Control and Computation

ETH Activities

Manfred Morari

Members: Mato Baotic
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Fachgruppen

System Dynamics & Control Group

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Control Engineering Group

Walter Schaufelberger
Adolf H. Glattfelder
Franta J. Kraus

Administration
Infrastructure
Teaching Program

People (November 2000)

		FTE
Professors	2 (+1)	2
Researchers	11	10
Graduate Students	14	14
Visitors	3	3
Administrative/Technical Staff	7	4.6
TOTAL	37	33.6

System Dynamics & Control

Biomedical Systems	
- Rehabilitation Engineering	Popovic, Jezernik, Keller, Pappas
- Anesthesia	Glattfelder, Gentilini, Stadler
Active Noise and Vibration Control, Intelligent Materials	Kaiser, Seba
Nonlinear Dynamics of Distillation Columns	Bonanomi, Dorn, Ulrich
Hybrid Systems	Bemporad, Borrelli, Cuzzola, Ferrari, Kojima, Mignone, Torrisi
- Analysis: Controllability, Verification, Stability, ...	
- Synthesis: Control, Estimation, Fault Detection	
- Applications	
Model Development for Control	Pearson
- Nonlinear Model Reduction	
- Nonlinear System Identification	

Role of ETH in CC

- Controller Synthesis via Optimization
- ABB Case Study
- Coordination of Case Studies

From Algebraic Equalities to Mixed-Integer Linear Inequalities

	Algebraic equalities	MI linear inequalities
Propositional logic (Williams, 1977)	Truth value operator: $\lfloor X \rfloor \in \{0,1\}$ $\lfloor P(X_1,\ldots,X_n) \rfloor = 1$	$\bar{\delta} = [\delta_1,\ldots,\delta_n]' \in \{0,1\}^n$ $A\bar{\delta} \leq B$
Mixed product (Glover, 1975) (Witsenhausen, 1966)	$z = \delta x$ $\begin{cases} \delta \in \{0,1\} \\ x \in [m, M] \end{cases}$	$\begin{cases} z \leq M\delta \\ z \geq m\delta \\ z \leq x - m(1-\delta) \\ z \geq x - M(1-\delta) \end{cases}$
Threshold condition	$\lfloor x \leq 0 \rfloor = \delta$	$\begin{cases} x \leq M(1-\delta) \\ x \geq \epsilon + (m+\epsilon)\delta \end{cases}$

MLD Hybrid Models

Mixed Logical Dynamical (MLD) form (Bemporad, Morari, *Automatica*, March 1999)

$$x(t+1) = Ax(t) + B_1u(t) + B_2\delta(t) + B_3z(t)$$
$$y(t) = Cx(t) + D_1u(t) + D_2\delta(t) + D_3z(t)$$
$$E_2\delta(t) + E_3z(t) \leq E_4x(t) + E_1u(t) + E_5$$

$$x, y, u = \begin{bmatrix} \star_c \\ \star_\ell \end{bmatrix}, \star_c \in \mathbf{R}^{n_c}, \star_\ell \in \{0,1\}^{n_\ell}, \quad z \in \mathbf{R}^{r_c}, \quad \delta \in \{0,1\}^{r_\ell}$$

- Well-Posedness Assumption :

$$\delta(t) = F(x(t), u(t))$$
$$z(t) = G(x(t), u(t))$$

$\longrightarrow$

$$\{x(t), u(t)\} \rightarrow \{x(t+1)\}$$
$$\{x(t), u(t)\} \rightarrow \{y(t)\}$$

are single valued

Well posedness allows defining trajectories in *x*- and *y*-space

Major Advantage of PWA/MLD Framework

All problems of analysis:

- Stability
- Verification
- Controllability / Reachability
- Observability

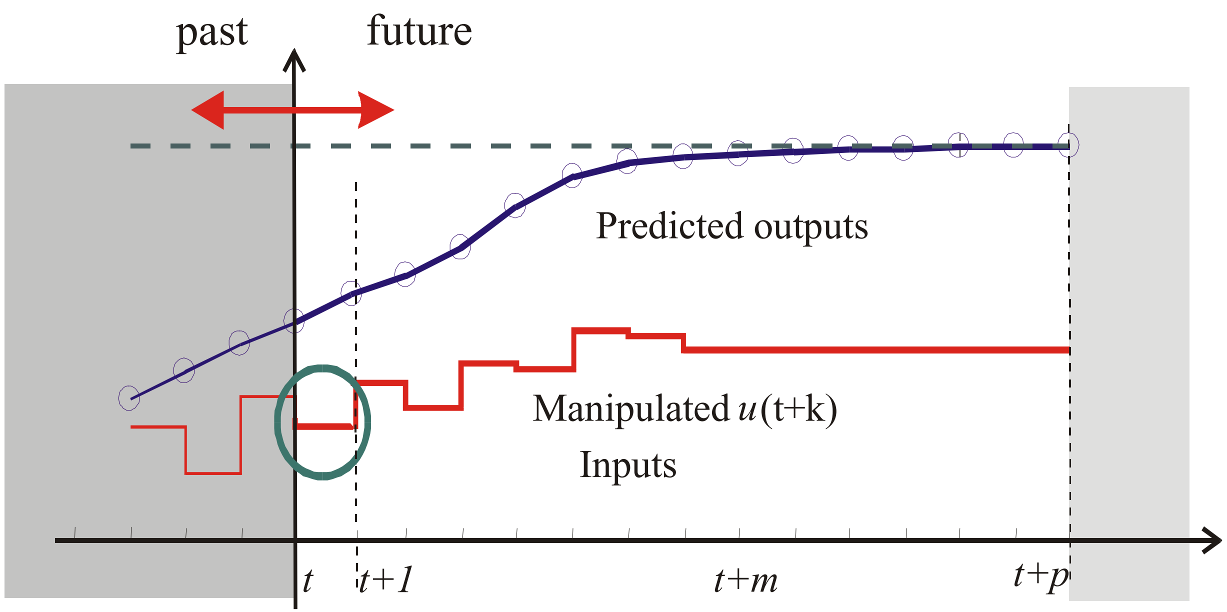
All problems of synthesis:

- Controller Design
- Filter Design / Fault Detection & Estimation

can be expressed as (mixed integer) *mathematical programming* problems for which many algorithms and software tools exist.

However, all these problems are NP-hard.

Finite Time Optimal Control



$$J_{opt} = \min_U J(U, x(t)) := \sum_{k=0}^{N} \|Q(x(t+k|t) - \bar{x})\| + \|Ru(t+k)\|$$

subject to $u_{\min} \leq u(t+k) \leq u_{\max}$

$x_{\min} \leq x(t+k|t) \leq x_{\max}$

system dynamics

$U := \{u(t), u(t+1), \ldots, u(t+N_u)\}$

Finite Time Optimal Control

$$J_{opt} = \min_U J(U, x(t)) := \sum_{k=0}^{N} \|Q(x(t+k|t) - \bar{x})\| + \|Ru(t+k)\|$$

$$\begin{aligned} \text{subject to} \quad & u_{\min} \leq u(t+k) \leq u_{\max} \\ & x_{\min} \leq x(t+k|t) \leq x_{\max} \\ & \text{system dynamics} \\ & U := \{u(t), u(t+1), \ldots, u(t+N_u)\} \end{aligned}$$

Model	**Perf. index**	**Method**
Linear	$\|\cdot\|_2$	**Quadratic Program**
Linear	$\|\cdot\|_\infty$	**Linear Program**
Hybrid	$\|\cdot\|_2$	**Mixed Integer Quadr. Program**
Hybrid	$\|\cdot\|_\infty$	**Mixed Integer Linear Program**

Feedback Control Solution

$$J_{opt} = \min_U J(U, x(t)) := \sum_{k=0}^{N} \|Q(x(t+k|t) - \bar{x})\| + \|Ru(t+k)\|$$

$$\text{subject to} \quad u_{\min} \leq u(t+k) \leq u_{\max}$$

$$x_{\min} \leq x(t+k|t) \leq x_{\max}$$

system dynamics

$$U := \{u(t), u(t+1), \ldots, u(t+N_u)\}$$

Theorem: The solution of the finite time optimal control problem yields a piecewise affine state feedback control law

Computation of Feedback Control Law

$$J_{opt} = \min_U J(U, x(t)) := \sum_{k=0}^{N} \|Q(x(t+k|t) - \bar{x})\| + \|Ru(t+k)\|$$

$$\text{subject to} \quad u_{\min} \leq u(t+k) \leq u_{\max}$$

$$x_{\min} \leq x(t+k|t) \leq x_{\max}$$

system dynamics

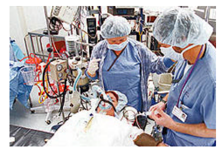
$$U := \{u(t), u(t+1), \ldots, u(t+N_u)\}$$

- The feedback controller can be computed explicitly
- Off-line optimization: optimize **for all** *x(t)*

Multi-parametric Program

Solvers for LP, QP, MILP and MIQP? are available

Applications

- Traction control (Ford Research Center)
- Gas supply system (Kawasaki Steel)
- Batch evaporator system (Esprit Project 26270)
- Anesthesia (Hospital Bern )
- Hydroelectric power plant (rittmeyer)
- Power generation scheduling (ABB)

Current and Future Projects

- **Algorithm Improvements (mpMIQP)**
- **Controller Characterization and Storage**
- **Controller Simplification**
  - **- Order reduction**
  - **- Decentralized structure**
- **Robustness**