Networked and Embedded Control Systems

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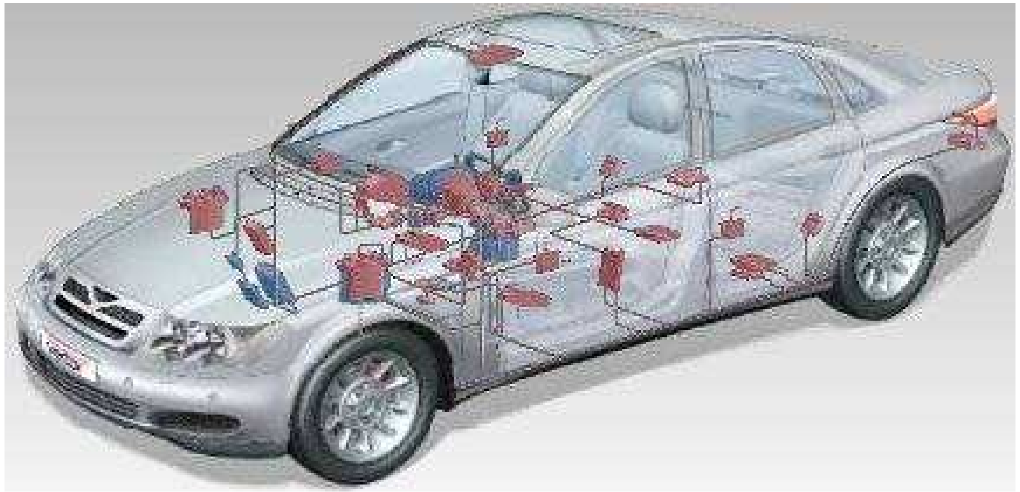
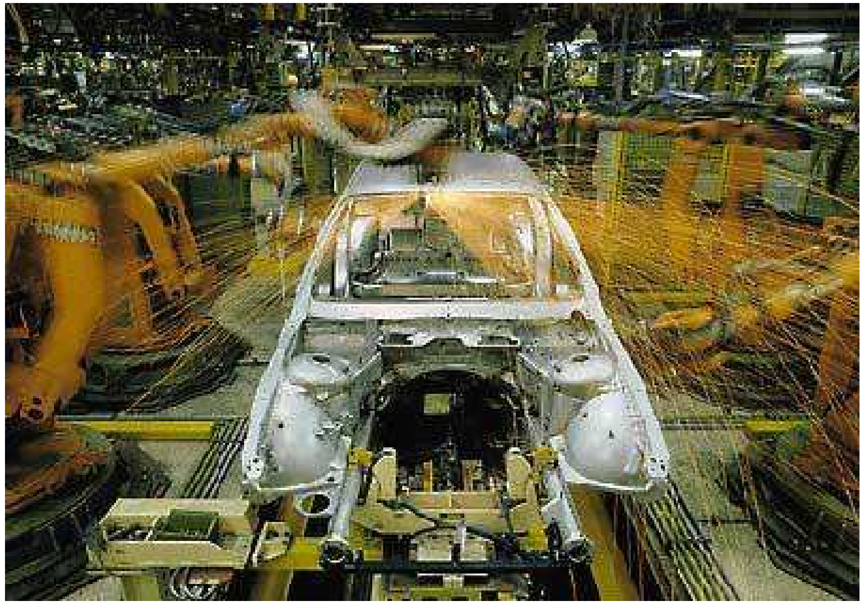
Application scenarios

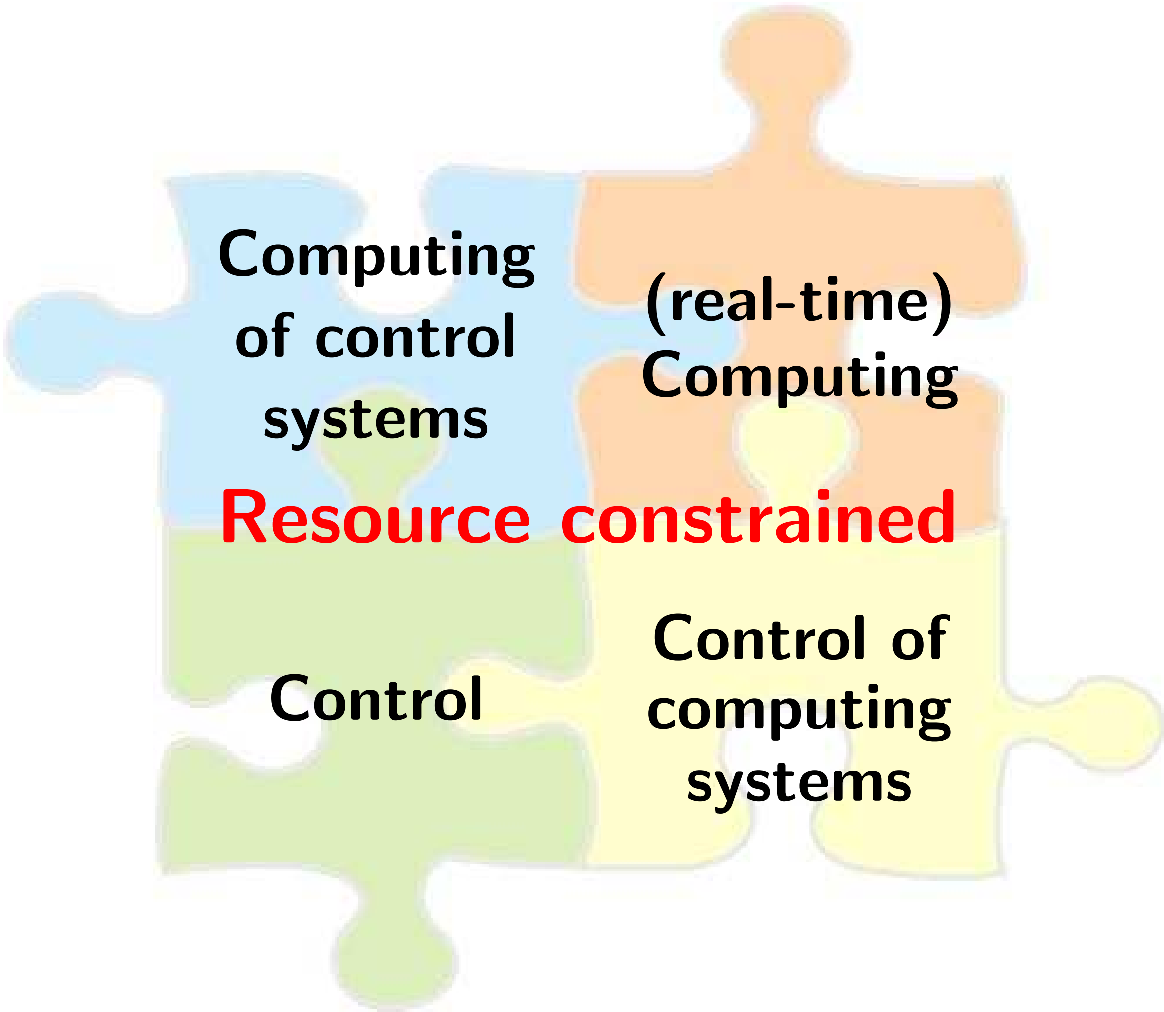




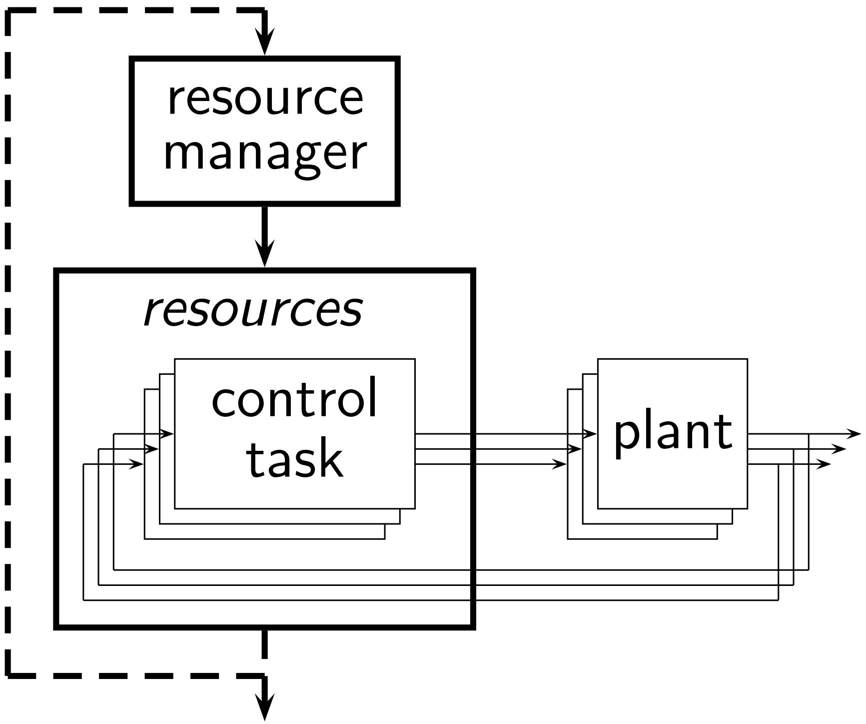
Common features

- Resource-constrained systems (mass-marked products subject to hard economic constraints)
- Often used in unpredictable environments
- The time when results are delivered is important
- Many simultaneously running control applications

Puzzle


Computing of control systems
(real-time) Computing
Resource constrained
Control
Control of computing systems

Understanding the puzzle

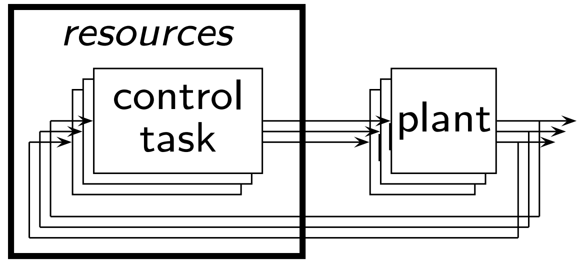


Building the puzzle

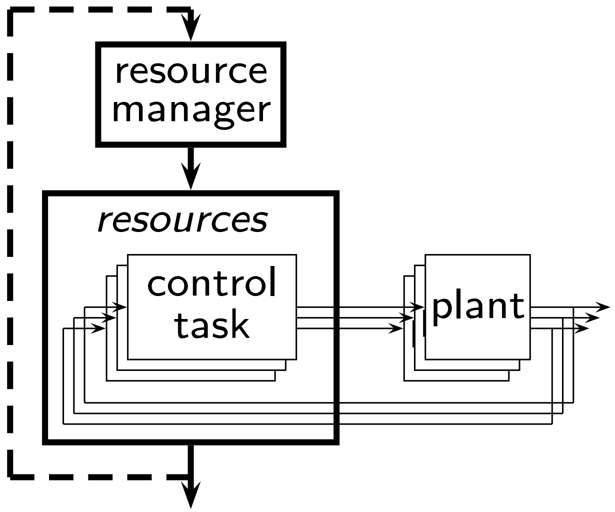
- Reality: gap between communities (real-time, control, ...)
- Why? Misconceptions
  - Real-time engineers assume hard deadlines for control algorithms
  - Control engineers assume determinism in the computing platform
- Need: Closer interaction between communities
- Today, emerging areas closing the gap
  - computing of control systems
  - control of computing systems

Contents

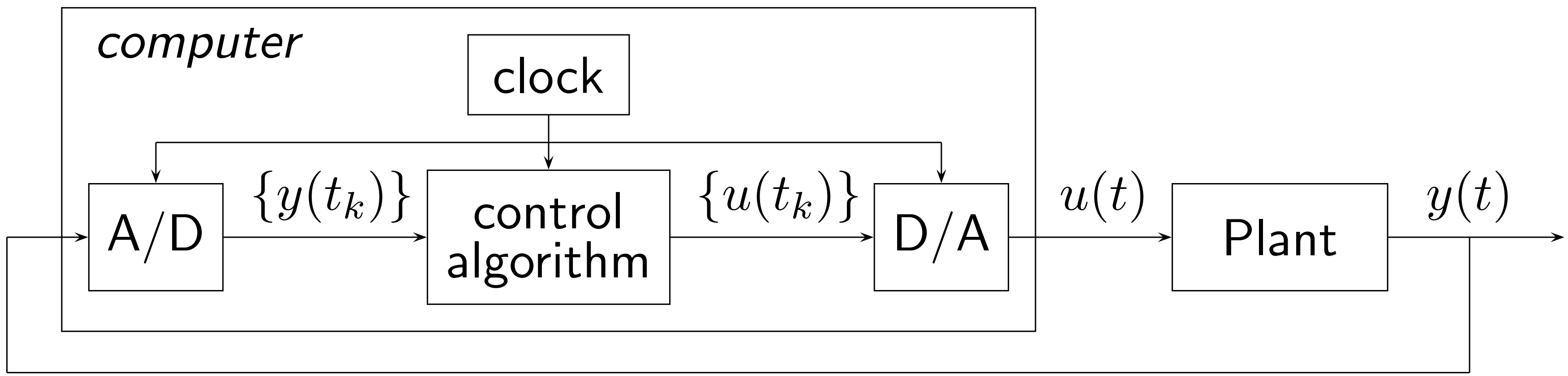
1. Real-time computing of control systems
   (a) Timing and implementation
   (b) Problems and solutions



2. Control of real-time control systems
   (a) Overview
   (b) Representative examples

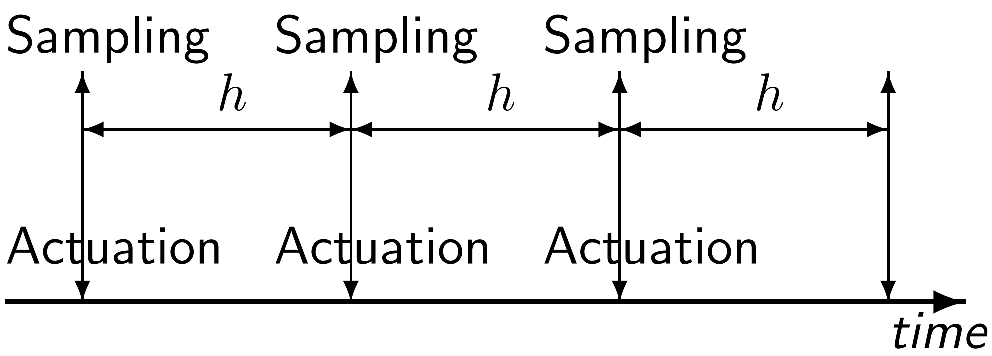


Timing and implementation



Simplest mathematical model with

- constant sampling period
- instantaneous input-output latency



Timing and implementation

Linear time-invariant continuous-time system state-space form

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

Discrete form, with sampling period h [1]

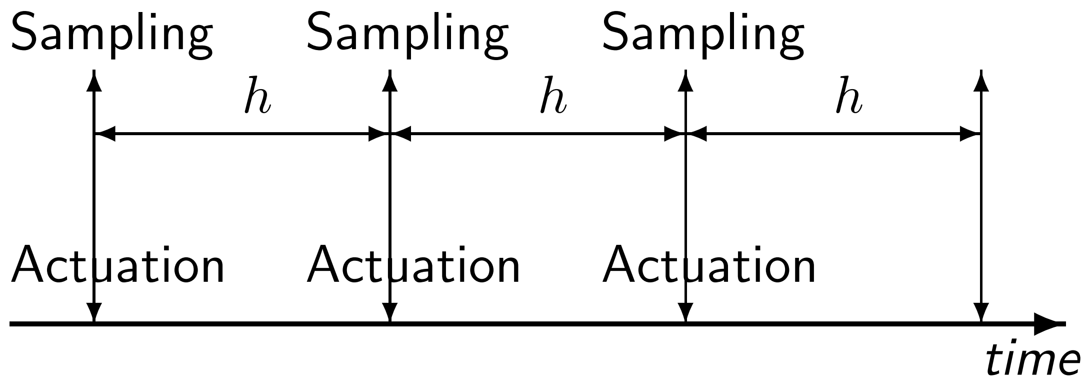
$$\begin{aligned} x_{k+1} &= \Phi(h)x_k + \Gamma(h)u_k \\ y_k &= Cx_k, \end{aligned} \tag{2}$$

where $\Phi(t)$ and $\Gamma(t)$ are obtained using the following

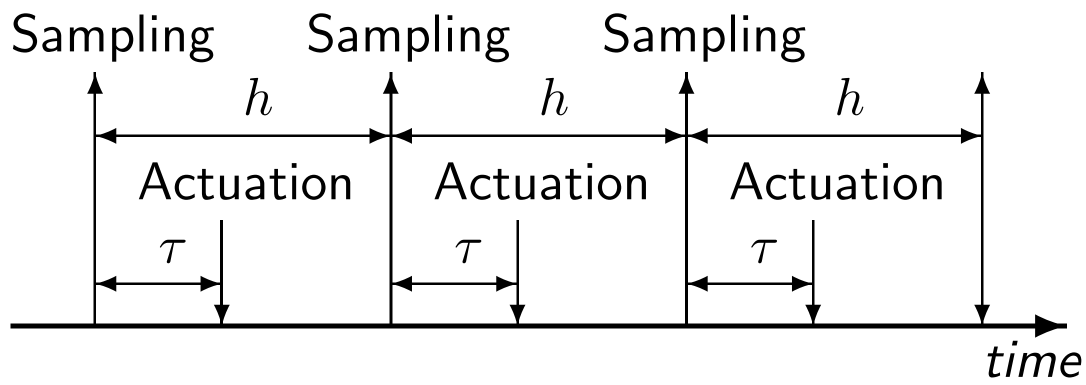
$$\Phi(t) = e^{At}, \quad \Gamma(t) = \int_0^t e^{As} B ds, \tag{3}$$

Timing and implementation

Timing of the basic model is not realistic



Adding a time delay to model an input/output latency due to the computation of the control algorithm or the insertion of a network



Timing and implementation

Continuous-time system with time delay τ

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bu(t-\tau) \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

Discrete form, with $\tau \leq h$

$$\begin{aligned} x_{k+1} &= \Phi(h)x_k + \Phi(h-\tau)\Gamma(\tau)u_{k-1} + \Gamma(h-\tau)u_k. \\ y_k &= Cx_k, \end{aligned} \tag{5}$$

where $\Phi(t)$ and $\Gamma(t)$ are also obtained using (3).

Timing and implementation

State-space form for (5), extended model:

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi(h) & \Phi(h-\tau)\Gamma(\tau) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} \Gamma(h-\tau) \\ I \end{bmatrix} u_k \qquad (6)$$

where $z_k \in \mathbb{R}^{m\times 1}$ represent past control signals.

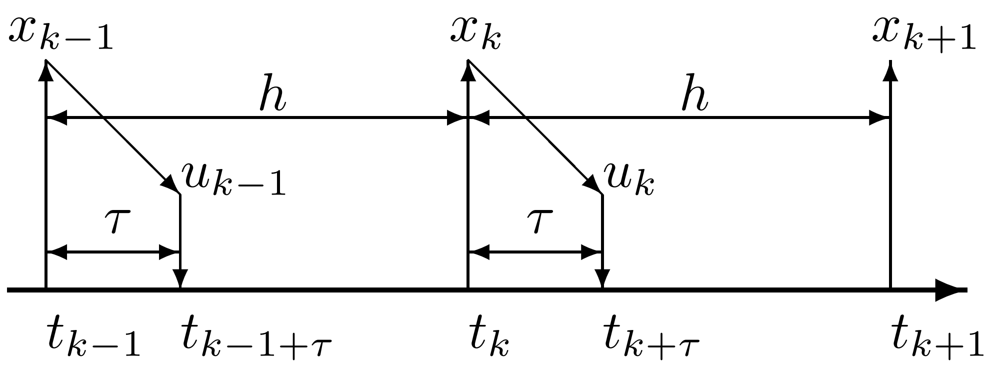
This notation slightly differs from conventional notation [1] to stress dependencies on h and τ.

The notation may be still misleading: u_k is applied τ time units after x_k is taken.

Timing and implementation

Notation issues:

k^{th} operation vs. timing



Let's obtain system (5) by looking at the dynamics from x_k to x_{k+1}. Denote the system state at time $t_{k+\tau}$ as $x_{k+\tau}$. Then

$$
\begin{aligned}
\text{From } x_k \text{ to } x_{k+\tau} \rightarrow x_{k+\tau} &= \Phi(\tau)x_k + \Gamma(\tau)u_{k-1} \\
\text{From } x_{k+\tau} \text{ to } x_{k+1} \rightarrow x_{k+1} &= \Phi(h-\tau)x_{k+\tau} + \Gamma(h-\tau)u_k \\
\text{All together} \rightarrow x_{k+1} &= \Phi(h-\tau)\left(\Phi(\tau)x_k + \Gamma(\tau)u_{k-1}\right) + \Gamma(h-\tau)u_k \\
&= \Phi(h-\tau)\Phi(\tau)x_k + \Phi(h-\tau)\Gamma(\tau)u_{k-1} + \Gamma(h-\tau)u_k \\
&= \Phi(h)x_k + \Phi(h-\tau)\Gamma(\tau)u_{k-1} + \Gamma(h-\tau)u_k \\
&= \text{model (5)}
\end{aligned}
$$

Timing and implementation

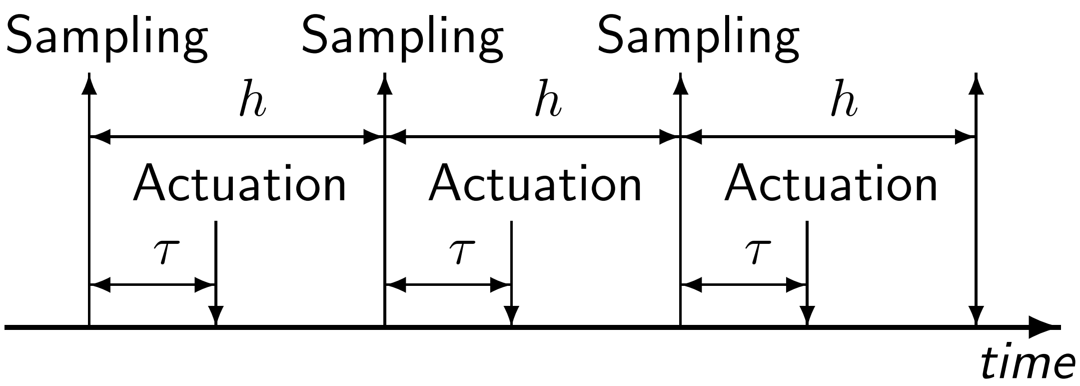
For closed loop operation of (6), given

$$u_k = -K(h,\tau)\begin{bmatrix} x_k \\ z_k \end{bmatrix} \tag{7}$$

where $K(h,\tau)$ is the state feedback gain, the system evolution is

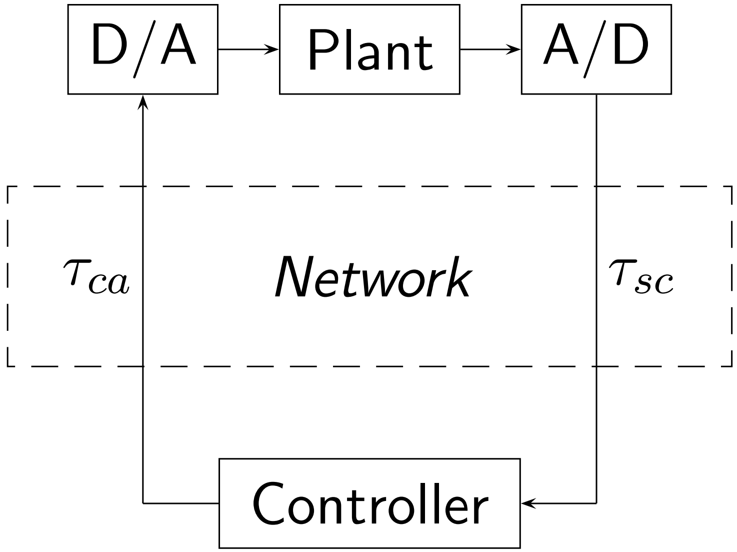
$$\begin{aligned}\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} &= \left(\begin{bmatrix} \Phi(h) & \Phi(h-\tau)\Gamma(\tau) \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \Gamma(h-\tau) \\ I \end{bmatrix} k(h,\tau)\right)\begin{bmatrix} x_k \\ z_k \end{bmatrix} = \\ &= \Phi_{cl}(h,\tau)\begin{bmatrix} x_k \\ z_k \end{bmatrix}\end{aligned} \tag{8}$$

Note: K depends on “future” parameters, h and τ



Timing and implementation

The extended form (6) also can model networked control systems



Delays controller-to-actuator τ_{ca} and sensor-to-controller τ_{sc} can be integrated into τ in (6).

Timing and implementation

Example. Double integrator differential equation:

$$\frac{d^2y}{dt^2} = u \tag{9}$$

If y and $\dot{y}$ are x_1 and x_2, a state space form is given by

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{aligned} \tag{10}$$

Discrete-time model, with period h and delay τ

$$x_{k+1} = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \tau\left(h - \frac{\tau}{2}\right) \\ \tau \end{bmatrix} u_{k-1} + \begin{bmatrix} \frac{(h-\tau)^2}{2} \\ h - \tau \end{bmatrix} u_k \tag{11}$$

Timing and implementation

If $h = 0.1$s and $\tau = 0.01$s, the state space form is given by

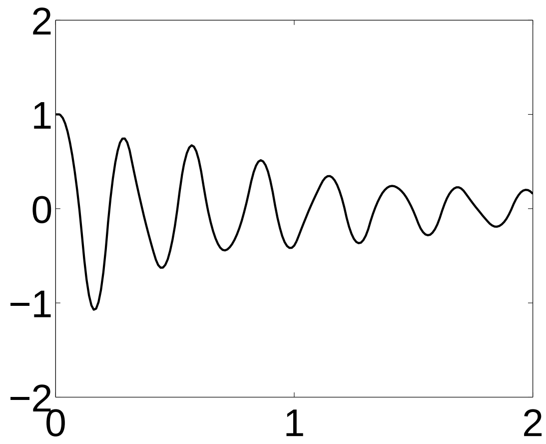
$$x_{k+1} = \begin{bmatrix} 1 & 0.1 & 0.001 \\ 0 & 1 & 0.01 \\ 0 & 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.004 \\ 0.09 \\ 1 \end{bmatrix} u_k \qquad (12)$$

Closing the loop with $u_k = -\begin{bmatrix} 271.7 & 21.86 & 0.23 \end{bmatrix} x_k$, the closed loop poles are $\lambda_1 = -0.3,\ \lambda_2 = -0.1,\ \lambda_3 = -0.9$

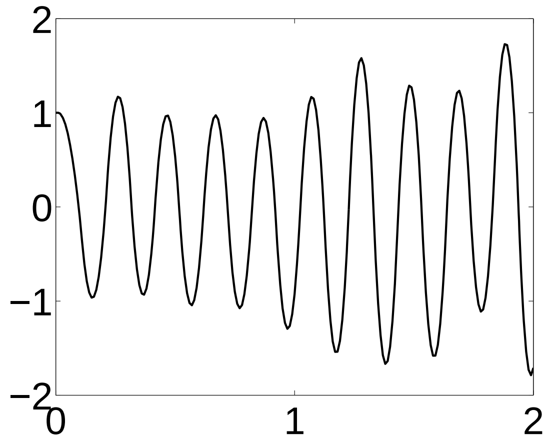
- With a faster micro $\tau = 0.005$, poles go at $\lambda_1 = 0.0082 - 0.2850i,\ \lambda_2 = 0.0082 + 0.2850i,\ \lambda_3 = -1.5513$
- With a slower micro $\tau = 0.02,\ \lambda_1 = -0.4910 + 0.9516i,\ \lambda_2 = -0.4910 - 0.9516i,\ \lambda_3 = 0.1316,$ with $|\lambda_1| = |\lambda_2| = 1.0708$

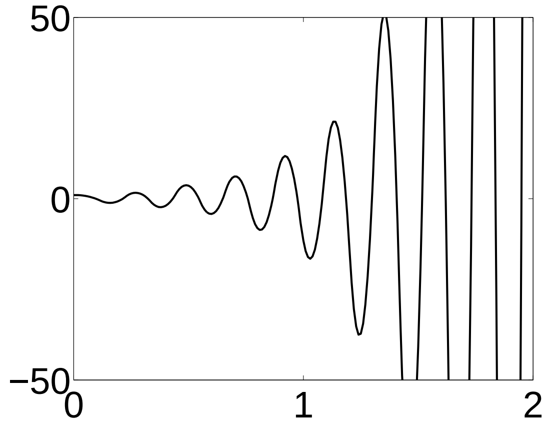
Timing and implementation

Random delay (with $\tau \in \left[\ 0.005 \quad 0.015\ \right]$, where $\tau_d = 0.01$)



Random period (with $h \in \left[\ 0.05 \quad 0.15\ \right]$, where $h_d = 0.1$)





Timing and implementation

- Timing is a key aspect !!!!
- The extended model (5) permits to model timing aspects.
- And the implementation should also enforce the timing.

Timing and implementation

Let's implement a controller: an infinite loop with a periodic activity to be executed every sampling period h

Periodic activity:

read_input(y_k); //assuming that $y_k = x_k$
$u_k = -Lx_k$;
write_output(u_k);

Which is the right code?

Timing and implementation

Let's implement a controller: an infinite loop with an algorithm to be executed every sampling period h. First attempt:

loop
  PeriodicActivity;
  WaitTime(h);
end loop

The computation time of *PeriodicActivity* is not accounted for.

Timing and implementation

Let's implement a controller: an infinite loop with an algorithm to be executed every sampling period h. Second attempt:

loop
Start = CurrentTime();
PeriodicActivity;
Stop = CurrentTime();
C := Stop - Start;
WaitTime(h - C);
end loop

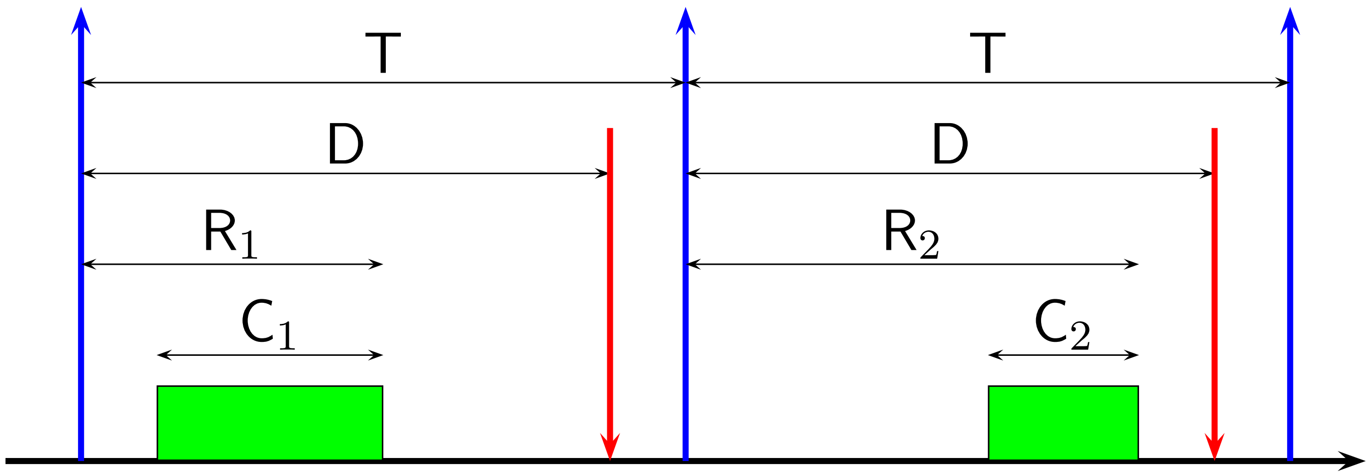
An interrupt causing suspension may occur between the assignment and *WaitTime*; or overrun problem.

Timing and implementation

- Methodologies for guaranteeing timeliness are required.
- Real-time technology ([2] or [3]) is the candidate: *in real-time computing the correctness of the system depends not only on the logical result of the computation but also on the time at which the results are produced* [4].

Timing and implementation

Task (or message) basic parameters:



Tasks: periodic, sporadic, aperiodic
hard, soft, best effort
time or event triggered
pre-emptive, not pre-emptive

Timing and implementation

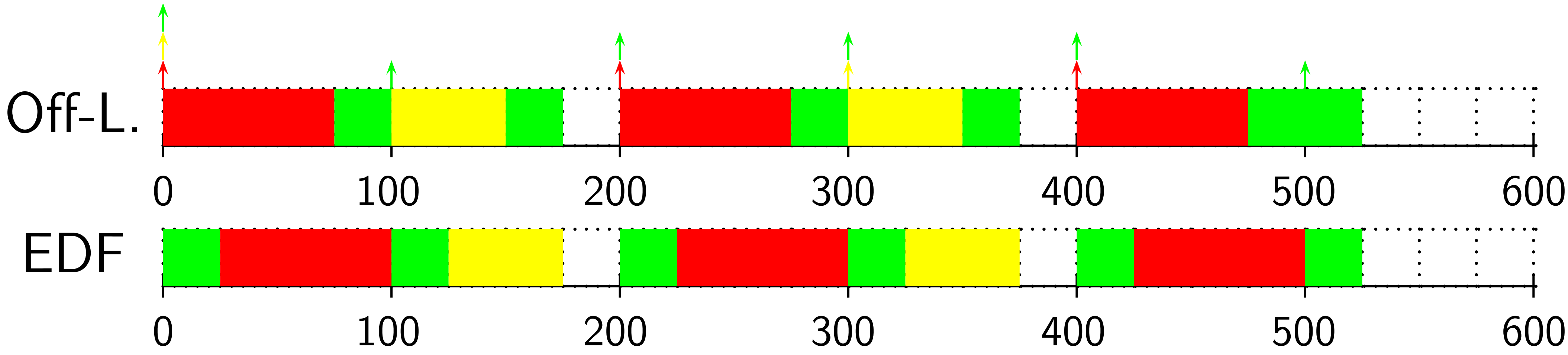
- Scheduling problem: how to assign tasks to the processor/network such that the set of (timing) constraints is met.

- Scheduling approaches:
  - offline scheduling: the time axis is divided in intervals of equal length (time slots), each task is statically allocated in a slot in order to meet the desired request rate, and the execution in each slot is activated by a timer.
  - online scheduling: each task is assigned a priority, scheduling feasibility is verified using analytical techniques, and tasks are executed on a priority-based kernel.

Timing and implementation

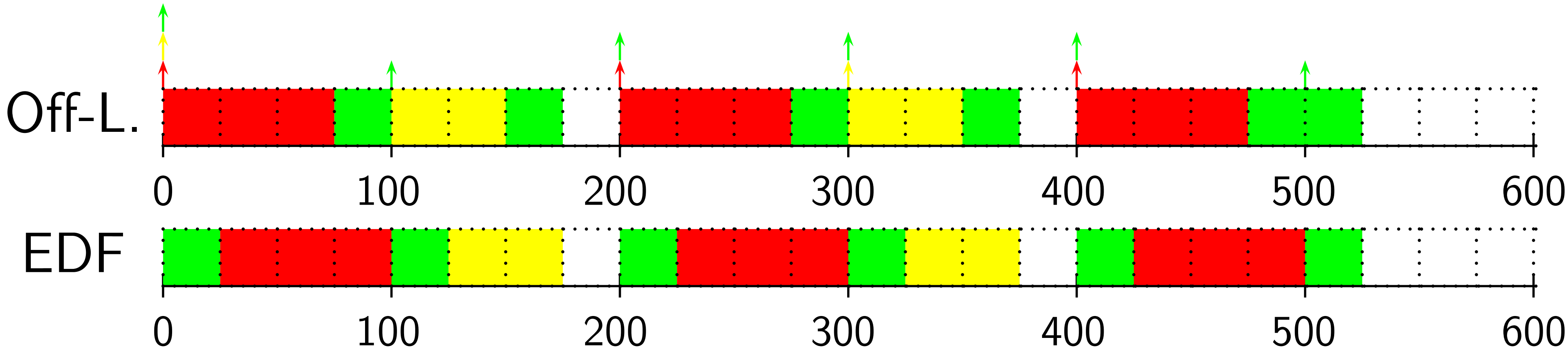
Example $(D = h)$

task	h	C
A	200 ms	75 ms
B	300 ms	50 ms
C	100 ms	25 ms

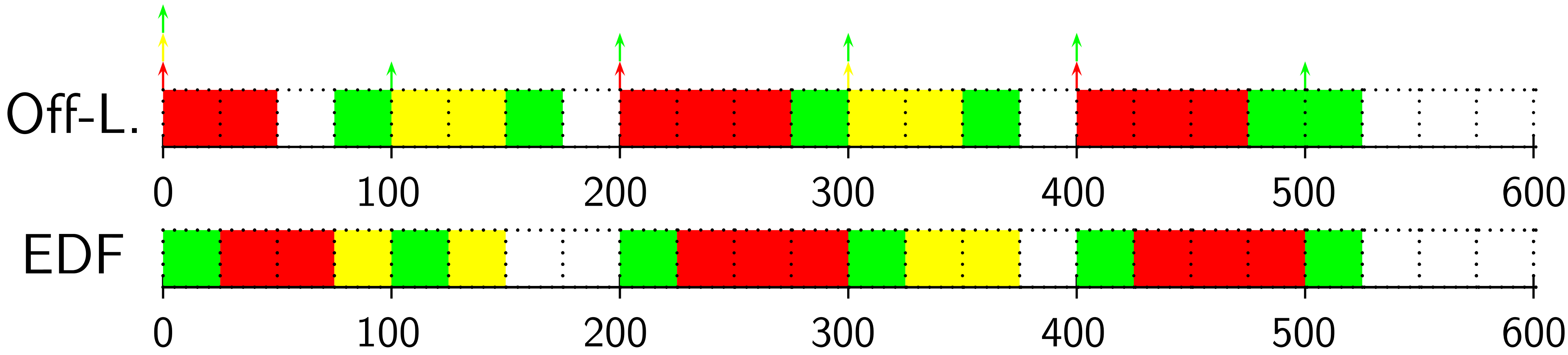


What happens if tasks execute less than C?

Timing and implementation



Run-time schedule with execution time of first job of A is 2



Timing and implementation

Observations on off-line scheduling

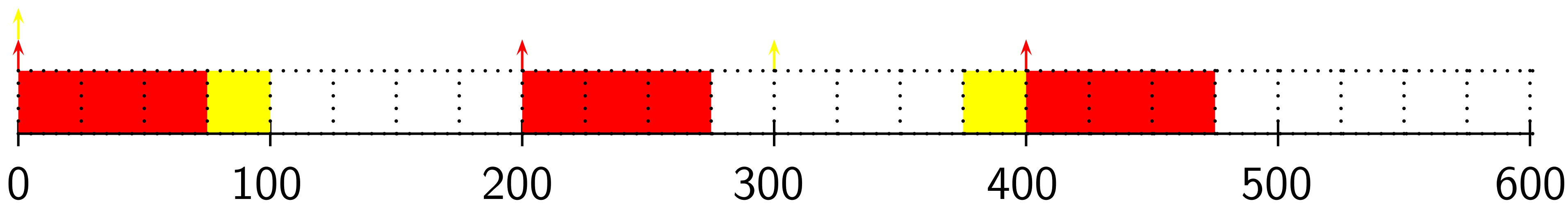
- Disadvantages
  - Keeping strict periodic execution is not possible
  - Hard to build, modify or expand
  - Lacks flexibility to adapt to resource availability or varying application demands
- Advantages
  - Simple implementation enforcing precise timing (no real-time operating system required)
  - Low run-time overhead

Timing and implementation

Example of what strict timing may mean....

Game: Let's try drawing a time line to execute tasks, keeping a constant distance between consecutive job start-times

task	h	C
A	200 ms	75 ms
B	300 ms	25 ms



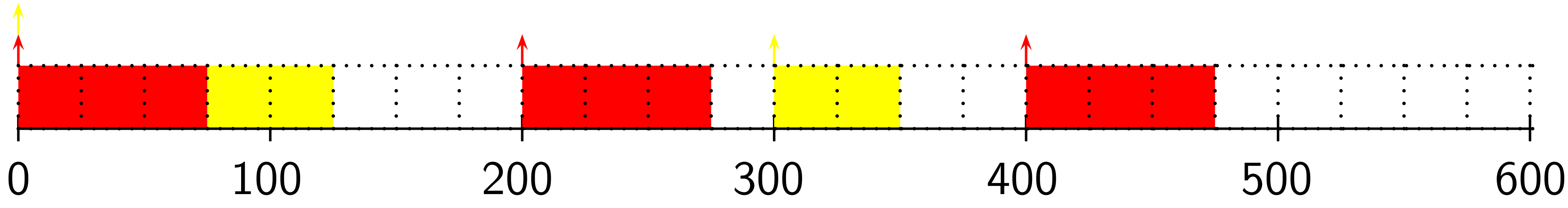
Assume **A** takes longer, e.g., C$= 100$ ms.

Can we still keep the constant distance?

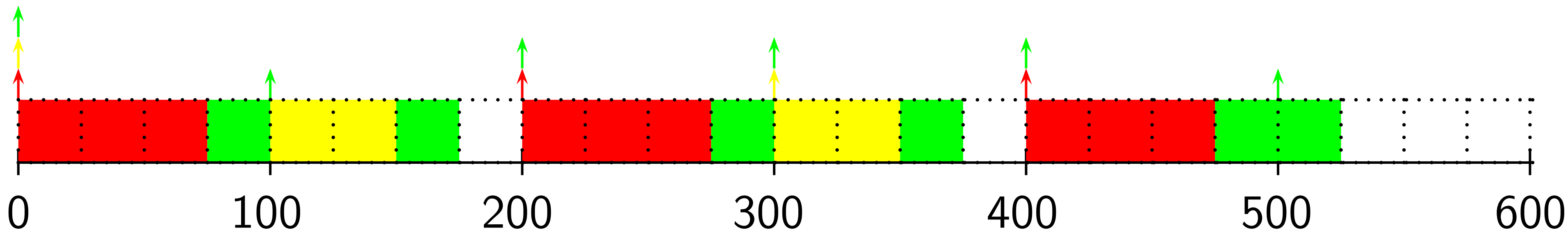
Timing and implementation

Example of *hard to modify*: Let's draw a time-line to execute

task	h	C
A	200 ms	75 ms
B	300 ms	50 ms

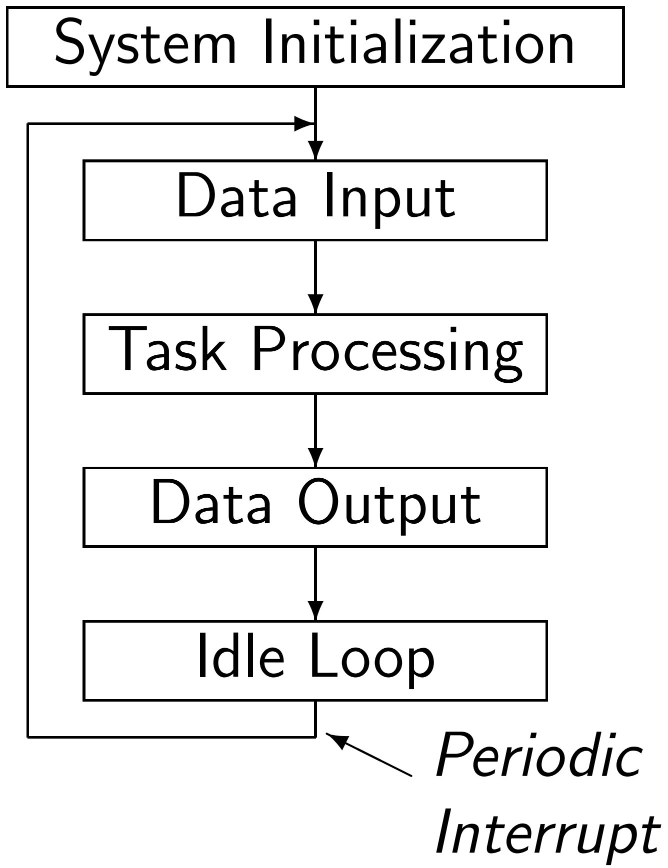


Assume a new task **C** with $h = 100$ ms and C$= 25$ ms. We have to rebuild the whole schedule !!!!



Timing and implementation

Note on offline scheduling: cyclic executives are the traditional offline scheduling approach for control applications [6], and widely used in industry.



Timing and implementation

Observations about on-line scheduling

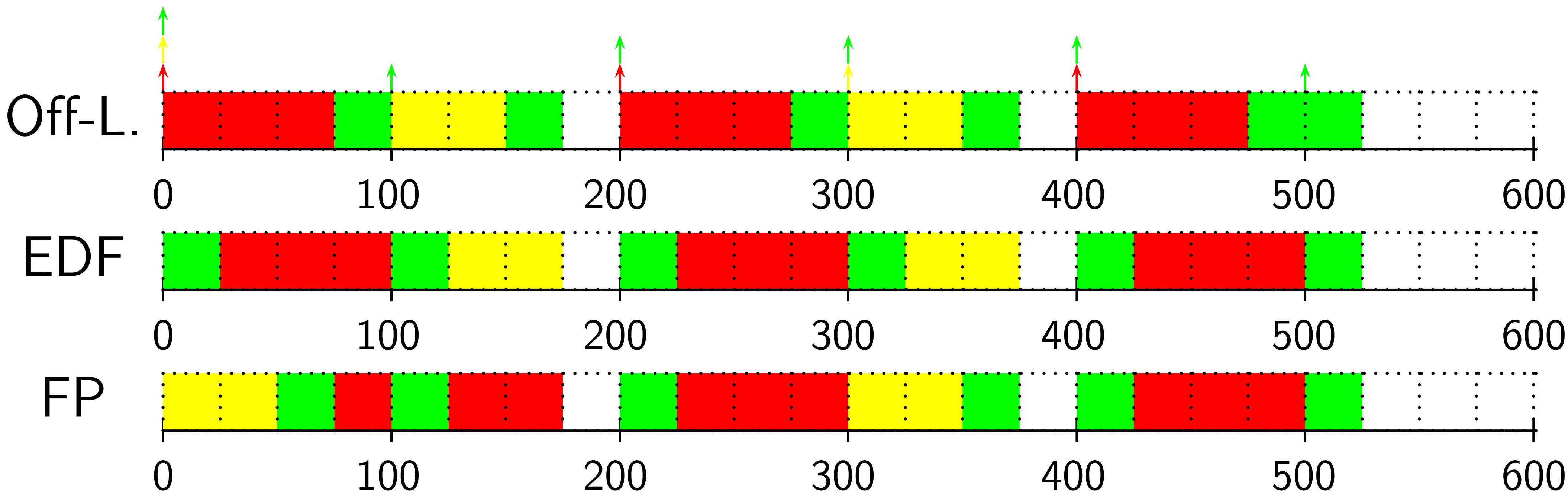
- Disadvantages
  - Keeping **strict** periodic execution is not possible
  - Needs operating system support (**overhead**)

- Advantages
  - Easy to build, analyze, modify or expand
  - Provides **flexibility** to adapt to resource availability or varying application demands

Timing and implementation

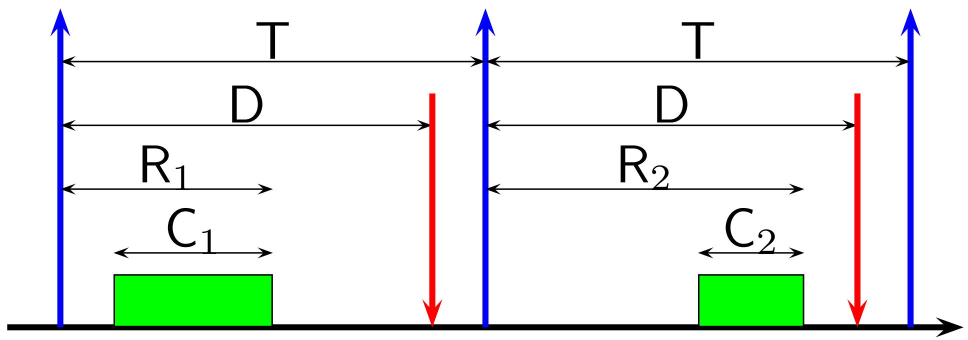
Previous example with a new schedule given by Fixed Priority online scheduling. Which is the priority assignment?

task	h	C
A	200 ms	75 ms
B	300 ms	50 ms
C	100 ms	25 ms

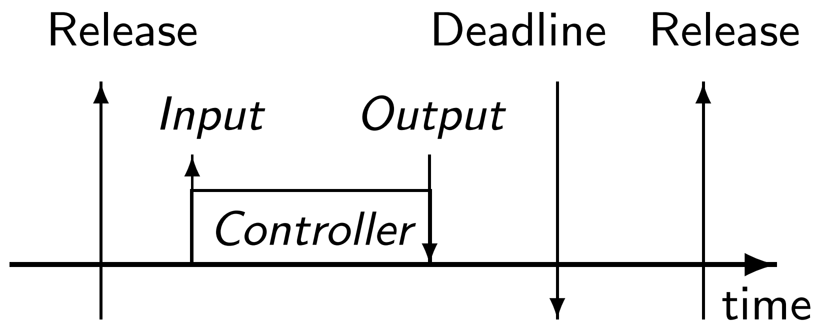


Timing and implementation

Enforcing the timing assumed in mathematical models using real-time technology



Hard real-time periodic task model



Naif control task model

- task period equal to sampling period T$=h$
- sampling and actuation occur at the beginning and termination of each job execution
- and task deadline bounding the time delay, D$\geq \tau$

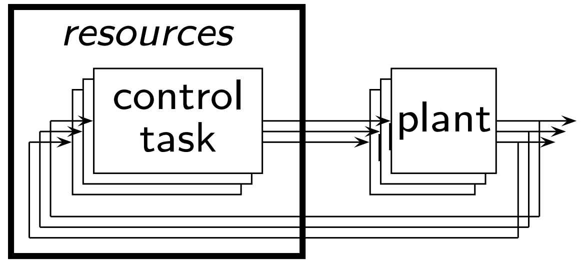
Timing and implementation

In the previous model for implementing controllers,

- if D$= \tau$: the expected timing from the model in closed loop operation (8) is perfectly kept !!! However, task set schedulability is severely reduced !!!
- if D$> \tau$: task set schedulability is increased (more tasks can be executed) at the expenses of introducing time uncertainty in sampling and actuation operations.

Contents

1. Real-time computing of control systems
   (a) Timing and implementation
   (b) Problems and solutions



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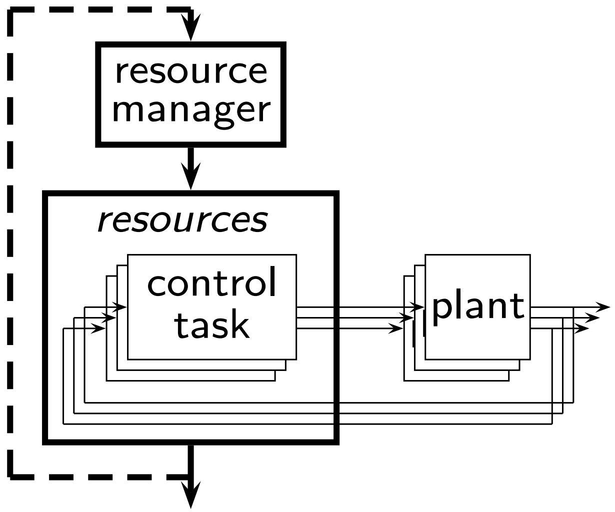
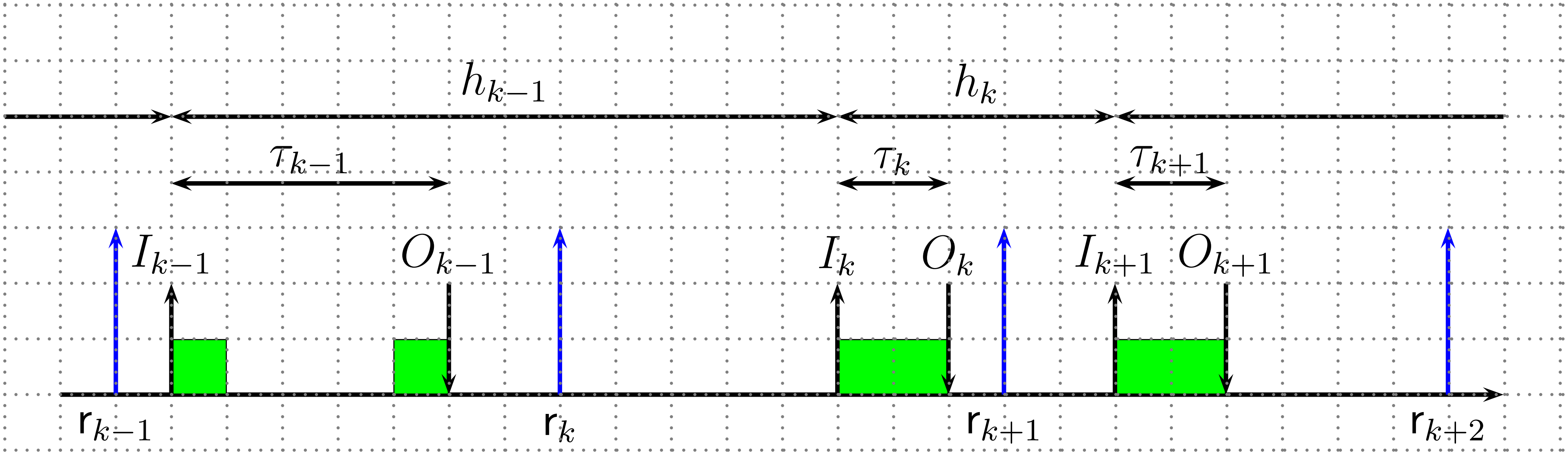
   (a) Overview
   (b) Representative examples



Problems and solutions

(RT) Computing introduces time uncertainty in periods and delays



- Job released at $r_k = kh$
- Sampling jitter: $\{h_k\}$
- Latency jitter: $\{\tau_k\}$

Problems and solutions

(RT) Computing introduces time uncertainty in periods and delays

Approaches:

- Ignore the problem
- Design the controller to be robust against time uncertainty
- Design the computing to minimize or eliminate the time uncertainty

Problems and solutions

Traditional approaches to time uncertainty

- Smith predictor
- Modified z-transform
- State-space lifting techniques

Limitation: ideal delays or mutirate (periodic) systems.

Alternative solutions

Problems and solutions

Remember the standard model (6)

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi(h) & \Phi(h-\tau)\Gamma(\tau) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} \Gamma(h-\tau) \\ I \end{bmatrix} u_k$$

where $x_{k+1} = x(kh+h)$ and $z_{k+1} = z(kh+h)$

If h and τ vary at each job execution, the model is given by

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi(h_k) & \Phi(h_k-\tau_k)\Gamma(\tau_k) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} \Gamma(h_k-\tau_k) \\ I \end{bmatrix} u_k \quad (13)$$

where $x_{k+1} = x(\sum_{i=0}^{k+1} h_i)$ and $z_{k+1} = z(\sum_{i=0}^{k+1} h_i)$

Problems and solutions

(13) is a family of models. Observations:

- Given the $\{h_k, \tau_k\}$ values, a specific system is obtained and it can be analyzed
- Controllability and observability
- Stability

Problems and solutions

Given the $\{h_k, \tau_k\}$ values, a specific system is obtained and it can be analyzed. Example.

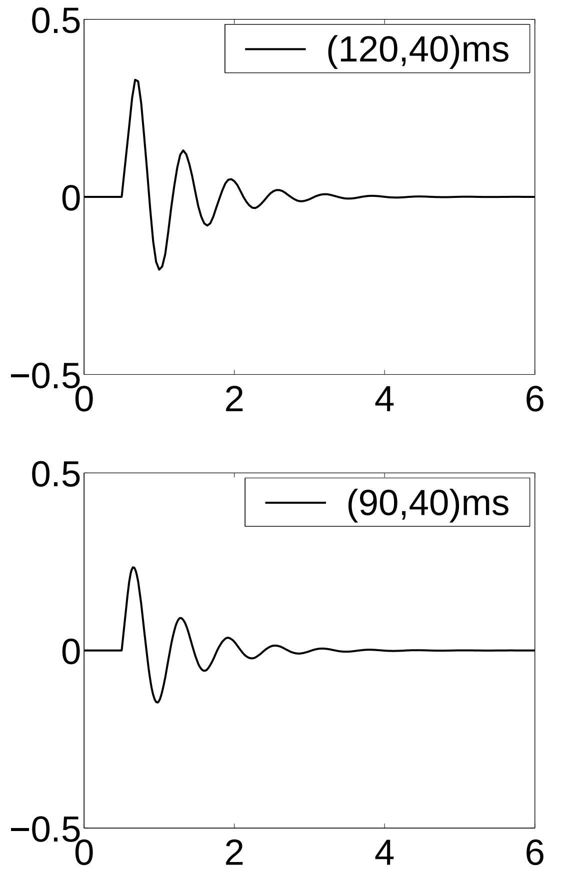
$$\frac{d^2y}{dt^2} = u$$

Let's locate the continuous closed loop poles at $\lambda_{1,2} = -1.5 \pm 10 * i$.

task	h	C=τ
A	120 ms	40 ms
B	90 ms	40 ms

$$K_{120,40} = \begin{bmatrix} 75.8572 & 10.1051 & 0.3435 \end{bmatrix}$$
$$K_{90,40} = \begin{bmatrix} 83.5998 & 9.7351 & 0.3225 \end{bmatrix}$$

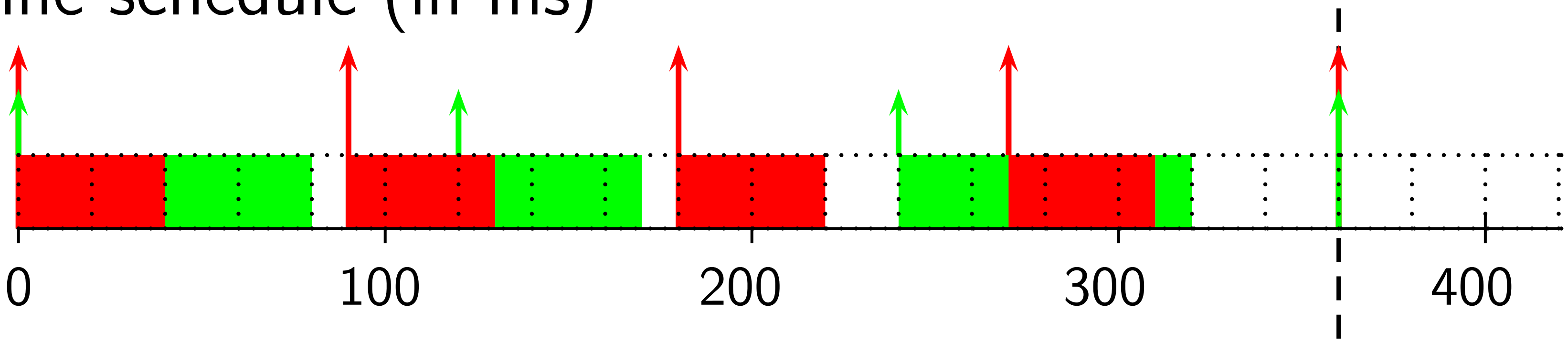
In isolation

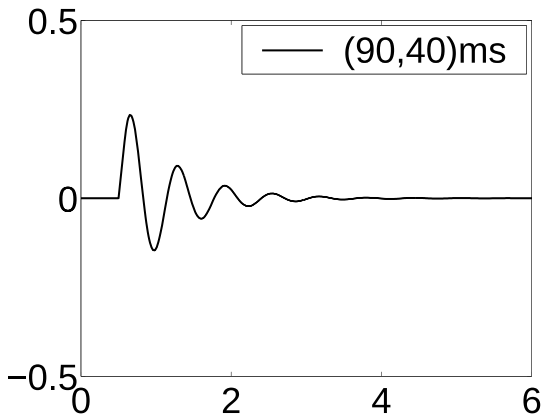


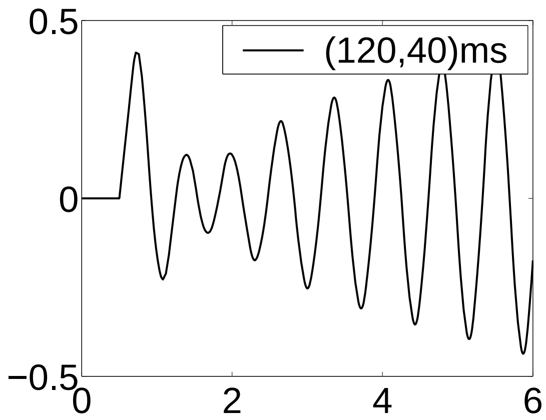
Problems and solutions

Given the $\{h_k, \tau_k\}$ values, a specific system is obtained and it can be analyzed. Example.

Offline schedule (in ms)

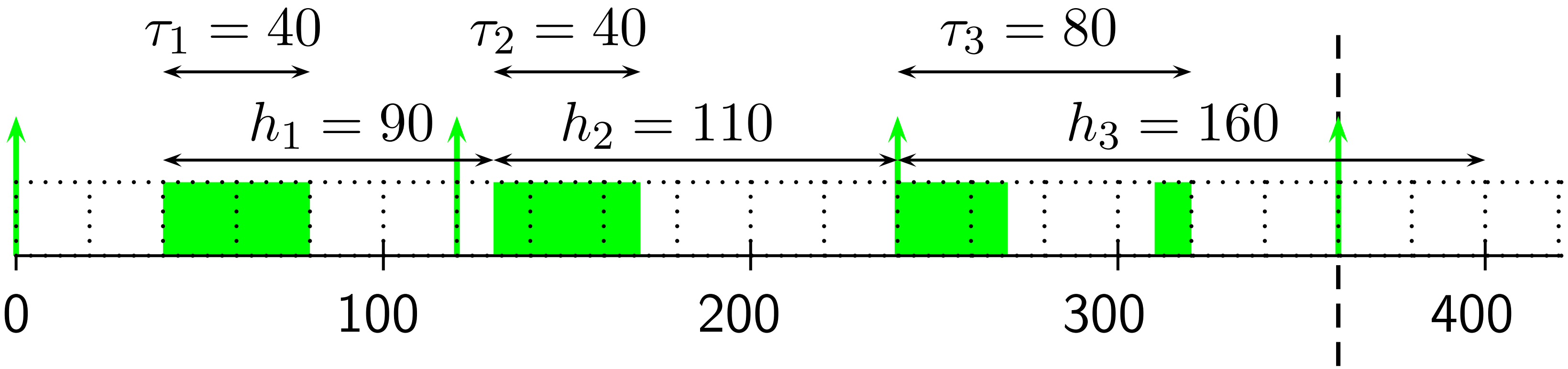






Problems and solutions

Given the $\{h_k, \tau_k\}$ values, a specific system is obtained and it can be analyzed. Example.

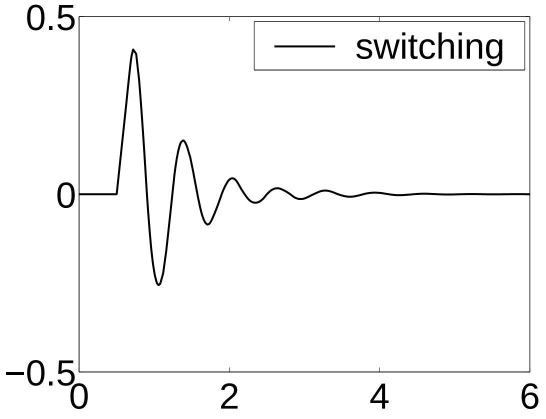


Let's approach A as a switched system:

$$\Phi_{cl}(h,\tau) \in \{\Phi_{cl}(90,40), \Phi_{cl}(110,40), \Phi_{cl}(160,80)\}$$

with

$$K_{90,40} = \left[\begin{array}{ccc} 83.5998 & 9.7351 & 0.3225 \end{array}\right]$$
$$K_{110,40} = \left[\begin{array}{ccc} 78.4894 & 10.0117 & 0.3377 \end{array}\right]$$
$$K_{160,80} = \left[\begin{array}{ccc} 65.0282 & 12.7871 & 0.8149 \end{array}\right]$$



Problems and solutions

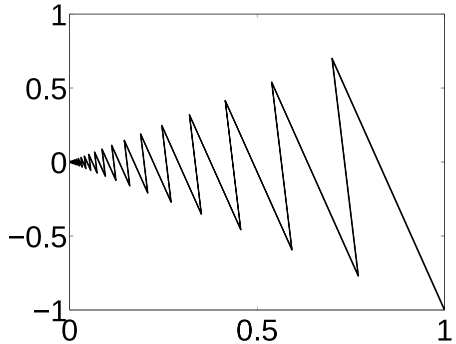
Given the $\{h_k, \tau_k\}$ values, a specific system is obtained and it can be analyzed. Is the previous analysis enough?

Example of unstable switched sequence $A_2A_1A_2A_1\ldots$ where each subsystem A_i is stable. Given

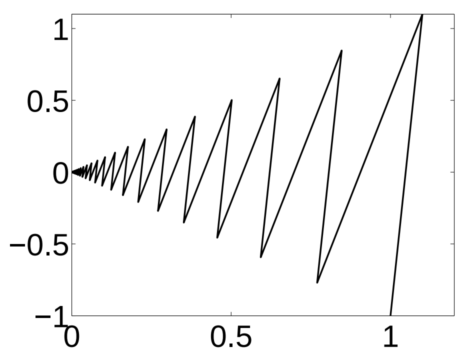
$$x_{k+1} = Ax_k, \quad k \geq 0, \quad x_0 = x0, \quad A \in \{A1, A2\} \tag{14}$$

with $A_1 = \begin{bmatrix} 0.9 & 0.2 \\ -0.2 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9 & -0.2 \\ 0.2 & -0.9 \end{bmatrix}, \quad x0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

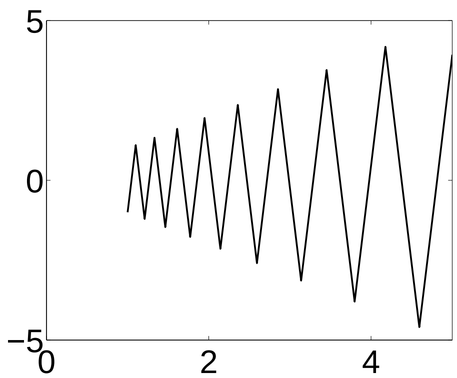
A_1^k



A_2^k



$(A_2A_1)^k$



Problems and solutions

Controllability

- Is (13) controllable? Yes. (See proof in appendix 1)
- Therefore, we can find $\{u_k\}$ to bring the system to x_{eq}
- However, to compute $\{u_k\}$, we need to know beforehand $\{h_k, \tau_k\}$
- ... which in the general case, it's not known !!!

Observation (feasibility problem): to compute u_k we need to know $\{h_k, \tau_k\}$

Problems and solutions

Observability

- Is (13) observable? Yes, if the output matrix outputs the additional variable. (See proof in appendix 2)
- No feasibility problems exist (past h_k, τ_k and u_k are known).

Remark

Although (13) is controllable and observable, the admissible time variability is not known.

Problems and solutions

Stability. Looking at matrices

- Single closed loop matrix

$$\text{stable} \Leftrightarrow \rho(\Phi_{cl}(h,\tau)) < 1 \tag{15}$$

- Sequence of closed-loop matrices

$$\text{stable} \Leftrightarrow \rho(\Phi_{cl}(h_1,\tau_1)\cdot\Phi_{cl}(h_2,\tau_2)\cdot\ldots\cdot\Phi_{cl}(h_n,\tau_n)) < 1 \tag{16}$$

- Closed-loop matrices randomly taken from a finite set Ω [7]

$$\Omega \text{ stable} \Leftrightarrow \exists P > 0 : \Omega^T P \Omega - P < 0,\ \forall \Omega \in \Omega^K, K \geq 0 \tag{17}$$

Problems and solutions

Stability. ... References

- Delays: For time-varying but bounded delays, simply checked in a Bode plot [8]
- Sampling periods: For uncertain sampled data systems, treated as hybrid system and using Lyapunov functions with discontinuities [9]

Problems and solutions

(RT) Computing introduces time uncertainty in periods and delays

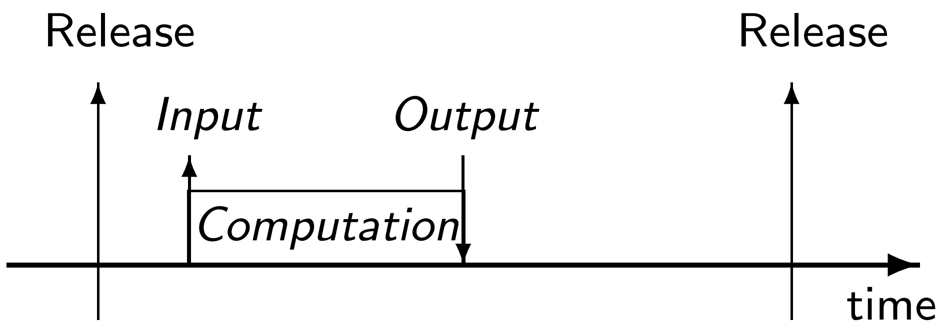
Approaches:

- Ignore the problem
- Design the controller to be robust against time uncertainty
- Design the computing to minimize or eliminate the time uncertainty

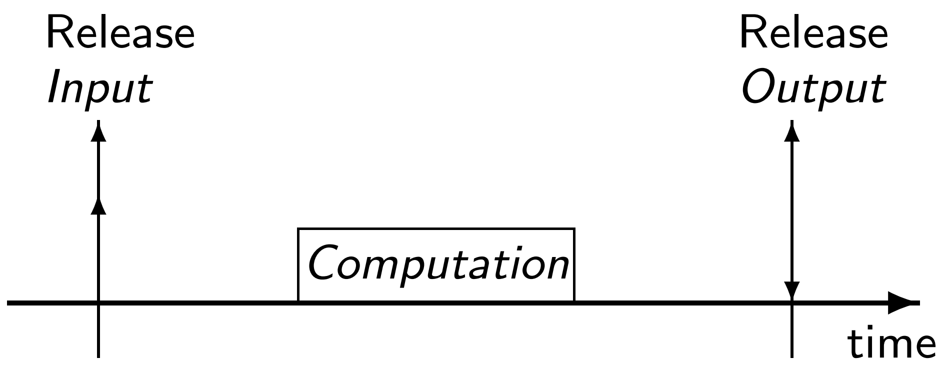
Problems and solutions

Design the computing to minimize/eliminate the time uncertainty

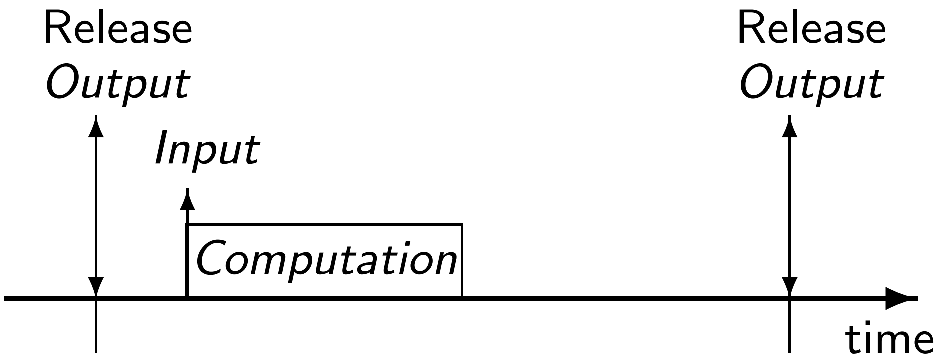
Naif approach [10]



One-sample approach [5], [11]



One-shot approach [12]



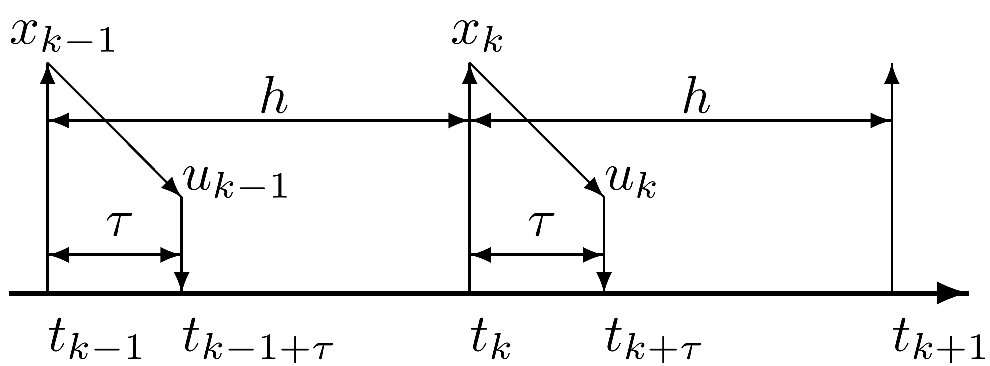
Problems and solutions

Design the computing to minimize/eliminate the time uncertainty

Model (6) in closed-loop form is based on two synchronization points, on a time reference given by the sampling instants.

$$\begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} \Phi(h) & \Phi(h-\tau)\Gamma(\tau) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} + \begin{bmatrix} \Gamma(h-\tau) \\ I \end{bmatrix} u_k$$

$$u_k = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} = K_1 x_k + K_2 z_k \quad \text{with} \quad K_1 \in \mathbb{R}^{1\times n}, \quad K_2 \in \mathbb{R}^{1\times m}$$



Problems and solutions

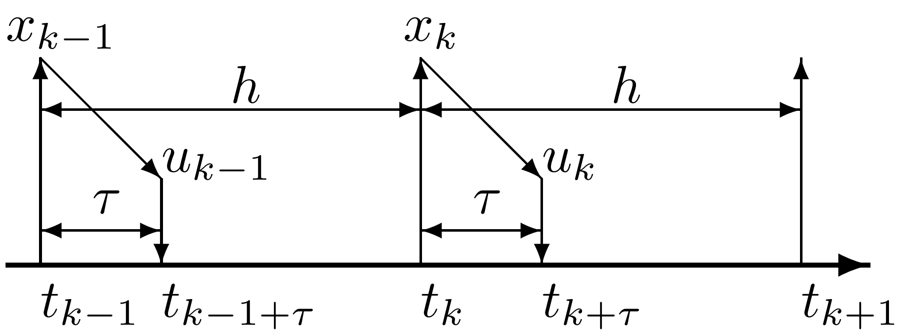
Design the computing to minimize/eliminate the time uncertainty

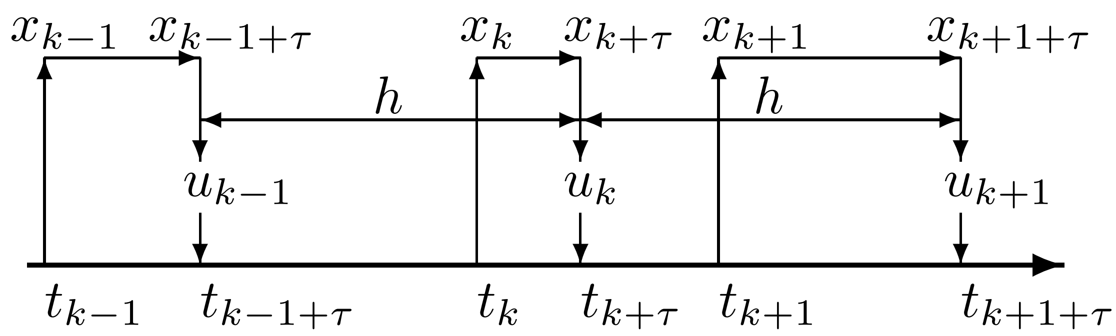
Constructing the one-shot task model: changing time coordinates to a time reference given by the actuation instants.

$$x_{k+\tau+1} = \Phi(h)x_{k+\tau} + \Gamma(h)u_k, \quad \text{with } u_k = Kx_{k+\tau} \quad \text{with } L \in \mathbb{R}^{1\times n}. \tag{18}$$

$x_{k+\tau}$ has to be predicted from x_k:

$$x_{k+\tau} = \Phi(\tau)x_k + \Gamma(\tau)u_{k-1}. \tag{19}$$



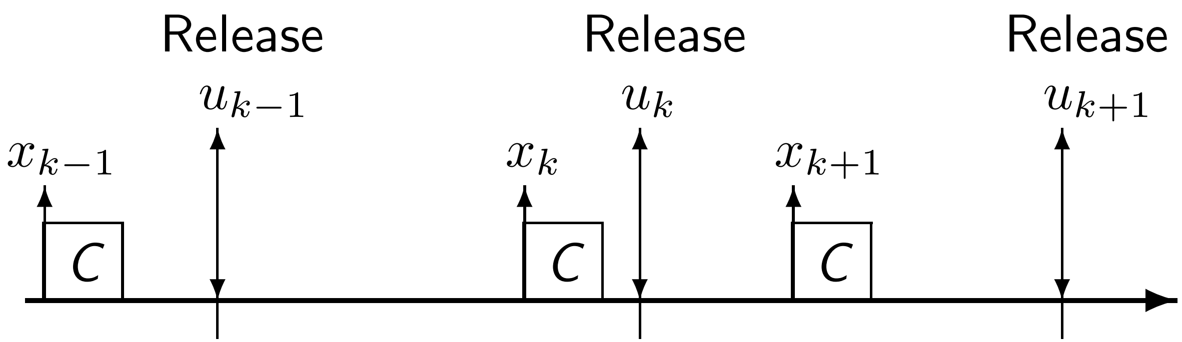


Problems and solutions

Design the computing to minimize/eliminate the time uncertainty

- All closed loop dynamics given by (18) and (19) (i.e., one-shot) can be obtained by (6) (standard).
- All closed loop dynamics given by (6) can be obtained by (18) and (19) if $m = n$.

The standard model is more general.... but one-shot admits irregular sampling

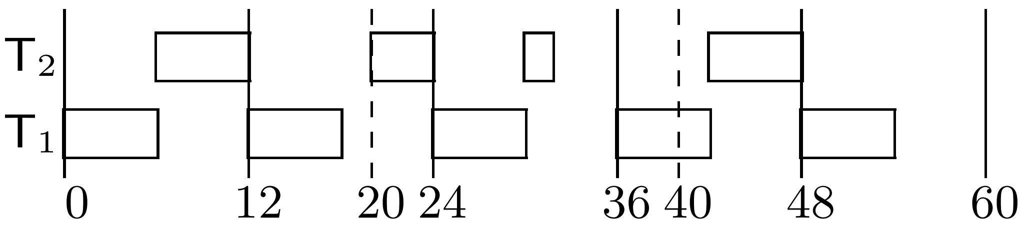


Problems and solutions

Design the computing to minimize/eliminate the time uncertainty

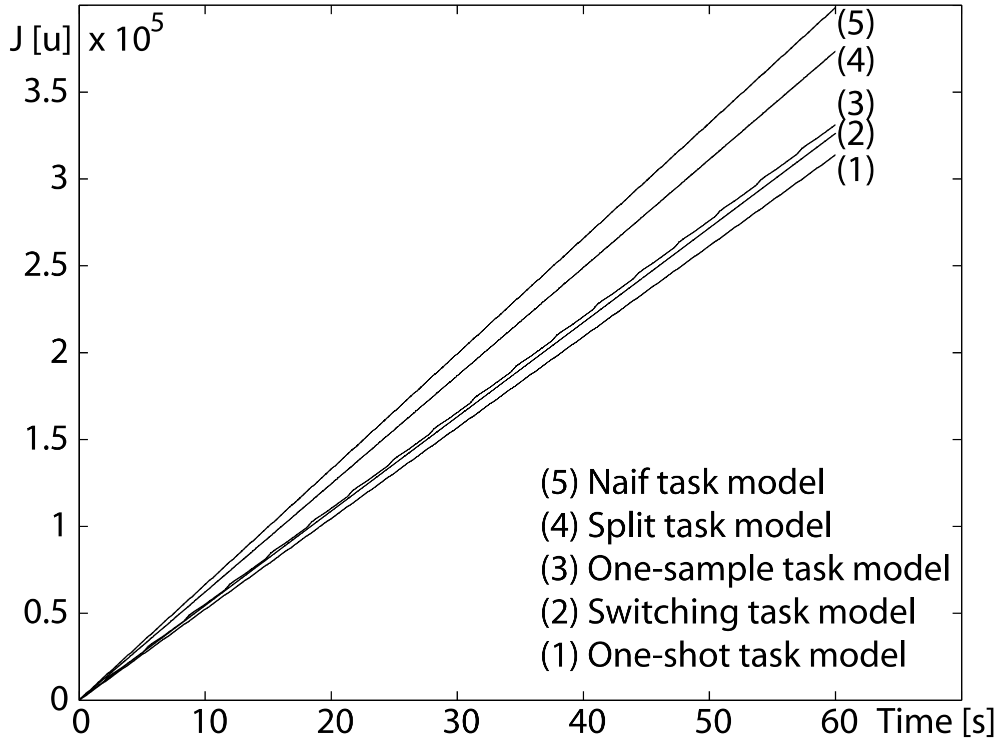
Evaluation: Naif, one-sample, switching, one-shot, split [13]

	h	C
T_1	12	6
T_2	20	6



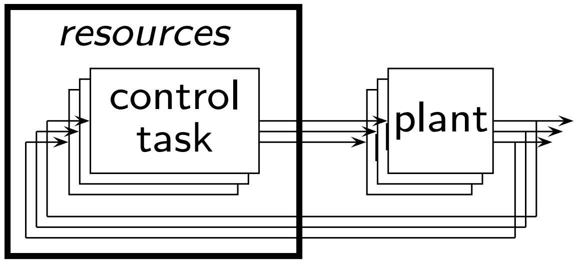
Voltage stabilizer (RCRC circuit)

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -918.2 & -90.9 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 918.2 \end{bmatrix} u(t)$$

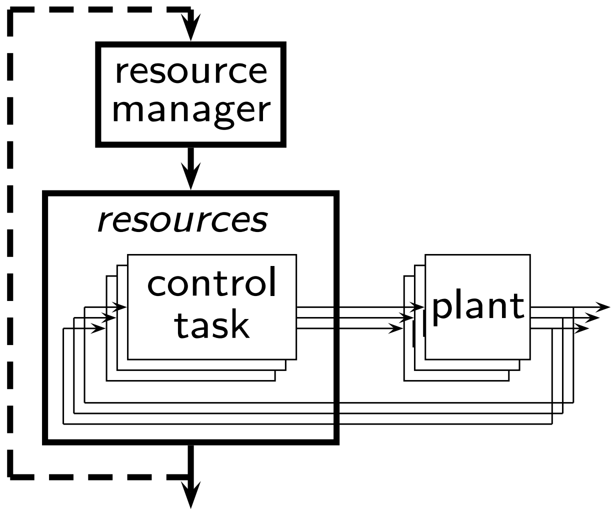


Contents

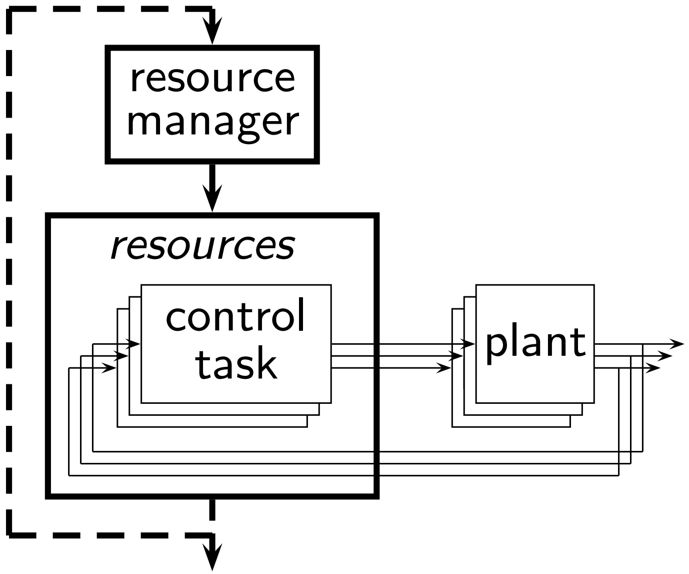
1. Real-time computing of control systems
   (a) Timing and implementation
   (b) Problems and solutions



2. Control of real-time control systems
   (a) Overview
   (b) Representative examples



Overview



Control of real-time control systems, also known as

- (Optimal) Sampling period selection
- Feedback scheduling
- Event-based scheduling

Overview

Objective: to efficiently use resources when control loops share limited resources. Main flavors:

- Maximize aggregated control loop performance by fully and cleverly exploiting the available resources

$$\Downarrow$$

Feedback scheduling (FS)

- Minimize resource utilization while bounding inter-sampling dynamics

$$\Downarrow$$

Event-based scheduling (ES)

Overview * Feedback scheduling

Common formulation: optimization problem

minimize (maximize):	penalty (benefit) on control performance
with respect to:	sampling periods / job execution
subject to:	closed loop stability
	task set schedulability

Two type of results

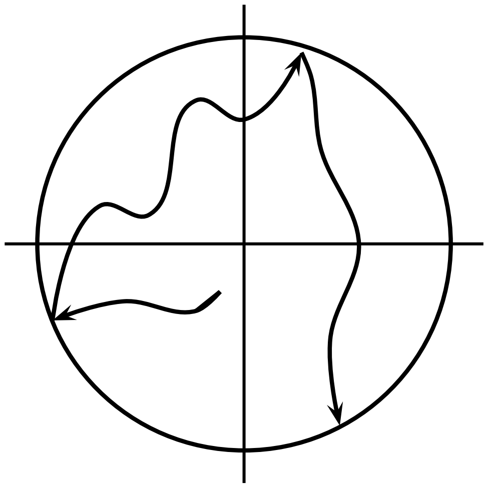
- Optimal sampling periods (e.g., [14], [15],[16],[17],[18])
- Optimal job sequence (e.g., [19], [20][1], [21])

[1] Based on bounding the inter-sampling dynamics.

Overview * Event-based scheduling

Common idea: to bound the inter-sample dynamics or to ensure stability (e.g., [22], [23], [24], [25], [26], [27]). Approaches:

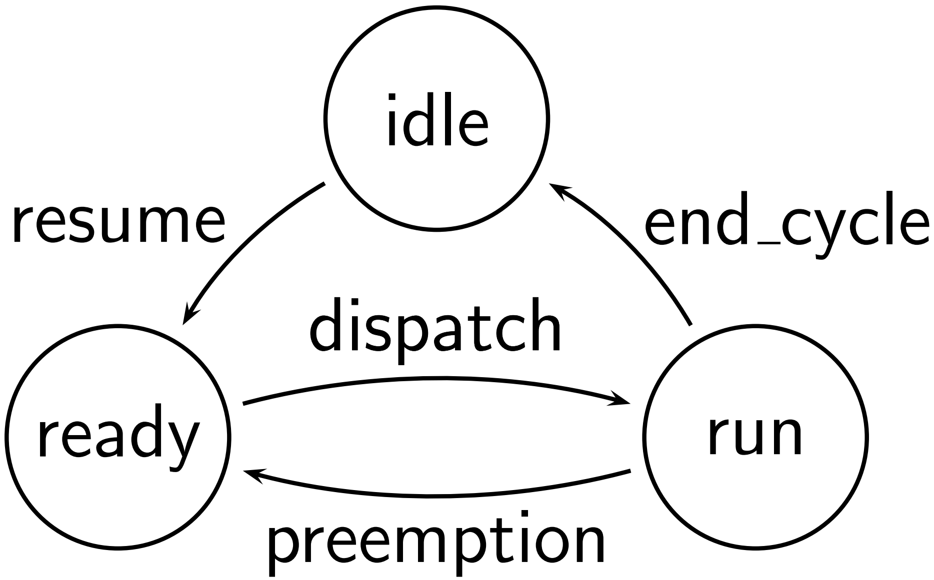
- Integrate an analog event detector, e.g. [22] or [24]
- Assume a coordinator aware of all plant states, e.g., [23]
- Enforce a minimum inter-execution time, e.g., [26]
- Observe the occurrence of the event (self-triggered), e.g., [25], [27]



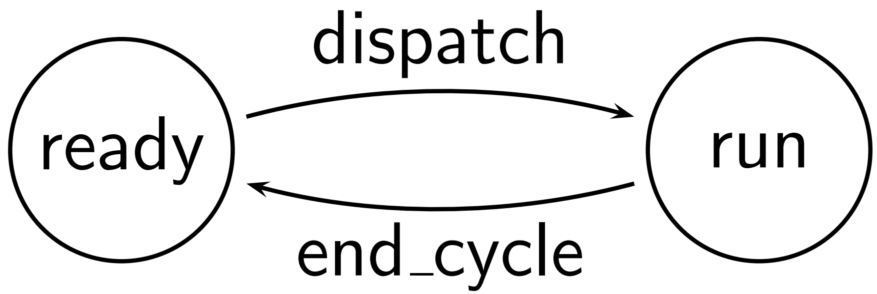
Overview * Taxonomy

	Which	What	Who	When	How		
	Criterion			it is solved	Solution	Timing Constraints	Sched.
[14] Set96	Optimizat.	TT	Coord.	Offline	Periods	Static periodic	EDF
[22] Arz99	Bound d.	ET	Task	Online	Periods	Aperiodic ET	Missing
[23] Zha99	Bound d.	ET	Coord.	Online	Job	Aperiodic ET	EFS
[15] Eke00	Optimizat.	TT	Coord.	Online	Periods	Varying periodic	EDF
[19] Reh00	Optimizat.	TT	Coord.	Offline	Sequences	Static pseudo periodic	Cyc. Ex.
[20] Hri01	Bound d.	TT	Coord.	Offline	Sequences	Static pseudo periodic	Cyc. Ex.
[16] Mar04	Optimizat.	TT	Coord.	Online	Periods	Varying periodic	EDF
[17] Pal05	Optimizat.	TT	Coord.	Offline	Periods	Static periodic	EDF
[18] Hen05	Optimizat.	TT	Coord.	Online	Periods	Varying periodic	EDF
[21] Ben06	Optimizat.	TT	Coord.	Online	Sequences	Dynamic pseudo per.	Flex.C.E
[25] Tab06	Bound d.	ET	Task	Online	Periods	Aperiodic TT	WiP
[26] Joh07	Bound d.	ET	Task	Online	Periods	Sporadic TT	Spor.
[27] Lem07	Bound d.	ET	Task	Online	Periods	Aperiodic TT	Elastic S.

Overview * FS vs. ES * Kernel



time-driven operation



control-driven operation

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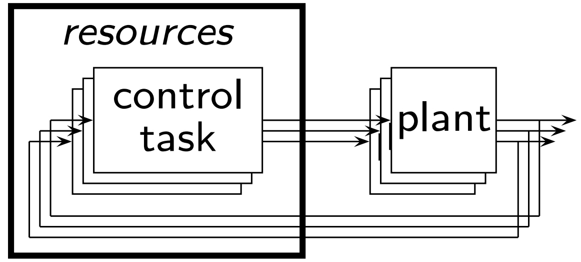
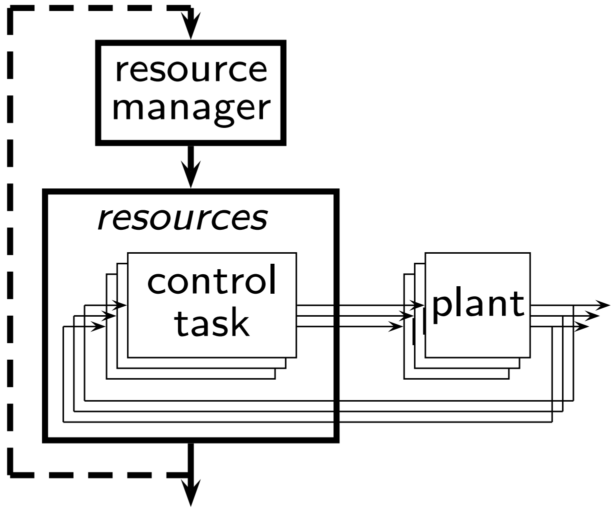


2. Control of real-time control systems
   (a) Overview
   (b) Representative examples
      - periods (optimization)
      - sequences (optimization)
      - event-based

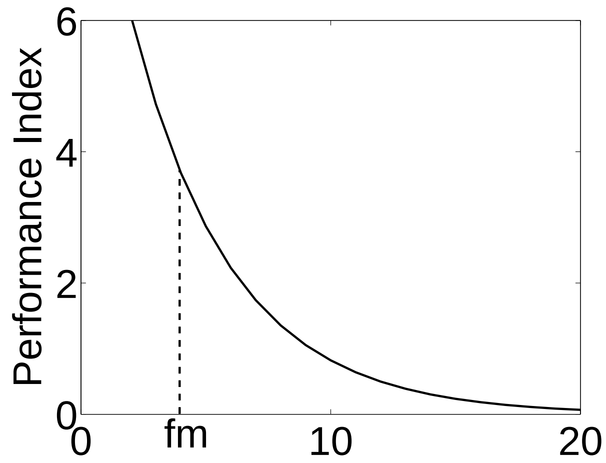


Representative examples (periods)

[14] On task schedulability in real-time control systems

- Set of n control tasks sharing a CPU
- Performance index for each control task (cost):

  $\Delta J(f_i) = J_D(f_i) - J$
- A minimum frequency for each task f_{mi} must be guaranteed



Representative examples (periods)

[14] On task schedulability in real-time control systems

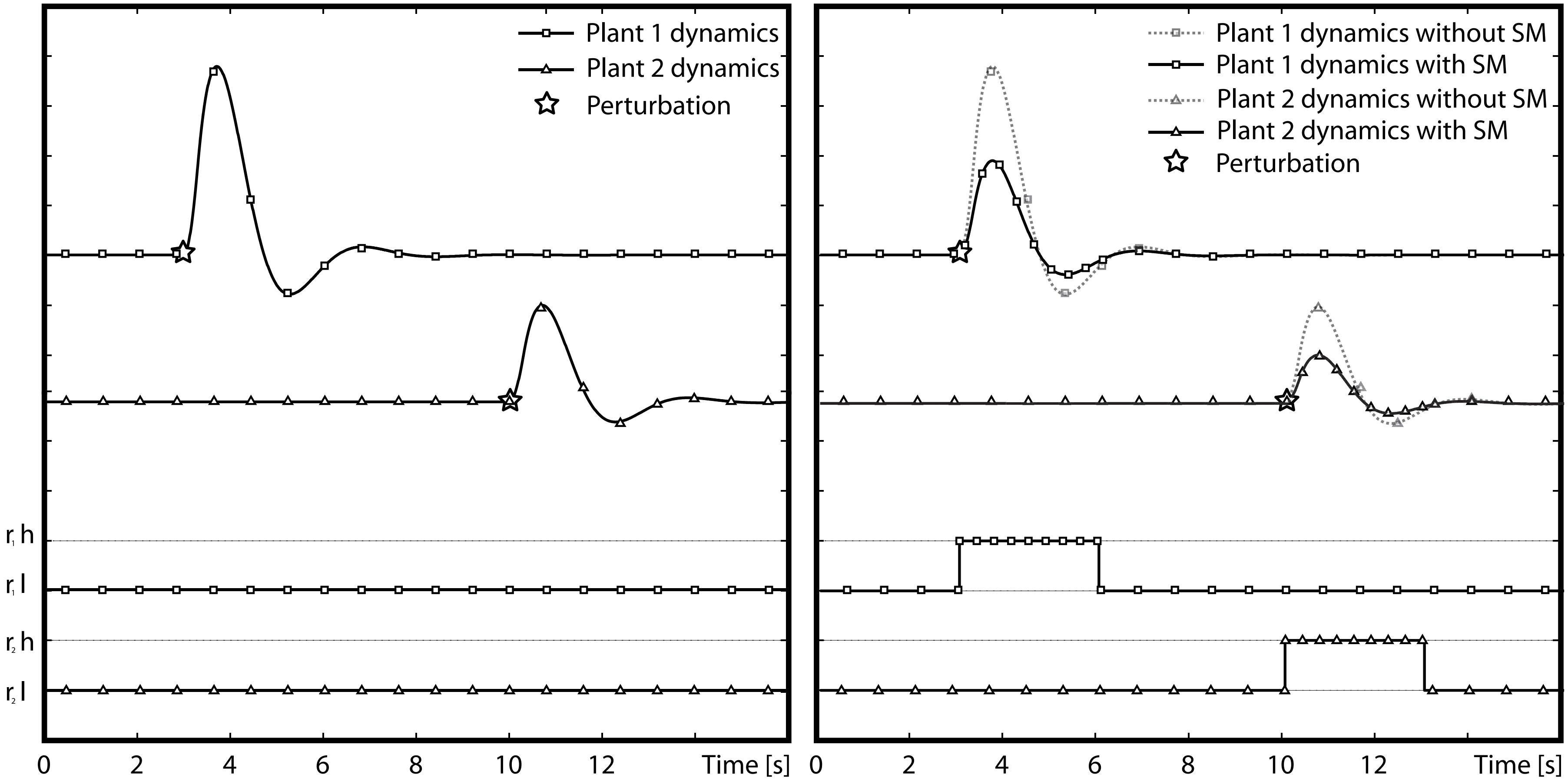
$$
\begin{aligned}
\text{min} \quad & \Delta J = \sum_{i=1}^{n} \omega_i \Delta J_i = \sum_{i=1}^{n} \omega_i \alpha_i e^{-\beta_i f_i} \\
\text{with respect to} \quad & f_1, f_2, \ldots, f_n \\
\text{subject to} \quad & \sum_{i=1}^{n} C_i f_i \leq A, \quad 0 < A \leq 1 \\
& f_i \geq f_{mi}, \quad i = 1, \ldots, n
\end{aligned} \tag{20}
$$

Solution:

It statically sets several frequencies $\geq f_{mi}$ and the rest $= f_{mi}$

Representative examples (periods)

[16] Optimal state feedback based resource allocation for resource-constrained control tasks. Key observation:



Representative examples (periods)

[16] Optimal state feedback based resource allocation for resource-constrained control tasks

- Set of n control tasks sharing a CPU
- A minimum resource share is guaranteed per task: $r_{i,min} = \frac{c_i}{h_{i,max}}$
- Performance index for each control task (benefit): $\alpha_i r_i + \beta_i$
- Instantaneous feedback: $e_i = |x_k|$

Representative examples (periods)

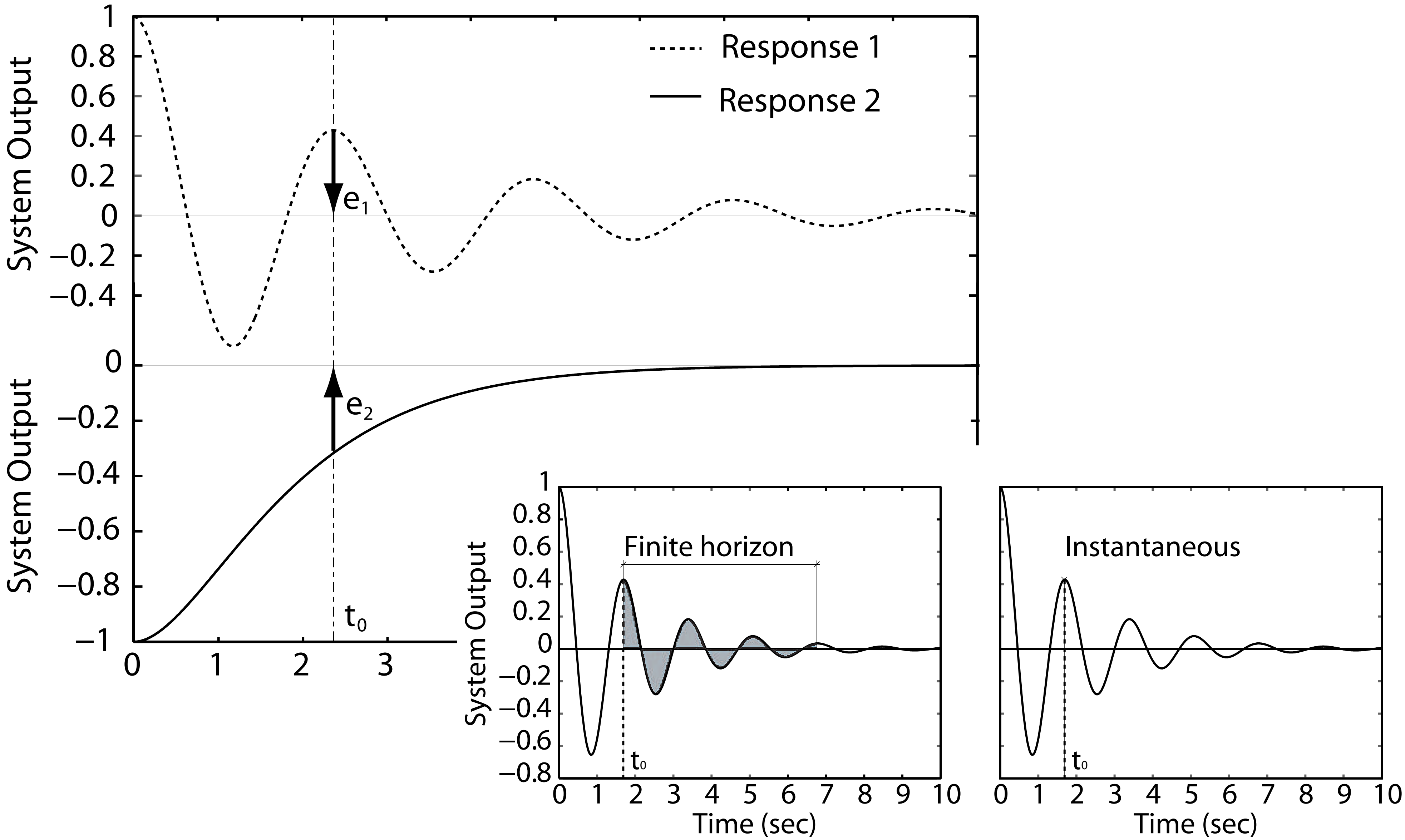
[16] Optimal state feedback based resource allocation for resource-constrained control tasks

$$
\begin{aligned}
\text{max} \quad & \sum_{i=1}^{n} \omega_i p_i(r_i) e_i = \sum_{i=1}^{n} \omega_i (\alpha_i r_i + \beta_i) |x_k| \\
\text{with respect to} \quad & r_1, r_2, \ldots, r_n \\
\text{subject to} \quad & \sum_{i=1}^{n} \Delta r_i \leq U_s(t) \quad \text{and} \quad \Delta r_i \geq 0, \quad i = 1, \ldots, n \\
\text{where} \quad & r_i = r_{i,min} + \Delta r_i \text{ and } U_s(t) = \text{available\_slack}(t)
\end{aligned} \tag{21}
$$

Solution: Assign all slack to the task whose plant has the largest error, where slack is the unused and thus available resources

Drawback: Instantaneous feedback, e.g. $|x_k|$, may be not helpful in certain scenarios

Representative examples (periods)



Representative examples (periods)

[18] Optimal on-line sampling period assignment for real-time control tasks based on plant state information

- Set of n control tasks sharing a CPU
- Performance index for each control task based on a finite horizon prediction (cost):

$$J(x_0, h, T_{fbs}) = x_0^T S x_0 + T_{fbs} \bar{J} \tag{22}$$

where

- $\bar{J} = \frac{1}{h}\big(\mathrm{tr}\ S(h) R_1(h) + J_v(h)\big)$ is the stationary cost per time unit
- $x_0^T S x_0$ is the transient cost, where S is the solution to:

Representative examples (periods)

[18] Optimal on-line sampling period assignment for real-time control tasks based on plant state information

- the algebraic Riccati equation (23) for optimal controllers providing the optimal cost (24) for a standard quadratic cost function (25)

$$S = \Phi^T S \Phi + Q_1 - (\Phi^T S \Gamma + Q_{12})(\Gamma^T S \Gamma + Q_2)^{-1}(\Gamma^T S \Phi + Q_{12}^T) \tag{23}$$

$$J = x_0^T S x_0 + \sum_{k=0}^{N-1} \Big(\mathrm{tr}\ S(h) R_1(h) + J_v(h) \Big) \tag{24}$$

$$J = \mathrm{E}_v \Big\{ \sum_{k=0}^{N-1} \Big(x(kh)^T Q_1 x(kh) + 2x(kh)^T Q_{12} u(kh) + u(kh)^T Q_2 u(kh) + J_v(h) \Big) \Big\} \tag{25}$$

- the Lyapunov equation (26) for an arbitrary state feedback control law $u(kh) = -Kx(kh)$, to be evaluated in (24)

$$S = (\Phi - \Gamma L)^T S (\Phi - \Gamma L) + Q_1 - Q_{12} L - L^T Q_{12}^T + L^T Q_2 L \tag{26}$$

Note that Φ, Γ, Q_1, Q_{12}, Q_2, J_v, R_1, and S all depend on the sampling interval h.

Representative examples (periods)

[18] Optimal on-line sampling period assignment for real-time control tasks based on plant state information

$$
\begin{aligned}
\text{min} \quad & \sum_{i=1}^{n} J_i(x_i(t_0), h_i, T_{fbs}) \\
\text{with respect to} \quad & h_1, h_2, \ldots, h_n \\
\text{subject to} \quad & \sum_{i=1}^{n} \frac{C_i}{h_i} \leq 1 \\
& h_i \geq 0, \quad i = 1, \ldots, n
\end{aligned} \tag{27}
$$

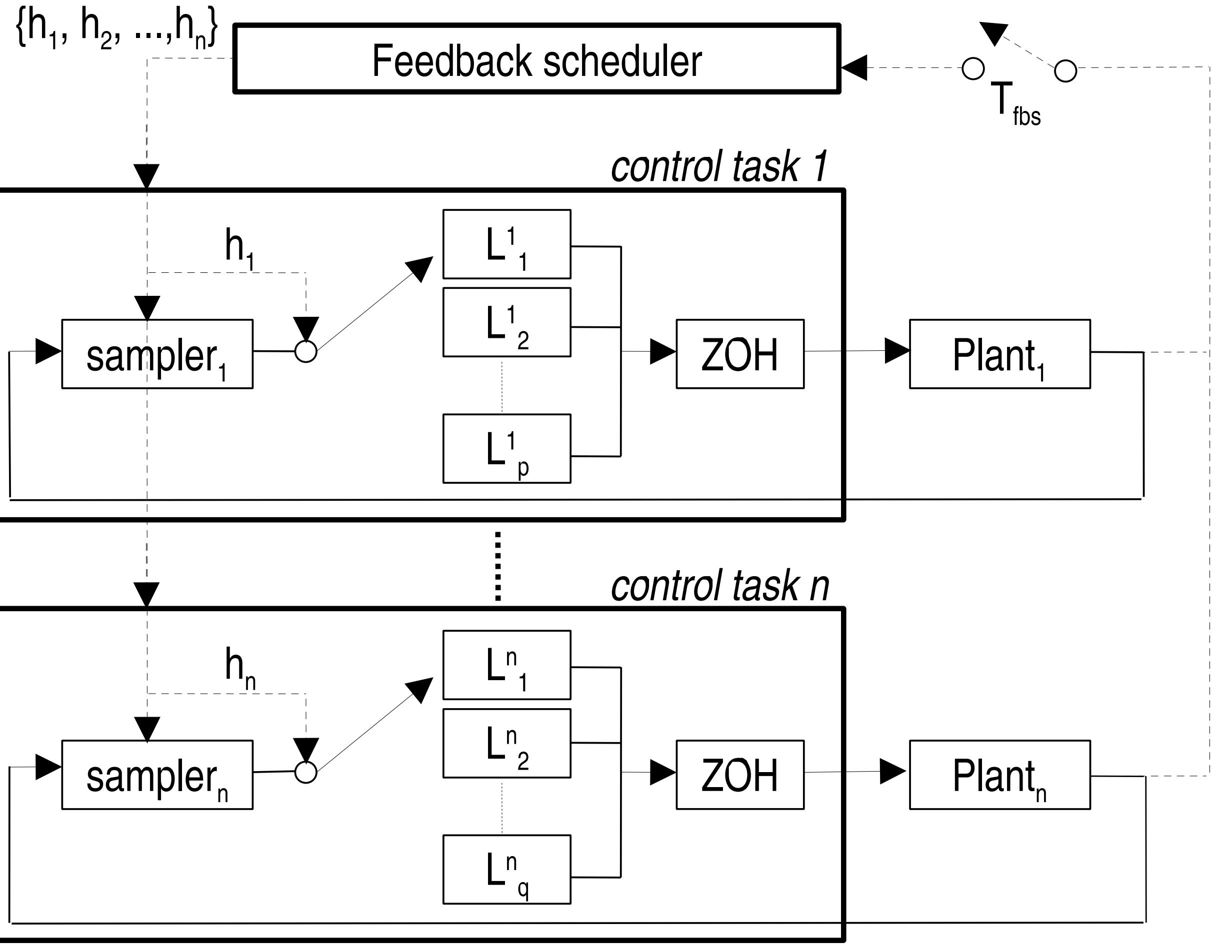
Solution:

Finding an analytical solution in the general case is not possible.

But an approximate general solution exists and works [28]

Representative examples (periods)

Operation of [16] ($T_{fbs} \rightarrow h_i$) or [18] ($T_{fbs} >> h$).



Representative examples (sequences)

[19] Integration of off-line scheduling and optimal control

- Set of n control tasks sharing a CPU
- Repeated cycle divided into p slots, cycle of length T_p
- Tasks execute within slots
- LQ controllers

$$\min_{\hat{u}_i} \quad E\left[\bar{x}_i^T \bar{S}_i \bar{x}_i(n) + \sum_{i=1}^{n} \begin{bmatrix} \bar{x}_i \\ \bar{u}_i \end{bmatrix}^T \begin{bmatrix} \bar{Q}_1 & \bar{Q}_{12} \\ \bar{Q}_{12}^T & \bar{Q}_2 \end{bmatrix}_i \begin{bmatrix} \bar{x}_i \\ \bar{u}_i \end{bmatrix}\right] \qquad (28)$$

$$\text{such that} \quad \bar{x}_i(k+1) = \bar{A}_i \bar{x}_i(k) + \bar{B}_i \bar{u}_i(k) + \bar{G}_i \bar{v}_i(k)$$

- Let s denote a scheduling sequence and S_p a set of schedules

Representative examples (sequences)

[19] Integration of off-line scheduling and optimal control

Finding the optimal schedule formulated as a combinatorial optimization problem

$$
\begin{array}{rl}
\text{min} & f^{per}(s,p) \\
\text{when} & s \in S_p \\
 & p = 1 \dots T_p
\end{array}
\qquad (29)
$$

where $f^{per}(s,p)$ is a performance measure derived from (28)

Solution: A periodic schedule $\hat{s}(t) = s_1 s_2 \dots s_p s_1 \dots,$ where $\hat{s}(t)$ indicates the controller run at time t, and a periodic linear feedback law such that $u_{\hat{s}(t)} = K_t x_{\hat{s}(t)}(t)$

Representative examples (event-based)

[23] Stable and real-time scheduling of a class of hybrid dynamic systems

- N continuous dynamic plants

$$\dot{x}_i = A_i x_i + b_i u_i, \quad i = 1, \ldots, N \tag{30}$$

- Discrete-event scheduling

$$Event(i, T_k) = \begin{cases} 1 & \text{if } \|x_i(T_k)\| = \max_{j=1,\ldots,N} \|x_j(T_k)\| \text{ at } T_k \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

During $t \in [\ T_k \quad T_k + h\ )$ *plant*$_i$ runs in closed loop (rest in open loop)

- Objective: to ensure stability

Representative examples (event-based)

[23] Stable and real-time scheduling of a class of hybrid dynamic systems

- Control design specification: to ensure asymptotical and exponential stability for all plants
- Outcome:
  - Sufficient conditions
  - Stabilizing feedback gains

Observation: similar to previous feedback-scheduling approaches but using an event-based scheduling and single feedback gains.

Representative examples (event-based)

[25] Preliminary results on state-triggered scheduling of stabilizing control tasks

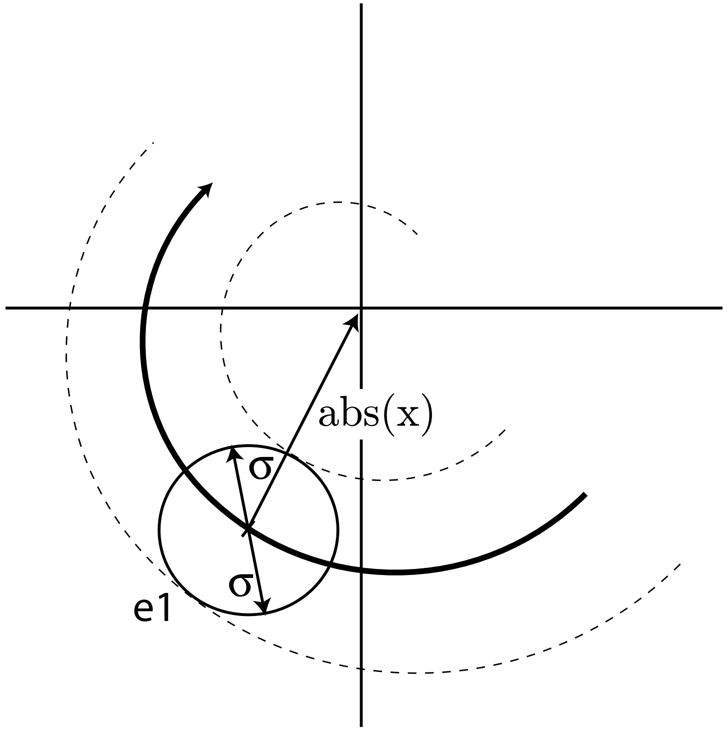
- Closed loop continuous time system with discrete controller

$$\dot{x} = f(x, k(x+e)) \quad \text{where} \quad e(t) = x(t_i) - x(t) \tag{32}$$

- Event-triggered executions:

  $|e(t)| \leq \sigma |x(t)|$

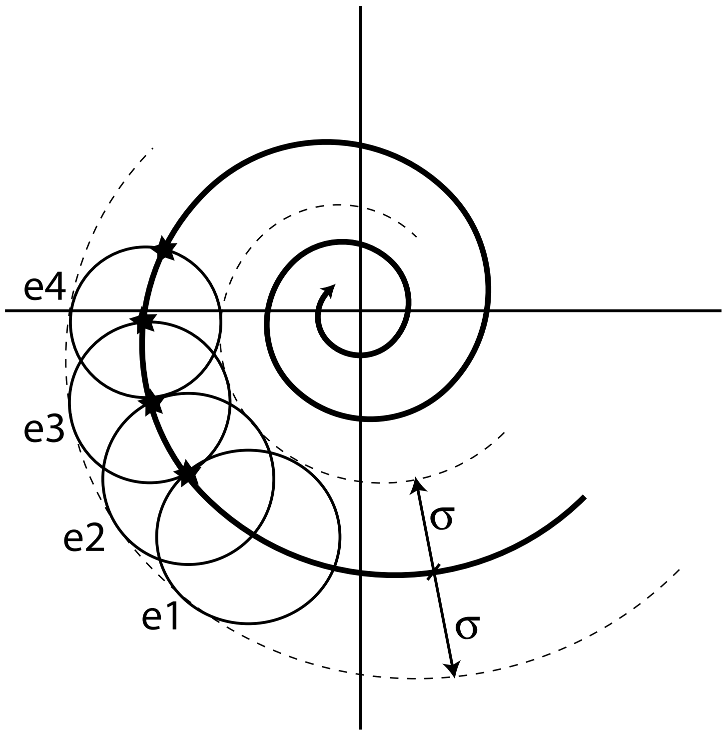
  to enforce stability



Representative examples (event-based)

[25] Preliminary results on state-triggered scheduling of stabilizing control tasks

- avoids accumulation points
- provides estimates of the time between consecutives executions



Miscellaneous

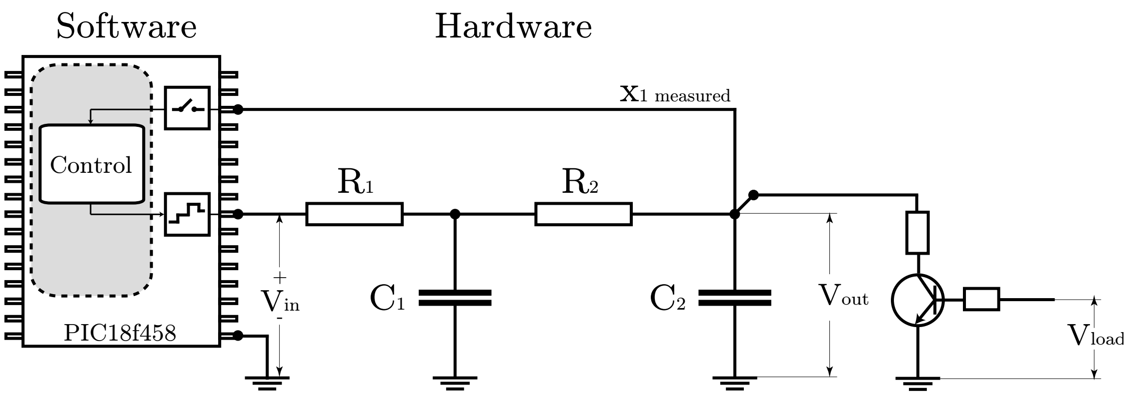
Miscellaneous - Jitters

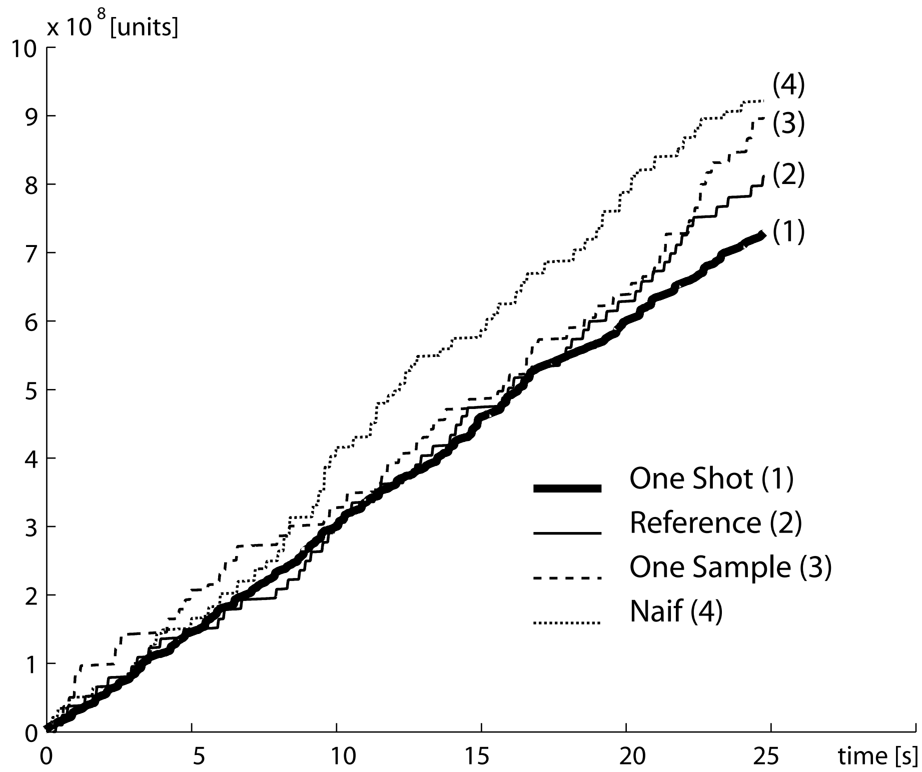
Simulation: Some feedback scheduling approaches vs. jitters. Three control tasks controlling RCRC circuits. Evaluation using a quadratic cost function.

Approach	Original	Indep. Proc.
Static approach	109.05	105.82
Off-line RM [14]	121.85	96.59
On-line FS [15]	99.92	98.74
On-line instantaneous FS [16]	90.63	64.41
On-line finite horizon FS [28]	100.61	86.99
Heuristic on-line cyclic scheduling [19]	62.43	62.48

Miscellaneous - No jitters

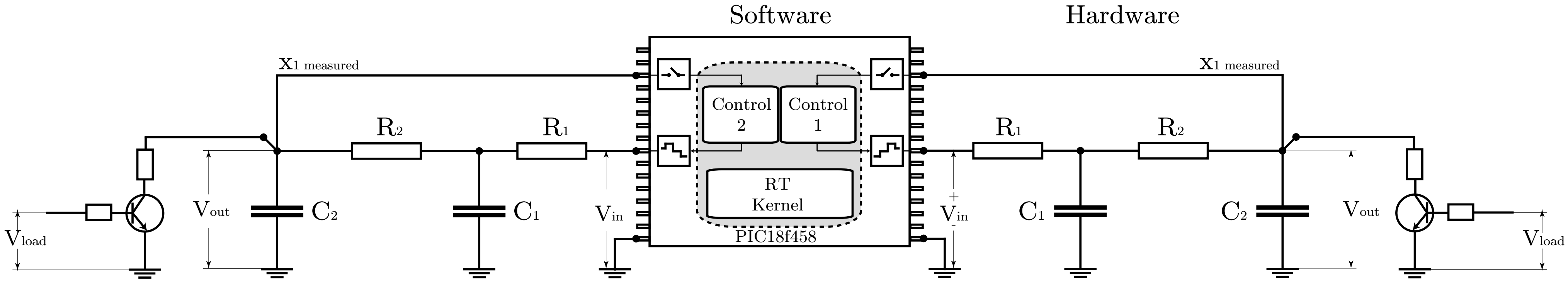
Implementation: one-shot task model.

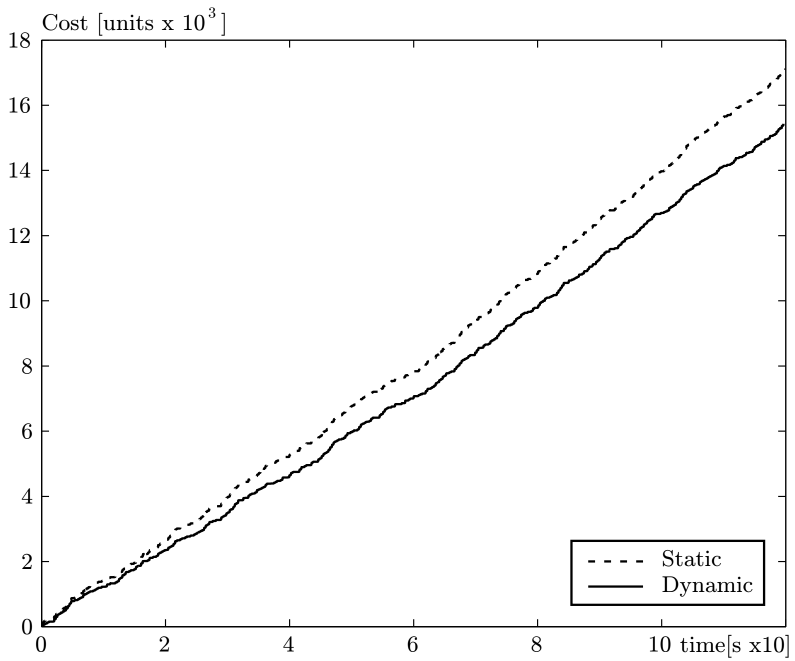


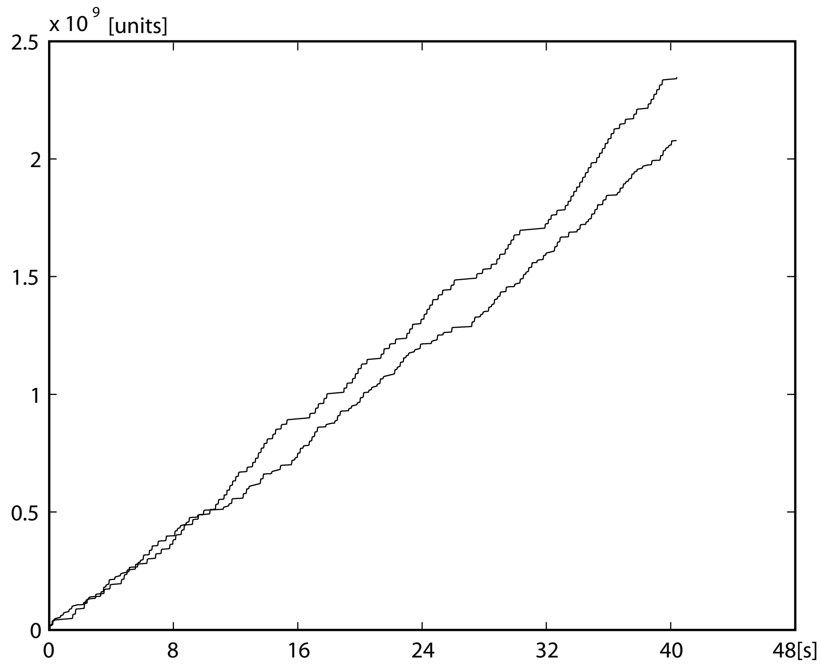


Miscellaneous - Feedback scheduling

Implementation: FS - [16] (left) and [18]+[28] (right)







Miscellaneous - Simulation tool

TrueTime (http://www.control.lth.se/truetime/)

Simulation of Networked and Embedded Control Systems

- Matlab/Simulink-based simulator for real-time control systems.
- Facilitates co-simulation of controller task execution in real-time kernels, network transmissions, battery-powered devices, and continuous plant dynamics.

Summary

- Networked and embedded control systems are everywhere
  - Resources
  - Timing
  - Dynamic behavior
- Overcoming separation of concerns
  - Real-time computing of control systems
  - Control of real-time control systems

Appendix 1 (controllability)

Controllability. Is (13) controllable? Yes.

Proof. We assume that the standard system (2) is controllable

$$W_c = \det\left(\begin{bmatrix}\Gamma \; \Phi\Gamma \; \cdots \; \Phi^{n-1}\Gamma\end{bmatrix}\right), \quad \det(W_c) \neq 0 \tag{33}$$

Let us define

$$\phi_a(h_k, \tau_k) = \begin{bmatrix} \Phi(h_k) & \Phi(h_k - \tau_k)\Gamma(\tau_k) \\ 0 & 0 \end{bmatrix} \tag{34}$$

$$\Gamma_a(h_k, \tau_k) = \begin{bmatrix} \Gamma(h_k - \tau_k) \\ I \end{bmatrix} \tag{35}$$

$$x_a(k) = \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} \tag{36}$$

Appendix 1 (controllability)

Let the system state at $k = n$ be

$$x(n) = \prod_{i=1}^{n} \phi_a(h_{n-i+1}, \tau_{n-i+1}) x(0) + W_c U \tag{37}$$

with

$$\begin{aligned} W_c &= \left[\underbrace{\Gamma_a(h_n, \tau_n)}_{\text{for } j=n} \; \ldots \; \underbrace{\left(\prod_{i=1}^{n-1} \phi_a(h_{n-i+1}, \tau_{n-i+1}) \right) \Gamma_a(h_1, \tau_1)}_{\text{for } j=1} \right] \\ U &= \left[u^T(n-1) \; \ldots \; u^T(0) \right]^T \end{aligned} \tag{38}$$

Appendix 1 (controllability)

Subtituting (34) and (35) into (38) we obtain

$$W_c = \begin{bmatrix} \overbrace{\Gamma_0(h_n \tau_n)}^{j=n} & \cdots & \overbrace{\left(\prod_{i=1}^{n-2} \Phi(h_{n-i+1})\right)\Gamma_1(h_2, \tau_2) + \left(\prod_{i=1}^{n-1} \Phi(h_{n-i+1})\right)\Gamma_0(h_1, \tau_1)}^{j=1} \\ I & \cdots & 0 \end{bmatrix} \tag{39}$$

For MIMO systems, (13) is controllable if $\det(W_c) \neq 0$. Developing the determinant from the last row, and setting $\tau_k = 0$ and $h_k = h$, we obtain condition (33)

Appendix 1 (controllability)

$\det(W_c)$ is a continuous function of a continuous variable

$$\begin{array}{rcl} det(W_c)\ :\ \mathbf{R}^{nxn} & \rightarrow & \mathbf{R} \\ (h_1, h_2, \ldots, h_n, \tau_1, \tau_2, \ldots, \tau_n) & \rightarrow & det(W_c\left[h_1, h_2, \ldots, h_n, \tau_1, \tau_2, \ldots, \tau_n\right]) \end{array}$$

If the original system (2) is controllable, then

$$\exists (h_1, h_2, \ldots, h_n, \tau_1, \tau_2, \ldots, \tau_n) \quad | \quad det(W_c) \neq 0$$

And due to continuity

$$\exists B((h_1, h_2, \ldots, h_n, \tau_1, \tau_2, \ldots, \tau_n), \delta) \quad | \quad det(W_c) \neq 0$$

$\square$

Appendix 2 (observability)

Observability. Is (13) observable? Yes, if the output matrix outputs the additional variable.

Proof. We assume that the standard system (2) is observable

$$W_o = \det\left(\begin{bmatrix} C \\ C\Phi(h) \\ \vdots \\ C\Phi^{n-1}(h) \end{bmatrix}\right), \quad \det(W_o) \neq 0 \tag{40}$$

and we use definitions (34), (35) and (36), and we set as output matrix

$$C_a = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \tag{41}$$

Appendix 2 (observability)

Without losing generality, if $u_k = 0$, the initial state can be observed in n steps, being n the order of (13)

$$\begin{aligned} y_a(0) &= C_a x_a(0) \\ y_a(1) &= C_a x_a(1) = C\phi_a(h_1, \tau_1) x_a(0) \\ &\vdots \\ y_a(n) &= C_a \prod_{i=1}^{n-1} \phi_a(h_{n-i+1}, \tau_{n-i+1}) x_a(0) \end{aligned} \tag{42}$$

From (42), the obserbability matrix is

$$W_o = \begin{bmatrix} C_a \\ \vdots \\ C_a \prod_{i=1}^{n-1} \phi_a(h_{n-i+1}, \tau_{n-i+1}) \end{bmatrix} \tag{43}$$

Appendix 2 (observability)

Subtituting (41) and (34) into (43) we obtain

$$W_o = \begin{bmatrix} C & 0 \\ 0 & I \\ \vdots & \vdots \\ C\prod_{i=1}^{n-1}\Phi(h_{n-i+1}) & C\prod_{i=1}^{n-2}\Phi(h_{n-i+1})\Phi(h_1-\tau_1)\Gamma(\tau_1) \\ 0 & 0 \end{bmatrix} \quad (44)$$

For MIMO systems, $W_o \in \mathbb{R}^{2n\times n}$. Therefore, we can construct W_o^* with n rows of W_o. Then, (13) is observable if $\det(W_o^*) \neq 0$.

Appendix 2 (observability)

For W_o^* we pick rows $2, 3, 5, 7, \ldots, n-1$ of W_o

$$W_o^* = \begin{bmatrix} 0 & I \\ C\Phi(h_1) & C\Phi(h_1 - \tau_1)\Gamma(\tau_1) \\ \vdots & \vdots \\ C\prod_{i=1}^{n-1}\Phi(h_{n-i+1}) & C\prod_{i=1}^{n-2}\Phi(h_{n-i+1})\Phi(h_1 - \tau_1)\Gamma(\tau_1) \end{bmatrix} \quad (45)$$

With constant period and $\tau = 0$ we obtain

$$W_o^* = \begin{bmatrix} 0 & I \\ C\Phi & 0 \\ \vdots & \vdots \\ C\Phi^{n-1} & 0 \end{bmatrix} \quad (46)$$

Appendix 2 (observability)

Developing the determinant of (46) by the first row

$$\det(W_o^*) = \pm\det\left(\begin{bmatrix} C\Phi \\ \vdots \\ C\Phi^{n-1} \end{bmatrix}\right) = \pm\det\left(\begin{bmatrix} C \\ \vdots \\ C\Phi^{n-2} \end{bmatrix}\right)\det(\Phi) \quad (47)$$

Note: $det(\Phi) \neq 0$ and recall (40) $\Rightarrow \det(W_o^*) \neq 0$.

As before, $\det(W_o)$ is a continuous function of a continuous variable. If the original system (2) is controllable, then

$$\exists(h_1, h_2, \ldots, h_n, \tau_1, \tau_2, \ldots, \tau_n) \quad | \quad det(W_o^*) \neq 0$$

And due to continuity

$$\exists B((h_1, h_2, \ldots, h_n, \tau_1, \tau_2, \ldots, \tau_n), \delta) \quad | \quad det(W_o^*) \neq 0$$

□

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