

Design of Observers for Hybrid Systems

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Outline

- ✱ Motivations and Previous Results
- ✱ FSM Observer Design
- ✱ Continuous System Observer Design
- ✱ Hybrid Observer Design
 - ✱ Current-location Observable Hybrid Systems
 - ✱ The general case
- ✱ Conclusions

Motivations : Power-train control

- ★ Power-train control was formulated as a hybrid control problem:
 - ★ Cut-off control [Automatica, 1995]
 - ★ Fast force transient [CDC, 1998]
- ★ Control algorithms require full state feedback
- ★ It is not economically feasible or even possible to measure the complete state of the system.
- ★ HENCE ...we need a hybrid observer!

Previous Results on Observer Design

✱ Continuous Systems

- ✱ Luenberger [TAC 1971]: Introduction to observers
- ✱ Kalman [ASME 1960]: Optimal disturbance rejection observers
- ✱ Liberzon Hespanha Morse [CDC 1999]: Stability of switched systems

✱ Discrete Systems

- ✱ Ramadge [CDC 1986]: Current-state observability
- ✱ Caines et al. [CDC 1988]: Current-state tree
- ✱ Ozveren and Willsky [TAC 1989]: Observability with a delay

✱ Hybrid Systems

- ✱ Ackerson and Fu [TAC 1970], Alessandri and Coletta [HSCC 2001]: Assuming location knowledge
- ✱ Mosterman and Biswas [HSCC 1999]: Model abstractions
- ✱ Morari [TAC 2000]: Hybrid observers

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- ✧ **FSM Observer Design**
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- ✧ Hybrid Observer Design
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- ✧ Conclusions and Future Work

FSM Observability

- ★ An FSM (alive)

$$q(k+1) \in \varphi(q(k), \sigma(k+1))$$

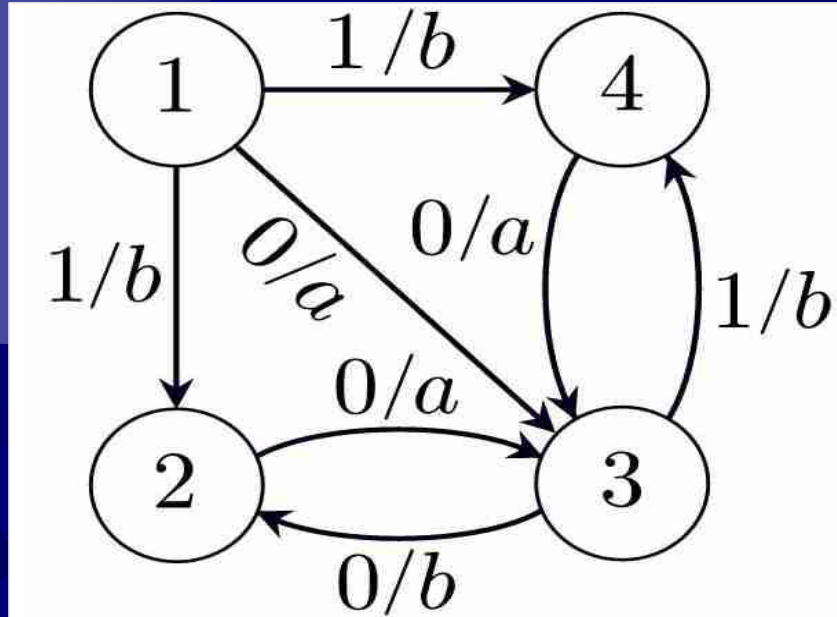
$$\sigma(k+1) \in \phi(q(k))$$

$$\psi(k+1) \in \eta(q(k), \sigma(k+1), q(k+1))$$

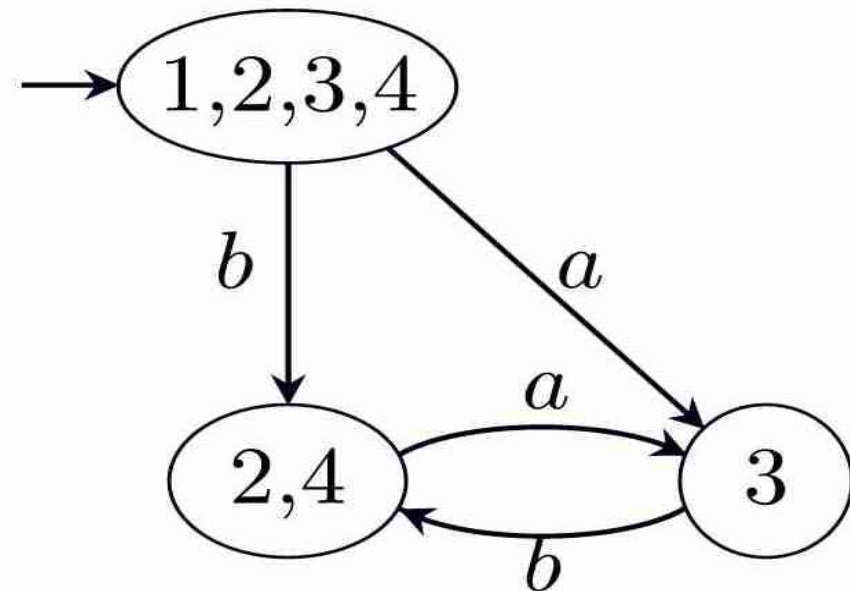
is said to be **current-state observable** if there exists an integer K such that

- ★ for any unknown initial state $q(0)$ and
 - ★ for any input sequence $\sigma(k)$
- the state $q(i)$ can be determined for every $i > K$ from the observation sequence $\psi(k)$ up to i .

Observers for FSMs



FSM

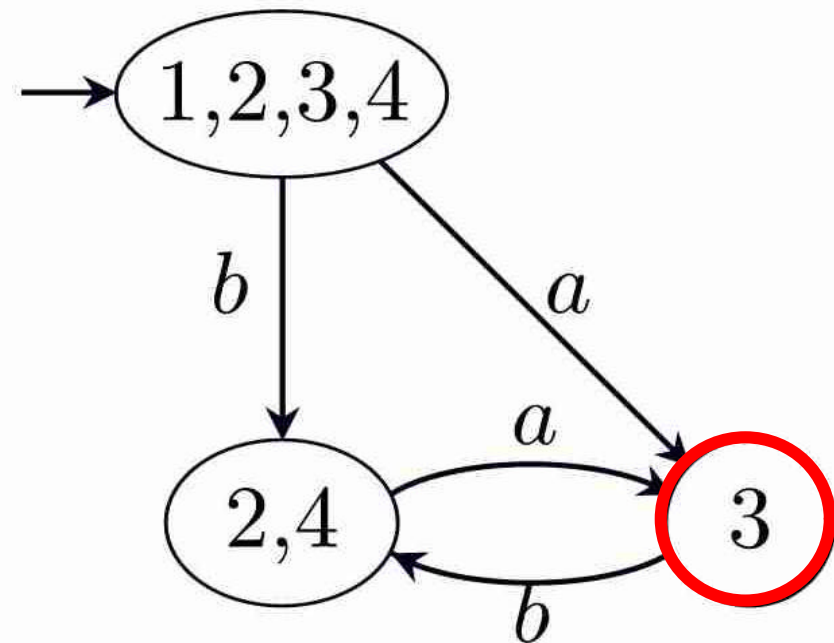
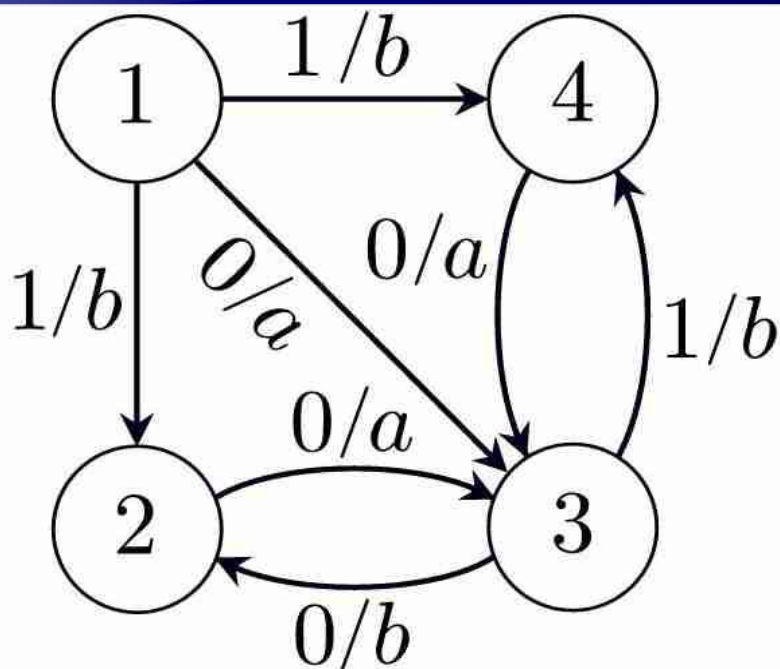


Observer

Theorem: An alive FSM is **current-state observable** iff there exists a nonempty subset E_O of singletons in the observer FSM such that

- ✱ E_O is invariant
- ✱ all cycles are contained in E_O

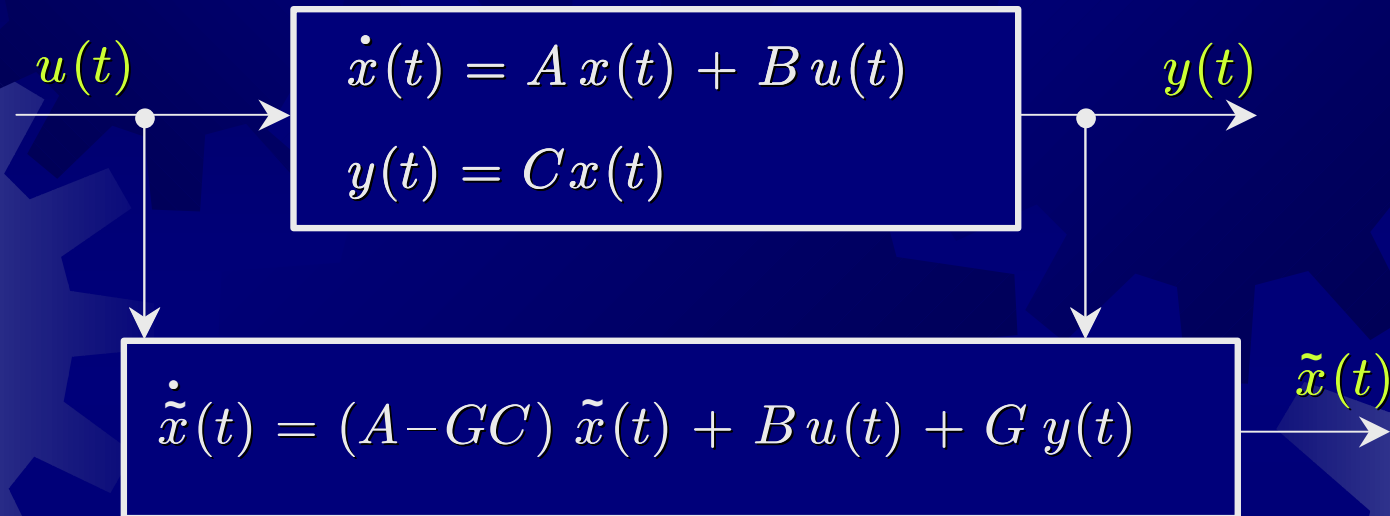
$Q \cap Q_0$ is not invariant



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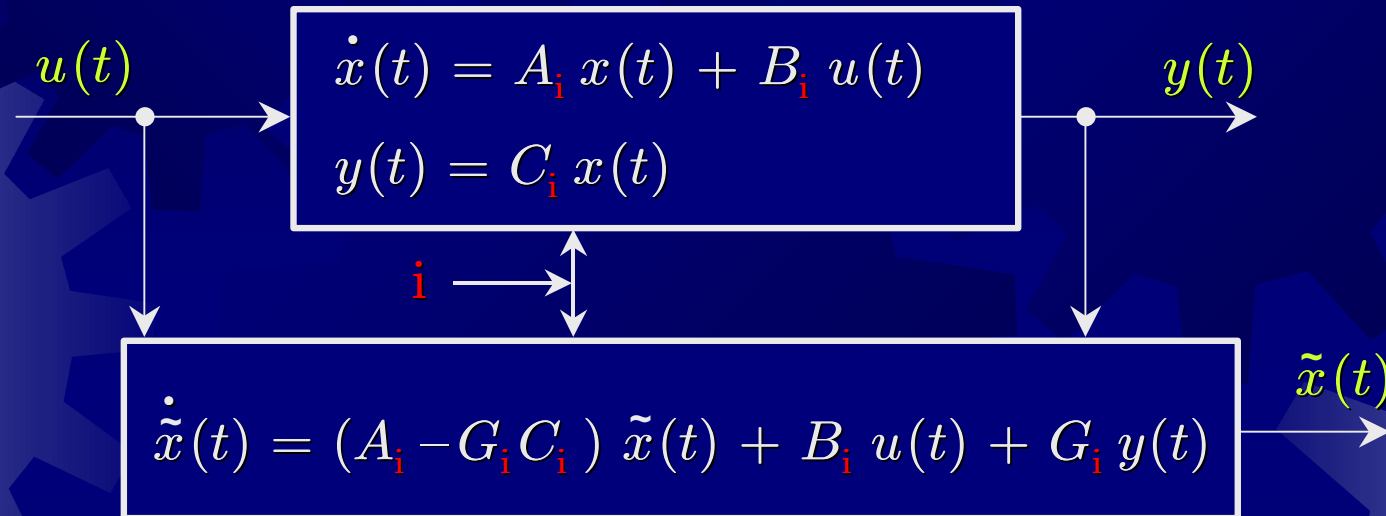
Continuous Observers for LTI Systems



$$\tilde{x}(t) - x(t) = e^{(A - GC)t} [\tilde{x}(0) - x(0)]$$

- ✦ Assuming (A, C) observable, the matrix G determines the speed of convergence of the state estimate

Switching Observers for LTI Systems



The switching observer is globally exponentially stable, if

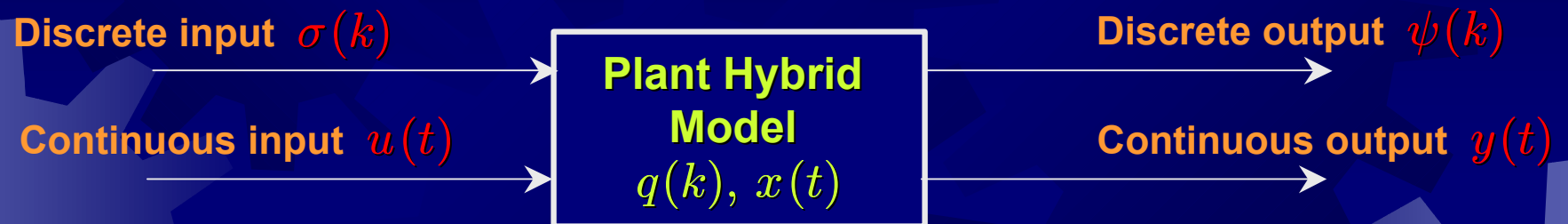
- ★ either the Lie algebra $\{F_i = A_i - G_i C_i\}$ is solvable
 - pairwise commuting \Rightarrow nilpotent \Rightarrow solvable Lie algebra

- ★ or there is a dwell time $t > \sup_i \left\{ \frac{\log c_i}{m_i} \right\}$, where $\|e^{F_i t}\| \leq c_i e^{m_i t}$

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Hybrid Observer Scheme



The plant hybrid model is

$$q(k+1) \in \varphi(q(k), \sigma(k+1))$$

$$\sigma(k+1) \in \phi(q(k), x(t_{k+1}^-), u(t_{k+1}^-))$$

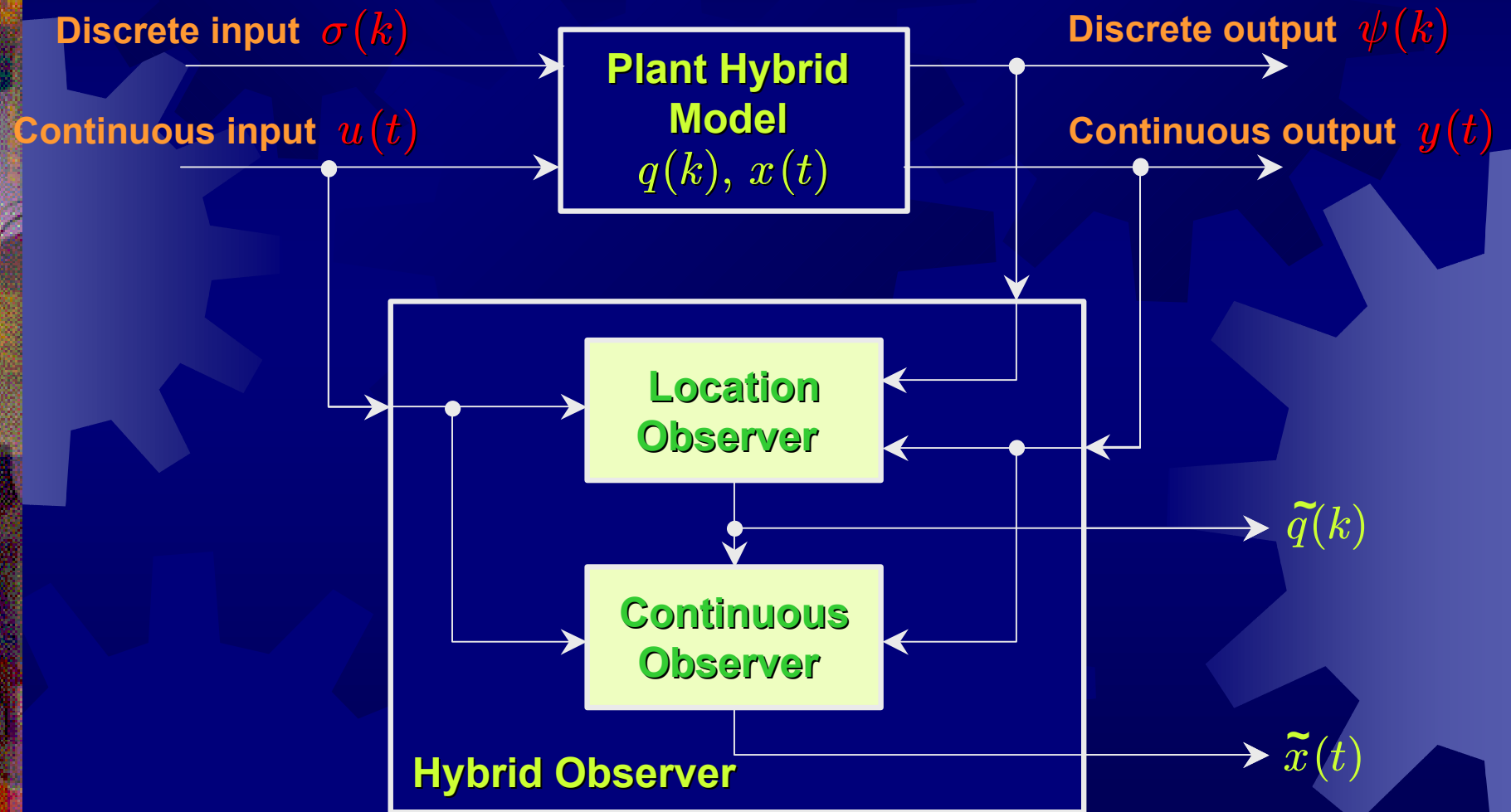
$$\psi(k+1) \in \eta(q(k), \sigma(k+1), q(k+1))$$

$$\dot{x}(t) = A_{q(k)} x(t) + B_{q(k)} u(t)$$

$$y(t) = C_{q(k)} x(t)$$

$$x(t_k) := R^1_{q(k), q(k+1)} x(t_k^-) + R^0_{q(k), q(k+1)}$$

Hybrid Observer Scheme



Exponential Ultimate Boundedness

A hybrid observer is said to be **exponentially ultimately bounded** if there exist a positive integer K and constants $c \geq 1$, $\mu > 0$ and $b \geq 0$, such that

$$\tilde{q}(k) = q(k), \quad \forall k \geq K$$

$$\|\tilde{x}(t) - x(t)\| \leq c e^{-\mu(t - \hat{t}_K)} \|\tilde{x}(\hat{t}_K) - x(\hat{t}_K)\| + b, \quad \forall t > \hat{t}_K$$

for any hybrid initial state and plant inputs.

- μ is the rate of convergence
- b is the ultimate bound, if $b = 0$, the observer is said to be **exponentially convergent**

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Design of Observer for Current-location Observable Hybrid Systems

Fact: If the FSM

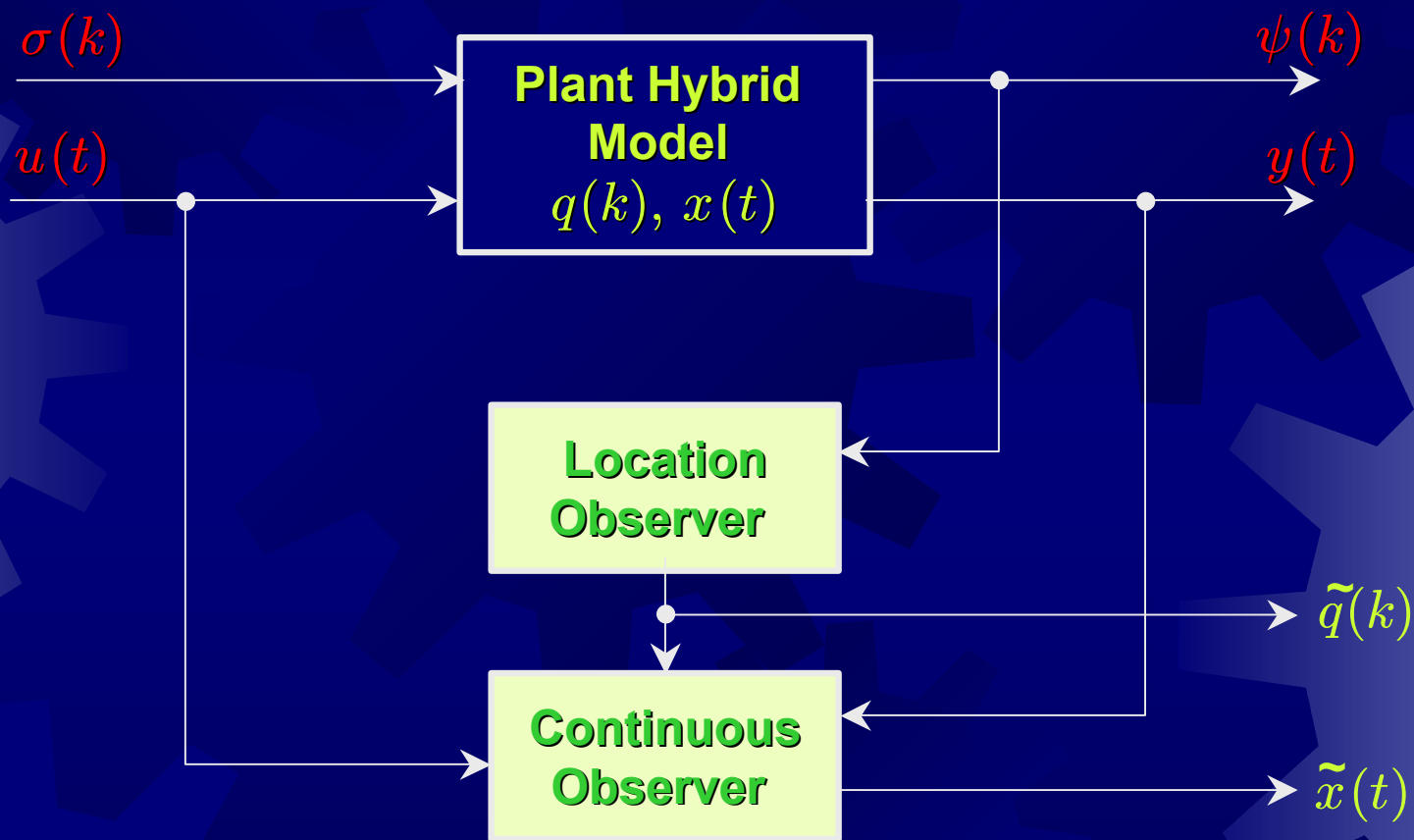
$$q(k+1) \in \varphi(q(k), \sigma(k+1))$$

$$\sigma(k+1) \in \bigcup_{\substack{x \in X \\ u \in U}} \phi(q(k), x, u)$$

$$\psi(k+1) \in \eta(q(k), \sigma(k+1), q(k+1))$$

associated to the hybrid plant is alive and **current-state observable** then the current location of the hybrid plant can be identified from the discrete output only.

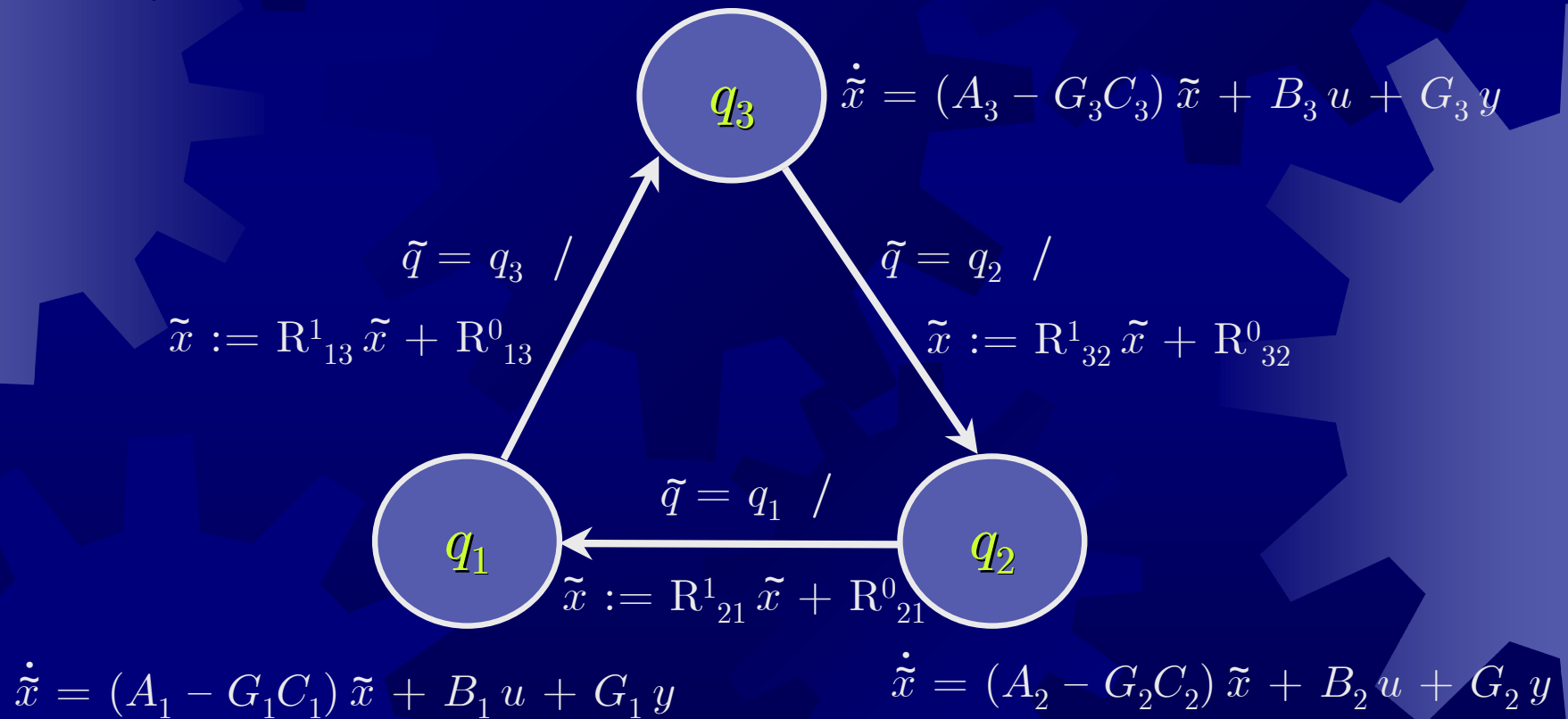
If the Location Observer identifies the discrete input and discrete output only ...



...the synthesis of the Location Observer and of the Continuous Observer can be done separately!

Location Observer : Observer for the FSM associated to the hybrid plant

Continuous Observer : A bank of Luenberger's with resets and switchings controlled by the location



Exponential Convergence

Theorem: Given a hybrid system with dwell time $D > 0$, if there exist matrices G_i such that

- ★ $A_i - G_i C_i$ have distinct eigenvalues

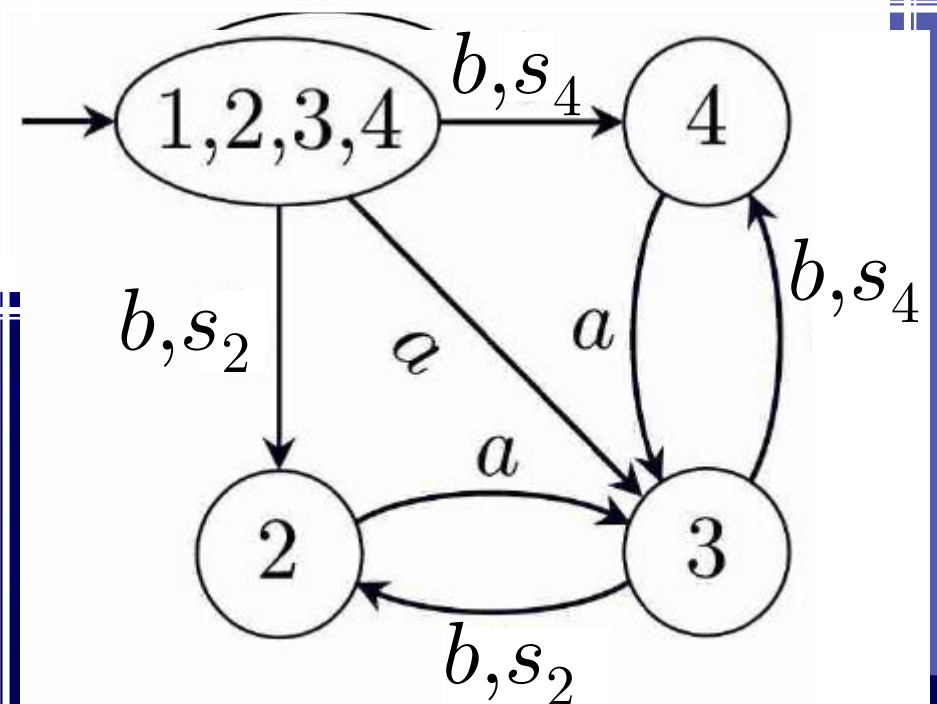
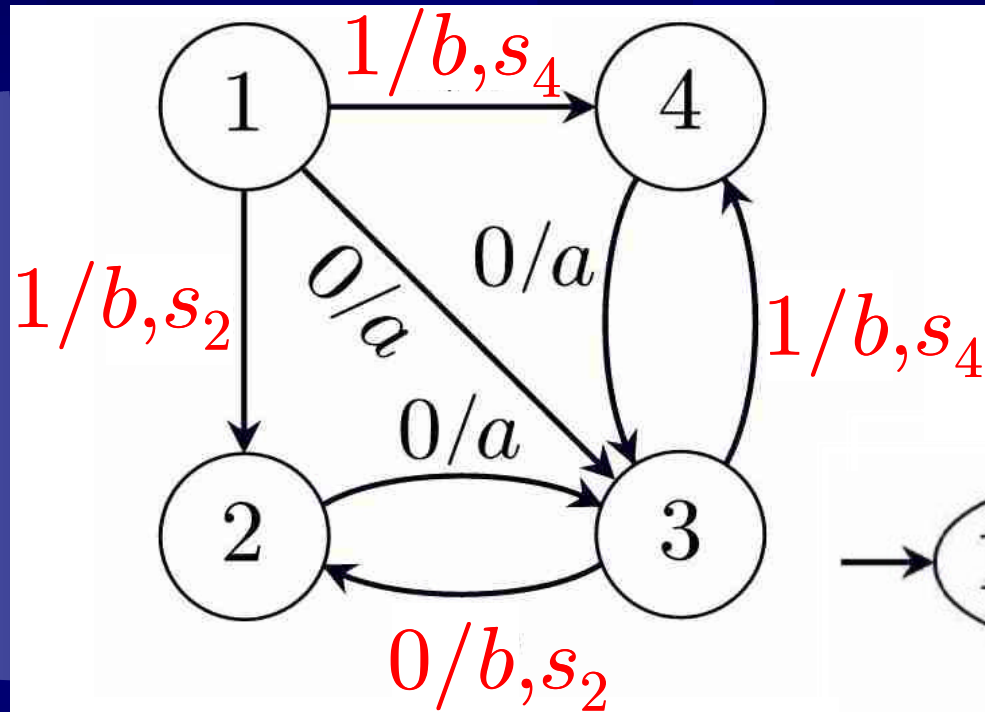
- ★ $\alpha(A_i - G_i C_i) + \frac{\max \{ 0, \log (r_i^1 k (A_i - G_i C_i)) \}}{D} \leq -\mu$

where $r_i^1 = \max \{ \| R_{ij}^1 \| \}$, then **the hybrid observer is exponentially convergent** with rate of convergence greater than μ .

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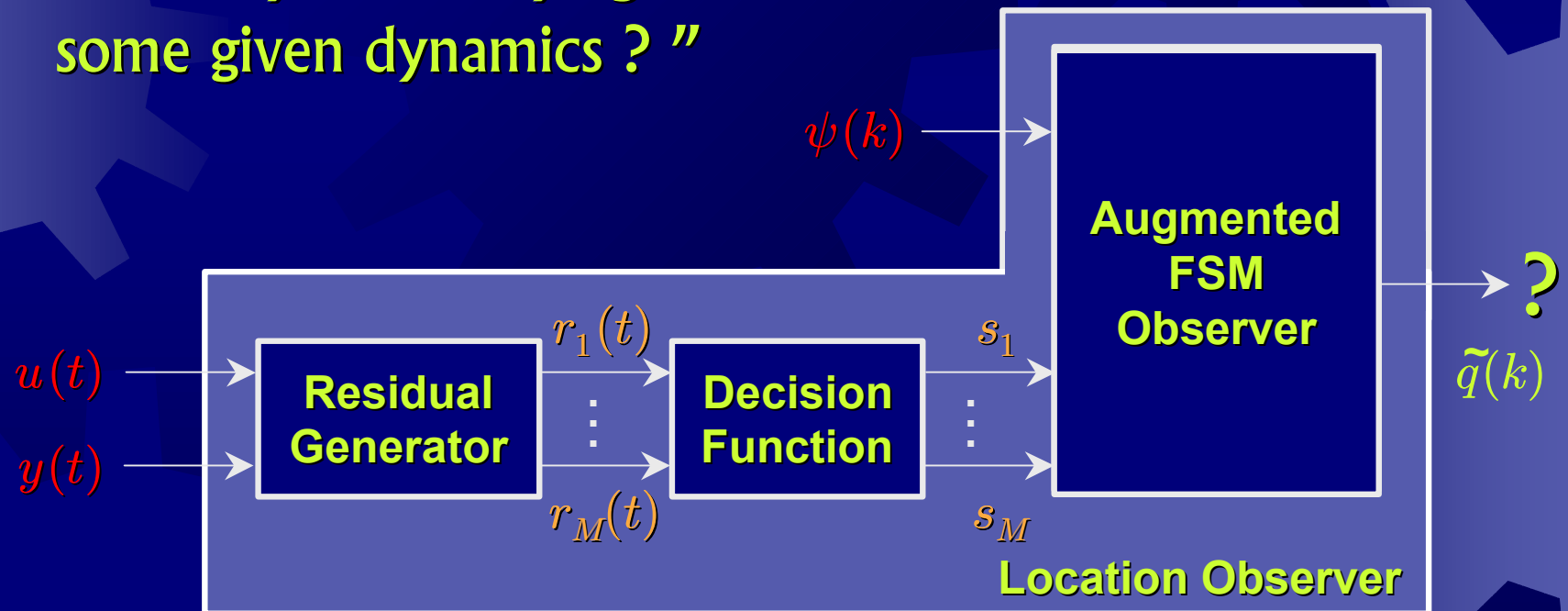
Location Observer Design: the general case



Location Observer Scheme

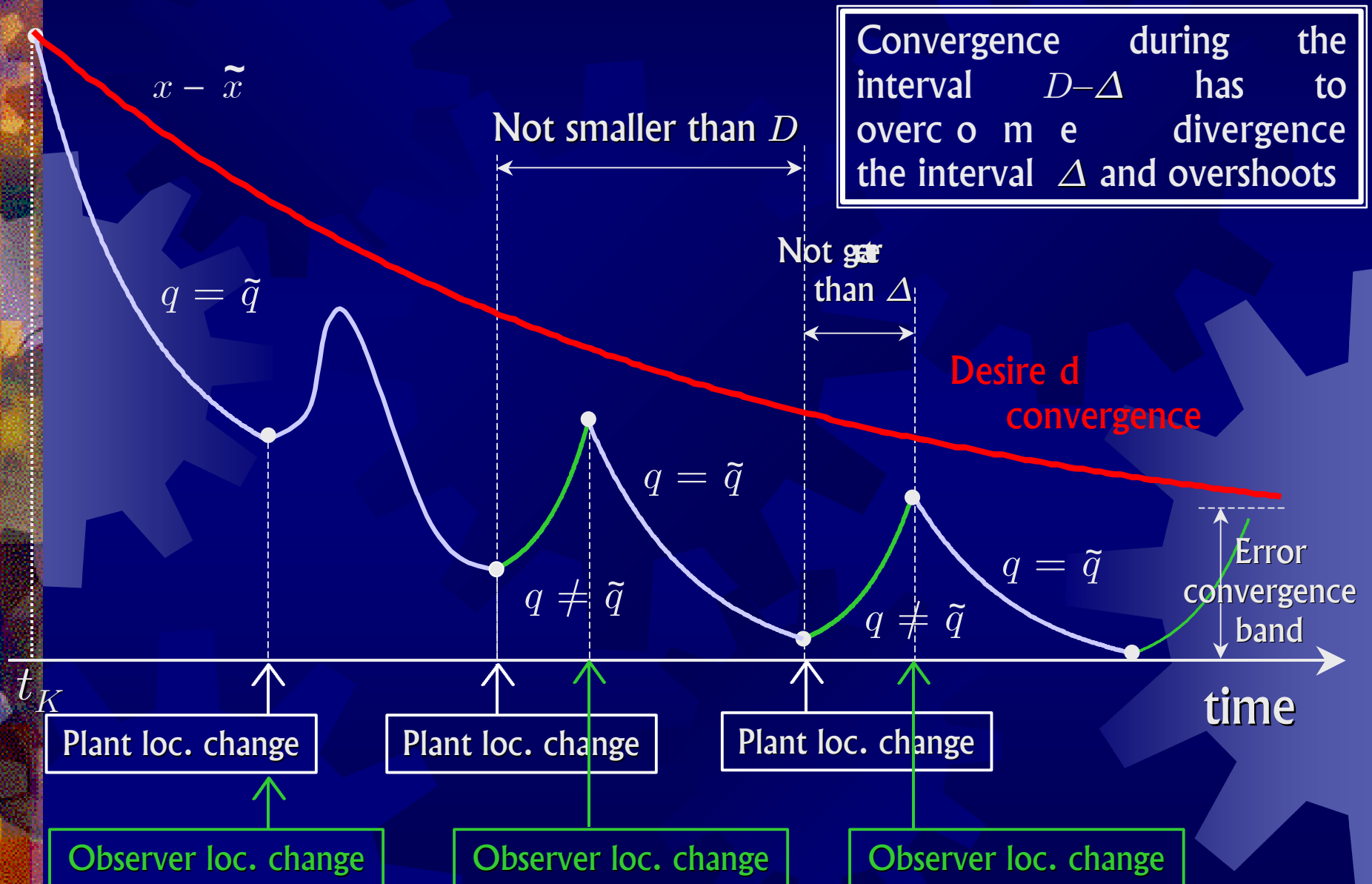
Residual Generation is similar to the Failure Detection and Identification problem:

“ is the system obeying some given dynamics ? ”



[Massoumnia, Verghese, Willsky, TAC 1989]

Exponential Ultimate Boundness



Theorem: Given a hybrid system with dwell time $D > 0$, if there exist matrices G_i such that

- ★ $A_i - G_i C_i$ have distinct eigenvalues
- ★ the location observer identifies changes in the plant location with a delay less than Δ with $0 \leq \Delta < D$
- ★ $\alpha(A_i - G_i C_i) + \frac{\max\{0, \log(r_i^1 k(A_i - G_i C_i))\}}{D - \Delta} \leq -\mu$

where $r_i^1 = \max\{\|R_{ij}^1\|\}$, then the hybrid observer is exponentially ultimately bounded with rate of convergence greater than μ .

If there are no resets, the hybrid observer is exponentially convergent.

Conclusions and Future Work

- ✱ A design methodology for exponentially convergent hybrid observers has been proposed
- ✱ This methodology has been recently extended to hybrid plant with **subject to continuous disturbances**
- ✱ Techniques exploiting information associated to **discrete transitions detection** has been investigated in order to
 - ✱ improve continuous state convergence
 - ✱ estimate unobservable continuous state components at transition times