Discrete-event modelling and diagnosis of quantised systems

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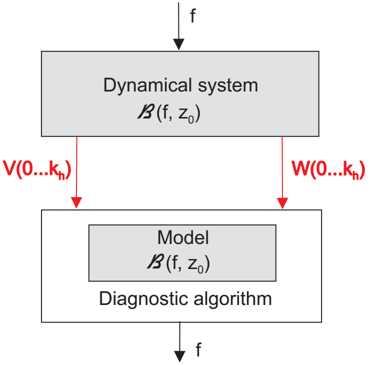
Survey

Model-based diagnosis



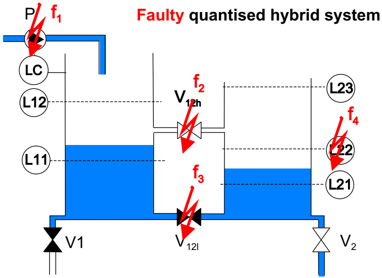
Diagnostic problem

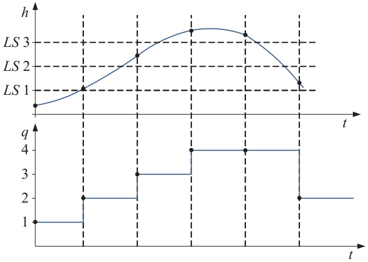
Given: Model depending on f and z_0
Measured I/O pair (V, W)

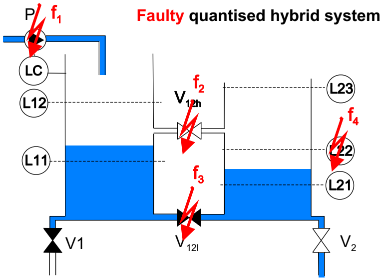
Consistency-based diagnosis:

Can the system subject to fault f generate the output W if it obtains the input V?

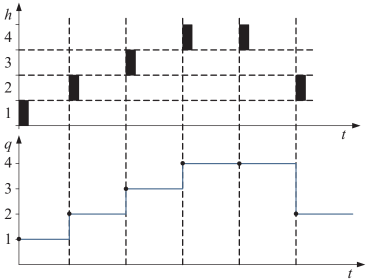
Model-based diagnosis
Example







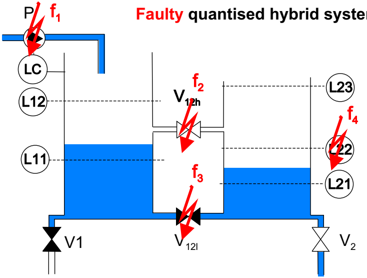
Faulty quantised hybrid system
P
f1
LC
L12
L11
f2
V12h
L23
f4
L22
L21
f3
V1
V12l
V2

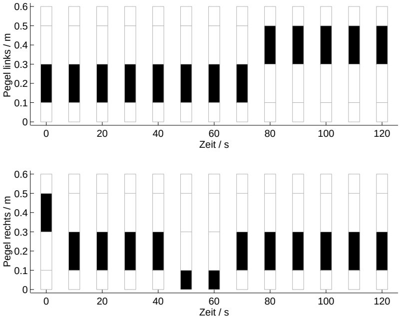


h
4
3
2
1
t
q
4
3
2
1
t

Model-based diagnosis
Example

Faulty quantised hybrid system

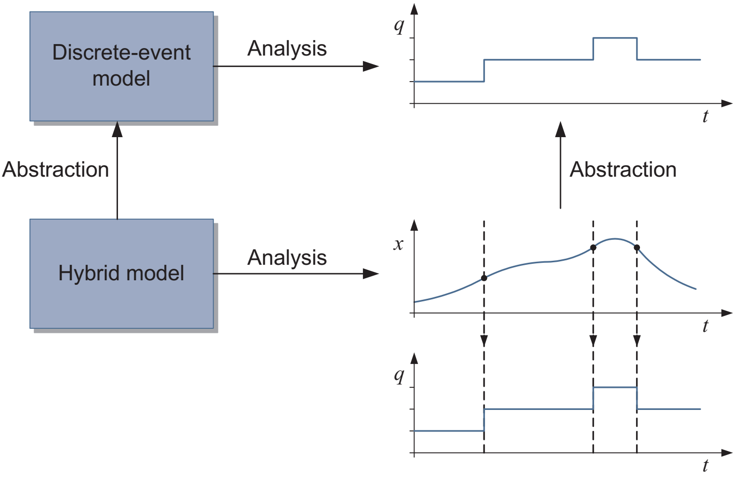




Is the tank system faulty?

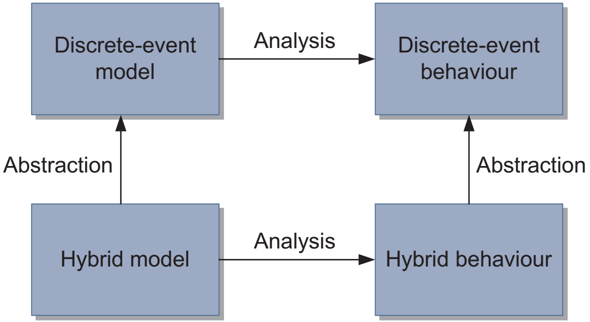
Survey

Abstraction-based modelling



Idea:
Ignore unimportant details
preserve important properties

Abstraction-based modelling



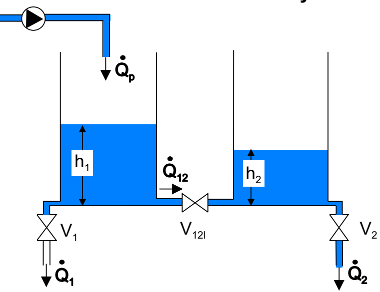
Properties of the abstract model:

- Homomorphism (Automata theory)
- Isomorphism
- Bisimulation (Pappas 2000, 2003)

in all cases: the reachability property is retained.

Example

Nonlinear System



$$
\begin{aligned}
\dot{h}_1 &= \frac{1}{A_1}\left(\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12}\right) \\
\dot{h}_2 &= \frac{1}{A_2}\left(\dot{Q}_{12} - \dot{Q}_2\right) \\
\dot{Q}_1 &= Pos(V_1)\; S_v \sqrt{2gh_1} \\
\dot{Q}_{12} &= Pos(V_{12l})\; S_v \,\mathrm{sgn}(h_1 - h_2)\, \sqrt{2g|h_1 - h \\
\dot{Q}_2 &= Pos(V_2)\; S_v\, \sqrt{2gh_2} \\
\dot{Q}_p &= p(t)\dot{Q}_{p0}
\end{aligned}
$$

with

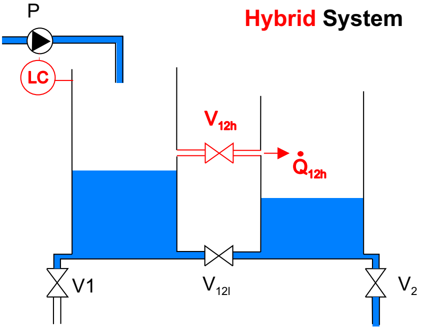
$$
\begin{aligned}
0 &\leq h_1(t), h_2(t) \leq h_{max} \\
0 &\leq p(t) \leq 1
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\
\mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t))
\end{aligned}
$$

Example

Hybrid System



$$\dot h_1 = \frac{1}{A_1}\left(\dot Q_p - \dot Q_1 - \dot Q_{12} - \dot Q_{12h}\right)$$

$$\dot h_2 = \frac{1}{A_2}\left(\dot Q_{12} + \dot Q_{12h} - \dot Q_2\right)$$

$$\dot Q_1 = \begin{cases} Pos(V_1)\, S_v \sqrt{2gh_1} & \text{if } \; h_1 > 0 \\ 0 & \text{else} \end{cases}$$

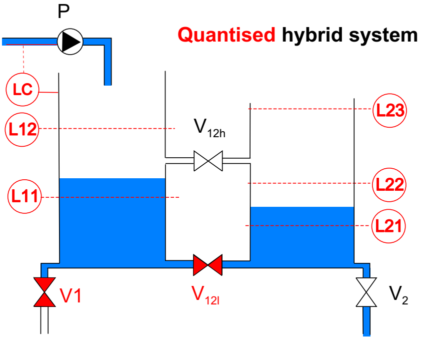
$$\dot Q_{12} = Pos(V_{12l})\, S_v \operatorname{sgn}(h_1 - h_2)\, \sqrt{2g|h_1 - h}$$

$$\dot Q_{12h} = \begin{cases} Pos(V_{12h})\, S_v \operatorname{sgn}(h_1 - h_2)\, \sqrt{2g|h_1 - h_2|} & \text{if } \; h_1, h_2 > h_v \\ Pos(V_{12h})\, S_v \sqrt{2g|h_1 - h_v|} & \text{if } \; h_1 > h_v, h_2 \le h_v \\ -Pos(V_{12h})\, S_v \sqrt{2g|h_2 - h_v|} & \text{if } \; h_2 > h_v, h_1 \le h_v \\ 0 & \text{if } \; h_1, h_2 \le h_v \end{cases}$$

$$\dot Q_2 = \begin{cases} Pos(V_2)\, S_v \sqrt{2gh_2} & \text{if } \; h_2 > 0 \\ 0 & \text{else} \end{cases}$$

$$\dot Q_p = \begin{cases} p(t)\dot Q_{p0} & \text{if } \; h_1 < h_{1max} \\ 0 & \text{if } \; h_1 \ge h_{1max} \end{cases}$$

Example



$$
\begin{aligned}
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\dot{Q}_{12} &= Pos(V_{12l})\; S_v\, \mathrm{sgn}(h_1 - h_2)\, \sqrt{2g|h_1 -}\\
\dot{Q}_{12h} &= \begin{cases} Pos(V_{12h})\; S_v\, \mathrm{sgn}(h_1 - h_2)\, \sqrt{2g}\\ Pos(V_{12h})\; S_v\, \sqrt{2g|h_1 - h_v|}\\ -Pos(V_{12h})\; S_v\, \sqrt{2g|h_2 - h_v|}\\ 0 \end{cases}\\
\dot{Q}_2 &= \begin{cases} Pos(V_2)\; S_v \sqrt{2gh_2} & \text{if } h_2 > 0\\ 0 & \text{else}\end{cases}\\
\dot{Q}_p &= 0 \quad \text{if } h_1 \geq h_{1max}
\end{aligned}
$$

$$\dot{Q}_p \in [0.8\,\dot{Q}_{p0}, \dot{Q}_{p0}] \quad \text{if } h_1 < h_{1max}$$

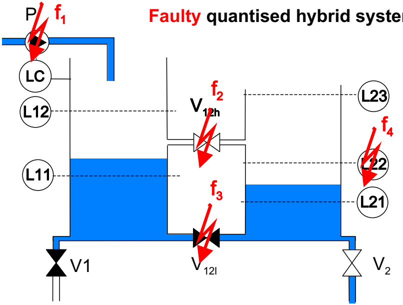
$$L_{11}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{11}\\ 0 & \text{else}\end{cases} \qquad L_{12}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{12}\\ 0 & \text{else}\end{cases}$$

$$L_{21}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{21}\\ 0 & \text{else}\end{cases} \qquad L_{22}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{22}\\ 0 & \text{else}\end{cases}$$

$$L_{23}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{23}\\ 0 & \text{else}\end{cases}$$

Example

Faulty quantised hybrid system



$$\dot h_1 = \frac{1}{A_1}\left(\dot Q_p - \dot Q_1 - \dot Q_{12} - \dot Q_{12h}\right)$$

$$\dot h_2 = \frac{1}{A_2}\left(\dot Q_{12} + \dot Q_{12h} - \dot Q_2\right)$$

$$\dot Q_1 = \begin{cases} Pos(V_1)\ S_v\sqrt{2gh_1} & \text{if } h_1 > 0 \\ 0 & \text{else} \end{cases}$$

$$\dot Q_{12} = (1-f_3)Pos(V_{12l})\ S_v\,\mathrm{sgn}(h_1 - h_2)\ \sqrt{}$$

$$\dot Q_{12h} = \begin{cases} (1-f_2)Pos(V_{12h})\ S_v\,\mathrm{sgn}(h_1 - h \\ (1-f_2)Pos(V_{12h})\ S_v\ \sqrt{2g|h_1 -} \\ -(1-f_2)Pos(V_{12h})\ S_v\ \sqrt{2g|h_2} \\ 0 \end{cases}$$

$$\dot Q_2 = \begin{cases} Pos(V_2)\ S_v\sqrt{2gh_2} & \text{if } h_2 > 0 \\ 0 & \text{else} \end{cases}$$

$$\dot Q_p = \begin{cases} (1-f_1)p(t)\dot Q_{p0} & \text{if } h_1 < h_{1max} \\ 0 & \text{if } h_1 \geq h_{1max} \end{cases}$$

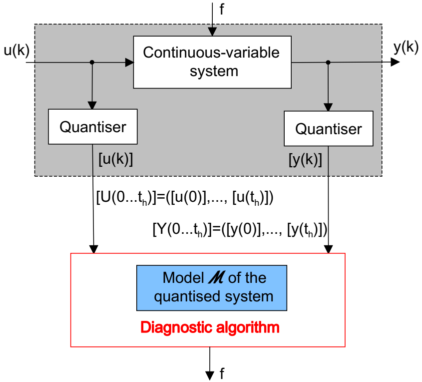
$$f_1, f_2, f_3, f_4 \in \{0, 1\}$$

$$L_{11}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{11} \\ 0 & \text{else} \end{cases} \qquad L_{12}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{12} \\ 0 & \text{else} \end{cases}$$

$$L_{21}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{21} \\ 0 & \text{else} \end{cases} \qquad L_{22}(t) = \begin{cases} (1-f_4) & \text{if } h_1(t) > l_{22} \\ 0 & \text{else} \end{cases}$$

$$L_{23}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{23} \\ 0 & \text{else} \end{cases}$$

Outline of the lecture

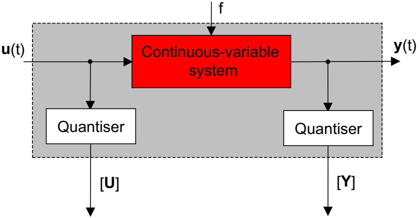


Solution steps

1. **Modelling**
   Find a discrete-event representation of the quantised system
2. **Diagnosis**
   Find a method to decide whether the quantised system behaves like the discrete-event model

Survey

Quantised systems



Continuous–variable system:

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k)), \qquad \mathbf{x}(0) = \mathbf{x}_0 \qquad (*)$$

$$\mathbf{y}(k) = \mathbf{g}(\mathbf{x}(k), \mathbf{u}(k)) \qquad (**)$$

For given $\mathbf{x}_0$ and

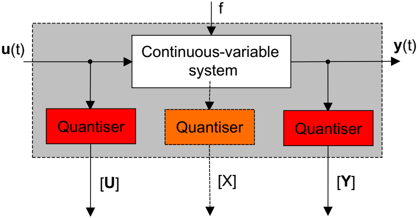
$$\mathbf{U}(0...t_h) = (\mathbf{u}(0), \mathbf{u}(1), \mathbf{u}(2), ..., \mathbf{u}(t_h))$$

the system generates

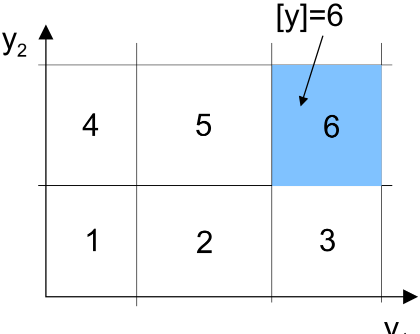
$$\mathbf{X}(0...t_h) = (\mathbf{x}(0), \mathbf{x}(1), \mathbf{x}(2), ..., \mathbf{x}(t_h))$$

$$\mathbf{Y}(0...t_h) = (\mathbf{y}(0), \mathbf{y}(1), \mathbf{y}(2), ..., \mathbf{y}(t_h))$$

Quantised systems

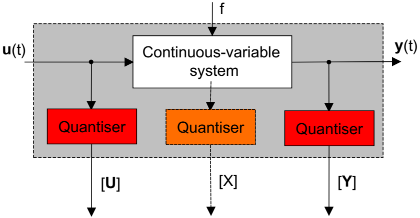


Output quantisation:



$$\begin{aligned} [\mathbf{y}(k)] &= w \\ \text{if } \mathbf{y}(k) &\in \mathcal{Q}_y(w) \\ \mathcal{N}_w &= \{0, 1, 2, \ldots, R\} \end{aligned}$$

Quantised systems



Input and state quantisation:

$$[\mathbf{u}(k)] = v \quad \text{if} \quad \mathbf{u}(k) \in \mathcal{Q}_u(v) \qquad \mathcal{V} = \{0, 1, 2, ..., M\}$$

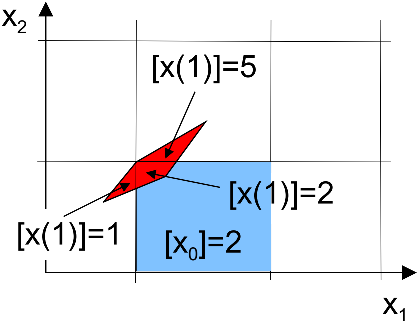
$$[\mathbf{x}(k)] = z \quad \text{if} \quad \mathbf{x}(k) \in \mathcal{Q}_x(z) \qquad \mathcal{Z} = \{0, 1, 2, ..., N\}$$

Consequence: The quantised system has a discrete–event behaviour.

Nondeterminism of the quantised system behaviour

For given quantised initial state $[\mathbf{x}(0)]$ and quantised input $[\mathbf{u}(0)]$ the system may generate more than one quantised successor state $[\mathbf{x}(1)]$

$$\mathbf{x}(1) = \mathbf{f}(\mathbf{x}(0), \mathbf{u}(0)),$$



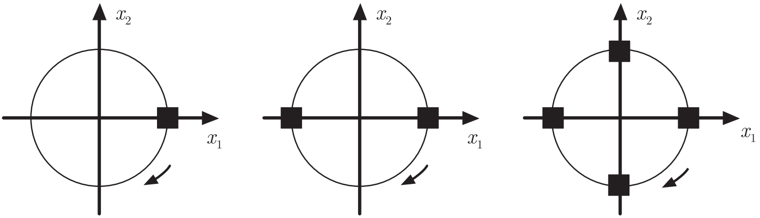
Consequence:
Discrete-event models of quantised systems have to be nondeterministic.

Nondeterminism of the quantised system behaviour

Exception: (Lunze, *Automatica* 1994)

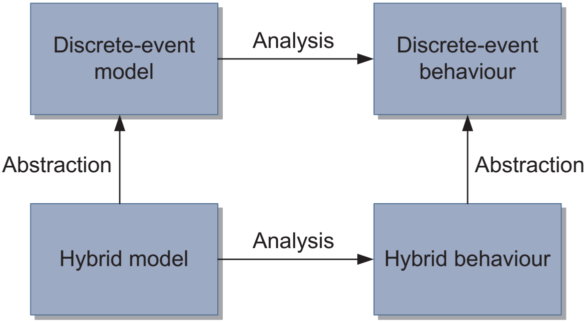
- linear autonomous system $\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k)$
- equidistant partioning with resolution q_{xi}
- $\mathbf{A} = \mathrm{diag}\; q_{xi}\; \mathrm{diag}\; (2n_i + 1)^{-1}\; \mathbf{P}\; \mathrm{diag}\; q_{xi}^{-1}$

Then the quantised system is deterministic.



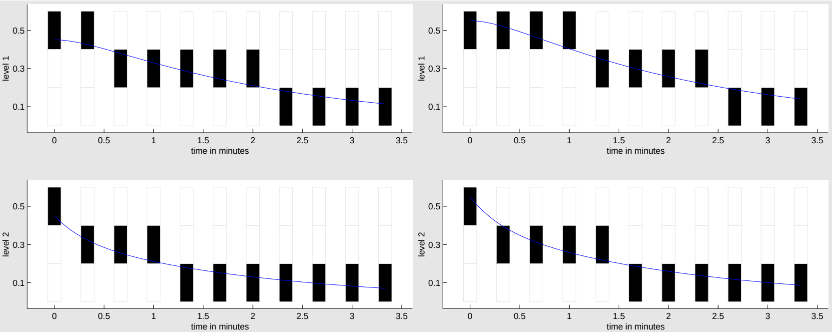
a) $A_\mathrm{d} = I$ b) $A_\mathrm{d} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ c) $A_\mathrm{d} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Nondeterminism of the quantised system behaviour



A bisimilar discrete-event representation of quantised systems exists only in exceptional cases.

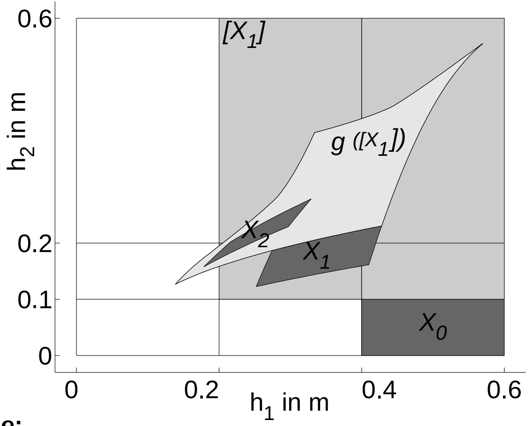
Nondeterminism of the quantised system behaviour
Behaviour of the quantised tank system



Nondeterminism of the quantised system behaviour

Quantised systems do **not** possess the Markov property

$$\mathrm{Prob}\left([\mathbf{x}(k+1)] \mid [\mathbf{x}(k)], [\mathbf{x}(k-1)], ..., [\mathbf{x}(0)]\right) = \mathrm{Prob}\left([\mathbf{x}(k+1)] \mid [\mathbf{x}(k)]\right)$$



Consequence:

No representation form, which possesses the Markov property, can precisely describe a quantised system

Survey

Modelling problem

Given: Quantised system

Find: Model with the following property:

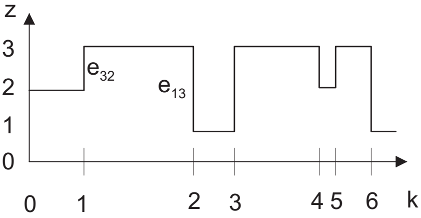
$$\begin{array}{ccc} \text{Set of model trajectories} & \supseteq & \text{Set of system trajectories} \\ \mathbf{Z}([\mathbf{x}(0)],[\mathbf{U}]) & \supseteq & [\tilde{\mathbf{X}}([\mathbf{x}(0)],[\mathbf{U}])] \end{array}$$

- Such a model is called complete.
- Spurious solutions = Model trajectories that the quantised system cannot follow

Basics of automata theory

Discrete signal spaces
$$\begin{aligned} v &\in \mathcal{N}_v = \{1, 2, ..., M\} \\ z &\in \mathcal{N}_z = \{1, 2, ..., N\} \\ w &\in \mathcal{N}_w = \{1, 2, ..., R\} \end{aligned}$$

Event = change of the input, state or output



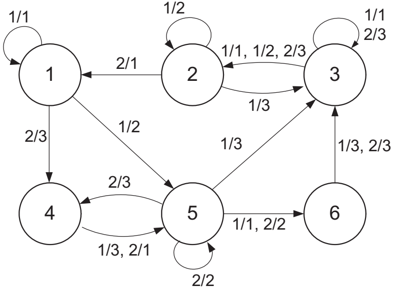
State sequence $Z(0...6) = (2, 3, 1, 3, 2, 3, 1)$

Nondeterministic automaton

$$\mathcal{N}(\mathcal{Z}, \mathcal{V}, \mathcal{W}, L, z(0))$$

State transition relation $L : \mathcal{Z} \times \mathcal{W} \times \mathcal{Z} \times \mathcal{V} \to \{0, 1\}$

$L(z', w, z, v) = 1 \Rightarrow$ automaton may jump from $z(k) = z$ towards $z(k+1) = z'$ while producing the output $w(k) = w$ for input $v(k) = v$



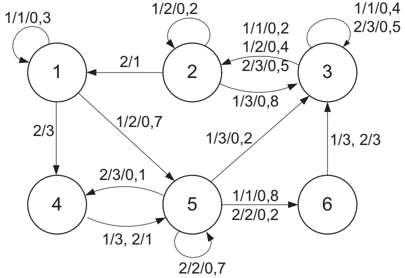
Stochastic automaton

$$\mathcal{S}(\mathcal{Z}, \mathcal{V}, \mathcal{W}, L, \mathrm{Prob}\,(z(0)))$$

State transition probability distribution

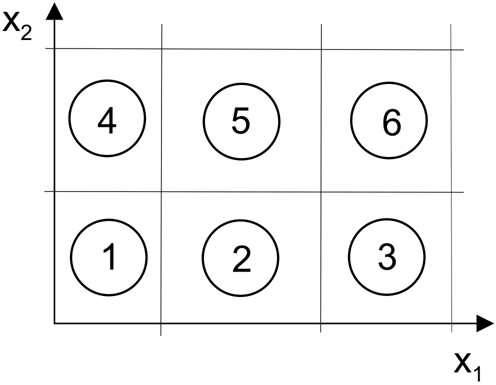
$$L : \mathcal{N}_z \times \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \;\longrightarrow\; [0, 1]$$

$$L(z', w \mid z, v) = \mathrm{Prob}(Z(1) = z', W(0) = w \mid Z(0) = z, V(0) = v)$$



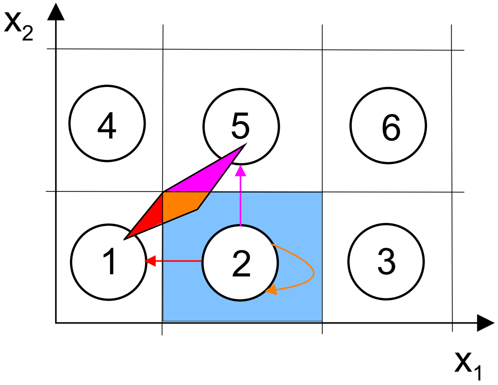
$\mathcal{S}(\mathcal{Z}, \mathcal{V}, \mathcal{W}, L, \mathrm{Prob}\,(z(0)))$

- $\mathcal{Z}$ - set of quantised state symbols
- $\mathcal{V}$ - set of quantised input symbols
- $\mathcal{W}$ - set of quantised output symbols



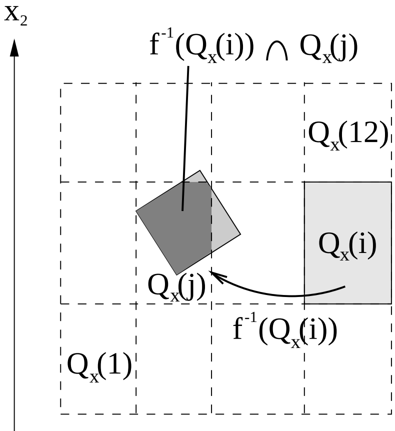
$$L(z', w \mid z, v) = \mathrm{Prob}\left([\mathbf{x}(1)] = z', [\mathbf{y}(0)] = w \mid [\mathbf{x}(0)] = z, [\mathbf{u}(0)] = v\right)$$

$$\mathrm{Prob}\,(z_0) > 0 \;\; \text{for} \;\; z_0 = [\mathbf{x}_0]$$

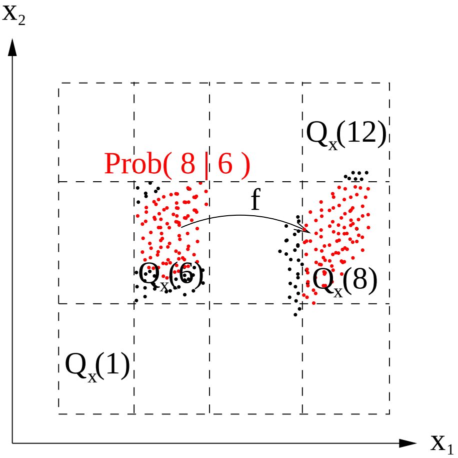


Abstraction formula

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k)) \qquad L(i \,|\, j) = \frac{\lambda\left(\mathbf{f}^{-1}(\mathcal{Q}_x(i)) \cap \mathcal{Q}_x(j)\right)}{\lambda(\mathcal{Q}_x(j))}$$



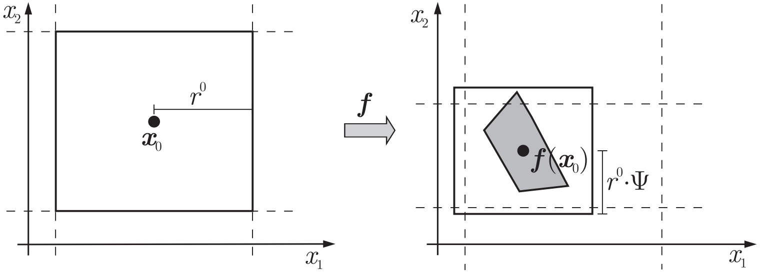
Point-based cell-to-cell mapping



The automaton obtained is, in general, **incomplete**.

Hyperbox mapping (Lunze, Schröder, ECC 2001)

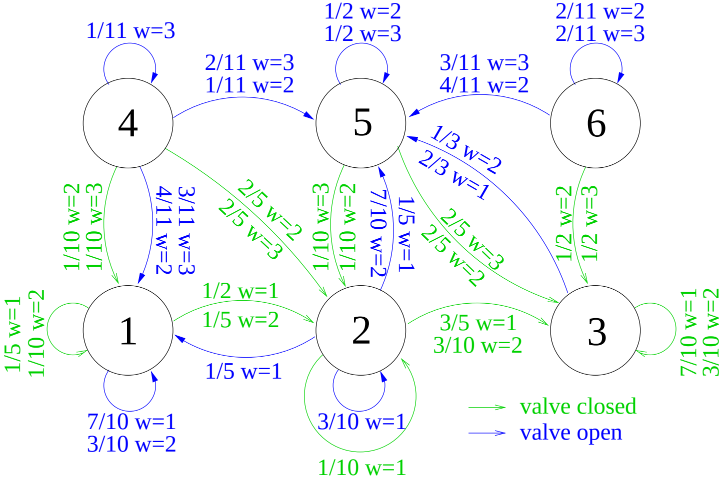
Assumption: System satisfies a Lipschitz condition:
$||(\mathbf{f}(\mathbf{x}_1) - \mathbf{f}(\mathbf{x}_2)||_\infty \leq \psi \cdot ||\mathbf{x}_1 - \mathbf{x}_2||_\infty$



The automaton is complete.

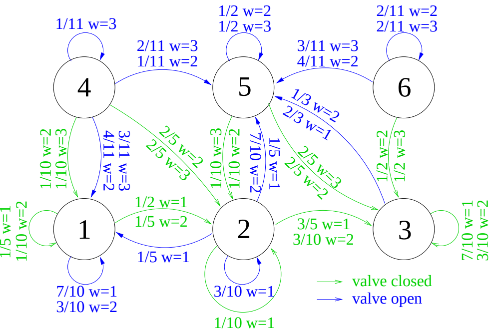
Modelling of quantised systems by stochastic automata

Model of the tank system for fault f_1



Modelling of quantised systems by stochastic automata

$$\mathbf{Z}([\mathbf{x}(0)],[\mathbf{U}]) \supseteq [\tilde{\mathbf{X}}([\mathbf{x}(0)],[\mathbf{U}])]$$



$$\dot{h}_1 = \frac{1}{A_1}\left(\dot{Q}_p - \dot{Q}_1 - \dot{Q}_{12} - \dot{Q}_{12h}\right)$$

$$\dot{h}_2 = \frac{1}{A_2}\left(\dot{Q}_{12} + \dot{Q}_{12h} - \dot{Q}_2\right)$$

$$\dot{Q}_1 = \begin{cases} Pos(V_1)\ S_v\sqrt{2gh_1} & \text{if } h \\ 0 & \text{else} \end{cases}$$

$$\dot{Q}_{12} = (1-f_3)Pos(V_{12l})\ S_v\,\mathrm{sgn}(h_1 -$$

$$\dot{Q}_{12h} = \begin{cases} (1-f_2)Pos(V_{12h})\ S_v\,\mathrm{sgn}(h \\ (1-f_2)Pos(V_{12h})\ S_v\sqrt{2g|} \\ -(1-f_2)Pos(V_{12h})\ S_v\sqrt{2} \\ 0 \end{cases}$$

$$\dot{Q}_2 = \begin{cases} Pos(V_2)\ S_v\sqrt{2gh_2} & \text{if } h \\ 0 & \text{else} \end{cases}$$

$$\dot{Q}_p = \begin{cases} (1-f_1)p(t)\dot{Q}_{p0} & \text{if } h_1 < l \\ 0 & \text{if } h_1 \geq l \end{cases}$$

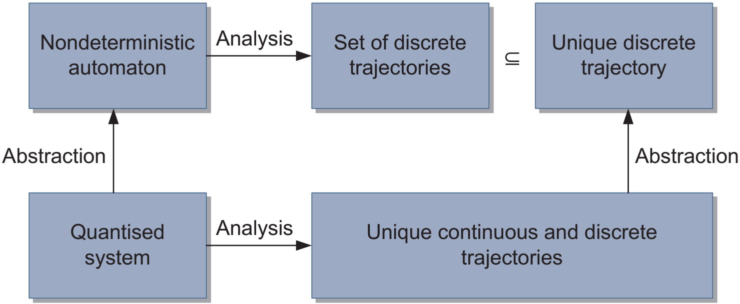
$$L_{11}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{11} \\ 0 & \text{else} \end{cases}$$

$$L_{12}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{12} \\ 0 & \text{else} \end{cases}$$

$$L_{21}(t) = \begin{cases} 1 & \text{if } h_1(t) > l_{21} \\ 0 & \text{else} \end{cases}$$

$$\begin{cases} (1-f_4) & \text{if } h_1(t) > l_{22} \end{cases}$$

Abstraction-based modelling



Properties of the abstract model:

The stochastic automaton is a complete model.

Given: $\mathrm{Prob}\,(Z(0) = z)$ for all $z \in \mathcal{Z}$

$$[\mathbf{U}] = ([\mathbf{u}(1)], [\mathbf{u}(2)], ..., [\mathbf{u}(k_h)])$$

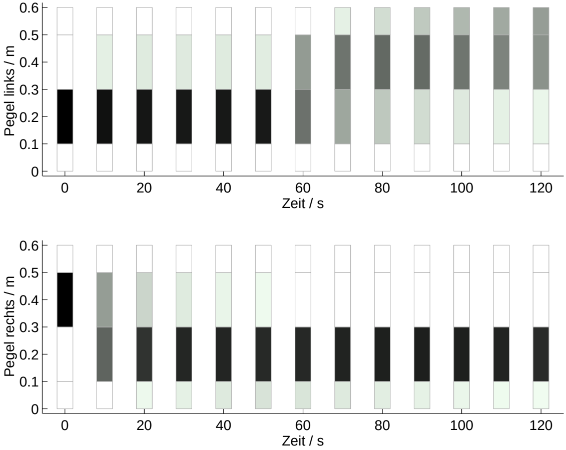
Determine:

$$\mathrm{Prob}\,(Z(k+1) = z') = \sum_{w \in \mathcal{W}} \sum_{z \in \mathcal{Z}} L(z', w \mid z, [\mathbf{u}(k)]) \cdot \mathrm{Prob}\,(Z(k) = z)$$

(Chapman-Kolmogorov equation)

Modelling of quantised systems by stochastic automata

Simulation:



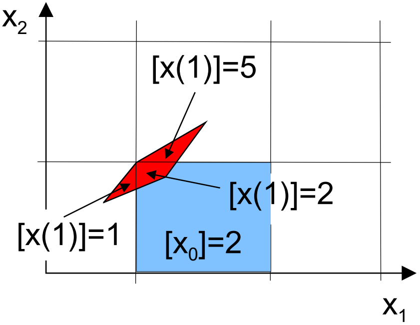
Survey

The state partitioning problem
(Lunze 2004)

Blondel/Megretski: *Open problems in Systems and Control Theory*, Princeton Univ. Press 2004

Under what conditions is the discrete-event behaviour deterministic?

$$\forall i \in \mathcal{Z} \;\; \exists j : \mathbf{f}(\mathbf{x}) \in \mathcal{Q}(j) \text{ for all } \mathbf{x} \in \mathcal{Q}(i)$$



The state partitioning problem

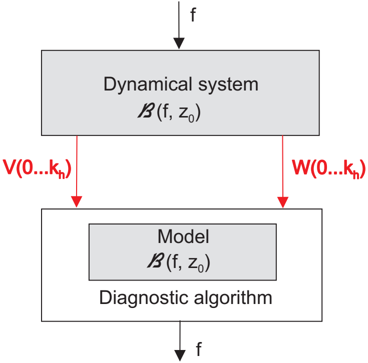
Problem A
Given: $\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k))$
Partition $\mathcal{Q}_x$

Find: Conditions on $\mathbf{f}$, $\mathcal{Q}_x$ such that
the discrete-event behaviour is deterministic

Problem B
Given: $\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k))$

Find: Partition $\mathcal{Q}_x$ such that
the discrete-event behaviour is deterministic

Survey

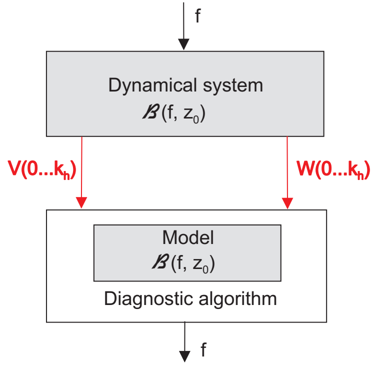


Diagnostic problem

Given: Model depending on f and z_0
Measured I/O pair (V, W)

Consistency-based diagnosis:
Can the system subject to fault f generate the output W if it obtains the input V?

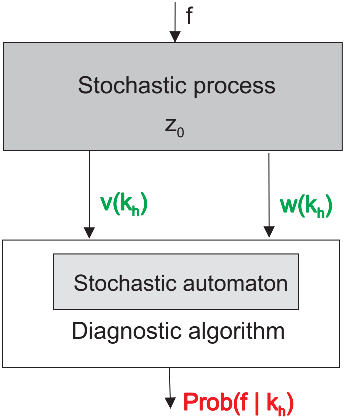
Model-based diagnosis



Consequences:

- Diagnostic problems include observation problems.
- **Fault detection:**
  Inconsistency with the faultless system
- **Fault identification:**
  Consistency with the system subject to fault f
  $\longrightarrow$ f is a fault candidate

Diagnosis of automata



Diagnostic problem

Given: Stochastic automaton $\mathcal{S}$
$V(0...k_h), W(0...k_h)$

Find: Fault f

State observation of nondeterministic automata

Main idea of model-based diagnosis:
Check the consistency of the I/O pair with the automaton-

Given: I/O pair

$$\begin{aligned} V(0...k_h) &= (v(0), v(1), ..., v(k_h)) \\ W(0...k_h) &= (w(0), w(1), ..., w(k_h)) \end{aligned}$$

The I/O pair is consistent with the automaton if there exists a state sequence $Z(0...k_h)$ such that

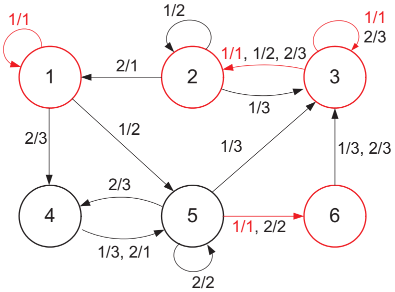
$$L(z(k+1), w(k), z(k), v(k)) = 1 \quad \text{for all } k$$

The consistency check includes a state observation problem.

State observation of nondeterministic automata

A-priori information: $\mathcal{Z}(0\,|\,-1) = \{1,2,3,4,5,6\}$

Measurement: $v(0) = 1, w(0) = 1$



Information about the state obtained by the measurement:

$$\mathcal{Z}(0\,|\,0) = \{1,3,5\}$$
$$\mathcal{Z}(1\,|\,0) = \{1,2,3,6\}$$

State observation of nondeterministic automata

$$\begin{aligned} \mathcal{Z}(k_h \,|\, k_h) &= \{z \in \mathcal{Z}(k_h \,|\, k_h - 1) \,:\, \exists z' : L(z', w, z, v) = 1\} \\ \mathcal{Z}(k_h + 1 \,|\, k_h) &= \{z' \,:\, \exists z \in \mathcal{Z}(k_h \,|\, k_h) : L(z', w, z, v) = 1\} \end{aligned}$$

Consistency check:
The I/O pair is consistent with the automaton if and only if

$$\mathcal{Z}(k_h \,|\, k_h) \neq \emptyset \quad \text{for all} \quad k_h$$

State observation algorithm

Given: Nondeterministic automaton $\mathcal{N}$
Initial state set $\mathcal{Z}(0 \mid -1)$

Init.: $\mathcal{Z}' = \mathcal{Z}(0 \mid -1)$
$k_h = 0$

Do: 1. Measure the I/O pair (v, w)

2. Determine
$\mathcal{Z} := \{z \in \mathcal{Z}' \mid \exists z' : \; L(z', w, z, v) = 1\}$

3. **Consistency check:**
If $\mathcal{Z} = \emptyset$, stop the algorithm
(inconsistent I/O pair or wrong initial state set)

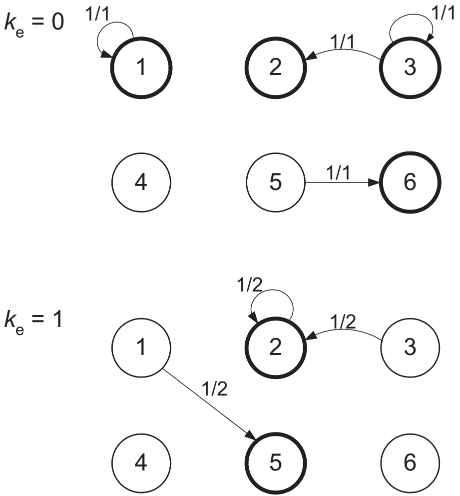
4. Determine
$\mathcal{Z}' := \{z' \mid \exists z \in \mathcal{Z} : \; L(z', w, z, v) = 1 \}$

5. $k_h := k_h + 1$
Continue with Step 1

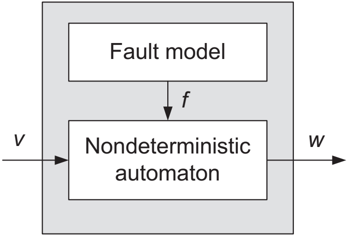
Result: $\mathcal{Z} = \mathcal{Z}(k_h \mid k_h)$ for increasing time horizon k_h

State observation of nondeterministic automata

I/O pair: $(V, W) = ((1, 1, 2, 2),\ (1, 2, 2, 3))$



Fault detection:
If the I/O sequence is inconsistent with the model of the faultless system, then a fault has occurred.



Fault model: $\mathcal{N}_{\mathrm{F}}(\mathcal{N}_f, G_{\mathrm{F}}, z_{\mathrm{F0}})$

Nondeterministic automaton including the fault model: $\tilde{z} = \binom{z}{f}$

State transition relation:

$$\tilde{L}\left(\begin{pmatrix} z' \\ f' \end{pmatrix}, w, \begin{pmatrix} z \\ f \end{pmatrix}, v\right) = L(z', w, z, v, f) \cdot G(f', f),$$

Diagnosis: Consistency check of the I/O pair and the extended automaton.

Diagnostic algorithm

Algorithm *Diagnosis of nondeterministic automata*

Given: Nondeterministic automaton $\mathcal{N}$, Fault model $\mathcal{N}_{\mathrm{F}}$
Initial state set $\mathcal{Z}(0\,|\,-1)$
Initial fault set $\mathcal{F}(0\,|\,-1)$

Init.: $\tilde{\mathcal{Z}}' = \mathcal{Z}(0\,|\,-1) \times \mathcal{F}(0\,|\,-1)$
$k_h = 0$

Diagnostic algorithm

Do: 1. Measure the I/O pair (v, w)

2. Determine
$\tilde{\mathcal{Z}} := \{(z, f) \in \tilde{\mathcal{Z}}' \,|\, \exists (z', f') : \; L(z', w, z, v, f) \cdot G(f', f) = 1\}$

3. **Consistency check:**
If $\tilde{\mathcal{Z}} = \emptyset$, stop the algorithm
(wrong initial state set or initial fault set)

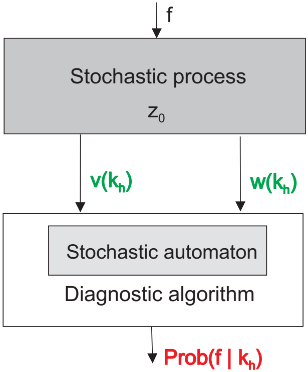
4. Determine
$\mathcal{Z}' := \{(z', f') \,|\, \exists (z, f) \in \tilde{\mathcal{Z}}_k : \; L(z', w, z, v, f) \cdot G(f', f) = 1\}$

5. Determine $\mathcal{F} = \{f \; : \; (z, f) \in \tilde{\mathcal{Z}}_k\}$

6. $k_h := k_h + 1$
Continue with Step 1

Result: $\mathcal{F} = \mathcal{F}(k_h \,|\, k_h)$ for increasing time horizon k_h

Diagnosis of stochastic automata

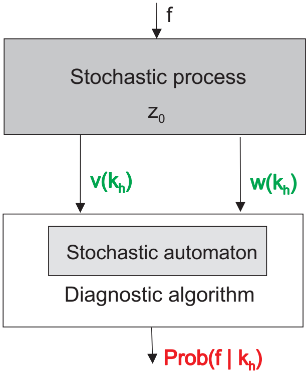


Diagnostic problem

Given: Stochastic automaton $\mathcal{S}$
$V(0 \ldots k_h), W(0 \ldots k_h)$

Find: Fault f

Diagnosis of stochastic automata



Result: $\mathrm{Prob}\,(f \mid V(0...k_h), W(0...k_h)) =: \mathrm{Prob}\,(f \mid k_h)$

$$\mathcal{F}(k_h) = \{f \ : \ \mathrm{Prob}\,(f \mid k_h) > 0\}$$

Diagnosis of stochastic automata
(Lunze, Schröder, *Discrete Event Dynamic Systems*, 2001)

Init. $\mathrm{Prob}\,(f, z(0) \,|\, -1) = \mathrm{Prob}\,(f, z(0))$
$k_h := 0$

Do: 1. Measure $v(k_h)$, $w(k_h)$

2. Determine $L(k_h) = L(z(k_h+1), w(k_h) \,|\, z(k_h), v(k_h), f)$

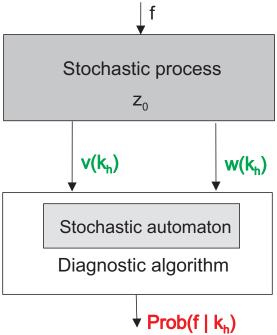
3. $$\mathrm{Prob}\,(f|k_h) = \frac{\sum_{\substack{z(k_h+1)\\ z(k_h)}} L(k_h) \cdot \mathrm{Prob}\,(f, z(k_h) \,|\, k_h - 1)}{\sum_{\substack{z(k_h+1)\\ z(k_h), f}} L(k_h) \cdot \mathrm{Prob}\,(f, z(k_h) \,|\, k_h - 1)}$$

4. $\mathcal{F}(k_h) = \{f \mid \mathrm{Prob}\,(f \,|\, k_h) > 0\}$

5. $$\mathrm{Prob}\,(f, z(k_h+1) \,|\, k_h) = \frac{\sum_{z(k_h)} L(k_h) \cdot \mathrm{Prob}\,(f, z(k_h) \,|\, k_h - 1)}{\sum_{\substack{z(k_h+1)\\ z(k_h), f}} L(k_h) \cdot \mathrm{Prob}\,(f, z(k_h) \,|\, k_h - 1)}$$

6. $k_h := k_h + 1$
Continue with Step 1

Diagnosis of stochastic automata

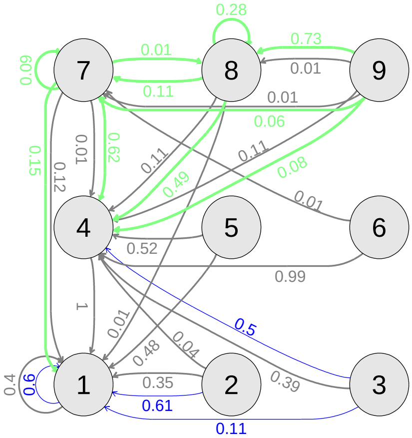


Result: $\text{Prob}\,(f \mid k_h)$

$$\mathcal{F}(k_h) = \{f \;:\; \text{Prob}\,(f \mid k_h) > 0\}$$

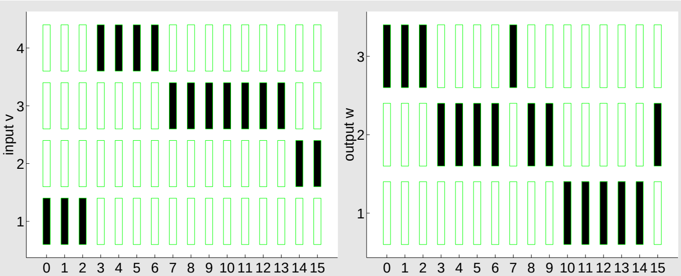
- The system is subject to some fault $f \in \mathcal{F}(k_h)$.
- **Fault detection:**
  If $f_0 \notin \mathcal{F}(k_h)$ holds, the system is known to be subject to some fault.
- **Fault identification:**
  If $\mathcal{F}(k_h) = \{f_i\}$ is a singleton, the system is known to be subject to fault f_i.

BRIDGE Benchmark problem

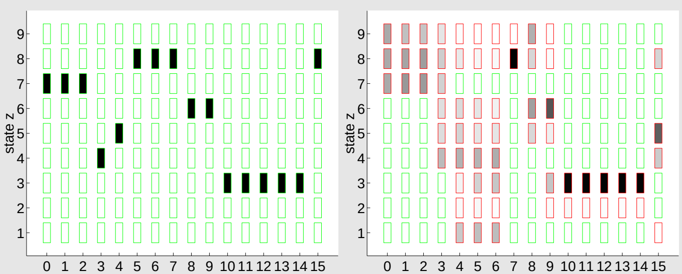


BRIDGE Benchmark problem

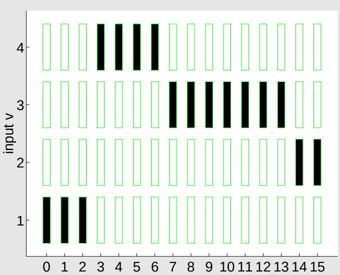
I/O pair:

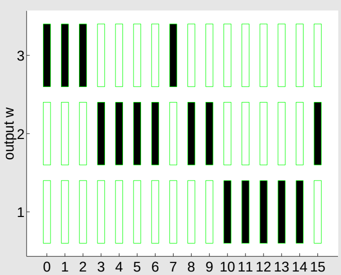


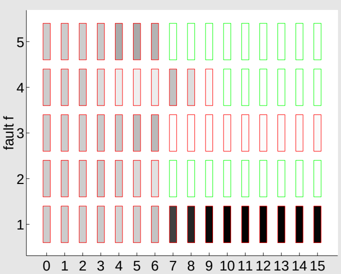
State observation:



BRIDGE Benchmark problem

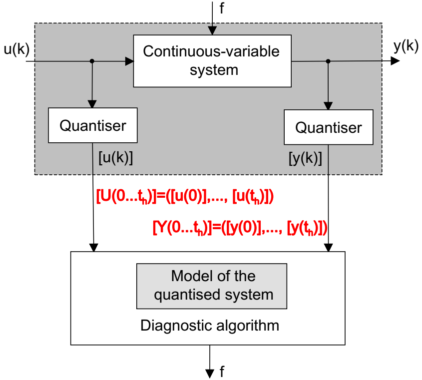
I/O pair: 



Solution: 

Survey

Diagnosis of quantised systems



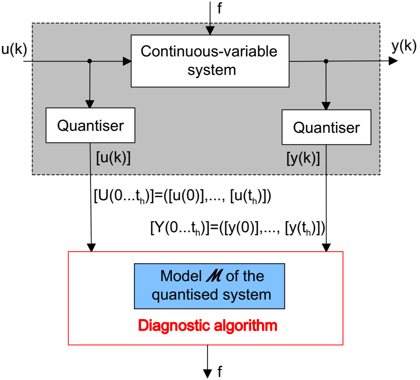
Diagnostic problem:

Given: Quantised system

I/O pair $([\mathbf{U}(0 \ldots t_h)], [\mathbf{Y}(0 \ldots t_h)])$

Find: Fault f

Diagnosis of quantised systems



Solution steps

1. **Modelling**
   Determine a complete discrete-event model
2. **Diagnosis**
   Use the diagnostic method for stochastic automata to check whether $([\mathbf{U}], [\mathbf{Y}])$ is consistent with the discrete-event model

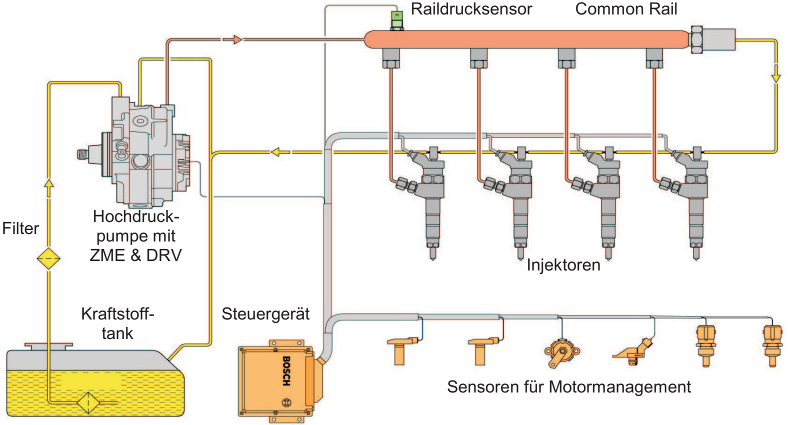
Diagnosis of quantised systems

Diagnostic results:
The results obtained for the discrete-event model hold for the quantised system because the model is complete.

- **Fault detection:**
  If $([\mathbf{U}], [\mathbf{Y}])$ is inconsistent with the model, a fault does exist.
- **Fault identification:**
  If $([\mathbf{U}], [\mathbf{Y}])$ is consistent with the model that holds for fault f, f is a fault candidate.

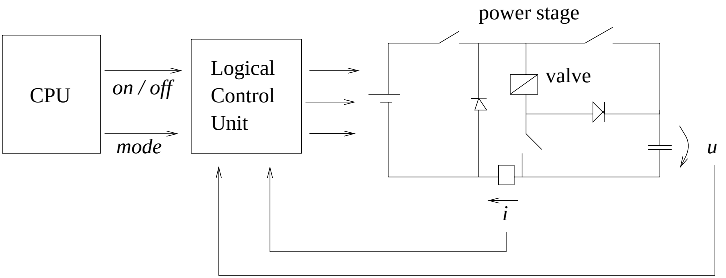
Survey

Diagnosis of the common rail diesel injection system
(Förstner, Lunze 1999)



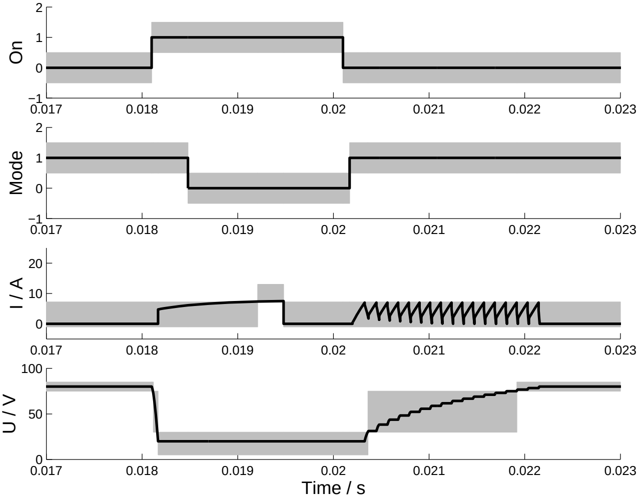
Diagnosis of the common rail diesel injection system

Injector block



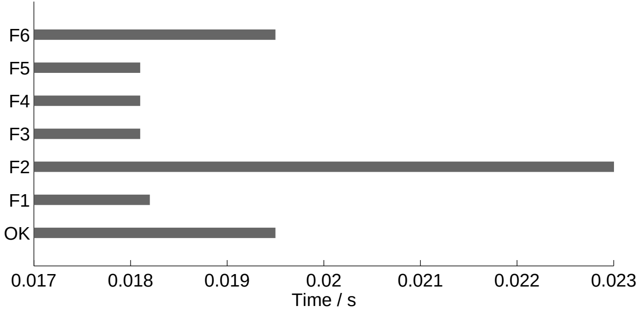
Diagnosis of the common rail diesel injection system

Quantised measurements



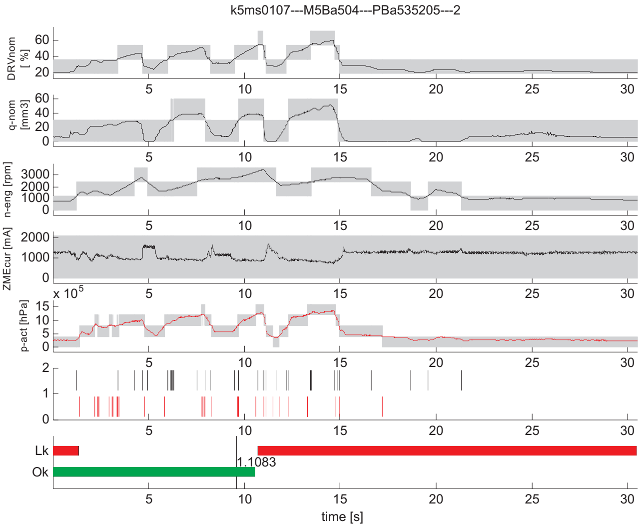
Diagnosis of the common rail diesel injection system

Diagnostic result for the injector

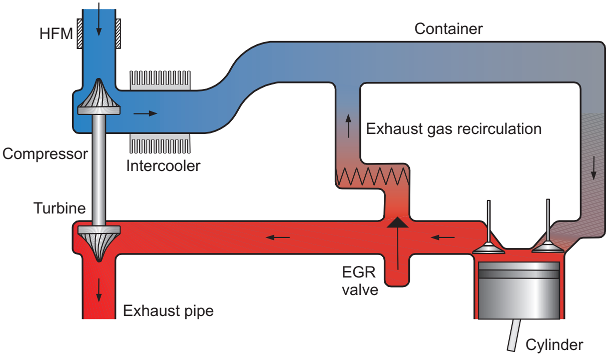


Diagnosis of the common rail diesel injection system

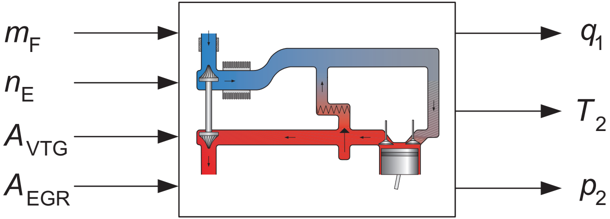
Diagnostic result for the pressure control system:



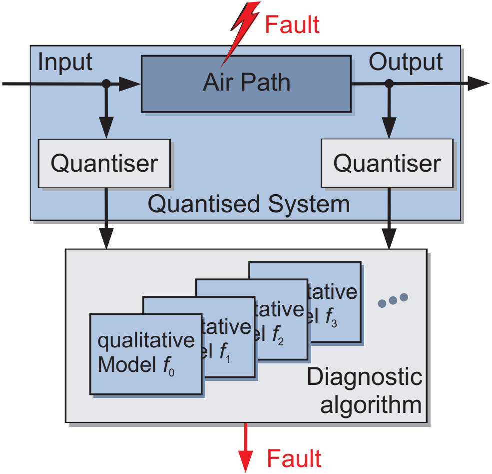
Diagnosis of the air path of a diesel engine
(Falkenberg, Neidig, Lunze, Fritz, *ATP international* 2006)



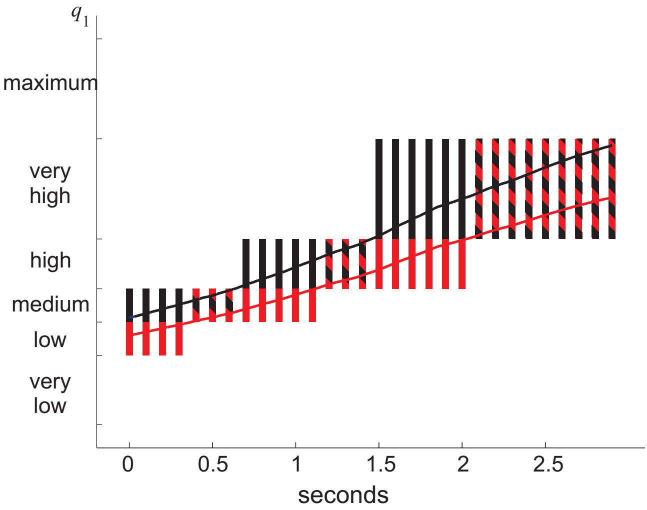
Diagnosis of the air path of a diesel engine



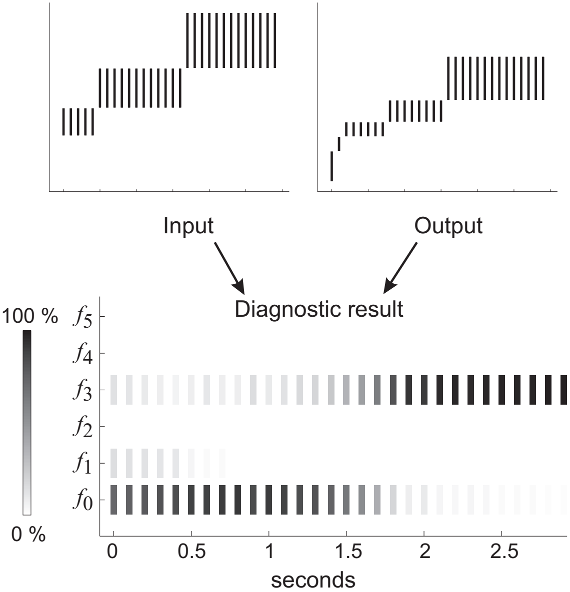
Diagnosis of the air path of a diesel engine



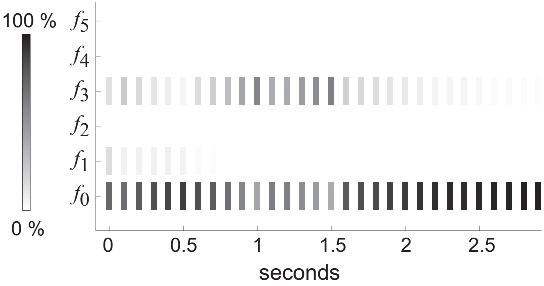
Diagnosis of the air path of a diesel engine



Diagnosis of the air path of a diesel engine

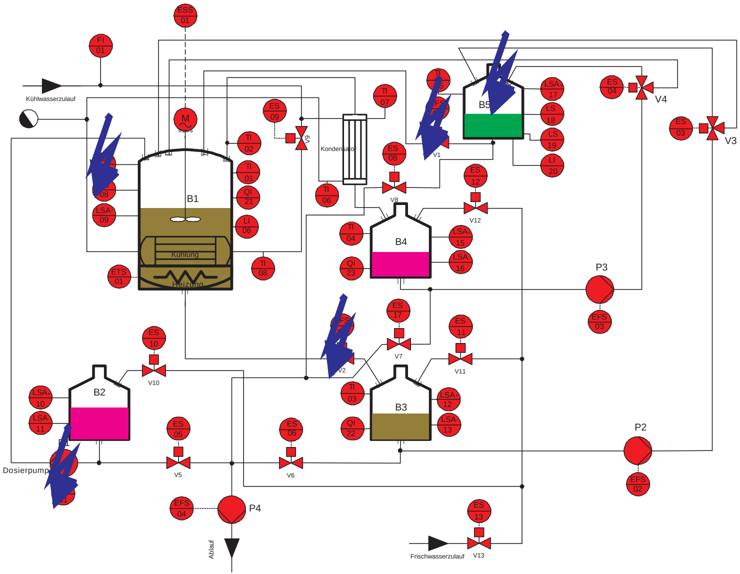


Example: Diagnosis of the air path of a diesel engine



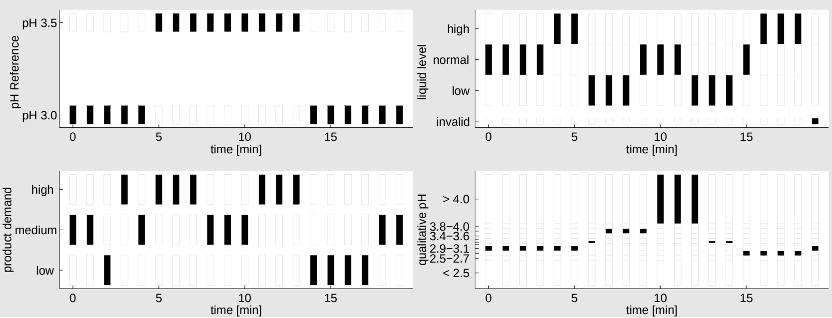
Diagnosis of a neutralisation process
(Lunze, Schröder 1999; Schröder 2003)

Diagnosis of a neutralisation process



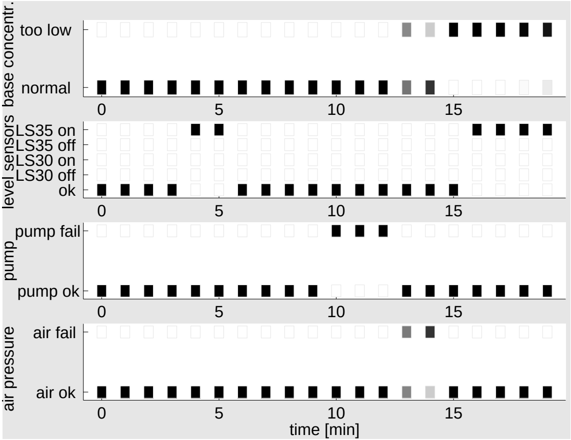
Diagnosis of a neutralisation process

Quantised measurements



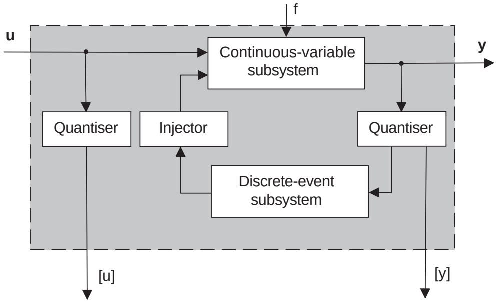
Diagnosis of a neutralisation process

Diagnostic results



Survey

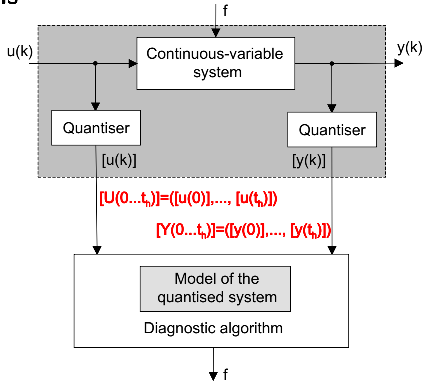
Conclusions



- Quantisers occur naturally within dynamical systems
- Quantisers reflect the uncertainties of the inputs and measurements
- Quantisers are introduced to reduce the information used during the diagnosis
    - Reduce the measurement information
    - Reduce the model complexity

Conclusions

Process diagnosis



To solve process supervision tasks,
ignore as many details as possible:

→ Use discrete-event models of the hybrid system

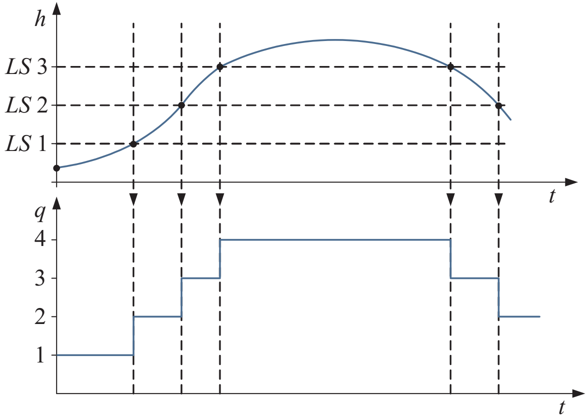
Conclusions

- The dynamics of quantised systems can be described by event sequences
- Complete discrete-event representations of quantised systems can be obtained by abstraction.
- Diagnosis means to test whether the measured I/O pair is consistent with the model.
- Methods and algorithms are available for diagnosing quantised systems

The theory on quantised systems bridges the gap between continuous systems theory and discrete systems theory

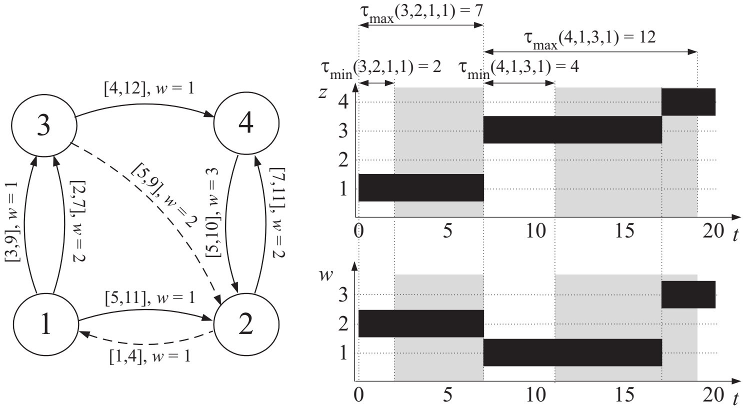
Outlook

Discrete-event quantised systems



Outlook

Timed discrete-event models



(a) (b)

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- J. Lunze, J. Raisch:
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