Reliability and Efficiency for Market Parties in Power Systems

Laura Puglia, Panagiotis Patrinos, Daniele Bernardini, and Alberto Bemporad
IMT Institute for Advanced Studies
Lucca, Italy
Email: laura.puglia@imtlucca.it

***Abstract*—In this paper we present control strategies for solving the problems of risk-averse bidding on the electricity markets, focusing on the Day-Ahead and Ancillary Services market, and of optimal real-time power dispatch from the point of view of a market participant, or *Balance Responsible Party* (BRP). For what concerns the bidding problem, the proposed algorithms are based on two-stage stochastic programming and are aimed at finding the optimal allocation of production between the day-ahead exchange market and the ancillary services market. For the real-time power dispatch problem, we devised a two–level hierarchical control strategy, where the upper–level computes economically optimal power set-points for the generators, and the lower level tracks them while considering constraints and dynamical models of the plant. Simulation results based on realistic data modeling the Dutch transmission network are shown to evaluate the effectiveness of the approach.**

***Index Terms*—Distributed power generation, Optimization, Power generation planning, Predictive control, Smart grids**

I. Introduction

Liberalization and deregulation of electricity markets have led to a competitive environment consisting of market participants, termed as *Balance Responsible Parties* (BRPs), that are legally entitled to trade electricity on the various markets in order to satisfy their loads and ensure safe and reliable operation of the national grid.

On the other hand, adoption levels of renewable resources are continuously increasing due to the need for a decrease of production costs and greenhouse emissions from electricity generation by conventional fossil-fueled power plants (e.g., coal, gas, etc.). Efficient integration of intermittent generation into the existing power grid is a major bottleneck due to high variability and low predictability of renewable resources, especially wind [1].

In this paper we summarize the control schemes for optimal bidding on the Day-Ahead markets and efficient real-time operation of BRPs, developed within the activities of the EU project "E–Price: Price-based Control of Electrical Power Systems". The concept of E–Price is to devise efficient algorithms, market architectures and ICT interfaces in order to increase BRPs responsibility for the reliability of the power system while allowing flexibility and efficiency of the markets. In particular, BRPs are provided economical incentives to guarantee the correct and safe operation on the grid.

Although market structures vary with respect to each country, they share some common characteristics. Specifically, in the market considered here, participant BRPs place their bids on the *Day-Ahead* (DA) energy market and the *Ancillary Services* (AS) market regarding energy delivery and capacity availability for each *Program Time Unit* (PTU) of the following day. At the end of the day-ahead auction, the *Transmission System Operator* (TSO) selects the accepted and rejected bids according to some clearing mechanism [2] and publishes the future prices and volumes, for each PTU of the following day. Subsequently, each BRP receives its *Energy Program* (E-Program) from the TSO. The E-Program describes the amount of energy supplied or consumed by the BRP at every hour of the following day. Moreover, due to uncertainties in power demand and generation, a *imbalance market* (or *real-time market*) operated by the TSO helps counteract real-time global energy imbalances [3]. Due to unforeseen fluctuations of renewable sources or time-varying loads, the TSO can activate the bids previously placed on the AS market and, as a consequence, the BRP which submitted that specific bid must adjust its E-Program in real time accordingly.

For what concerns optimal bidding on the DA time scale, in Section II we formulate scenario-based control problems based on two-stage stochastic optimization. The most relevant difference in the DA market design with respect to current market structure is the presence of a double-sided AS market, by which BRPs can also submit request values, thus giving an estimate of their possible deviation from the E–Program, and giving the market the possibility to arrange the required reserves in advance, avoiding large imbalance. For the economically optimal management of real-time operations of BRPs, in Section III we propose a hierarchical MPC algorithm based on a temporal decomposition of the problem in two time scales (energy and power). The proposed control strategies are tested in simulation with realistic data modeling the Dutch transmission network. Finally, conclusions are given in Section IV.

This work was supported by the European project E–Price: Price-based Control of Electrical Power Systems, FP7-IST contract no. 249096.

II. OPTIMAL BIDDING ON ENERGY MARKETS

A BRP has several ways to sell the energy it produces. The most common option nowadays is to stipulate long-term bilateral contracts with external retail or distribution companies, where the price of energy is determined in advance. BRPs can also offer the produced energy to the spot market. To that purpose, a BRP must submit portfolio-based, daily *bid curves* in which they define the price at which they are willing to sell as a function of the produced energy for each hour of the following day. Furthermore, a BRP can address part of the capacity not sold at the day-ahead market to the reserve capacity market, which will be used by the TSO in real-time for regulation purposes (we neglect other markets such as the intra-day market for clarity).

Bidding curves are formulated as non-decreasing piecewise constant curves functions, and defined by a set of couples $\{(x_1, \lambda_1), \ldots, (x_K, \lambda_K)\}$, where λ_i is the minimum price requested to deliver the amount of energy x_i, $i = 1, \ldots, K$. One day before the delivery, the TSO collects bid curves from all BRPs and operates the clearing of the market by crossing the aggregated day-ahead bid curve with the aggregated load profile (which is usually price-unelastic). The *clearing price* and *volume* for the spot market are detected by the intersection of the two curves. The clearing price is then applied to every transaction on the market. In the E–Price framework, the day-ahead and the ancillary services sessions are assumed to be executed one after another, so that coupling between the problems of bidding on the day-ahead and on the ancillary markets can be neglected [4].

The architecture of ancillary services markets differ from country to country. For instance, the AS market can be *pay-as-bid*, that is, payments are executed as indicated by the bid curve. One of the innovative aspects considered in E–Price is to take into account *double-sided* AS markets. Each BRP gives an estimation of the possible deviation from the E-Program (due to uncertain events like wind production or elastic load) so that the market can be prepared and store the necessary reserve capacity. Deviations from the E-Program can be partly covered internally, by activating their spare regulating capacity, or they can be solved on the market. The part of regulating capacity that the BRP deems not economically convenient to cover for internally is included in the bidding curve under the form of *request curves*.

Therefore, each BRP submits supply energy curves S and request curves R. In particular, the supply S indicates the residual capacity a BRP wants to sell and be paid for by the TSO. It implies a positive cash flow. In particular, with S^+ we denote positive supply (the BRP is paid for additional *production*), and with S^- we denote negative supply (the BRP is paid for additional *absorption*). Request curves, on the other hand, imply a negative cash flow, i.e., the BRP is paying money to the TSO. In particular, with R^+ we denote positive request (BRP expects to be "long" and is willing to pay for additional *injection*), and with R^- we denote negative request (BRP expects to be "short" and is willing to pay for additional *absorption*).

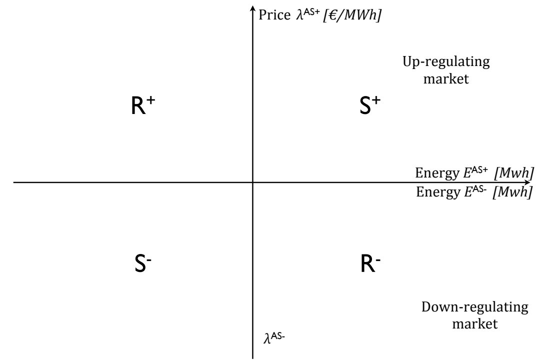


Fig. 1. Power flow directions and price signs for double-sided AS markets

Based on the collected bids, which can be submitted up to one hour before delivery, the TSO operates the clearing mechanism similarly as for the day-ahead market, intersecting the aggregated supply and request curves. From this process, a BRP can result as either a *supplier* or as a *requestor*, depending if the cleared capacity is positive or negative, respectively. Namely, we denote with $E^{AS+}(k) \in \mathbb{R}$ the up-regulating cleared capacity, i.e., BRP power surplus injected into the grid, and with $E^{AS-}(k) \in \mathbb{R}$ the down-regulating cleared capacity, i.e., BRP power shortage absorbed from the grid, at the PTU $k \in \mathbb{Z}_+$. Moreover, let $\lambda^{AS+}(k) \geq 0$ be the price for up-regulating energy, and $\lambda^{AS-}(k) \leq 0$ the price for down-regulating energy, in €/MWh (see Figure 1).

After clearing, awarded BRPs are obliged to reserve the assigned capacity for real-time purposes, and payments to/from the BRPs for allocating capacity reserves are proportional to the corresponding AS price. Therefore, the BRP daily revenue for reserve capacity allocation is given by

$$R_{CA} \triangleq a \sum_{k=1}^{N_{PTU}} \left(\lambda^{AS+}(k) E^{AS+}(k) + \lambda^{AS-}(k) E^{AS-}(k)\right) \quad (1)$$

where N_{PTU} is the number of PTUs in one day (we assume a PTU of 15 min, hence $N_{PTU} = 96$), and the coefficient $a > 0$ is a market design parameter. Choosing it too small will reduce the incentive of a BRP for bidding to supply AS, and choosing it too large will reduce the incentive of a BRP for bidding to request AS. An optimal value of a exists, which yields the largest liquidity in the cleared volumes. A large liquidity means, in general, more efficiency and therefore lower prices.

During real-time operations and with a sampling time T_{P}, the TSO can activate the bids and send to the BRP a request for increasing or decreasing its power set–point. These requests are defined by an AGC signal $\delta_p(t) \in \mathbb{R}$, and derive from real-time fluctuations of power generation and consumption. The AGC signal is distributed among BRPs based on their cleared capacity. The resulting daily revenue for the BRP on the AS market is

$$R_{AS} \triangleq \sum_{t=1}^{N}(\lambda^{AS+}(t)[\delta_p(t)]_+ - \lambda^{AS-}(t)[-\delta_p(t)]_+) \qquad (2)$$

where N is the number of T_{P} intervals in a day and $[x]_+ \triangleq \max\{x, 0\}$ is the positive part of x. BRPs deviating from their scheduled E-Program more than their requested capacity have to pay the excess deviation at a price $\lambda_{IM} = \phi\lambda^{AS+}$ in up-regulating mode, and $\lambda_{IM} = \phi\lambda^{AS-}$ in down-regulating mode, where $\phi \geq 1$ is a market parameter. Hence, by design, creating imbalance in a double-sided AS market is more expensive than in traditional single-sided markets. Regulating capacity is in most cases overpaid, as far as the regulating power requested by the TSO is within the cleared capacity.

A. Day-Ahead bidding strategy for BRPs

BRPs can be thought as aggregated companies producing and consuming energy, satisfying a certain amount of internal loads, and maximizing their own profit while taking into account risks due uncertain market behavior. BRP profit is given by the revenues due to bilateral contracts and trading on the DA exchange and AS markets, minus costs due to energy generation and imbalance. Therefore, optimal bidding strategies are needed to support the decision making process that leads to the DA energy production planning. It includes the following tasks:

1) Submit bid curves to the DA exchange market,
2) Plan an approximate production profiles for each generator by calculating the unit commitment,
3) Submit bid curves to the AS market.

When submitting the DA energy exchange bid curve, a BRP must take into account the uncertainty deriving from renewable production, loads, and AS prices (which are not disclosed yet). Since energy cannot be stored, in fact, energy offered on the DA exchange is no more available for trading on AS market.

For the formulation of optimal bids curves for the DA exchange and the AS market we propose a control approach based on numerical optimization, and in particular on the formulation of two-stage stochastic optimization problems where constraints on minimum and maximum power set–points of generators are taken into account, and a risk measure called Conditional Value at Risk (CVaR) is minimized to ensure that profits are maximized while risks of economical losses, depending on the realization of a stochastic parameter, are taken into account. In the DA exchange problem, the stochastic parameter models uncertainty on AS prices, that have not been disclosed yet. In the AS problem, the stochastic parameter represents uncertain load and generation for renewables. These optimization problems are formulated as linear programming problems (LP) and are suitable for real-time implementation. Detailed mathematical formulation of the proposed optimization problems is not shown here for space reasons.

B. Simulation results

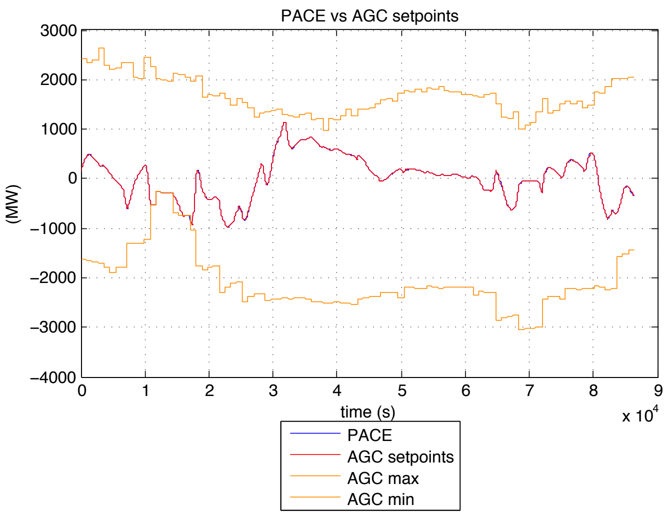
The proposed day-ahead bidding strategy has been tested in a simulation environment reproducing a national power system. The environment includes 7 BRPs with different production portfolios and risk attitudes, a TSO and a set of prosumers with elastic demand. Generators consist of gas turbines and wind farms. Data about installed capacities, historical prices, observed wind production and load have been provided by TenneT, the Dutch TSO and KEMA (omitted here for brevity). The double-sided architecture has been compared to the current single-sided structure for reserve markets, where no request bids can be submitted and imbalances are completely solved on the market based on a bid ladder. Comparisons are made in terms of net profit, generation cost, and profit deriving from AS trade. Robustness with respect to wind uncertainty has been tested by running simulations over 8 data sets, each with different wind forecasts (perfect and imperfect) and wind generation capacity (medium and large). The market parameter in (1) is set as $a = 0.05$, and the imbalance penalty is $\phi = 0.1$. Numerical results over a time interval of one day (96 PTUs) are reported in Table I. Notice that in the simulations the imperfect wind forecasts imply that more wind than expected blows in the system. For this reason production costs are lower in imperfect forecast conditions: underestimation of renewable production leads to lower day-ahead programs. On the other hand, in real-time BRPs decrease the power set-points of their plants and save production costs. As a consequence, also net profits are higher in imperfect forecast conditions, due to the avoided production costs and to the higher liquidity of the AS trade. This effect is caused by the day-ahead forecast errors, which BRPs exploit to their advantage by selling downward regulating capacity at a high price. Consistently with our hypothesis, imbalance costs are lower in case of perfect forecast because in general the proper amount of energy has been allocated beforehand. In each setting the double-sided architecture shows better performance with respect to profit and imbalance costs. By construction, AS prices are higher in the double-sided market and this encourages BRPs to allocate resources in a more efficient way. BRPs providing regulating capacity are more rewarded, thus increasing the social welfare. To give an idea of the regulating capacity actually activated in real time we refer to Figure 2. The red line is the aggregated AGC signal sent by the TSO to BRPs for regulating power. The trend of the signal is basically the same in the two settings, but in the double-sided case (Figure 2(b)) it is for most part included between the two green lines delimiting the cleared AS capacity. As far as the signal keeps between the two green lines, the system is prepared to react by delivering the previously allocated power. In case there is no sufficient cleared AS capacity, the TSO must resort to the un-cleared bids. If not even this capacity suffices, the TSO must ask the help of neighboring countries.

III. Real-time operations of market parties

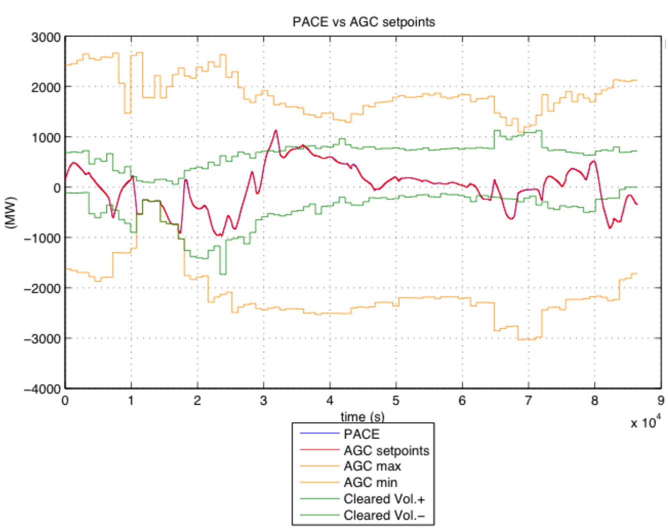
In real time a BRP must fulfill its E-Program in order to avoid internal imbalance and imbalance costs, and it has

TABLE I
ECONOMIC PERFORMANCE METRICS IN SIMULATED CASE STUDIES

	Market type	Wind production	Wind forecast	Production cost (€)	AS profit (€)	Imbalance costs (€)
#1	Single-sided	medium	perfect	1,821,846	285,294	36,602
#2	Double-sided	medium	perfect	1,824,743	366,895	16,899
#3	Single-sided	large	perfect	1,639,333	329,327	84,365
#4	Double-sided	large	perfect	1,631,739	617,481	66,597
#5	Single-sided	medium	imperfect	1,749,858	347,151	105,577
#6	Double-sided	medium	imperfect	1,750,263	452,491	98,024
#7	Single-sided	large	imperfect	1,595,605	903,332	706,271
#8	Double-sided	large	imperfect	1,594,743	902,984	438,066



(a) Single-sided market



(b) Double-sided market

Fig. 2. AGC signal and allocated upward/downward capacity with medium wind production and imperfect forecast

to cope with uncertainties induced by intermittent generation from renewable sources, time-varying loads and imbalance prices, as well as perturbations of its E-Program due to AS bids activated by the TSO. If the TSO calls for a specific AS bid, the BRP responsible for this bid is asked to deliver the requested energy by adjusting its E-Program accordingly. In addition, imbalance prices are characterized by large volatility and can also be negative, i.e., incentives for the BRP to deviate from the E-Program to compensate possible energy surplus or shortfall in the grid [5]. The real-time BRP control problem is further complicated by the coupling of energy between consecutive PTUs, due to the bounds on the rate of change of the power set–points of the generators (ramp-rate constraints).

In this section we present an algorithm for efficient and reliable real-time operation of a BRP, based on hierarchical Model Predictive Control (MPC). MPC has emerged in the last decades as the leading technology for advanced control of highly complex and multivariable processes. Its success is mainly due to its ability to handle constraints on the system (e.g., bounds on selected variables and their rates of change) while taking into account the system dynamics and optimizing a given objective function (e.g., minimizing costs and risks). The system model, constraints, and objective function define an optimal control problem over a finite time-horizon in the future, that is solved on-line to obtain an optimal sequence of future control moves. Only the first of the moves is applied to the process, as at the next time step a new optimal control problem is solved, to exploit the information coming from fresh new measurements. In this way, an open-loop design methodology is transformed into a feedback one [6].

The overall goal of the controller presented here is to determine power set–points for the controllable generators so as to minimize generation costs by utilizing intermittent resources as much as possible, and economically track the E-Program assigned by the TSO, meaning that it may be profitable to deviate either upwards or downwards from the E-Program, depending on the current imbalance prices. In the proposed two-level hierarchical control architecture, the upper–level MPC operates on the *energy time scale*, whose sampling period is typically in the minutes range, and computes power and energy set–points based on predictions of the uncertain exogenous inputs (AS bids activated by TSO, imbalance prices, intermittent generation and load) by minimizing BRP costs. The lower–level MPC operates on the *power time scale*, whose sampling period is typically in the seconds range, and tries to track the set–points received by upper level by taking into account the detailed dynamics of the generators. The main advantages of the proposed scheme are the real-time calculation of economically optimal set–points that allow to exploit possible favorable imbalance prices, the effective energy set–points tracking and the ramp-rate constraints handling that enable smooth transitions between PTUs.

A. Basic setup and notation

We consider a BRP consisting of $n_{\rm p}$ controllable generators, $n_{\rm r}$ uncontrollable generators and uncontrollable load. To simplify the presentation, in the following we assume that all controllable generators of the BRP are always turned on, and that no plant trips occur. The proposed strategies are easily extended to the case where generators can be switched on and turned off in real-time [7].

Before presenting the proposed control scheme we need to introduce some notation. The length of a PTU is denoted by $T_{\rm PTU}$ [s], while $T_{\rm E}$ [s] is the sampling time of the energy time scale and $T_{\rm P}$ [s] is the sampling time of the power time scale. We assume that $T_{\rm PTU} \geq T_{\rm E} \geq T_{\rm P}$ and that $r_{\rm ET} \triangleq \frac{T_{\rm PTU}}{T_{\rm E}}$, $r_{\rm PE} \triangleq \frac{T_{\rm E}}{T_{\rm P}}$, and $r_{\rm PT} \triangleq \frac{T_{\rm PTU}}{T_{\rm P}}$, are positive integers. We use symbols n, k and $t \in \mathbb{Z}_+$ to index time in the PTU, energy and power time scale respectively. Let $\kappa_{\rm ET}(k) \triangleq \left\lfloor \frac{k}{r_{\rm ET}} \right\rfloor$ denote the PTU active at time k in the energy time scale. Similarly, we define $\kappa_{\rm PE}(t) \triangleq \left\lfloor \frac{t}{r_{\rm PE}} \right\rfloor$, $\kappa_{\rm PT}(t) \triangleq \left\lfloor \frac{t}{r_{\rm PT}} \right\rfloor$. Consider now the PTU $n \in \mathbb{Z}_+$. Then, the intervals on the energy time scale that are active along n are $k \in \mathbb{N}_{[k_{\min}(n), k_{\max}(n)]}$, where $k_{\min}(n) \triangleq r_{\rm ET} n$, $k_{\max}(n) \triangleq r_{\rm ET}(n+1) - 1$. Similarly, for $k \in \mathbb{Z}_+$ in the energy time scale, the set of intervals on the power time scale that are active along k are $t \in \mathbb{N}_{[t_{\min}(k), t_{\max}(k)]}$, where $t_{\min}(k) \triangleq r_{\rm PE} k$, $t_{\max}(k) \triangleq r_{\rm PE}(k+1) - 1$.

The trajectories of controllable power injections, uncontrollable power injections and load on the power time scale are denoted by $p^i(t)$, $i = 1, \ldots, n_{\rm p}$, $r^i(t)$, $i = 1, \ldots, n_{\rm r}$, and $d(t)$, respectively. Furthermore, let $p(t) \triangleq (p^1(t) \cdots p^{n_{\rm P}}(t))$, $r(t) \triangleq (r^1(t) \cdots r^{n_{\rm r}}(t))$. Let $\bar{p}^i(k)$ denote the average controllable power injection as a variable at some future energy time interval k. Similarly, let $\bar{r}^i(k)$, $\bar{d}(k)$ denote the predicted values of the average uncontrollable power injections and load, respectively, at some future energy time interval k. We define $\bar{p}(k) \triangleq (\bar{p}^1(k), \ldots, \bar{p}^{n_{\rm P}}(k))$, $\bar{r}(k) \triangleq (\bar{r}^1(k) \cdots \bar{r}^{n_{\rm r}}(k))$.

Now we need to introduce some quantities related to energy production and forecast on the energy and the power time scales. For $k \in \mathbb{Z}_+$ and $t \in \mathbb{N}_{[t_{\min}(k), t_{\max}(k)+1]}$, let $e_k^{\rm p}(t)$ and $e_k^{\rm rd}(t)$ denote the accumulated actual energy produced by controllable generators, and by uncontrollable generators and load, respectively, along energy time interval k and up to time instant $tT_{\rm P}$, based on the actual power injections of the BRP on the power time scale. For $k \in \mathbb{Z}_+$, $e_k^{\rm p} \triangleq e_k^{\rm p}(t_{\max}(k)+1)$ and $e_k^{\rm rd} \triangleq e_k^{\rm rd}(t_{\max}(k)+1)$ are the corresponding accumulated energy produced within the energy time interval k. Accordingly, for $n \in \mathbb{Z}_+$ and $k \in \mathbb{N}_{[k_{\min}(n), k_{\max}(n)+1]}$, let $e^{\rm p}(n;k)$ and $e^{\rm rd}(n;k)$ denote the accumulated actual energy produced by controllable generators, and by uncontrollable generators and load, respectively, along PTU n, up to time instant $kT_{\rm E}$, based on the real power injections of the BRP. We denote with $\bar{e}^{\rm p}(n;k)$, $\bar{e}^{\rm rd}(n;k)$ the predicted values of $e^{\rm p}(n;k)$, $e^{\rm rd}(n;k)$, respectively. For $n \in \mathbb{Z}_+$, $e^{\rm p}(n) \triangleq e^{\rm p}(n; k_{\max}(n)+1)$ and $e^{\rm rd}(n) \triangleq e^{\rm rd}(n; k_{\max}(n)+1)$ denote the accumulated energy produced by controllable generators, and by uncontrollable generators and load, respectively, along PTU n, and $\bar{e}^{\rm p}(n)$, $\bar{e}^{\rm rd}(n)$, denote the predictions of the corresponding quantities. All energy quantities are expressed in MWh, while powers are in MW.

B. MPC on the energy time scale

Given the E-program assigned by the TSO and the set of activated AS bids, the upper–level MPC performs economic optimization deciding the power and energy set–points for the lower–level MPC, based on predictions of the uncertain load, uncontrollable generation and activated AS bids, so as to minimize production and imbalance costs. Models for generation and imbalance costs and the upper–level MPC problem are presented next.

1) Generation costs: It is assumed that individual generation costs of controllable generators are modeled by convex quadratic functions, i.e., $\ell_i^{\rm p}(\bar{p}^i) \triangleq a_i(\bar{p}^i)^2 + b_i\bar{p}^i + c_i$, $a_i \geq 0$, $i \in \mathbb{N}_{[1,n_{\rm p}]}$. The total generation cost $\ell^{\rm p}(\bar{p})$ of the BRP consists of the sum of production costs related to the controllable generators, i.e., $\ell^{\rm p}(\bar{p}) \triangleq \sum_{i=1}^{n_{\rm p}} \ell_i^{\rm p}(\bar{p}^i)$.

2) Imbalance costs: For $n \in \mathbb{Z}_+$, $e^{\rm prog}(n)$ is the energy that the BRP is commited to supply to (or absorb from) the TSO at PTU n, according to its E-Program, determined on the day ahead. On the power time scale (usually 4 s), the TSO sends to each BRP actively participating in the secondary control arrangements a delta power signal $\delta_p(t) \in \mathbb{R}$ corresponding to the AS bids activated by the TSO, which the BRP has to realize by changing the power set–point of the selected units. Then, the BRP E-Program is offset with the requested energy resulting in the final E-Program $e^{\rm final}(n)$:

$$e^{\rm final}(n) \triangleq e^{\rm prog}(n) + \textstyle\sum_{\{t \in \mathbb{Z}_+ | \kappa_{\rm PT}(t) = n\}} \delta_p(t). \quad (3)$$

We also define the energy due for secondary control in the current interval of the energy time scale as

$$e^{\rm sc}(k) \triangleq \tfrac{T_P}{3600} \textstyle\sum_{t=t_{\min}(k)}^{t_{\max}(k)} \delta_p(t), \quad (4)$$

and denote with $\bar{e}^{\rm sc}(k)$ its prediction. Moreover, the imbalance $\Delta e(n)$ of the BRP at PTU n is the difference between the actual energy produced by the BRP at PTU n and $e^{\rm final}(n)$, i.e., $\Delta e(n) \triangleq e(n) - e^{\rm final}(n)$.

If the BRP has a *surplus of energy* ($\Delta e(n) > 0$), then the TSO buys this energy at the *surplus imbalance price* $\lambda_{\rm IM}^+(n)$. The price $\lambda_{\rm IM}^+(n)$ can be negative, in which case the BRP is the one who pays the TSO. On the contrary, if the BRP has a *shortfall of energy* ($\Delta e(n) < 0$), then it buys energy from the TSO at the *shortfall imbalance price* $\lambda_{\rm IM}^-(n)$. The price $\lambda_{\rm IM}^-(n)$ can be negative, in which case the TSO is the one who pays the BRP. Therefore, the imbalance cost for the BRP is defined as $\ell_{\rm IM}(\Delta e) \triangleq -\lambda_{\rm IM}^+[\Delta e]_+ + \lambda_{\rm IM}^-[-\Delta e]_+$. Since $[x]_+ = \frac{1}{2}(|x| + x)$, we have

$$\ell_{\rm IM}(\Delta e) = \frac{1}{2}(\lambda_{\rm IM}^- - \lambda_{\rm IM}^+)|\Delta e| - \frac{1}{2}(\lambda_{\rm IM}^- + \lambda_{\rm IM}^+)\Delta e. \quad (5)$$

Here, we assume that $\lambda_{\rm IM}^- \geq \lambda_{\rm IM}^+$. The assumption is substantiated by historical data[1] regarding on year of imbalance prices obtained by TenneT, the Dutch TSO, where only 0.008% of the PTUs have $\lambda_{\rm IM}^- < \lambda_{\rm IM}^+$. Under this assumption, $\ell_{\rm IM}$ is a convex piecewise–affine (PWA) function.

3) Upper–level MPC formulation: According to the MPC philosophy, at every time instant $k \in \mathbb{Z}_+$ on the energy time scale we formulate and solve a finite–horizon optimal control problem, where the goal is find a sequence of power and energy set–points for the generators so as to minimize the expected costs along the energy prediction horizon $N_{\rm E}$. The problem is stated in (6) and described next.

$$
\begin{aligned}
\min \; & \sum_{j=k}^{k+N_{\rm E}-1} \ell_{\rm p}(\bar{p}(j)) + \sum_{n=\underline{n}(k)}^{\overline{n}(k)-1} \ell_{\rm IM}(\bar{e}^{\rm p}(n) + \bar{e}^{\rm rd}(n) - \bar{e}^{\rm final}(n)) \\
& + \ell_{\rm IM}(s_k^{N_{\rm E}}(\bar{e}^{\rm p}(\overline{n}(k)) + \bar{e}^{\rm rd}(\overline{n}(k))) - \bar{e}^{\rm final}(\overline{n}(k))) \qquad \text{(6a)} \\
\text{s.t.} \; & \bar{p}(k-1) = \tfrac{T_{\rm P}}{T_{\rm E}} \sum_{t=t_{\min}(k-1)}^{t_{\max}(k-1)} p(t), \; \bar{e}^{\rm p}(\underline{n}(k)) = e^{\rm p}(\underline{n}(k); k), \qquad \text{(6b)} \\
& \bar{e}^{\rm rd}(\underline{n}(k)) = e^{\rm rd}(\underline{n}(k); k), \; \bar{e}^{\rm final}(\underline{n}(k)) = e^{\rm final}(\underline{n}(k)), \qquad \text{(6c)} \\
& \bar{p}^i_{\min} \leq \bar{p}^i(j) \leq \bar{p}^i_{\max}, \; i \in \mathcal{I}, \; j \in \mathcal{J}, \qquad \text{(6d)} \\
& \tfrac{T_{\rm E}}{60} \Delta p^i_{\min} \leq \frac{\bar{p}^i(j)}{\bar{p}^i(j-1)} - 1 \leq \tfrac{T_{\rm E}}{60} \Delta p^i_{\max}, \; i \in \mathcal{I}, \; j \in \mathcal{J}, \qquad \text{(6e)} \\
& \bar{e}^{\rm p}(\underline{n}(k)) = \bar{e}^{\rm p}(\underline{n}(k)) + \tfrac{T_{\rm E}}{3600} \sum_{j=k}^{k_{\max}(\underline{n}(k))} \sum_{i=1}^{n_{\rm p}} \bar{p}^i(j), \qquad \text{(6f)} \\
& \bar{e}^{\rm rd}(\underline{n}(k)) = \bar{e}^{\rm rd}(\underline{n}(k)) + \tfrac{T_{\rm E}}{3600} \sum_{j=k}^{k_{\max}(\underline{n}(k))} \sum_{i=1}^{n_{\rm r}} (\bar{r}^i(j) - \bar{d}(j)), \qquad \text{(6g)} \\
& \bar{e}^{\rm p}(n) = \tfrac{T_{\rm E}}{3600} \sum_{j=k_{\min}(n)}^{k_{\max}(n)} \sum_{i=1}^{n_{\rm p}} \bar{p}^i(j), \; n \in \mathcal{N}, \qquad \text{(6h)} \\
& \bar{e}^{\rm rd}(n) = \tfrac{T_{\rm E}}{3600} \sum_{j=k_{\min}(n)}^{k_{\max}(n)} \sum_{i=1}^{n_{\rm r}} (\bar{r}^i(j) - \bar{d}(j)), \; n \in \mathcal{N}, \qquad \text{(6i)} \\
& \bar{e}^{\rm p}(\overline{n}(k)) = \tfrac{T_{\rm E}}{3600} \sum_{j=k_{\min}(\overline{n}(k))}^{k+N_{\rm E}-1} \sum_{i=1}^{n_{\rm p}} \bar{p}^i(j), \qquad \text{(6j)} \\
& \bar{e}^{\rm rd}(\overline{n}(k)) = \tfrac{T_{\rm E}}{3600} \sum_{j=k_{\min}(\overline{n}(k))}^{k+N_{\rm E}-1} \sum_{i=1}^{n_{\rm r}} (\bar{r}^i(j) - \bar{d}(j)), \qquad \text{(6k)} \\
& \bar{e}^{\rm final}(\underline{n}(k)) = \bar{e}^{\rm final}(\underline{n}(k)) + \sum_{j=k}^{k_{\max}(\underline{n}(k))} \bar{e}^{\rm sc}(j), \qquad \text{(6l)} \\
& \bar{e}^{\rm final}(n) = \bar{e}^{\rm prog}(n) + \sum_{j=k_{\min}(n)}^{k_{\max}(n)} \bar{e}^{\rm sc}(j), \; n \in \mathcal{N}, \qquad \text{(6m)} \\
& \bar{e}^{\rm final}(\overline{n}(k)) = \bar{e}^{\rm prog}(\overline{n}(k)) + \sum_{j=k_{\min}(\overline{n}(k))}^{k+N_{\rm E}-1} \bar{e}^{\rm sc}(j), \qquad \text{(6n)} \\
& \mathcal{I} \triangleq \mathbb{N}_{[1,n_{\rm p}]}, \; \mathcal{J} \triangleq \mathbb{N}_{[k,k+N_{\rm E}-1]}, \; \mathcal{N} \triangleq \mathbb{N}_{[\underline{n}(k)+1,\overline{n}(k)-1]}. \qquad \text{(6o)}
\end{aligned}
$$

The objective function (6a) allows to minimize the expected generation and imbalance costs along the prediction horizon.

[1] Available at http://www.tennet.org/english/operational_management

Notice that the PTUs that are active during the energy prediction horizon and starting from the time instant k are $n \in \mathbb{N}_{[\underline{n}(k),\overline{n}(k)]}$, where $\underline{n}(k) \triangleq \kappa_{\rm ET}(k)$, $\overline{n}(k) \triangleq \kappa_{\rm ET}(k+N_{\rm E}-1)$.

Equations (6b) and (6c) provide the initial conditions on power and energy for the MPC problem. Bounds on the power output of each controllable generator are imposed by (6d). The power profiles are also subject to downward and upward ramping limits $\Delta p_{\min}$ and $\Delta p_{\max}$ [%/min]. Satisfaction of such ramp–rate constraints is enforced by (6e). Finally, (6f)–(6k) model the energy balances inside each PTU of the prediction horizon, and (6l)–(6n) define the energy set–point given by the deviated E-Program.

Problem (6) is a convex quadratic program, that can be solved efficiently with off-the-shelf software tools.

C. MPC on the power time scale

The lower–level MPC acts on the power time scale and tracks the reference power and energy signals obtained by the upper–level MPC by taking into account detailed generators dynamics. The models of controllable generators and the lower-level problem formulation are described next.

1) Dynamics of controllable generators: Generators are modeled using the model developed in [8]. It consists of two parts: the first describes the fast power changes of primary reserve activation, and the second concerns the relatively slow variations that take place in secondary reserve activation. The *fast* model consists of a low and a high pass filter connected in series:

$$
p^i_{\rm fast}(s) = \frac{\tau^i_{\rm H} s}{\tau^i_{\rm H} s + 1} \frac{K^i}{\tau^i_{\rm L} s + 1} p^i_{\rm prim}(s), \qquad (7)
$$

where $p^i_{\rm prim}$ is the power for primary control. The *slow* model is given by

$$
p^i_{\rm sl}(s) = \frac{e^{-T^i_{\rm delay} s}}{\tau^i s + 1} (u^i(s) + p^i_{\rm prim}(s)), \qquad (8)
$$

where u^i is the power set–point of the generator. The primary control of the i–th generator is described by

$$
p^i_{\rm prim}(s) = -\frac{100 p^i_{\max}}{f_{\rm nom} c^i_{dr}} \delta_f(s), \qquad (9)
$$

where δ_f is the change in frequency with respect to the nominal frequency $f_{\rm nom}$ (50 Hz), and c^i_{dr} is the droop [%]. The power output of the i–th generator is given by

$$
p^i(s) = p^i_{\rm fast}(s) + p^i_{\rm sl}(s). \qquad (10)
$$

Using a first–order Padé approximation for the time delay and a ZOH discretization with sampling time $T_{\rm P}$, (7)–(10) are represented in state–space as

$$
\begin{aligned}
x^i(t+1) &= A^i x^i(t) + B^i u^i(t) + E^i \delta_f(t), \qquad \text{(11a)} \\
y^i(t) &= C^i x^i(t), \qquad \text{(11b)}
\end{aligned}
$$

where $x(t)^i \in \mathbb{R}^4$, $u(t)^i \in \mathbb{R}$, and $y^i(t) = [p^i(t) \; p^i_{\rm sl}(t)]'$ are the i–th generator's state, input (i.e., the power set–point) and output vectors. By collecting the models (11) for all generators

$i = 1, 2, \ldots, n_{\mathrm{p}}$, the aggregated dynamics of the BRP are written in the compact form

$$x(t+1) = Ax(t) + Bu(t) + E\delta_f(t), \qquad (12a)$$
$$y(t) = Cx(t). \qquad (12b)$$

2) Lower–level MPC formulation: At every $t \in \mathbb{Z}_+$ on the power time scale, the lower level MPC tries to track the power and energy reference signals $\bar{p}$, $\bar{e}^{\mathrm{p}}$ obtained from the upper level, along the power prediction horizon N_{P}. The problem is stated in (13), where we use the notation $\bar{e}^{\mathrm{p}}_k$, p^i_j, x_j, $\delta_{f,j}$ to denote *predictions* for e^{p}_k, $p^i(j)$, $x(j)$, $\delta_f(j)$.

$$\min \sum_{j=t}^{t+N_{\mathrm{P}}} ||p_j - \bar{p}(\kappa_{\mathrm{PE}}(j))||^2 + \sum_{k=\underline{k}(t)}^{\overline{k}(t)} q_k(\Delta e_k)^2 \qquad (13a)$$
$$\text{s.t. } p_t = p(t),\ \delta_{f,t} = \delta_f(t),\ x_t = \hat{x}(t), \qquad (13b)$$
$$x_{j+1} = Ax_j + Bu_j + E\delta_{f,j},\ j \in \mathcal{J}, \qquad (13c)$$
$$p_j = C_1 x_j,\ p_{\mathrm{sl},j} = C_2 x_j,\ j \in \mathcal{J}, \qquad (13d)$$
$$p^i_{\min} \le u^i_j \le p^i_{\max},\ i \in \mathcal{I},\ j \in \mathcal{J}, \qquad (13e)$$
$$\tfrac{T_{\mathrm{P}}}{60}\Delta p^i_{\min} \le \frac{u^i_j}{u^i_{j-1}} - 1 \le \tfrac{T_{\mathrm{P}}}{60}\Delta p^i_{\max},\ i \in \mathcal{I},\ j \in \mathcal{J}, \qquad (13f)$$
$$p^i_{\min} \le p^i_{\mathrm{sl},j} \le p^i_{\max},\ i \in \mathcal{I},\ j \in \mathcal{J}, \qquad (13g)$$
$$\tfrac{T_{\mathrm{P}}}{60}\Delta p^i_{\min} \le \frac{p^i_{\mathrm{sl},j}}{p^i_{\mathrm{sl},j-1}} - 1 \le \tfrac{T_{\mathrm{P}}}{60}\Delta p^i_{\max},\ i \in \mathcal{I},\ j \in \mathcal{J}, \qquad (13h)$$
$$\bar{e}^{\mathrm{p}}_{\underline{k}(t)} = e^{\mathrm{p}}_{\underline{k}(t)}(t) + \tfrac{T_{\mathrm{P}}}{3600} \sum_{j=t}^{t_{\max}(\underline{k}(t))} \sum_{i=1}^{n_{\mathrm{p}}} p^i_j, \qquad (13i)$$
$$\bar{e}^{p}_k = \tfrac{T_{\mathrm{P}}}{3600} \sum_{j=t_{\min}(k)}^{t_{\max}(k)} \sum_{i=1}^{n_{\mathrm{p}}} p^i_j,\ k \in \mathcal{K}, \qquad (13j)$$
$$\bar{e}^{p}_{\overline{k}(t)} = \tfrac{T_{\mathrm{P}}}{3600} \sum_{j=t_{\min}(\overline{k}(t))}^{t+N_{\mathrm{P}}-1} \sum_{i=1}^{n_{\mathrm{p}}} p^i_j, \qquad (13k)$$
$$\Delta e_k = \bar{e}^{\mathrm{p}}_k - \bar{e}^{\mathrm{p}}(\kappa_{\mathrm{ET}}(k); k+1),\ k \in \mathcal{K}, \qquad (13l)$$
$$\Delta e_{\overline{k}(t)} = \bar{e}^{\mathrm{p}}_{\overline{k}(t)} - s^{N_{\mathrm{P}}}_t \bar{e}^{\mathrm{p}}(\kappa_{\mathrm{ET}}(\overline{k}(t)); \overline{k}(t)+1), \qquad (13m)$$
$$\mathcal{I} \triangleq \mathbb{N}_{[1,n_{\mathrm{p}}]},\ \mathcal{J} \triangleq \mathbb{N}_{[t,t+N_{\mathrm{P}}-1]},\ \mathcal{K} \triangleq \mathbb{N}_{[\underline{k}(t),\overline{k}(t)-1]}. \qquad (13n)$$

The cost function (13a) penalizes deviations from the power and energy set–points. The coefficients $q_k > 0$ can be set to decrease with k, in order to put more emphasis on the current PTU and take advantage of good short term predictions. It is however important to penalize the energy tracking error also for future PTUs, in order to compute optimal power profiles that respect ramp–rate constraints. Notice that the energy time intervals that are active during the prediction horizon and starting from the power time interval t are $k \in \mathbb{N}_{[\underline{k}(t),\overline{k}(t)]}$, where $\underline{k}(t) \triangleq \kappa_{\mathrm{PE}}(t)$ and $\overline{k}(t) \triangleq \kappa_{\mathrm{PE}}(t + N_{\mathrm{P}})$.

Equations (13b) provides the initial conditions of the problem, that is, current power injections, frequency deviations and estimated states of the generators. In fact, since a full state measurement is not available, we use an estimation $\hat{x}(t)$ of the true state vector $x(t)$, obtained through an observer such as a Kalman filter.

TABLE II
COMPARISON OF HMPC AND SPT FOR REAL-TIME POWER DISPATCH

Controller	Generation cost (€)	Imbalance cost (€)	Total cost (€)
HMPC	578,674	-49,057	529,617
SPT	549,257	-3,285	545,973

Equations (13c)–(13d) model the dynamics of the controllable generators, while (13e)–(13f) and (13g)–(13h) impose bounds and ramp-rate constraints on the power set–points and on the slow powers, respectively. Moreover, (13i)–(13k) describe the energy balances in the power time scale. Finally, (13l)–(13m) define the tracking error on the energy reference trajectory provided by the upper level controller, where $s^{N_{\mathrm{P}}}_t \triangleq \mathrm{mod}\,(\frac{(t+N_{\mathrm{P}}+1)T_{\mathrm{P}}}{T_{\mathrm{E}}})/T_{\mathrm{E}}$.

By solving problem (13) we obtain the power set–points $u^i(t) = u^i_t$, $i \in \mathbb{N}_{[1,n_{\mathrm{p}}]}$, which are applied to the generators at every time $t \in \mathbb{Z}_+$ on the power time scale. Similarly to (6), problem (13) is a convex quadratic program and is suitable for efficient real-time implementation.

D. Case study

The proposed Hierarchical MPC (HMPC) scheme is tested in simulation on a BRP consisting of a set of 10 controllable generators, most of which are Combined Cycle Gas Turbines (CCGT), and one uncontrollable generator representing a wind farm. The considered generators include small, medium and large-sized plants, with maximum output power ranging from 53 MW to 1675 MW, maximum efficiency between 38% and 59%, and maximum allowed ramp rate between 1.5%/min and 5%/min, emulating a typical BRP in the Dutch power system. Realistic data for wind generation, wind forecast, E-Program, imbalance prices, frequency deviation, volume of AS bids activated in real-time were obtained from KEMA and TenneT (detailed data are omitted here due to space limitations).

Simulation were carried out for 16 PTUs, where the length of one PTU is 15 minutes. The energy sampling time is $T_{\mathrm{E}} = 60$ s, while the power sampling time is $T_{\mathrm{P}} = 4$ s. The prediction horizons for the upper and lower level MPCs are $N_{\mathrm{E}} = 16$ and $N_{\mathrm{P}} = 10$, respectively. The proposed HSMC scheme was compared against a more basic control policy which is supposed to reflect, to some extent, the current practice. In this algorithm, power set–points for the generators for each PTU are computed on the day-ahead in an open-loop fashion, while the real-time controller tries to reach the set–points along the current PTU in a static way, without taking into account plants dynamics. We call this algorithm *Static set–point Tracking* (SPT).

The two schemes are compared in terms of generation costs, imbalance costs, and total costs along the 16 PTUs. Numerical results are shown in Table II. HMPC exhibits clearly superior performance with respect to SPT, and this is mainly due to its capability of exploiting favorable imbalance prices, taking into account uncontrollable generation forecasts, and handling ramp-rate constraints safely. Figures 3 and 4 depict the power profiles of the generators for HMPC and SPT, respectively.

The abrupt changes on the power profiles in Fig. 3 clearly shows SPT inability to handle ramp-rate constraints and allow smooth transitions between PTUs. On the other hand, due to explicit ramp-rate constraint handling and energy integral action, HMPC exhibits smooth transitions between PTUs, as shown in Fig. 4.

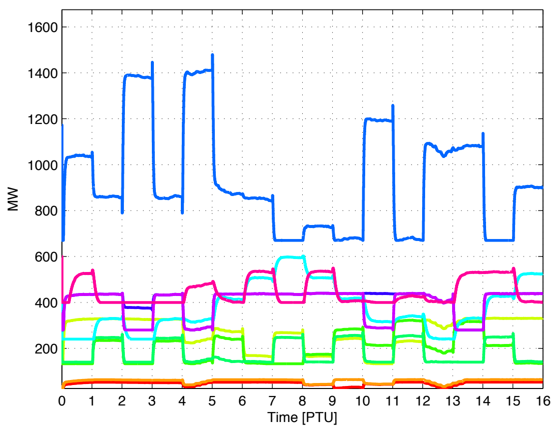


Fig. 3. Power profile of BRP plants obtained with SPT.

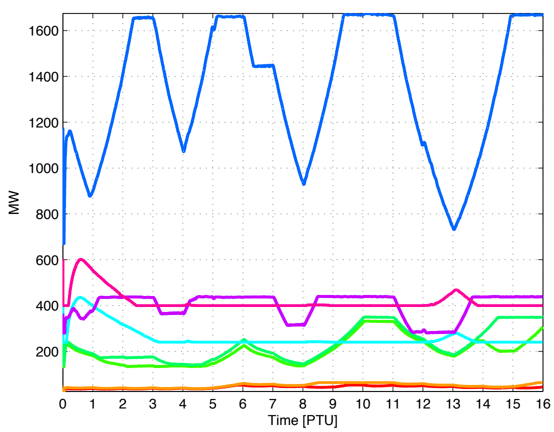


Fig. 4. Power profile of BRP plants obtained with HMPC.

IV. Conclusions and Future Work

In this paper we summarized the control approaches developed within the E–Price project for solving the problems of optimal bidding on the energy markets and optimal real-time power dispatch, from the point of view of a market participants, or BRP. For what concerns the bidding problem, the proposed algorithms are based on two-stage stochastic programming and are aimed at finding the optimal allocation of production between the day-ahead exchange market and the ancillary services market. For the real-time power dispatch problem, we devised a two–level hierarchical control strategy, where the upper–level computes economically optimal power set-points for the generators, and the lower level tracks them while considering constraints and dynamical models of the plant. Simulation results based on realistic data modeling the Dutch transmission network show that the approach provides a reliable and valid solution for integration of renewable energy sources, solving their most crucial problems related to intermittence and forecast errors, by computing in real-time economically optimal set–points based on accurate predictions.

Ongoing research work aims at extending the proposed algorithms to a broader stochastic framework, using results of [9], so as to further improve both performance and robustness of the controller, as well as incorporating price-elastic prosumers into the BRP model [10].

References

[1] B. Ummels, M. Gibescu, E. Pelgrum, W. Kling, and A. Brand, "Impacts of wind power on thermal generation unit commitment and dispatch," *IEEE Transactions on Energy Conversion*, vol. 22, no. 1, pp. 44–51, 2007.

[2] M. Shahidehpour, H. Yamin, and Z. Li, *Market operations in electric power systems*. Wiley, 2002.

[3] A. Jokic, "Price-based optimal control of electrical power systems," Ph.D. dissertation, Technische Universiteit Eindhoven, 2007.

[4] A. Virag, A. Jokic, R. Hermans, and P. Van den Bosch, "Combined bidding at power and ancillary service markets," in *8th International Conference on the European Energy Market (EEM)*, Zagreb, Croatia, 2011, pp. 568–573.

[5] M. Sewalt and C. de Jong, "Negative prices in electricity markets," *Commodities Now*, 2003. [Online]. Available: www.commodities-now.com

[6] A. Bemporad, "Model-based predictive control design: New trends and toolsl," in *Proc. 45th IEEE Conf. on Decision and Control*, San Diego, CA, 2006, pp. 6678–6683.

[7] P. Patrinos, D. Bernardini, A. Maffei, A. Jokic, and A. Bemporad, "Two-time-scale MPC for economically optimal real-time operation of balance responsible parties," in *8th IFAC Symposium on Power Plant and Power System Controls*, Toulouse, France, 2012.

[8] B. Roffel and W. de Boer, "Analysis of power and frequency control requirements in view of increased decentralized production and market liberalization," *Control engineering practice*, vol. 11, no. 4, pp. 367–375, 2003.

[9] P. Patrinos, S. Trimboli, and A. Bemporad, "Stochastic MPC for real-time market-based optimal power dispatch," in *50th IEEE Conf. on Decision and Control and European Control Conf., Orlando, FL*, 2011, pp. 7111–7116.

[10] P. Patrinos, D. Bernardini, A. Jokic, G. Dekker, and A. Bemporad, "Two-time-scale stochastic MPC for economically optimal real-time operation of balance responsible parties," submitted for journal publication, 2013.