Hybrid Command Governors for Idle Speed Control in Gasoline Direct Injection Engines

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Abstract: The design of an idle speed controller for automotive GDI engines is considered. A hybrid model of a GDI engine operating in stratified mode is presented. The idle speed control problem is formalized as a constrained optimal control problem where fuel consumption has to be minimized. A sub-optimal but effective and easily implementable solution is obtained by resorting to the Command Governor methodology for a discrete-time abstraction of the hybrid model. Simulation results of the hybrid closed-loop system are presented.

Keywords: Engine control, hybrid systems, command governors, optimal control.

1 Introduction

The main targets of the design of 4-stroke gasoline engines for passenger cars are: improvement of safety, driveability and comfort, minimization of fuel consumption and compliance with the emissions standards. Besides the direct economic benefit for customers, reduction of fuel consumption results in reduction of the combustion product $CO_2$, which is a critical issue due to the well known effects of $CO_2$ on global environmental warming. High fuel economy, as well as high driving performances, can be achieved by modern Gasoline Direct Injection (GDI) engines (see [1] for an extensive description). Direct injection is characterized by: 1) low pumping and heat losses, which increase thermal efficiency, 2) low temperature of charge air, producing high volumetric efficiency and anti-knock characteristics and 3) high response and superior transient driveability (due to direct fuel injection into the cylinder).

Traditional no-GDI gasoline engines operate around stoichiometric air/fuel ratio for any value of load and engine speed. As a consequence, for such engines fuel consumption is typically high in the part-load range where the actual requested mechanical power is low. In GDI engines, the possibility of using lean mixtures (i.e. high air/fuel ratio) in the stratified mode allows one to reduce fuel consumption by 20-25% at low loads. In a typical driving cycle, the most significant reduction of fuel consumption can be obtained when the engine is in the idle operation mode ([2, 3, 4, 5, 6]), which is activated when the gear is set to neutral and the gas pedal is released. Naturally enough, in idle speed control problems the main objective is the minimization of fuel consumption while preventing the engine to stall. The difficulty of the problem lies in the load variations coming from the possibly intermittent use of devices powered by the engine, such as the air conditioning system and the steering wheel servo-mechanism, in addition to other permanent time-varying torque losses.
A survey on different engine models and control design methodologies for idle control is given in [3]. Both time–domain (e.g. [4]) and crank–angle domain (e.g. [5]) models have been proposed in the literature to solve the idle management problem. These models deal with the average value of engine speed. More recently, cycle-accurate models, which describe periodic speed variations due to torque fluctuations, have been investigated [7]. Several design techniques have been applied to the idle control problem, such as multi-variable control [8], \( \ell_1 \) control [4], \( H_\infty \) control [9], \( \mu \)-synthesis [10], sliding mode control [11] and LQ-based optimization [12]. More recently, hybrid control strategies have been proposed in [2] and [13] where the idle control problem was solved by computing analytically the set of all hybrid states for which there exists a hybrid control strategy meeting the specifications. The class of all “safe” controllers obtained by this procedure is referred to as “maximal controller” and the implementation of a particular controller extracted from the maximal controller, e.g. the one which minimizes fuel consumption, is quite expensive in terms of computing time and memory.

Here, a simpler effective solution is presented by resorting to the Command Governor (CG) approach. A CG is a nonlinear device which is added to a pre-compensated control system. The latter, in the absence of the CG, is designed so as to perform satisfactorily in the absence of constraint violations. Whenever necessary, the CG modifies the reference to the closed-loop system so as to avoid violation of constraints. The basic idea is that of maintaining the closed-loop system close to its nominal (linear) regime, where all nominal closed-loop properties are preserved. It is worth pointing out that a system equipped with a CG takes a special simplified structure at the cost typically of performance degradation with respect to more general approaches, e.g. direct hybrid control approach or constrained predictive control. CG usage can be however justified in industrial applications wherein a massive amount of flops per sampling time is not allowed, and/or one is typically only commissioned to add to existing standard PID-like compensators peripheral units which, as CG’s, do not change the primal compensated control system. Specific merits of the CG approach in dealing with constraints are that it can handle absolute and incremental constraints on input and state-related variables of the plant and that the numerical burdens of the on-line computation can be modulated according to the available computing power, ranging from solving on-line convex multidimensional optimization problems to consulting look-up tables. Studies along these lines are appeared e.g. in [14]-[19]. For a specific application to the idle control problem see [18].

The paper is organized as follows: in Section 2) we present a hybrid model for a GDI engine operating in stratified mode that describes the behavior of the engine in the idle speed operation mode. The adoption of a hybrid formalism allows us to describe the cyclic behavior of the engine, thus capturing the effect of each fuel ignition on the generated torque and the interaction between the discrete torque generation and the continuous power-train and air dynamics. In Section 3) we formalize the idle speed control problem as a fuel consumption minimization problem subject to constraints on engine speed and air-to-fuel ratio. In Sections 4)-5) an implementable solution is obtained via the use of a hybrid CG device which essentially, on the basis of the closed-loop system state, changes the nominal set-point (the engine speed and the air-to-fuel ratio) in order to impose at each time instant the lowest possible engine speed compatible with the fulfillment of all prescribed constraints. Simulation results of the closed-loop hybrid system model are reported in Section 6) and some conclusions end the paper.

2 Hybrid model of a Gasoline Direct Injection (GDI) engine

In this section a nonlinear hybrid model of a 4–stroke 4–cylinder GDI engine is presented. The proposed model has been developed and identified in collaboration with Magneti Marelli Powertrain (Italy) on the basis of the experimental data obtained from a 2–liter GDI engine. Extensive simulations of the engine model have been performed in Matlab/Simulink\textsuperscript{TM}.

We report here the description of the model for the stratified charge. The control inputs are: the command to the throttle valve, referred to as \( \alpha \) - which is used to control the amount of air loaded by the cylinders during the intake stroke-, and the mass of fuel injected in each engine cycle, referred to as \( q_b \). Since we assume to operate in stratified charge, spark ignition is synchronized with fuel injection and cannot be used as a control input.

The GDI engine hybrid model is composed of four interacting subsystems: the throttle valve, the intake manifold, the cylinders and the crankshaft. See Figure 1.
The dynamic of the throttle valve is modelled by a first-order lag with input delay:

\[ \dot{\alpha}_e(t) = \frac{1}{\tau_\alpha} \alpha_e(t) + \frac{1}{\tau_\alpha} \alpha(t - d_\alpha) \]  

(1)

where \( \alpha \) and \( \alpha_e \) denote, respectively, the throttle command and the throttle angle, and \( d_\alpha \) models the actuator delay.

The intake manifold dynamics is described in terms of the manifold pressure \( p \) as follows:

\[ \dot{p}(t) = K_{\text{gas}} (F_{\text{far}}(\alpha_e(t)) - F_{\text{cil}}(p(t), n(t))) \]  

(2)

In (2) the evolution of the manifold pressure depends on the difference between the input air–flow into the manifold \( F_{\text{far}} \) and output air–flow \( F_{\text{cil}} \) (the latter being a function of the manifold pressure \( p \) and the crankshaft speed \( n \)).

The end of a stroke and the beginning of the subsequent one in the four–stroke engine cycle\(^1\) is captured by dead center events produced when the pistons reach either the top or bottom position. We denote by \( k \) the sequence of dead center events and by \( t_k \) the sequence of times at which they occur. The amount of air \( q_a \) loaded by a cylinder during the \( k \)-th intake stroke is obtained by integrating the air–flow \( F_{\text{cil}} \) between two dead centers, i.e.

\[ q_a(k) = \int_{t_{k-1}}^{t_k} F_{\text{cil}}(p(t), n(t)) \, dt \]  

(3)

The cylinders hybrid model describes the generation of the engine torque. The engine torque \( T_{\text{eng}}(t) \) is modeled as a piece–wise constant signal synchronized with the dead center events. For the \( k \)-th expansion stroke, the amount of the engine torque \( T_{\text{eng}}(k) \) depends in a nonlinear fashion on: the mass of injected fuel \( q_b(k) \), the normalized air–to–fuel ratio \( \lambda(k) \) of the loaded mixture, and the value of the engine speed at the beginning of the stroke \( n(t_k) \):

\[ T_{\text{eng}}(t) = T_{\text{eng}}(k) = T_{\text{eng}}(q_b(k), \lambda(k), n(t_k)) \quad \text{for} \quad t \in [t_k, t_{k+1}). \]  

(4)

The normalized air–to–fuel ratio \( \lambda(k) \) of the mixture during the \( k \)-th expansion stroke is defined has

\[ \lambda(k) = \frac{q_a(k-1)}{q_{a0}} \]  

(5)

where \( \frac{q_a}{q_{a0}} \) stands for the stoichiometric air–to–fuel ratio. Notice that in (5), the normalized air–to–fuel ratio \( \lambda(k) \) depends on the amount of air \( q_a(k-1) \) loaded in the cylinder during the previous intake stroke: the one step delay models the compression stroke which is located between intake and expansion. The timing of the engine internal variables is depicted in Figure 2.

Finally, the crankshaft block describes the evolution of the crankshaft revolution speed \( n \), whose acceleration depends on the difference between the engine torque \( T_{\text{eng}} \) and the load torque \( T_{\text{load}} \):

\[ \ddot{n}(t) = K_J (T_{\text{eng}}(t) - T_{\text{load}}(t)) \]  

(6)

\(^1\)Intake, compression, expansion and exhaust strokes.
Figure 2: Evolution of the intake air flow $F_{cil}$, loaded air $q_a$, injected fuel $q_b$, generated torque $T_{eng}(t)$.

It is worth noticing that the load torque $T_{load}$ consists essentially of three distinct amounts: the pumping torque $T_p$, the friction torque $T_a$, and the torque $T_d(t)$ due to the auxiliary subsystems powered by the engine (e.g. air conditioning compressor, steering pump, electric generator, etc.). In turn, for reasons that will be clear in the following, it is convenient to split $T_d$ as follows

$$T_d(t) = T_{pr}(t) + T_{ump}(t)$$

where $T_{ump}$ collects all unpredictable but bounded torques, whereas represents predictable amounts, usually larger than $T_{ump}$, and

$$T_{ump}(t) \in D_1, \quad T_{pr}(t) \in D_2, \quad \forall t.$$

As an example, the air conditioning subsystem switching on/off generates a load which can be considered predictable. In fact, we can assume to know both the time instants of the switching on/off and the value of the load and exploit such an information in order to achieve less conservative results.

3 Problem formulation

The goal of this paper is the design of an idle speed control for GDI engines, which minimizes fuel consumption, maintains system variables within prescribed operative constraints and prevents engine stall. Fuel minimization should be achieved both in steady-state conditions and during transients caused by disturbance torques acting on the crankshaft. Usually, the specification for the idle speed control is to maintain the engine speed around a nominal reference value $n_r$. In steady-state fuel consumption is strictly related to the nominal value $n_r$, in that the lower $n_r$ the lower the fuel consumption. Further, $n_r$ is a fixed reference value that is determined by trading-off between economical considerations (consumption) and the need of avoiding the engine to stall during transients due to load disturbances. Then, because fixed, in some situations $n_r$ could be remarkable higher than strictly necessary to keep the engine running. To optimize fuel consumption, we allow the engine speed to vary in an interval around the nominal value.

The specification for idle speed control design is formalized as follows:

$$\min_{\alpha(t), \lambda(k)} \sum_{k=0}^{\infty} q_b(k)$$

- $n(t) \in [710, 790] \text{ (rpm)}$
- $q_b(k) \geq 1 \text{ (mg)}$
- $\alpha(t) \geq 0 \text{ (degree)}$
- $\dot{\alpha}(t) \in [-5, 5] \text{ (degree/s)}$
- $\lambda(k) \in [0.8, 3.5]$
- $T_{ump}(t) \in D_1 = [3, 8] \text{ (Nm)}$
- $T_{pr}(t) \in D_2 = [0, 12] \text{ (Nm)}$
The lower-bound of 710 rpm on \( n(t) \) is imposed to prevent the engine from stalling, whereas the upper bound is dictated by economical considerations. The constraints on \( q, \alpha, \dot{\alpha}, \lambda \) are imposed by constructive details.

4 The proposed approach

The hybrid model presented in Section 2 is linearized about the operating point corresponding to the nominal idle speed \( n_0 = 750, \text{ (rpm)} \) and disturbance torque \( T_d = T_{d0} \). Then, the obtained model is discretized using a period equal to the throttle control sampling time \( T_c = 10 \text{ ms} \).

The time between to subsequent dead centers is approximated with its value at the nominal engine speed \( n_0 = 750 \text{ rpm} \), i.e. 40 ms, and expressed as 4 times \( T_c \), synchronizing the engine cycle and the throttle control. Furthermore, to take into account the drift between fuel injection commands, which have to be issued at dead center events, and throttle control commands, issued at fixed period \( T_c \), a delay of \( \alpha = 2T_c \) is introduced in the fuel control loop.

Thus, the linearized and discretized model is as follows:

\[
\begin{align*}
\{ x_p(t+1) &= Ax_p(t) + Bu(t) + B_d d(t) \\
y(t) &= Cx_p(t)
\end{align*}
\]

where

\[
x_p(t) = \begin{bmatrix}
n(t) - n_0 \\
p(t) - p_0 \\
\vdots \\
p(t-4) - p_0 \\
\vdots \\
\alpha(t) - \alpha_0 \\
q_0(t) - q_0 \\
\vdots \\
q_0(t-2) - q_0 \\
\alpha(t-1) - \alpha_0 \\
\vdots \\
\alpha(t-4) - \alpha_0
\end{bmatrix}
\]

\[
u(t) = \begin{bmatrix}
q_0(t) - q_0 \\
(\alpha(t) - \alpha_0) \\
\vdots \\
(\alpha(t-4) - \alpha_0)
\end{bmatrix}
\]

\[
d(k) = T_d(t) - T_{d0} \\
y(t) = \begin{bmatrix}
n(t) - n_0 \\
(\lambda(t) - \lambda_0)
\end{bmatrix}
\]

We proposed to use of a command governor (CG) for modifying on-line the desired value of engine speed and normalized air–to–fuel ratio, so that the prescribed constraints are never violated, irrespective of all possible load disturbance occurrences, and fuel consumption is optimized. For the engine at hand, it results that fuel consumption is minimized when

\[ n_r = 710 \text{ (rpm)}, \quad \lambda_r = 2 \]

Then, the basic strategy underlying the use of a CG will be that to apply the nominal reference values \( n_r \) and \( \lambda_r \) and let the CG to modify them on-line whenever their application would lead to constraint violation. Details on CG theory are reported in the next session.

5 The Command Governor (CG) approach

A CG control scheme, with plant, primal controller and CG device, is depicted in Fig. 3. Eq. (8) is a state-space description of the plant regulated by the primal controller.

\[
\begin{align*}
x(t+1) &= \Phi x(t) + Gg(t) + G_d d(t) \\
y(t) &= H_y x(t) \\
c(t) &= H_c x(t) + Lg(t) + L_d d(t)
\end{align*}
\]

In particular, \( x(t) \in \mathbb{R}^n \) is the state which includes plant and compensator states; \( g(t) \in \mathbb{R}^m \), which would be typically \( g(t) = r(t) \) if no constraints were present (no CG present), is the CG output, viz.
a suitably modified version of the reference signal $r(t) \in \mathbb{R}^m$; $d(t) \in \mathbb{R}^n$, an exogenous disturbance satisfying $d(t) \in \mathcal{D}$, $\forall t \in \mathbb{Z}_+$, with $\mathcal{D}$ a specified convex and compact set such that $0 \in \mathcal{D}$; $y(t) \in \mathbb{R}^m$ is the output, viz. a performance related signal which is required to track $r(t); c(t) \in \mathbb{R}^n$ the vector to be constrained, viz. $c(t) \in \mathcal{C}$, $\forall t \in \mathbb{Z}_+$, with $\mathcal{C}$ a specified convex and compact set. It is assumed that

1. System (8) is asymptotically stable;
2. System (8) is offset free, viz. $H_y(I - \Phi)^{-1}G = I_m$ (9)

CG problem consists of finding, at each time $t$, a command $g(t)$ as a function of the current state $x(t)$ and reference $r(t)$

$$g(t) := g(x(t), r(t))$$

in such a way that $g(t)$ is the best approximation of $r(t)$ at time $t$, under the constraint $c(t) \in \mathcal{C}$, $\forall t$, and all possible disturbance sequences $d(t) \in \mathcal{D}$. Moreover, it is required that: 1) $g(t) \to \hat{r}$ whenever $r(t) \to r$, with $\hat{r}$ the best feasible approximation of $r$; and 2) the CG have a finite settling time, viz. $g(t) = \hat{r}$ for a possibly large but finite $t$ whenever the reference stays constant after a finite time. By linearity, one is allowed to separate the effects of initial conditions and input from those of disturbances, e.g. $x(t) = \bar{x}(t) + \hat{x}(t)$, where $\hat{x}$ is the disturbance-free component (depending only on initial state and input) and $\bar{x}$ depending only on disturbances. Then, denote the disturbance-free steady-state solutions of (8), for a constant command $g(t) \equiv w$, as follows

$$\begin{align*}
\bar{x}_w &:= (I_n - \Phi)^{-1}Gw \\
\bar{y}_w &:= H_y(I_n - \Phi)^{-1}Gw \\
\bar{c}_w &:= H_c(I_n - \Phi)^{-1}Gw + Lw
\end{align*}$$

Consider next the following set recursion

$$\begin{align*}
\mathcal{C}_0 &= \mathcal{C} \sim L_d \mathcal{D} \\
\mathcal{C}_k &= \mathcal{C}_{k-1} \sim H_c \Phi^k G_d \mathcal{D} \\
\mathcal{C}_\infty &= \bigcap_{k=0}^{\infty} \mathcal{C}_k
\end{align*}$$

(12)

where $\mathcal{A} \sim \mathcal{E}$ is defined as $\{a \in \mathcal{A} : a + e \in \mathcal{A}, \forall e \in \mathcal{E}\}$. It can be shown that the sets $\mathcal{C}_k$ are non-conservative restrictions of $\mathcal{C}$ such that $c(t) \in \mathcal{C}_\infty, \forall t \in \mathbb{Z}_+$, implies $c(t) \in \mathcal{C}, \forall t \in \mathbb{Z}_+$. Thus, one can consider only disturbance-free evolutions of the system and adopt a “worst case” approach. Next consider, for a small enough $\delta > 0$, the sets:

$$\begin{align*}
\mathcal{C}^{\delta} &= \mathcal{C}_\infty \sim B_3 \\
\mathcal{W}^{\delta} &= \{w \in \mathbb{R}^n : \bar{c}_w \in \mathcal{C}^{\delta}\}
\end{align*}$$

(13)

where $B_3$ is the ball of radius $\delta$ centered at the origin. In particular, $\mathcal{W}^{\delta}$, which we assume non-empty, is the set of all commands whose corresponding steady-state solution satisfies the constraints with margin $\delta$. 

Figure 3: Command Governor structure
The main idea is to choose at each time step a constant virtual command \( v(\cdot) \equiv w \), with \( w \in \mathcal{W}_\delta \), such that the corresponding virtual evolution fulfills the constraints over a semi-infinite horizon and its distance from the constant reference of value \( r(t) \) is minimal. Such a command is applied, a new state is measured and the procedure is repeated. In this respect we define the set \( \mathcal{V}(x) \) as

\[
\mathcal{V}(x) = \{ w \in \mathcal{W}_\delta : \bar{c}(k,x,w) \in \mathcal{C}_k, \forall k \in \mathbb{Z}_+ \}
\]

where

\[
\bar{c}(k,x,w) := H_c \left( \Phi^k x(t) + \sum_{i=0}^{k-1} \Phi^{k-i-1} Gw \right) + Lw
\]

is to be understood as the disturbance-free virtual evolution at time \( k \) of \( c \) from the initial condition \( x \) at time 0 under the constant command \( v(\cdot) \equiv w \). As a consequence, \( \mathcal{V}(x) \subset \mathcal{W}_\delta \). Moreover, if non-empty, it represents the set of all constant virtual sequences in \( \mathcal{W}_\delta \) whose evolutions starting from \( x \) satisfies the constraints also during transients. Thus, taking as a selection index a quadratic cost, the CG output is chosen according to the solution to the following constrained optimization problem

\[
g(t) = \arg \min_{w \in \mathcal{V}(x(t))} \| w - r(t) \|_\Psi^2
\]

where \( \Psi = \Psi' > 0 \) and \( \| w \|_\Psi^2 := x' \Psi x \). It has been shown in [17] that the following properties hold for the above described CG.

**Theorem** - Let assumptions (9) be fulfilled. Consider system (8) along with the CG selection rule (16), and let \( \mathcal{V}(x(0)) \) be non-empty. Then:

1. The minimizer in (16) uniquely exists at each \( t \in \mathbb{Z}_+ \) and can be obtained by solving a convex constrained optimization problem, viz. \( \mathcal{V}(x(0)) \) non-empty implies \( \mathcal{V}(x(t)) \) non-empty along the trajectories generated by the CG command (16).

2. The set \( \mathcal{V}(x) \), \( \forall x \in \mathbb{R}^n \), is finitely determined, viz. there exists an integer \( k_0 \) such that if \( \bar{c}(k,x,w) \in \mathcal{C}_k \), \( k \in \{0,1,\ldots,k_0\} \), then \( \bar{c}(k,x,w) \subset \mathcal{C}_k \) \( \forall k \in \mathbb{Z}_+ \). Such a constraint horizon \( k_0 \) can be determined off-line.

3. The constraints are fulfilled for all \( t \in \mathbb{Z}_+ \).

4. The overall system is asymptotically stable; in particular, whenever \( r(t) \equiv r \), \( g(t) \) converges in finite time either to \( r \) or to its best steady-state admissible approximation,

\[
g(t) \to \hat{r} := \arg \min_{w \in \mathcal{W}_\delta} \| w - r \|_\Psi^2
\]

Consequently, by the offset free condition (9.2),

\[
\lim_{t \to +\infty} \tilde{y}(t) = \hat{r}.
\]

where \( \tilde{y} \) is the disturbance-free component of \( y \).

5.1 Hybrid CG design

In some situations the use of a single CG can be restrictive. This is the case e.g. when predictable disturbances are present and one wants to exploit such an information for achieving better results. Other cases comprise piecewise affine representation of nonlinear systems in specific state space regions and the enlargement of admissible set-point regions, which must be tracked without offset due to constraints on the system. Then, for hybrid CG we mean a bank of several CG’s, each one designed for dealing with a specific situation, and a supervisor in charge to identify what situation is going to occur and put the correct CG on-line. The forthcoming ideas are strongly inspired by [19], where such a technique was first suggested.
In order to be specific, consider the following set of reference set-points $r$ which are desired to be tracked without offset

$$r \in \Theta \subset \mathbb{R}^m$$

Assume for the problem at hand that $W^\delta$ is such that

$$\Theta \not\subset W^\delta$$

Then, the requirement that all set-points in $\Theta$ will be tracked without error cannot be accomplished. A way to overcome this limitation is that of covering the set $\Theta$ by a collection of sets $W^\delta_i$, $i = 1, ..., r$ with overlapping interiors corresponding to $r$ distinct $CG_i$ such that

$$\Theta \subset \bigcup_{i=1}^{r} W^\delta_i$$

interior\{$W^\delta_i \cap W^\delta_j$\} \neq 0, for at least a pair \((i, j)\) \in \{1, ..., r\}

Then, we can define the set $V: \{W^\delta_i\}_{i=1}^{r}$ of nodes of an oriented graph $G$ and the set $A$ of arcs connecting nodes of $G$

$$A := \{(i, j) : \text{interior} \{W^\delta_i \cap W^\delta_j\} \neq 0, \ i \neq j\}$$

Clearly, $CG_i$ operates properly when initial and final set-points belong to $W^\delta_i$. When the final set-point belongs to different set, say $W^\delta_j$, a procedure for switching between $CG_i$ to $CG_j$ and the time instant of switching have to be formulated. To this end, let $Z^\delta_i \subset \mathbb{R}^p \times \mathbb{R}^n$ denote the output admissible set for $CG_i$, that is the set of all pairs $[r \ x] \in Z^\delta_i$ whose evolutions (to be intended as the system response from initial state $x$ at time $t = 0$ to a set-point $r$) satisfy the constraints for all $t \in \mathbb{Z}_+$. The obvious relationship between $Z^\delta_i$ and $W^\delta_j$ is given by

$$W^\delta_i := \{w \in \mathbb{R}^m : [w \ x] \in Z^\delta_i \text{ for at least one } x \in \mathbb{R}^n\}$$

Correspondingly, we can define the set

$$X^\delta_i := \{x \in \mathbb{R}^n : [w \ x] \in Z^\delta_i \text{ for at least one } w \in \mathbb{R}^m\}$$

of all states which can be steered to feasible equilibrium points without constraint violations.

If \((i, j) \in A\) than also interior \{\(X^\delta_i \cap X^\delta_j\)\} \neq 0 and one can a-priori define a convenient transition reference $r_{ij} \in \text{interior} \{W^\delta_i \cap W^\delta_j\}$ such $\bar{x}_{r_{ij}} \in \text{interior} \{X^\delta_i \cap X^\delta_j\}$, where $\bar{x}_{r_{ij}}$ is the equilibrium state corresponding to $r_{ij}$ defined in (11). Then, $[r_{ij} \ \bar{x}_{r_{ij}}] \in Z^\delta_i \cap Z^\delta_j$ and the transfer strategy is simple.

Assume to be at time $\bar{t}$, be using $CG_i$ and let $r(\bar{t}) \in W^\delta_i$ and $r(\bar{t} + 1) \in W^\delta_j$ with $(i, j) \in A$. Then, a prototype switching logic for the transition between the $CG_i$ to $CG_j$ is as follows:

1. Solve and apply

$$g(\bar{t} + k) = \arg \min_{w \in V_i(x(\bar{t} + k))} \|w - r_{ij}\|_Q^2, \ k = 0, 1, ..., \bar{k}$$

when

$$x(t + \bar{k}) \in \text{interior} \{X^\delta_i \cap X^\delta_j\}$$

2. Switch to $CG_j$ and solve

$$g(t) = \arg \min_{w \in V_j(x(t))} \|w - r(t + 1)\|_Q^2, \ t \geq \bar{t} + \bar{k} + 1$$

The above scheme is motivated by the fact that for any $x \in \mathbb{R}^n$ the state evolution will enter in interior \{\(X^\delta_i \cap X^\delta_j\)\} within a finite number of time instants. An upper-bound to this integer can be pre-computed off-line with respect to all $x \in \mathbb{R}^n$ in a way similar to that used for determining the constraint horizon $k_0$ in step 2) of previous Theorem. Then, instead of checking the set-membership condition in step 1), one could determine such an upper-bound $\bar{k}$ off-line and exploit it during on-line operation by waiting for exactly $\bar{k}$ in step 1) before switching.

Of course, other possibilities exist which could be more effective for specific applications and the above switching logic has to be considered only from a conceptual point of view. In all cases, the adopting of the above guidelines allows one to retain in a hybrid general context the same stability and feasibility properties pertaining to the basic CG approach.
6 Controller synthesis

In this section the design of the proposed idle control is presented. The controller consists of two nested loops:

- a switching LQ controller in the inner loop, whose objective is the minimization of fuel consumption during transients;
- a CG in the outer loop, whose objective is the minimization of fuel consumption during steady states and the verification of the constraints.

The inner loop and the outer loop are, respectively, described in Section 6.1 and in Section 6.2 below. Simulation results of the closed-loop hybrid system are reported in Section 6.3.

6.1 Primal Control

The CG approach requires preliminarily the design, if not given, of a primal stabilizing controller which, because is supposedly to be used along with a CG, is designed without tacking into account the prescribed constraint. The one used here is depicted in Fig. 4. In order to have zero tracking error in steady-state we require the use of an integral action. This is done be resorting to the incremental model approach [20] which consists of rewriting the model described in Section 4 in terms of the extended state $x_c(t)$ and incremental input $\delta u(t) := u(t+1) - u(t)$

$$x_c(t+1) = \Phi x_c(t) + G \delta u(t), \quad x_c(t) := \begin{bmatrix} \delta x_p(t) \\ \varepsilon(t-1) \end{bmatrix}$$

$$\varepsilon(t) = H x_c(t)$$

where $\delta x_p(t) := x_p(t+1) - x_p(t)$ and $\varepsilon(t-1) = y(t-1) - g(t-1) - g(t-1)$, $g(t)$ being the reference signal. Then, optimal LQ state feedbacks of the form

$$\delta u(t) = -K_{Lq} x_c(t)$$

which minimizes the following quadratic cost

$$J = \sum_{t=0}^{\infty} \|\varepsilon(t)\|_{\Psi_{\varepsilon}}^2 + \|\delta u(t)\|_{\Psi_u}^2$$

with $\Psi_{\varepsilon} = \Psi_{\varepsilon}' \geq 0, \quad \Psi_u = \Psi_u' > 0$ can be easily determined. In particular, we have found convenient to determine two different LQ state feedback control laws, each one well suited to deal with a specific situation. Moreover, a supervisor ($K_{Lq}$ Selector) is in charge to identify when each specific controller has
to be put in the loop on the basis of the input $T_{dist}$ that indicates in advance the state of the (ON-OFF) predictable disturbance. Specifically, the two LQ control laws have been designed to work well during the occurrence of the following conditions: 1) “predictable disturbance on/off or off/on transitions” (Lq2) or “no predictable disturbance transitions” (Lq1). The main reason for using two state feedback control laws instead of a single one is that of having different gains during large transient occurrences and steady-state operations. This is convenient for trading-off between fuel consumption minimization and fast transients achievement. In fact, for fuel consumption minimization the weight $\Psi_u$ in the cost has to be chosen remarkably larger than $\Psi_\varepsilon$. Under small disturbances this choice ensures small fuel consumptions and the embedded integral action ensures zero tracking error in steady-state. However, sluggish responses result which cannot be acceptable especially after a large load disturbance change. In such a case, more active control actions are desired. It is worth pointing out that only the fuel consumption due to transients can be optimized by a suitable choice of the control law. On the contrary, the usually predominant amount necessary for supporting the engine during steady-states depends only by constructive details and actual loads which cannot be modified by a specific feedback.

Notice that the overall closed-loop stability can be verified by testing existence of a single symmetric positive definite matrix $P = P^T > 0$ which jointly satisfy [21]

$$\Phi_1^T P \Phi_1 - P < 0 \text{ and } \Phi_2^T P \Phi_2 - P < 0$$

with $\Phi_1 = \Phi + GK_{Lq1}$ and $\Phi_1 = \Phi + GK_{Lq1}$.

In order to clarify our strategy we consider the following simulative experiment corresponding to the torque disturbance profile depicted in Fig. 5. In the figure, a quite severe disturbances scenario is considered with predictable disturbances of 12 (Nm) occurring (superimposed to other disturbances in [3, 8] (Nm)) between the time intervals [5, 9] and [13, 17].

The responses under a dual switching LQ primal control structures, corresponding to reference signal $n_r = 740$ (rpm) and $\lambda_r = 2$, are reported in Fig. 6. The corresponding total fuel consumption was of 2861 (mg), out of which only 6.4 (mg) (0.22 %) due to transients.
Observe as the constraint on the engine speed is violated at time instant $t = 13$. It will be shown in next sections that the use of a well designed CG will allow one to enforce constraints for all time and regardless admissible load disturbances occurrence. Of course, feasible trajectories can be achieved by a different tuning of the dual LQ switching primal controller at the expense of a larger fuel consumption. If done for the disturbance sequence of Fig. 5, it is possible to achieve feasible trajectories under switching LQ strategies with total fuel consumption of 2875 (mg), out of which 20.4 (mg) (0.7 %) due to transients.

6.2 CG application (External Loop)

Accordingly to the above primal control structure, we have designed a bank of three CG’s (see Fig. 7), each one in charge to deal with a different situation. In particular:

CG1: Lq1 in the loop and predictable disturbance off;

CG2: Lq2 in the loop and predictable disturbance on/off or off/on transition;

CG3: Lq1 in the loop and predictable disturbance on.

The selection of the CG to be applied is handled by the block "CG selector" (see fig.7) that makes use of the input $T_{dinf}$.

The CG’s have been designed on the same incremental model for the plant (depicted in section 6.1) with the LQ primal controller specified above in the three cases. The only difference has regarded the assumed admissible load disturbance ranges. Specifically,

CG1: $D_1 \in [3, 8]$ (Nm) and $D_2 = \{0\}$ (Nm);

CG2: $D_1 \in [3, 8]$ (Nm) and $D_2 = [0, 12]$ (Nm);

CG3: $D_1 \in [3, 8]$ (Nm) and $D_2 = \{12\}$ (Nm);

The following steady-state admissible engine speed ranges (for $\lambda = 2$) and constraint horizons are resulted

CG1: $W_1 \cap \begin{bmatrix} \mathbb{R} \end{bmatrix} = [716, 785]$, $k_0 = 20$;

CG2: $W_2 \cap \begin{bmatrix} \mathbb{R} \end{bmatrix} = [720, 787]$, $k_0 = 10$;

CG3: $W_3 \cap \begin{bmatrix} \mathbb{R} \end{bmatrix} = [724, 770]$, $k_0 = 20$;

The set $\mathcal{A}$ (depicted in section 5) consists of the pairs $(1, 2)$ and $(3, 2)$ and the following transition references $r_{12} = 785$ (rpm) and $r_{32} = 725$ (rpm) have been chosen. For both “on/off” and “off/on” transitions, an upper-bound for $\bar{k}$ (depicted in section 5) was determined in 40 steps (0.4 sec.) and it has been used as a dwell time before switching.
6.3 Simulation

In this section we report some simulation results obtained applying the proposed LQ-CG hybrid control strategy, designed as illustrated in Sect. 5)-6) on the basis of the discrete-time model of Sect. 4), to the nonlinear hybrid model of the plant described in Section 2 under the load pattern of Fig. 5. Simulations show that the discrete-time approximation of the plant described in Section 4 is good enough since the performances of the hybrid closed loop system are satisfactory, both in terms of fuel consumption and constraints fulfillment.

Two simulations are here reported: the first consists of using the following constant set-point $r = \begin{bmatrix} 740 \\ 2 \end{bmatrix}$ at the CG input. This corresponds to the usual approach of setting a nominal idle speed in the middle of the allowed feasible band and let the feedback handling the transients due to load disturbances. The second corresponds to using the following constant set-point $r = \begin{bmatrix} 710 \\ 2 \end{bmatrix}$ and leave the CG to choose the lower possible set-point in every circumstance. The results of the simulations, both referred to the load disturbance sequence of Fig. 5, have been reported in Figs.8-11 for both experiments. For the first experiment we have measured a total fuel consumption of 2866 (mg), out of which 11.4 (mg) (0.40\%) due to transients. For the second experiment the total fuel consumption was of 2828 (mg), out of which 11.59 (mg) (0.41\%) due to transients. Notice as this large improvement has been obtained essentially via a lower consumption during steady-state phases corresponding to the imposition via the CG action of the lowest possible sustainable idle speed in steady-state compatible with the actual load.

Next Figs. 8-11 report the relevant signals during the simulation. The effect of the CG is especially relevant in the second experiment in which the set-point for the idle speed has been set to $n_r = 710$ (rpm). In fact, in Fig. 8 (below) the idle speed is always at the lowest level compatible with loads and constraints (compare upper and lower plots of Fig. 8 e.g. at time instants $t = 4, 14, 20$) and the constraints are satisfied for all time (compare with Fig. 6 where the constraint on the engine speed is violated and see also Fig. 9 for the constraint on the throttle angle). Relevant is also the injected fuel rate which allows one to compare the two strategies (see Fig. 10, upper and lower plots). Specifically, see as the injected fuel rate is never higher and strictly lower when possible in steady-state in the lower plot (e.g. at time $t = 14$) justifying the improvement in fuel consumption of the second strategy in steady-state.

![Figure 8: Engine speed: (Upper) first exp. ($n_r = 740$), (Lower) second exp. ($n_r = 710$)](image_url)
Figure 9: Throttle angle: (Upper) first exp. \((n_r = 740)\), (Lower) second exp. \((n_r = 710)\)

Figure 10: Injected fuel rate: (Upper) first exp. \((n_r = 740)\), (Lower) second exp. \((n_r = 710)\)

7 Conclusions

In this paper the idle speed control design problem for an automotive GDI engine has been considered where the main control objective was fuel consumption minimization while preventing engine stalls. Load variations and constraints fulfilment on relevant system variables have been explicitly taken into account in the control design as well as the requirement for computationally inexpensive and easily implementable solutions.

A highly accurate nonlinear hybrid model for the stratified mode of operation of a GDI engine has
been used to derive a low-dimensional linear discrete-time system used for control design purposes. The hybrid nature of the problem has reappeared in the linear discrete-time representation, essentially due to the presence of predictable load disturbances, and has led to the design of a hybrid CG unit for constraints fulfilment which uses two switching LQ optimal controllers as a primal control structure for ensuring nominal closed-loop stability and performance under linear regimes.

The CG approach has been instrumental not only for achieving lower fuel consumptions but also for improving the designer ability of explicitly taking care of prescribed constraints in the design phase, avoiding to heavy trade-off with optimality and extensively recurring to simulations for the assessment of the solution.

As a matter of fact, the use of the hybrid CG unit allowed fuel consumption to be reduced to 2828 \( (mg) \), out of which 11.41 \( (mg) \) due to transients, from 2875 \( (mg) \), out of which 20 \( (mg) \) due to transients, pertaining to the best admissible solution achieved by using only two switching LQ controllers (see Sect. 6.1). It results that the fuel consumption due to transients was reduced about of 50 \%, essentially for the freedom in tuning the primal LQ switching control structure without the need of taking into account the constraint. The overall consumption reduction was about 2 \% and the computational burden of the on-line part of the CG action computation, thanks to a careful implementation in Matlab, was limited to nearly 650 flops per step.

This allows us to conclude the proposed technique can achieve, over linear control methods, improvements on fuel consumption in the presence of constraints up to an extent which justifies the increase of computing burdens required by the hybrid CG algorithm.

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References


