on gradual mechanical power variations allows for reducing the complexity of the model and control strategy and for formulating a problem as a QP [3]. Furthermore, this approach allows one to consider the engine efficiency as a function of power only. In order to improve fuel consumption we use the main advantage of the series configuration, that is the mechanical decoupling between the engine and the drive axle. Once the desired mechanical power is determined, the engine operating point in terms of engine speed and torque is selected to be the most efficient for that desired power, i.e.

$$[\tau(k), \omega(k)] = f(P_{mec}(k)),$$

where  $f$  is the function that relates the mechanical power desired with the optimal engine operating point in terms of torque ( $\tau$ ) and engine speed ( $\omega$ ) and that account for power losses of the gear and generator. A low level controller regulates the engine to operate at that point. Hence the engine operates along the optimal power curve, except for the transients from one power level to another. However, if the variations of the mechanical power are limited and smooth over time, the transients are short and the engine operates almost always along the optimal curve. Further benefits of this strategy are the reduced power losses caused by the inertia of the engine.

By imposing limited power variations, we can restrict our attention to the efficiency of the engine as the optimal power curve, hence obtaining a one-dimensional engine efficiency map as a function of the ICE power. Linear or quadratic approximations of this map are possible, allowing the obtained optimization problem to be feasible for real-time solution. In this paper a quadratic approximation of the inverse of the efficiency is used

$$J_{\eta^{-1}}(P_{mec}) = \phi(P_{mec} - P^*)^2 + \gamma, \quad (2)$$

in order to minimize losses, where  $\phi = 5.70 \cdot 10^{-5}$ ,  $\gamma = 4.12 \cdot 10^{-2}$  and  $P^* = 15.87$  kW are estimated scalar parameters (see Figure 3). Hence the corresponding linear system is

$$\begin{cases} x(k+1) &= Ax(k) + B_1 \Delta P(k) + B_2 P_{req}(k) \\ y(k) &= Cx(k) + D_1 \Delta P(k) + D_2 P_{req}(k) \end{cases} \quad (3)$$

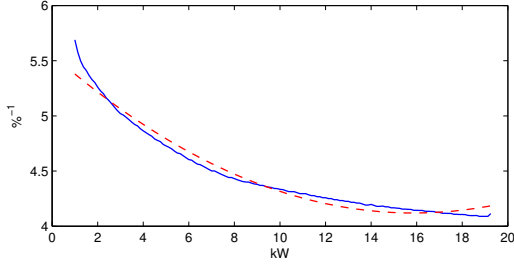


Fig. 3. Quadratic approximation  $J_{\eta-1}$  (red dashed line) of the inverted efficiency  $\frac{1}{\eta}$  (blue solid line) as a function of the ICE power  $P_{mec}$

where  $x(k) = [SoC(k) \ P_{mec}(k-1)]'$  is the state,  $u(k) = [\Delta P(k) \ P_{req}(k)]'$  is the input,  $y(k) = P_{el}(k)$  is the output, and

$$\begin{aligned} A &= \begin{bmatrix} 1 & KT \\ 0 & 1 \end{bmatrix}, & B_1 &= \begin{bmatrix} KT \\ 1 \end{bmatrix}, & B_2 &= \begin{bmatrix} -KT \\ 0 \end{bmatrix}, \\ C &= [0 \ -1], & D_1 &= -1, & D_2 &= 1. \end{aligned} \quad (4)$$

In (3)–(4) the electric power is defined as

$$P_{el}(k) = P_{req}(k) - P_{mec}(k), \quad \forall k \geq 0. \quad (5)$$

This implies that the controller is always supposed to enforce the power at wheels  $P_{mec}(k) + P_{el}(k)$  to be equal to the requested power  $P_{req}(k)$ . This underlying assumption is not seen as a restriction since it is a fundamental requirement for the powertrain operation.

In order to guarantee a prolonged battery life and to respect electro-mechanical limitations, the state, manipulated input and output of system (3) are subject to the constraints  $x(k) \in \mathbf{X}$ ,  $\Delta P(k) \in \mathbf{U}$ ,  $P_{el}(k) \in \mathbf{Y}$ ,  $\forall k \geq 0$ , where

$$\mathbf{X} \triangleq \{x : SoC_{min} \leq [1 \ 0]x \leq SoC_{max}, \quad 0 \leq [0 \ 1]x \leq P_{mec,max}\}, \quad (6a)$$

$$\mathbf{U} \triangleq [\Delta P_{min}, \Delta P_{max}], \quad (6b)$$

$$\mathbf{Y} \triangleq [P_{el,min}, P_{el,max}]. \quad (6c)$$

### III. STOCHASTIC MODELING OF POWER REQUEST

The power requested by the driver  $P_{req}$  is an input to the SMPC controller. Roughly speaking when the driver presses the throttle and the brake pedal these commands are interpreted as a power demand to the controller, that can be positive or negative. In model (3), we assume that the evolution of the requested power  $P_{req}(k)$  is driven by a discrete-time stochastic process. Unlike other approaches as DDP that exploit the knowledge of the driving cycle and therefore of the complete sequence of the power demand, we model the requested power as a Markov chain [5], [12]. This model is used to generate an estimated future power request, which is assumed to take a finite number of values  $P_{req} \in \{z_1, z_2, \dots, z_s\}$ . A discrete set of  $s$  values is used to approximate the actual continuum of values of requested power. The Markov Chain is defined by a transition

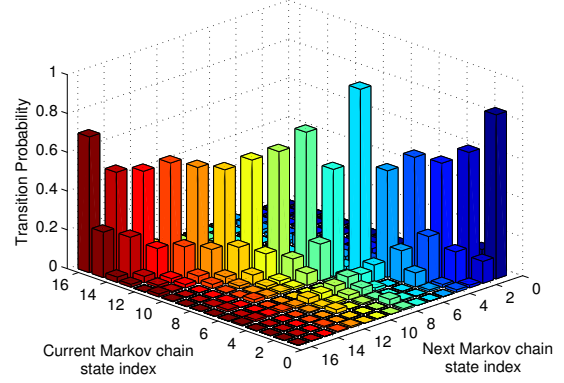


Fig. 4. Transition probability matrix

probability matrix  $T_M \in \mathbb{R}^{s \times s}$  such that

$$t_{ij} = \mathbf{P}[P_{req}(k+1) = z_j | P_{req}(k) = z_i], \quad (7)$$

where  $P_{req}(k)$  is the state of the Markov chain at time  $k$ ,  $t_{ij}$  is the  $(i, j)$ -th element of  $T_M$ , and  $\mathbf{P}[a]$  indicates the probability of the event  $a$ .

Specifying driving-cycle characteristics is equivalent to specifying the transition probabilities  $t_{ij}$ . Transition probabilities can be estimated from known cycles, such as past driving records and standard driving cycles. In this paper we estimate the number of states  $s$ , the transition probabilities  $t_{ij}$ , and the power values  $z_i$  from standard driving cycles selected to represent mixed city, suburban, and highway driving. From the speed profile,  $P_{req}$  is calculated through the quasi-static vehicle model. Transition probabilities are simply estimated by means of frequency analysis, where the observation data are counted as  $t_{ij} = \frac{m_{ij}}{m_i}$ , where  $m_{ij}$  is the number of occurrences of the transition from  $z_i$  to  $z_j$  and  $m_i = \sum_{j=1}^s m_{ij}$ , is total number of times that  $z_i$  has occurred. The number  $s = 16$  is selected to trade-off between the quality of the approximation and the complexity of the Markov chain. The estimated transition probabilities are shown in Figure 4, where it is evident a diagonal dominance. However, as shown later, modeling a non zero probability to change the power demand in prediction allows to improve the overall performance compared to a purely deterministic approach.

### IV. STOCHASTIC MODEL PREDICTIVE CONTROL DESIGN

Model predictive control (MPC) is a popular strategy which has been widely adopted in industry as an effective means of dealing with multivariable constrained control problems [13]. The idea behind MPC is to obtain the control signal by solving at each sampling time an open-loop finite-horizon optimal control problem based on a given prediction model of the process, by using the current state of the process as the prediction initial state. The control inputs are implemented in accordance with a receding horizon scheme. Classical MPC formulations do not provide a systematic way

to deal with model uncertainties and disturbances, which are often completely neglected in the prediction model. Robust MPC schemes that deal with the presence of disturbances are mostly based on the min-max approach, where the performance index to be minimized is computed over the worst possible disturbance realization [14]. However, on one hand nominal controllers which neglect the effect of disturbance may lead to poor performances when implemented in real processes, on the other hand, robust controllers provide a control law which is often too conservative.

In recent years *stochastic* MPC (SMPC) control schemes were formulated, where the available statistical information on the disturbance is exploited in order to minimize the expected value of the performance index [15]–[20]. In this paper, we adopt the SMPC approach of [21] based on scenario enumeration, which exploits ideas from multi-stage stochastic optimization. The knowledge of the disturbance model (i.e., the Markov chain described in Section III) is used by SMPC to possibly improve the closed-loop performance of the controlled system with respect to a standard deterministic MPC algorithm.

#### A. SMPC approach

The SMPC problem formulation is based on a maximum likelihood approach, where at every time-step an optimization tree is built using the updated information on the system state and on the Markov chain. Each node of the tree represents a predicted state which is taken into account in the optimization problem. Starting from the root node, which is defined by the current available measurements  $x(k)$  and  $P_{req}(k)$ , a list of candidate nodes is generated by considering all the possible future Markov states  $P_{req}(k+1|k)$ , together with their realization probability. Then, the node with maximum probability is added to the tree. This procedure is repeated iteratively, by generating at every step new candidates as children nodes of the last node added to the tree, until a desired number of nodes  $n_{max}$  is reached.

Hence, every node is identified by a different realization of the Markov chain (i.e., a scenario of power requests), and by a different input sequence. Causality of the resulting control law is enforced by allowing only one control move for every node, except leaf nodes (i.e., nodes with no successor) that have no associated control move. The reader is referred to [21] for further details on the tree design algorithm.

#### B. Problem formulation

In order to define the stochastic finite-time optimal control problem, let us introduce the following quantities:

- $\mathcal{T} = \{\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_n\}$ : the set of the tree nodes. Nodes are indexed progressively as they are added to the tree (i.e.,  $\mathcal{T}_1$  is the root node and  $\mathcal{T}_n$  is the last node added to the tree).
- $x_{\mathcal{N}}, \Delta P_{\mathcal{N}}, P_{el,\mathcal{N}}, P_{req,\mathcal{N}}$ : the state, the input, the output, the estimated power request associated with node  $\mathcal{N}$ , respectively.
- $pre(\mathcal{N})$ : the predecessor of node  $\mathcal{N}$ .

- $succ(\mathcal{N}, j)$ : the successor of node  $\mathcal{N}$  with state value  $z_j$  of the Markov Chain,  $j \in \{1, 2, \dots, s\}$ .
- $\pi_{\mathcal{N}}$ : the probability of reaching node  $\mathcal{N}$  from  $\mathcal{T}_1$ .  $\pi_{\mathcal{N}}$  is computed by means of the transition probability matrix  $T_M$ , i.e.,  $\pi_{succ(\mathcal{N},j)} = t_{ij}\pi_{\mathcal{N}}$ , if  $P_{req,\mathcal{N}} = z_i$ .
- $\mathcal{S} \subset \mathcal{T}$ : the set of the leaf nodes, defined as  $\mathcal{S} \triangleq \{\mathcal{T}_i \in \mathcal{T} : succ(\mathcal{T}_i, j) \notin \mathcal{T}, j \in \{1, 2, \dots, s\}\}$ .

In Figure 5 an illustrative optimization tree is shown to exemplify the notation. We present a control problem formulation where the objective function to be minimized relies on an approximation of the expected value of the closed-loop performance, evaluated as a quadratic function of the state and the input. This approximation can be made arbitrarily accurate by increasing the number of nodes  $n_{max}$ , at the expense of a higher computational load.

With a slight abuse of notation, in the following the abbreviate forms  $x_i, \Delta P_i, P_{el,i}, P_{req,i}, \pi_i, pre(i)$ , will be used to denote  $x_{\mathcal{T}_i}, \Delta P_{\mathcal{T}_i}, P_{el,\mathcal{T}_i}, P_{req,\mathcal{T}_i}, \pi_{\mathcal{T}_i}, pre(\mathcal{T}_i)$ , respectively. The SMPC problem at time  $k$  is formulated as

$$\begin{aligned} \min_{\{\Delta P_i\}} \quad & \sum_{i \in \mathcal{T} \setminus \{\mathcal{T}_1\}} \pi_i (x_i - x_{ref})' \begin{bmatrix} Q_{SoC} & 0 \\ 0 & \phi Q_J \end{bmatrix} (x_i - x_{ref}) + \\ & + \sum_{j \in \mathcal{T} \setminus \mathcal{S}} \pi_j Q_P \Delta P_j^2 \end{aligned} \quad (8a)$$

$$\text{s.t. } x_1 = x(k), \quad (8b)$$

$$P_{req,1} = P_{req}(k), \quad (8c)$$

$$\begin{aligned} x_i &= Ax_{pre(i)} + B_1 \Delta P_{pre(i)} + B_2 P_{req,pre(i)}, \\ & \forall i \in \mathcal{T} \setminus \{\mathcal{T}_1\}, \end{aligned} \quad (8d)$$

$$\begin{aligned} P_{el,i} &= Cx_i + D_1 \Delta P_i + D_2 P_{req,i}, \\ & \forall i \in \mathcal{T} \setminus \mathcal{S}, \end{aligned} \quad (8e)$$

$$x_i \in \mathbf{X}, \forall i \in \mathcal{T} \setminus \{\mathcal{T}_1\}, \quad (8f)$$

$$\Delta P_i \in \mathbf{U}, \forall i \in \mathcal{T} \setminus \mathcal{S}, \quad (8g)$$

$$P_{el,i} \in \mathbf{Y}, \forall i \in \mathcal{T} \setminus \mathcal{S}, \quad (8h)$$

where  $x_{ref} = [SoC_{ref} \ P_{mec,ref}]'$ ,  $Q_{SoC}, Q_J, Q_P$  are non-negative scalar weights, and  $\phi$  is defined as in (2). Note that by imposing  $P_{mec,ref} = P^*$  we have a term in the cost function to maximize an approximation of the engine efficiency  $\eta$ . The cost function (8a) is constrained by (8b)–(8h), where (8b) defines the initial state of the system. Equation (8c) constrains the input  $P_{req,1}$  of the first node of the optimization tree to be the current requested power. The power demands associated with the nodes,  $P_{req,i}, i > 1$ ,

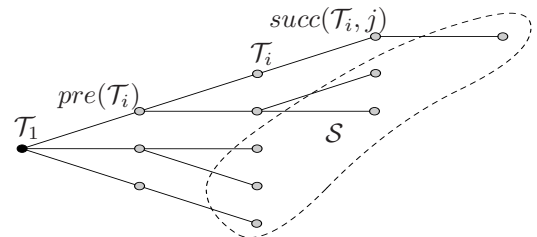


Fig. 5. Optimization tree diagram



are obtained from the Markov chain model. In other words, only the future power requests are quantized. Equations (8d)-(8h) are related to the evolution of the dynamical system (3) and to the electro-mechanical constraints defined in (6). Problem (8) can be cast as a standard quadratic programming problem (QP).

## V. FROZEN-TIME AND PRESCIENT MPC

In this section we introduce two deterministic control approaches based on the receding-horizon philosophy, namely the *frozen-time* MPC (FTMPC), and the *prescient* MPC (PMPC), that we will later compare to SMPC. FTMPC has no information the future, PMPC exploits an a priori knowledge of the requested power demand for a given future horizon window. In the FTMPC approach, no information on the Markov model is exploited, and the actual power demand value  $P_{req}(k)$  is assumed constant along the whole prediction horizon. In other words, the FTMPC can be seen as a special case of the SMPC where the Markov Chain transition matrix is an identity matrix,  $T_M = I_s$ , and the predicted state values are  $P_{req}(k+i|k) = P_{req}(k|k)$ ,  $\forall i, k$ .

In the PMPC approach, instead, the complete knowledge of the driving cycle is exploited: At time  $k$ , the PMPC solves an optimal control problem over a finite horizon of  $n_{max} - 1$  steps by knowing the future evolution of the requested power  $P_{req}(k+j|k)$  in advance ( $j = 0, \dots, n_{max} - 1$ ). The PMPC is optimal if  $n_{max}$  has the same length of the entire driving cycle, otherwise the solution is in general sub-optimal. FTMPC and PMPC can be seen as the upper and lower limits to SMPC, which instead uses a stochastic information to predict the future values of  $P_{req}$ .

## VI. SIMULATION RESULTS

### A. Simulation model

SMPC, FTMPC, and PMPC are simulated to validate their performance, using a nonlinear simulation model of the HEV. The nonlinear model is quasi-static and combines equations of different vehicle components. The HEV considered here derives from the series hybrid vehicle contained in the QSS toolbox [11]. The model has been modified to take into account the transients that affects the computation of fuel consumption. The quasi-static approach has been applied to various powertrain systems in [4]. The main drawback of such an approach formulation is that some dynamical effects are disregarded, and the physical causality is inverted. Therefore the driving cycle has to be known and the requested power needs always to be satisfied.

### B. Simulation results

The SMPC has been tested on several cycles by using the aforementioned nonlinear model. Although the performance of a HEV is usually evaluated on a standard pre-determined cycle, the Markov chain generating the requested power  $P_{req}$  is estimated off-line with data from several cycles, to be able to emulate diverse scenarios. We simulate the series hybrid vehicle model on the New European Driving Cycle (NEDC), which defines a vehicle speed reference profile to be tracked

TABLE I

Closed-loop results. Fuel consumption is expressed as the percentage improvement with respect to the deterministic FTMPC controller

	$\ \Delta P\ $	fuel cons. [kg]	fuel improv. [%]
FTMPC	5.7906	0.281	-
SMPC	3.1236	0.243	13.5
PMPC	0.8581	0.197	29.8

for a duration of around 20 minutes. The requested power profile for our case study is derived from the velocity profile prescribed by the NEDC.

The initial conditions are  $SoC(0) = 0.5$ ,  $P_{req}(0) = 0$ , and the following values for constraints (6) are used in simulation:  $SoC_{min} = 0.45$ ,  $SoC_{max} = 0.55$ ,  $P_{mec,max} = 20$  kW,  $\Delta P_{max} = -\Delta P_{min} = 5$  kW,  $P_{el,max} = -P_{el,min} = 40$  kW. The constraints on the  $SoC$  are set tight around the 50% of charge to preserve battery life-time. The weights in (8a) are  $Q_{SoC} = 500$ ,  $Q_J = 0.2$ ,  $Q_P = 0.4$ . The optimization tree which defines the optimal control problem is built with  $n_{max} = 100$  nodes and the state references in (8a) are  $SoC_{ref} = 0.5$  and  $P_{mec,ref} = P^* = 15.87$  kW. The simulation was run in SIMULINK<sup>®</sup>, and a comparison of performance is reported in Table I, where

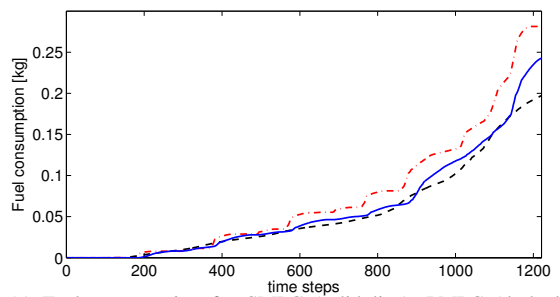
$$\|\Delta P\| = \left( \sum_{k=1}^{T_{sim}} \Delta P(k)^2 \right)^{\frac{1}{2}}$$

is the norm of the mechanical power variation along the whole simulation interval  $T_{sim} = 1220$  time steps. Figures 6–7 depict the trajectories obtained with the three proposed MPC controllers.

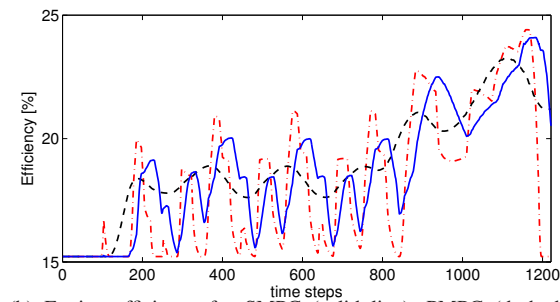
In Figure 6(a) one notices that fuel consumption of SMPC is better than FTMPC over the NEDC cycle, despite the fact that fuel consumption is not modeled in (3) and is not directly minimized. Indeed, fuel is indirectly minimized by (i) penalizing  $\Delta P$ , as fast variations of requested engine power represent one of the most fuel consuming phases, (ii) using the ICE as close as possible to its maximum efficiency point (see Figure 6(b)), and (iii) using the electric power to mitigate transients (see Figure 7(a)). The PMPC approach used here still provides a suboptimal solution, as the horizon is 100 seconds over the 1220 of the entire driving cycle. By exploiting the a priori knowledge of the cycle, PMPC can anticipate future power demand enough to maintain a high ICE efficiency and a smooth trajectory of the mechanical and the electric power, therefore providing a good benchmark for performance comparisons.

## VII. CONCLUSIONS

This paper has proposed a design method for the power management algorithm based on Markov chain modeling and stochastic model predictive control techniques. The driver power demand is modeled as a Markov chain to represent the future driver power request under different driving conditions. Contrary to PMPC, which is anticipative, the SMPC solution governs the engine, motor, and battery operations in a causal, time-invariant, state-feedback way. Simulation

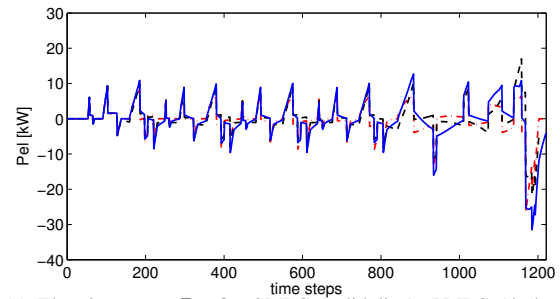


(a) Fuel consumption for SMPC (solid line), PMPC (dashed line), FTMP (dash-dotted line)

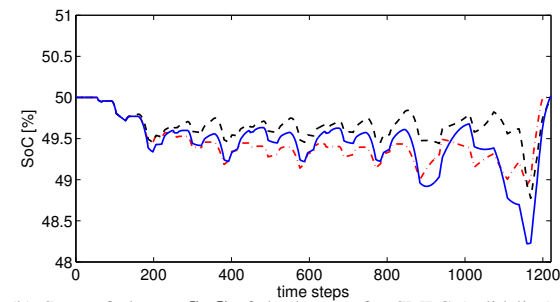


(b) Engine efficiency for SMPC (solid line), PMPC (dashed line), FTMP (dash-dotted line)

Fig. 6. Closed-loop trajectories: Fuel consumption and engine efficiency



(a) Electric power  $P_{el}$  for SMPC (solid line), PMPC (dashed line), FTMP (dash-dotted line)



(b) State of charge  $SoC$  of the battery for SMPC (solid line), PMPC (dashed line), FTMP (dash-dotted line)

Fig. 7. Closed-loop trajectories:  $P_{el}$  and  $SoC$

results indicate that the SMPC control strategy achieves improved performance compared with deterministic receding horizon techniques, close to the prescient approach, even if no exact knowledge of future power request is exploited. Future research efforts will involve testing the proposed approach on more realistic models of the HEV comparing with the globally optimal solution given by DP, and estimating online the Markov chain to adapt the controller to the driver's style and actual drive cycle.

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