Hybrid Control and Communication

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Outline

• Lecture I: Control under communication constraints
  – Motivating applications
  – Communication constraints
  – Compensation for delay and loss
  – Integrated design of control and communication
  – References

• Lecture II: Hybrid control of communication systems
  – Packet-switched networks
  – Hybrid model of congestion control
  – References

• [Lecture III (by L. Palopoli): Stabilization of quantized systems]
Lecture I: Control under communication constraints

- Classical control theory are based on perfect exchange of information
- Modern control systems are often networked
  + added flexibility
  + cheaper implementation
- Sensor and actuator data are then transmitted over a shared network resource
  - added uncertainty
  - higher complexity
Motivating applications

- Scania truck
- Volvo XC90
- SMART-1 spacecraft
- Power control in wireless system
- Congestion control in communication network
Networked control architecture of a Scania truck

- Control units connected through 3 controller area networks (CANs) coloured by criticality
- CAN is a standard introduced by Bosch 1986
Networked control architecture of a Volvo XC90

- 3 CAN networks connect up to 40 control units
- Example of control system using CAN is vehicle dynamics control (electronic stability program)

Powertrain and chassis
- TCM Transmission control module
- ECM Engine control module
- BCM Brake control module
- BSC Body sensor cluster
- SAS Steering angle sensor
- SUM Suspension module
- AUD Audio module

Infotainment/Telematics
- MP1.2 Media players 1 and 2
- PHM Phone module
- MMM Multimedia module
- SUB Subwoofer
- ATM Antenna tuner module

Body electronics
- CEM Central electronic module
- SWM Steering wheel module
- DDM Driver door module
- REM Rear electronic module
- PDM Passenger door module
- ICM Infotainment control
- UEM Upper electronic module
- DIM Driver information module
- AEM Auxiliary electronic
Networked control architecture of the SMART-1 spacecraft

- First European lunar mission, launched Sep 2003
- CAN networks for control system ("system") and for scientific experiments ("payload")
- Node and communication redundancies
Distributed power control in cellular systems

- Power control in each mobile station tries to keep signal-to-interference ratio (SIR) at a threshold value
- Disturbances from channel fluctuations and interfering traffic
- Control in mobile station based on quantized estimate of SIR communicated from base station
Congestion control in packet-switched data communication network

• Each sender regulates sending rate based on congestion information from receiver
• Variations in available bandwidth and traffic load
• Congestion indicated implicitly through missing acknowledgement packets
• Quantized control command

[Discussed in detail next lecture]
Examples of networked control architectures
What is hybrid in networked control?

- Networked control systems are inherently hybrid, not only because interaction of physical plant and computer control, but also because they have
  - mixture of event- and time-triggered communication protocols
  - asynchronous network nodes (no global clock)
  - quantized sensor data to limit network traffic
  - symbolic control commands to simplify design and operation

- Now on we mainly discuss modelling and compensating some communication constraints, cf., Mitter’s lecture
Control with constrained communication

- Limitations in the communication of sensor and actuator data impose constraints on the control system.

- Communication imperfections include:
  - Delay and jitter
  - Quantization [Lecture by Palopoli]
  - Packet loss
  - Bit error
  - Outage (lost connection)
Time delays

- Delays in communication due to buffering and propagation delays
- Delays are bad for control loops (avoid if possible)
- Delays can be fixed or varying, known (measurable) or unknown
- Data loss can be interpreted as infinity delay
**Theorem:** LTI feedback control system with phase margin $\varphi_m$ at cross-over frequency $\omega_c$ is stable, if communication adds fixed time delay

$$\tau < \varphi_m/\omega_c$$

**Proof:** Nyquist Criterion
Remark: Closed-loop gain needs to be sufficiently small. Easy to check through Bode plot.
Proof

Redraw block diagram:

Small Gain Theorem: \( \gamma(S_1)\gamma(S_2) < 1 \Rightarrow \text{stability} \)
gives the result with

\[
S_1(s) = \frac{sp(s)c(s)}{1+p(s)c(s)}, \quad S_2(s) = \frac{e^{-st} - 1}{s},
\]

\[
\gamma(S_1) = \sup_{\omega \in \mathbb{R}} \left| \frac{\omega p(i\omega)c(i\omega)}{1+p(i\omega)c(i\omega)} \right|, \quad \gamma(S_2) \leq \tau_{\text{max}}
\]
Relation to Nyquist Criterion

At $\omega = \omega_c$, we have

$$\left| \frac{P(i\omega)C(i\omega)}{1 + P(i\omega)C(i\omega)} \right| = \frac{1}{\left| 1 - e^{i\phi_m} \right|} \approx \frac{1}{\phi_m}$$

Lincoln inequality imposes

$$\frac{1}{\phi_m} < \frac{1}{\tau_{\text{max}} \omega_c}$$

Corresponds to the Nyquist Criterion

for fixed $\tau(t) = \tau_{\text{max}}$

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Unknown and varying delay jitter

Jitter \( \delta \tau \) is delay variation:

\[
\delta \tau = \tau_{\text{max}} - \tau_{\text{min}}
\]

**Theorem [Lincoln]:** LTI feedback control system with time-varying delay

\[
0 \leq \bar{\tau} - \frac{\delta \tau}{2} \leq \tau(t) \leq \bar{\tau} + \frac{\delta \tau}{2} \leq h
\]

is stable, if

\[
\left| \frac{P(i\omega)C(i\omega)}{1 + P(i\omega)C(i\omega)e^{-i\omega \bar{\tau}}} \right| < \frac{\sqrt{2}}{\delta \tau \omega}
\]
Known time delays can be compensated by **Smith predictor**

Closed-loop system with $\hat{P} = P$ and $\hat{\tau} = \tau$

$$y = \frac{PG}{1 + PG} e^{-s\tau} r$$

Design controller as if there were no time delay and then implement structure above
Time delay estimation

Delays can be estimated from time stamped data \((y(t), t)\) if nodes are synchronized.

- Major improvement in control performance if delays are known/measurable (or can be accurately estimated)

Example

- CAN protocol (discussed earlier) is event-triggered and does not give timing guarantees in general
- TTCAN (Time-triggered communication on CAN) is an extension to the CAN standard targeting the need for sampled-data control
Compensating known delays: State feedback controller

Plant with state output $P(s) = (sI - A)^{-1}B$

Sensor node sends $x(kh)$ at $t = kh$

Control node receives $x(kh)$ at $t = kh + \tau_k$
where $\tau_k = \tau(kh) < h$

In control node, compute state feedback

$$u(kh + \tau_k) = -L\bar{x}(kh + \tau_k)$$

$$\bar{x}(kh + \tau_k) = e^{A\tau_k}x(kh) + \int_{kh}^{kh+\tau_k} e^{A(kh+\tau_k-s)}Bu(s)\,ds$$
Compensating known delays:

**Output feedback controller**

Plant with $P(s) = C(sI - A)^{-1}B$

Sensor node sends $y(kh)$ at $t = kh$

Control node receives $y(kh)$ at $t = kh + \tau_k$

where $\tau_k = \tau(kh) < h$

In control node, compute estimate and feedback in the following order:

\[
\begin{align*}
\bar{x}(kh) &= \hat{x}(kh) + K[y(kh) - C\hat{x}(kh)] \\
\bar{x}(kh + \tau_k) &= e^{A\tau_k} \bar{x}(kh) + \int_{kh}^{kh+\tau_k} e^{A(kh+\tau_k-s)} Bu(s) ds \\
u(kh + \tau_k) &= -L\bar{x}(kh + \tau_k) \\
\hat{x}(kh + h) &= e^{A(h-\tau_k)} \bar{x}(kh + \tau_k) + \int_{kh+\tau_k}^{kh+h} e^{A(kh+h-s)} Bu(s) ds
\end{align*}
\]
Large delays and out-of-order delivery

Large known delays can be treated as before by extending the estimator state (one dim per extra sampling period delay)

- Buffers can handle out-of-order delivery, but may also increase delays
- Don’t wait for late data, but when they arrive use them to adjust old estimates
Packet loss

Channel between sensor node and control node modelled through stochastic binary variable $d(k)$:

$$d(k) = \begin{cases} 
1, & \text{lost packet } k \\
0, & \text{no loss}
\end{cases}$$

Modify traditional observer; simplest case:

$$\hat{x}(k+1) = \Phi \hat{x}(k) + \Gamma u(k) + \begin{cases} 
K[y(k) - C \hat{x}(k)], & d(k) = 0 \\
0, & d(k) = 1
\end{cases}$$

- Can be hard to handle packet loss: when decide that $d(k) = 1$?
  - E.g., TCP uses TimeOut variable to decide when a packet is lost
- It can be better to drop data, than use old information for control
Bit errors and outages

- **Bit errors** are due to rapid variations in the physical channel
- Unlikely in wired systems, but important in wireless systems
- Compensation through forward error correction (coding)

- **Outages** are sudden events when connection is lost
- A severe problem in most wireless systems
Integrated design of control and communication systems

- Up to here, communication has been considered as a disturbance or model imperfection affecting control
- What about jointly design control and communication, along the proposal in Mitter’s lecture?
- Let’s illustrate with an example
Example: Stabilization of networked systems

- Consider joint state feedback stabilization of a set of plants, when only one plant can utilize the bus at a time:

\[
\begin{align*}
    u_i &= K_i x_i \\
    \dot{x}_1 &= A_1 x_1 + B_1 u_1 \\
    \dot{x}_2 &= A_2 x_2 + B_2 u_2 \\
    \dot{x}_3 &= A_3 x_3 + B_3 u_3
\end{align*}
\]

Communication bus

- What control and communication policy should be adopted?
Hybrid system representation

- How choose the guard conditions of the hybrid automaton to stabilize the system?
“Largest state first”-policy

Theorem [Hristu-V. and Kumar]
For scalar unstable systems:

\[ |x_i| \to \epsilon \text{ if and only if } -\sum_{i=1}^{3} \frac{A_i}{B_i K_i} < 1. \]
Summary

- Communication and networking important in growing number of control applications
- Communication delays, quantizers, losses, errors and outages can be viewed as constraints imposed on the control system
- Can in many cases be suitably modelled as hybrid systems
- Design methodologies exist for certain classes of constraints, but much more remains to be done
- Desirable to jointly design control and communication system
  - Control algorithms need to adapt to changing network conditions
  - Communication protocols should be aware of control needs
  - But, other network applications set competing restrictions
References

• D. Hristu-Varsakelis and W. S. Levine, Ed.’s, Handbook of Networked and Embedded Control Systems, Birkhäuser, 2005
• 2E1245 Hybrid and Embedded Control Systems, http://www.s3.kth.se/control/kurser/2E1245

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Objective is to
• Give each user suitable service
• Utilize network resources efficiently

Obtained through two control mechanisms:

**Spatial control**
• route traffic short way through the network
• receiver address in header of each packet
• shortest-distance matrix in each router
• updated on a slow time scale

**Temporal control**
• adjust sending rate to available bandwidth
• base on info available in sender (end-to-end)
• implicit bandwidth estimate through ack’s
• updated on a faster time scale
Temporal congestion control

- Each network link has a limited capacity
- Variations in traffic is primarily handled by temporary storage in router buffers
- If a buffer gets full, it simply throws away incoming packets
- Hence, congestion leads to lost packets
- Acknowledgements (ack’s) indicate sender, who can take action
Transmission control protocol (TCP)

- TCP implements a congestion controller that regulates the sending rate.
- Control variable is the congestion window \( w \), which represents the number of outstanding (not-yet-acknowledged) packets.
- Control is based on implicit feedback information from ack’s.
- TCP follows additive increase multiplicative decrease (AIMD) strategy.
TCP congestion avoidance

Window $w$ is updated each round-trip time $RTT$

If no drops occur, then $w := w + 1$

If drop occurs, then $w := w/2$

Hybrid system is obtained by interpreting $w$ as a continuous-time real variable:

$$\dot{w} = \frac{1}{RTT}$$

$$w := \frac{w}{2}$$

Typical windows evolution:
Hybrid dynamics of a queue

Queue

Empty

Queue

Active

Queue

Full

\[ \dot{q} = 0 \]

\[ r \geq B \]

\[ q = 0 \]

\[ \dot{q} = r - B \]

\[ r \leq B, z := 0 \]

\[ \dot{z} = r - B \]

\[ z = 1 \]

\[ z := 0 \] drop

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Hybrid model of TCP over single link

\[ w = \frac{1}{RTT} \]

\[ w := \frac{w}{2} \]

\[ r = \frac{w}{RTT} \]

\[ RTT = T_{prop} + \frac{q}{B} \]

\[ \dot{q} = 0 \quad r \geq B \]

\[ \dot{q} = r - B \quad q = q_{\text{max}} \]

\[ \dot{q} = 0 \quad r \leq B, z := 0 \quad z = 1 \]

\[ z := 0 \quad \text{drop} \]

Sender \( r(t) \)

\( q(t) \)

B(t)

Receiver
The discrete states of TCP

- Congestion avoidance is the main state
- Timeout handles severe congestion
- Slow-Start gives faster growth initially

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Hybrid model of TCP

\[
ssth_r f := \infty, w_f := 1, \quad ssth_r f := \frac{w_f}{2}, w_f := 1, \quad t_{tim} < 0? \]

**slow-start:**
\[
\dot{w}_f = \frac{\log m}{RTT_f} w_f, \quad r_f = \frac{\beta w_f}{RTT_f}
\]

\(w_f > ssth_r f?\)
\[
t_{tim} := 1 \text{sec}
\]

**timeout:**
\[
\dot{t}_{tim} = -1, \quad r_f = 0
\]

---

**slow-start delay:**
\[
\dot{w}_f = \frac{\log m}{RTT_f} w_f, \quad t_{tim} = -1, \quad \dot{r}_f = \frac{\beta w_f}{RTT_f}
\]

\(t_{tim} < 0?\)
\[
t_{tim} := DDD_f^\ell
\]

---

**fast-recovery:**
\[
\dot{w}_f = 0, \quad \dot{t}_{tim} = -1, \quad \dot{r}_f = 0
\]

\(t_{tim} < 0, k > 0?\)
\[
t_{tim} := RTT_f, \quad k := k - 1, \quad r_f := 2r_f
\]

\(t_{tim} < 0, k \leq 0?\)

---

**cong.-avoidance delay:**
\[
\dot{w}_f = \frac{L}{RTT_f}, \quad \dot{t}_{tim} = -1, \quad \dot{r}_f = \frac{w_f}{RTT_f}
\]

\(t_{tim} < 0?\)
\[
t_{tim} := DDD_f^\ell
\]

---

**cong.-avoidance:**
\[
\dot{w}_f = \frac{L}{RTT_f}, \quad \dot{t}_{tim} = -1, \quad \dot{r}_f = \frac{w_f}{RTT_f}
\]

**\(E[n_{drop \ drops}]?\)**

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Alternative models of network traffic: Packet models and fluid models

**Packet models**
- Model each individual packet (event-driven)
- Accurate but computationally heavy

**Fluid models**
- Averaged fluid quantities (time-driven)
- Capture only steady-state and slow behaviors

- Hybrid model combines features of these traditional network models

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TCP over wireless links

Integration of Internet and cellular networks hard due to radio link variations

When used over wireless links, TCP cannot ensure a high link utilization

Packet drops, bandwidth and delay variations in radio link erroneously indicate network congestion to TCP

How do radio links affect TCP throughput?

Can we make the radio link and the cellular system “TCP friendly”?

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A hybrid cascade control problem

- Radio link transforms losses into random delays
- Key dynamics from cascaded feedback control loops
  - Inner and outer power controls
  - Link-layer retransmission
  - TCP
- Increased probability of spurious timeout gives reduced TCP throughput
- Adjust link layer properties to optimize TCP throughput

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New feedback protocols for wireless Internet

- Improved TCP throughput through new radio network feedback protocol
- Proxy between cellular system and Internet adapt sending rate to radio bandwidth variations obtained from radio network controller (RNC)
New feedback protocols for wireless Internet

- Hybrid controller in proxy regulates sending rate based on
  - Events generated by radio bandwidth changes obtained from RNC
  - Sampled measurements of queue length in RNC
- Improved time-to-serve-user and utilization compared to traditional end-to-end TCP
Challenges in network traffic control

- **Deployment**
  - Implementation in end computers

- **Distributed control**
  - Communication constraints
  - Implicit state information

- **Complex interacting dynamics**
  - Network, protocol and user dynamics
  - Large and varying time delays
  - Wired and wireless links
  - Packet loss

- **No clear optimality objective**
  - Network throughput
  - User throughput
  - Response time
  - Fairness
Summary

• Hybrid model of congestion control in packet-switched networks
• Combine event-driven packet models with time-driven fluid models
• Accurate on time-scale of the round-trip time
• Enables analysis and efficient simulations of congestion control

• Interactions between wireless links and TCP lead to performance loss
• Hybrid controller gives improved user experience and network utilization
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