Further Results on the Design of Hybrid Observers

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Outline

- Motivations and Previous Results
- FSM Observer Design
- Continuous System Observer Design
- Hybrid Observer Design
- Conclusions

Motivations: Power-train control

- Power-train control was formulated as a hybrid control problem:
  - Cut-off control [Automatica, 1995]
  - Fast force transient [CDC, 1998]
- Control algorithms require full state feedback
- It is not economically feasible or even possible to measure the complete state of the system.
- HENCE …we need a hybrid observer!

Previous Results on Observer Design

- Continuous Systems
  - Luenberger [TAC 1971]: Introduction to observers
  - Kalman [ASME 1960]: Optimal disturbance rejection observers
  - Liberzon Hespanha Morse [CDC 1999]: Stability of switched systems
- Discrete Systems
  - Ramadge [CDC 1986]: Current-state observability
  - Caines et al. [CDC 1988]: Current-state tree
  - Ozveren and Willsky [TAC 1989]: Observability with a delay
- Hybrid Systems
  - Ackerson and Fu [TAC 1970], Alessandri and Coletta [HSCC 2001]: Assuming location knowledge
  - Mosterman and Biswas [HSCC 1999]: Model abstractions
  - Morari [TAC 2000]: Hybrid observers

FSM Observability

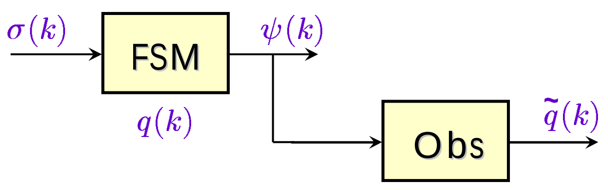
- An FSM (alive) $\mathcal{D} = (Q, \Sigma, \Psi, \varphi, \phi, \eta)$

$$\begin{aligned} q(k+1) &\in \varphi\left(q(k), \sigma(k+1)\right) \\ \sigma(k+1) &\in \phi\left(q(k)\right) \\ \psi(k+1) &\in \eta\left(q(k), \sigma(k+1), q(k+1)\right) \end{aligned}$$

- is said to be current-state observable if there exists an integer K such that
  - for any unknown initial state $q(0)$ and
  - for any input sequence $\sigma(k)$

the state $q(i)$ can be determined for every $i > K$ from the observation sequence $\psi(k)$ up to i.

Observers for FSMs

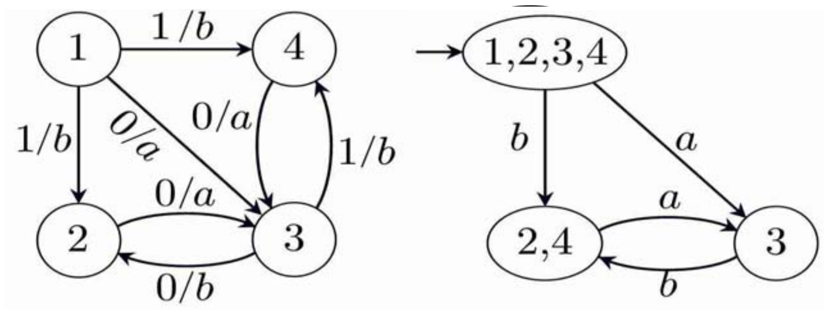


- Given an FSM, an observer is $\mathcal{O} = (Q_{\mathcal{O}}, \Sigma_{\mathcal{O}}, \Psi_{\mathcal{O}}, \varphi_{\mathcal{O}}, \phi_{\mathcal{O}}, \eta_{\mathcal{O}})$

with $Q_{\mathcal{O}} \subseteq 2^Q$, $\Sigma_{\mathcal{O}} = \Psi$, $\Psi_{\mathcal{O}} = Q_{\mathcal{O}}$ and $\eta_{\mathcal{O}} = \varphi_{\mathcal{O}}$

$$\begin{aligned} \tilde{q}(k+1) &= \varphi_{\mathcal{O}}\left(\tilde{q}(k), \psi(k+1)\right) \\ \psi(k+1) &\in \phi_{\mathcal{O}}\left(\tilde{q}(k)\right) \\ \tilde{\psi}(k+1) &= \varphi_{\mathcal{O}}\left(\tilde{q}(k), \psi(k+1)\right) = \tilde{q}(k+1) \end{aligned}$$

Example

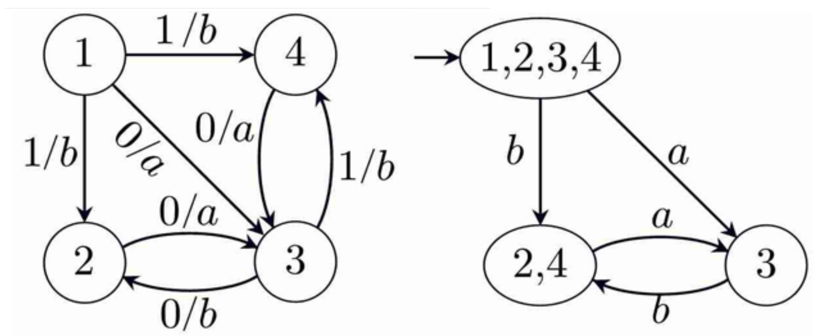


plant FSM *observer FSM*

Theorem: An alive FSM is current-state observable iff there exists a nonempty subset E_O of singletons in the observer FSM such that

- **E_O is invariant**
- **all cycles are contained in E_O**

$Q \cap Q_0$ is not invariant



Algorithm for Current-state Observability

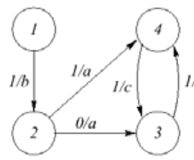
BEGIN
  IF $(S_O = \emptyset) \vee (C_O \not\subseteq S_O)$ THEN
    $\mathcal{D}$ is not current–state observable
    RETURN
  END IF
  $E_O = S_O$
  WHILE $(\mathrm{Pre}(\overline{E}_O) \cap E_O \neq \emptyset)$
    $E_O = E_O \setminus \mathrm{Pre}(\overline{E}_O)$
  END WHILE
  IF $C_O \subseteq E_O$ THEN
    $\mathcal{D}$ is current–state observable
  ELSE
    $\mathcal{D}$ is not current–state observable
  END IF
END

- **Denote by**
  - S_O **the subset of singletons**
  - C_O **the subset of states in cycles**
  - E_O **a subset of singletons**

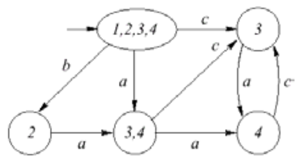
The algorithm

- **computes of the maximal set of singletons that is invariant**
- **tests if contains all the cycles**

Examples of Current State Observable FSMs



plant FSM

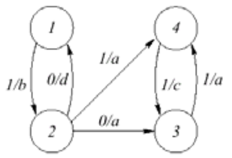


observer FSM

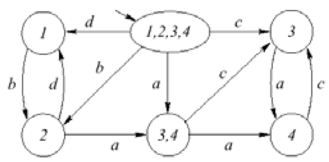
current-state obser.

- S_O = { {2}, {3},{4} }
- C_O = { {3}, {4} }
- E_O = { {3}, {4} }

E_O is contained in C_O



plant FSM



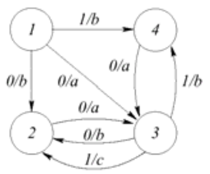
observer FSM

not current-state obser.

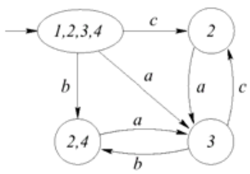
- S_O = {{1},{2},{3},{4}}
- C_O = {{1},{2},{3},{4}}
- E_O = { {3}, {4} }

E_O is NOT contained in C_O

Examples of Current State Observable FSMs



plant FSM

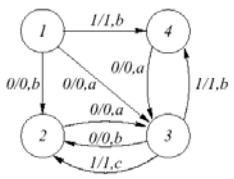


observer FSM

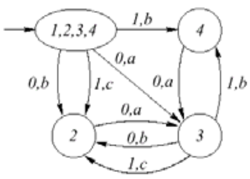
not current-state obser.

- S_O = { {2}, {3} }
- C_O = { {3}, {4}, {2,4} }
- E_O = { }

C_O is not contained in S_O



plant FSM



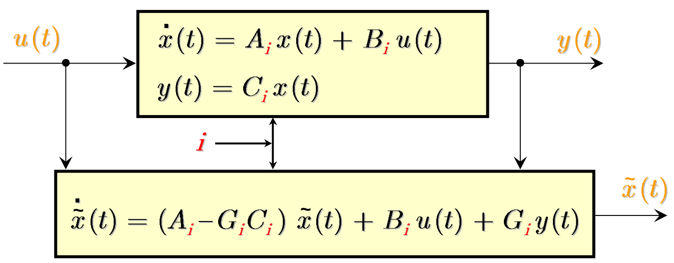
observer FSM

current-state obser. with inputs measurement

- S_O = {{2},{3},{4}}
- C_O = {{2},{3},{4}}
- E_O = {{2},{3},{4}}

E_O is contained in C_O

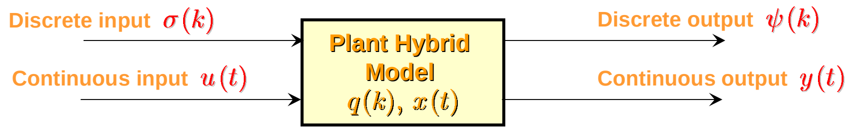
Switching Observers for LTI Systems



The switching observer is globally exponentially stable, if

- **either the Lie algebra $\{F_i = A_i - G_i C_i\}$ is solvable**
  - **pairwise commuting $\Rightarrow$ nilpotent $\Rightarrow$ solvable Lie algebra**
- **or there is a dwell time $\tau_D > \sup_{p \in \mathcal{P}} \left\{ \frac{\log c_p}{\mu_p} \right\}$ where $\|e^{A_p t}\| \leq c_p e^{-\mu_p t}$**

State Observation of Hybrid Systems



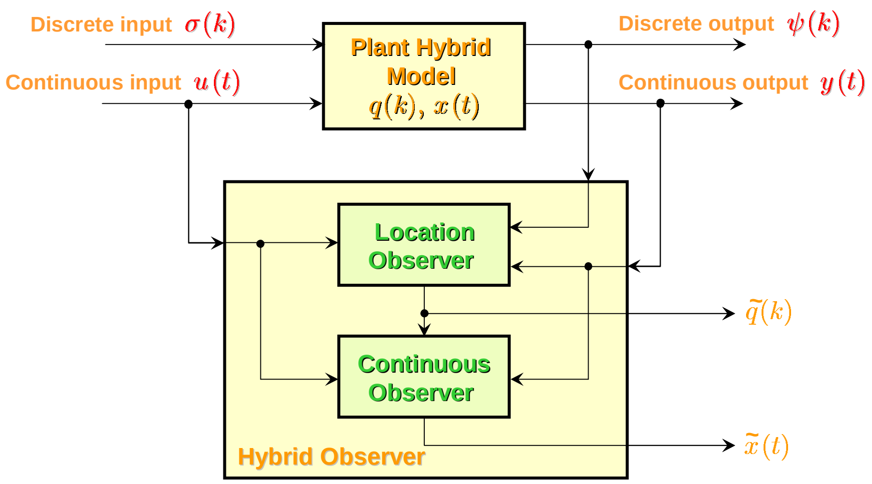
$$\mathcal{H} = (Q, \Sigma, \Psi, \varphi, \phi, \eta, X, U, \Omega, Y, f, h, r)$$

$$\begin{aligned} q(k+1) &\in \varphi\left(q(k), \sigma(k+1)\right) \\ \sigma(k+1) &\in \phi\left(q(k), x(t_{k+1}^-), u(t_{k+1}^-)\right) \\ \psi(k+1) &\in \eta\left(q(k), \sigma(k+1), q(k+1)\right) \end{aligned}$$

$$\begin{aligned} \dot{x}(t) &= f(q_i, x(t), u(t), w(t)) = A_i\, x(t) + B_i\, u(t) + w(t) \\ y(t) &= h(q_i, x(t)) = C_i\, x(t) \end{aligned}$$

$$x(t_k) = x(t_k^+) = r(q_i, q_j, x(t_k^-)) = R_{ij}^1\, x(t_k^-) + R_{ij}^0$$

Hybrid Observer Scheme



Specification: Exponential Ultimate Boundedness

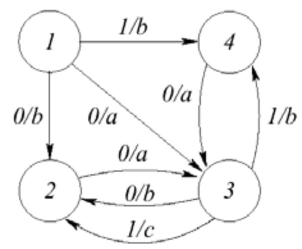
- A hybrid observer is said to be **exponentially ultimately bounded** if there exist a positive integer K and constants $c \geq 1$, $\mu > 0$ and $b \geq 0$, such that

$$
\begin{aligned}
\tilde{q}(k) &= q(k) && \forall k \geq K, \\
\|\tilde{x}(t) - x(t)\| &\leq c\|\tilde{x}(t_K) - x(t_K)\| \; e^{-\mu t} + b && \forall t > t_K.
\end{aligned}
$$

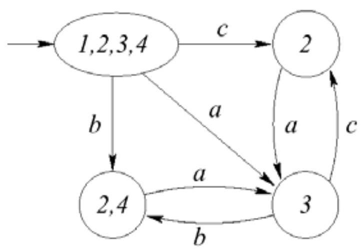
for any hybrid initial state and plant inputs.

  - μ is the **rate of convergence**
  - b is the **ultimate bound**
- if $b = 0$, the observer is said to be **exponentially convergent**.

Location Observer Design for Hybrid Plants



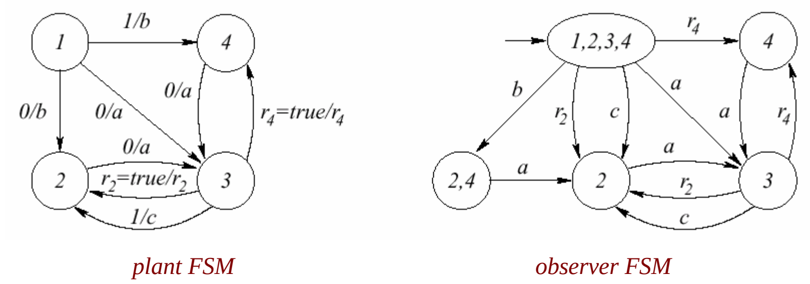
plant FSM



observer FSM

- The plant is not current-state observable: E_O is empty .
- Current state observability can be achieved if the **difference between the CT dynamics** in 2 and 4 can be identified
  - ▲ some additional **discrete outputs** can be obtained from the **CT evolution**
- Signatures are used to detect the CT dynamic parameters

Location Observer: Exploiting CT Plant Evolution

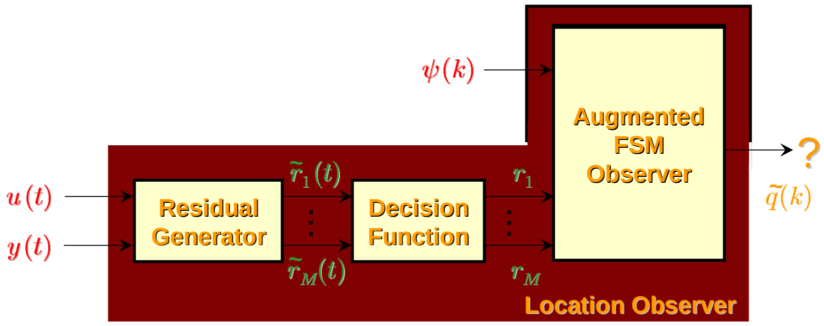


Processing of continuous plant signals cannot be done in zero time. Assume that

- the signature corresponding to the current location of the hybrid plant becomes true before the next transition of the hybrid plant
- all the other signatures (if any) associated to the outgoing arcs of the current observer location remain false

Location Observer Scheme

- Signature Generation is similar to the Failure Detection and Identification problem:
- “ is the system obeying some given dynamics ? ”



[Massoumnia, Verghese, Willsky, TAC 1989]

Residual Generator and Decision Function

Residual generator:

$$\dot{z}_j(t) = (A_j - L_j C_j) z_j(t) + B_j u(t) + L_j y(t)$$
$$\tilde{r}_j(t) = C_j z_j(t) - y(t)$$

Decision function:

$$r_j(t) = \begin{cases} true & \text{if } \|\tilde{r}_j(t)\| \leq \varepsilon \\ false & \text{if } \|\tilde{r}_j(t)\| > \varepsilon \end{cases}$$

Proposition 10 *For a given $\Delta > 0$, $\varepsilon > 0$ and a given upper bound Z_0 on $\|x - z_i\|$, if the estimator gains L_i in (23) are chosen such that $A_i - L_i C_i$ have distinct eigenvalues and*

$$\frac{\alpha(A_i - L_i C_i)}{k(A_i - L_i C_i)} < -\frac{\sqrt{n}\|C_i\| W}{\varepsilon} \qquad (26)$$

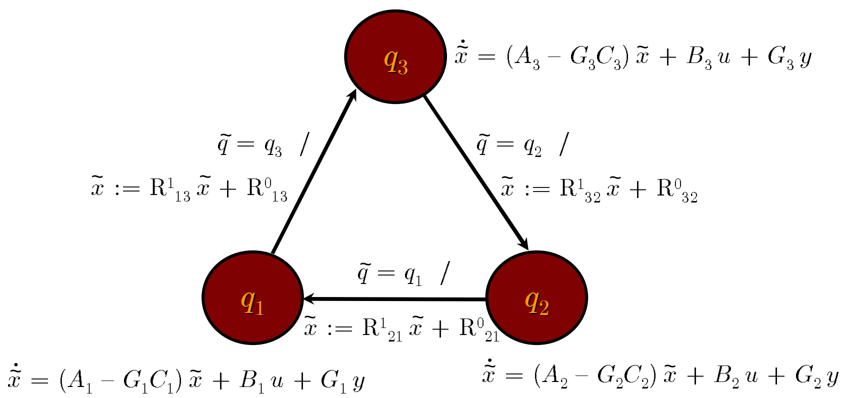
$$-\frac{1}{\alpha(A_i - L_i C_i)} \log \frac{k(A_i - L_i C_i)\,\|C_i\|\, Z_0}{\varepsilon + \sqrt{n}\|C_i\| \frac{k(A_i - L_i C_i)}{\alpha(A_i - L_i C_i)} W} \leq \Delta \qquad (27)$$

then r_i becomes true before a time Δ elapses after a change in the plant dynamics parameters to the values (A_i, B_i, C_i), and it remains true till the next transition of the hybrid plant.

Proposition 11 *Let $m = p$, if the matrix $(C_j - C_i)B_i + C_j(B_i - B_j)$ is invertible, with $i \neq j$, then for any hybrid plant initial condition, the class of plant inputs $u(t)$ that achieve $r_j(t) = true$ for all $t \geq t'$, with $t' < t_k + \Delta$, after a change in the plant dynamics parameters to (A_i, B_i, C_i) at some time t_k is not empty.*

Continuous Observer for Hybrid Plants

- A bank of Luenberger's observers with resets and switchings controlled by the identified plant location



Continuous Observer: Exploiting Discrete Plant Evolution

- Consider an event $\overline{\sigma}$ that can be instantaneously identified
  - location transition identified without signature
  - invertibility of $\psi(k) \in \eta(q_i, \sigma, q_j)$.
- If the system below admits unique solution

$$\begin{array}{rcl} C_i\, x & = & y(t_k^-) \\ \overline{\sigma} & \in & \phi(q_i, x, u(t_k^-)) \end{array}$$

then the continuous state can be instantaneously identified.

- For linear guards $D_{ij}x + E_{ij} = 0$, we have $\begin{bmatrix} D_{ij} \\ C_i \end{bmatrix} x = \begin{bmatrix} -E_{ij} \\ y(t_k^-) \end{bmatrix}$

Hybrid Observer

- FSM: Location observer
- CT: Continuous observer

$$\begin{cases} \dot{\tilde{x}}(t) = 0 & \text{if } \tilde{q} \in Q_{\mathcal{O}} \setminus E_{\mathcal{O}} \\ \dot{\tilde{x}}(t) = (A_i - G_i C_i)\, \tilde{x}(t) + B_i u(t) + G_i y(t) & \text{if } \tilde{q} = \{q_i\} \in E_{\mathcal{O}} \end{cases}$$

$\tilde{x}(\hat{t}_k) = R^1_{ij}\, x(\hat{t}_k^-) + R^0_{ij}$ — for CS instantaneous identification

$\tilde{x}(\hat{t}_k) = \tilde{x}(\hat{t}_k^+) = R^1_{ij}\, \tilde{x}(\hat{t}_k^-) + R^0_{ij}$ — otherwise

Plant + Observer Hybrid System

$$\dot{x}(t) = A_i\,x(t) + B_i\,u(t) + w(t) \qquad \text{if } q = q_i$$

$$\begin{cases} \dot{\zeta}(t) = F_i\,\zeta(t) - w(t) & \text{if } \tilde{q} = \{q_i\} \\ \dot{\zeta}(t) = F_j\,\zeta(t) + v_{ji}(t) - w(t) & \text{if } \tilde{q} \neq \{q_i\} \end{cases}$$

$$v_{ji}(t) = [(A_j - A_i) - G_j(C_j - C_i)]\,x(t) + (B_j - B_i)u(t) \qquad F_j = A_j - G_jC_j$$

▲ plant transition

$$(q_j, \{q_\ell\}) \rightarrow (q_i, \{q_\ell\})$$

$$\begin{aligned} x(t_k) &= x(t_k^+) = R_{ji}^1\,x(t_k^-) + R_{ji}^0 \\ \zeta(t_k) &= \zeta(t_k^+) = \zeta(t_k^-) - R_{ji}^0 + [I - R_{ji}^1]\,x(t_k^-) \end{aligned}$$

▲ observer transition

$$(q_\ell, \{q_j\}) \rightarrow (q_\ell, \{q_i\})$$

$$\zeta(\hat{t}_k) = \zeta(\hat{t}_k^+) = R_{ji}^1\,\zeta(\hat{t}_k^-) + R_{ji}^0 - [I - R_{ji}^1]\,x(\hat{t}_k)$$

▲ plant and observer transition

$$(q_j, \{q_j\}) \rightarrow (q_i, \{q_i\})$$

$$x(t_k) = x(t_k^+) = R_{ji}^1\,x(t_k^-) + R_{ji}^0$$

$$\zeta(t_k) = \zeta(t_k^+) = R_{ji}^1\,\zeta(t_k^-) \qquad \text{or} \qquad \zeta(t_k) = 0$$

Main Result

Theorem 12 *Given a hybrid system $\mathcal{H}_P$ as in (10–16) that is current–location observable via signatures, with dwell time D and such that matrices A_i in (13) have distinct eigenvalues for each i such that $\{q_i\} \in E_{\mathcal{O}}$, if for each $\{q_i\} \in E_{\mathcal{O}}$ there exists a gain matrix G_i such that*

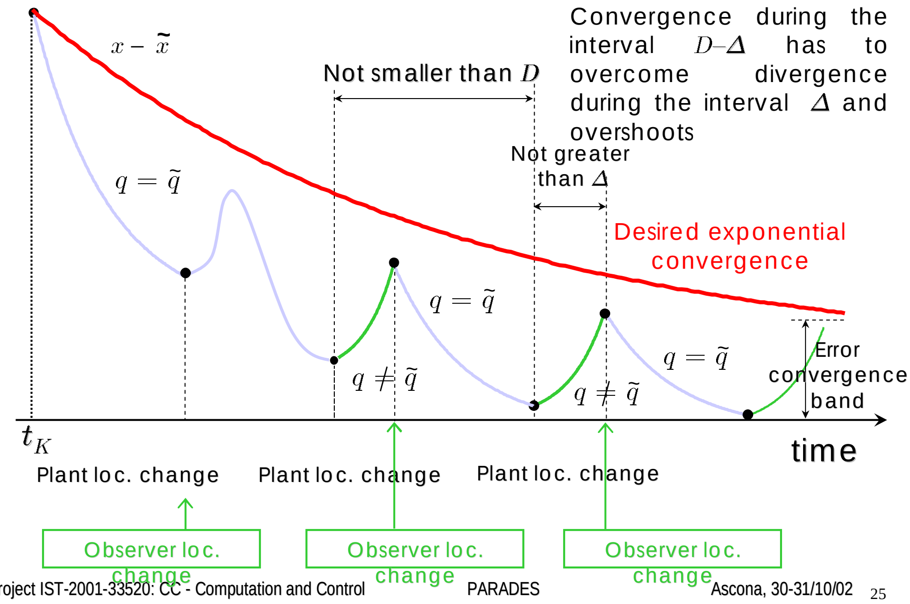
1. *$A_i - G_iC_i$ has distinct eigenvalues;*
2. *the location observer identifies a change in the hybrid system location within time Δ with $0 \leq \Delta \leq D$;*
3. $\alpha(A_i - G_iC_i) + \dfrac{\max\{0, \log[r_i^1\,k(A_i - G_iC_i)]}{D - \Delta} \leq -\mu < 0$

where $r_i^1 = \max_{q_j \in Reach(q_i)} \|R_{ij}^1\|$, then the hybrid observer $\mathcal{H}_O$ is exponentially ultimately bounded with rate of convergence μ.

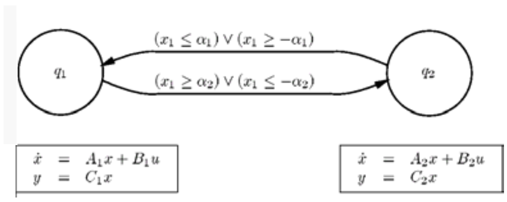
In the case of absence of disturbances and continuous state resets, any desired value for the ultimate bound can be achieved by choosing Δ small enough. Otherwise, the ultimate bound cannot be lower than a minimum threshold value.

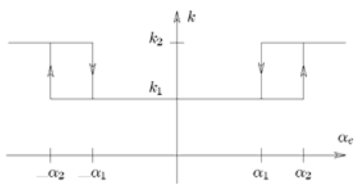
$$\|w(t)\|_\infty = \max_{i=1,\dots,n} \sup_{t \geq 0} |w_i(t)| \leq W$$

Exponential Ultimate Boundedness

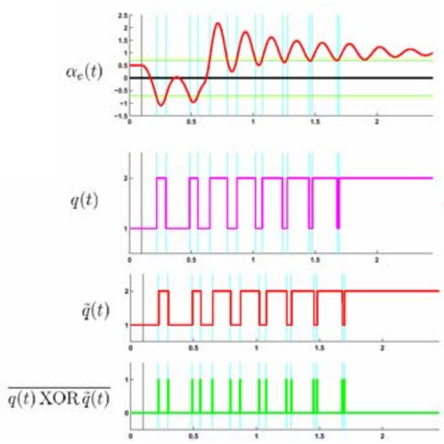


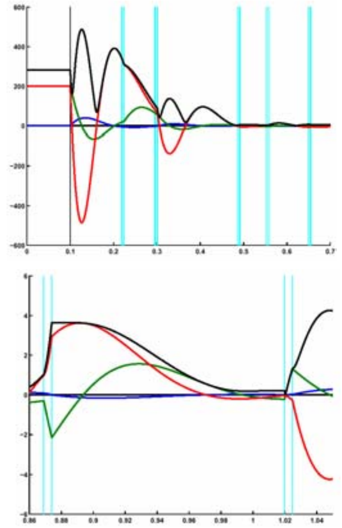
Engine Control Application





Simulation Results





Conclusions

- A design methodology for hybrid observers has been proposed
- This methodology has been recently extended to hybrid plant with subject to continuous state disturbances
- Techniques exploiting information associated to discrete transitions detection has been investigated in order to
  - improve continuous state convergence
  - estimate unobservable continuous state components at transition times
- Simulation results for automotive driveline estimation have been obtained.