ABB Case Study: Optimal Control and Analysis

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ABB

Outline

1. Overview
2. Modeling
3. Optimal Control Problem
4. Sensitivity Analysis
5. Conclusions and Outlook

1. **Overview**
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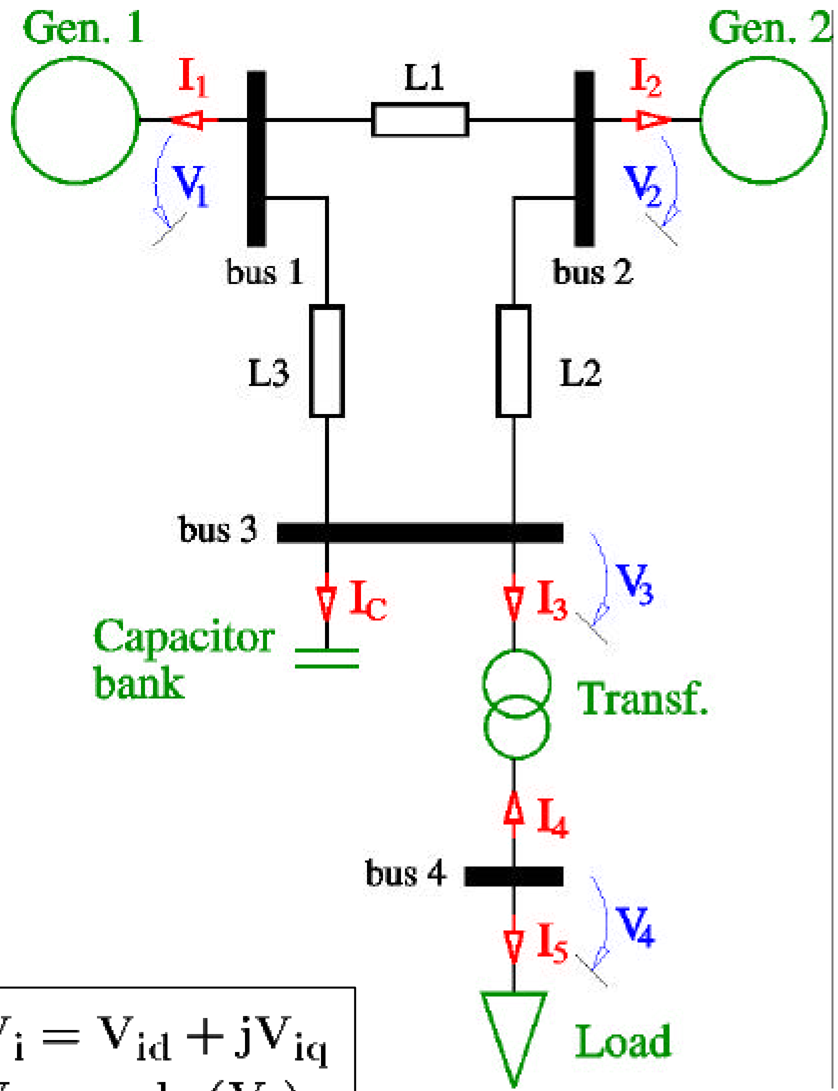
ABB Case Study: Overview



$$V_i = V_{id} + jV_{iq}$$
$$V_{im} = \mathrm{abs}(V_i)$$

Generator 1:
- infinite bus

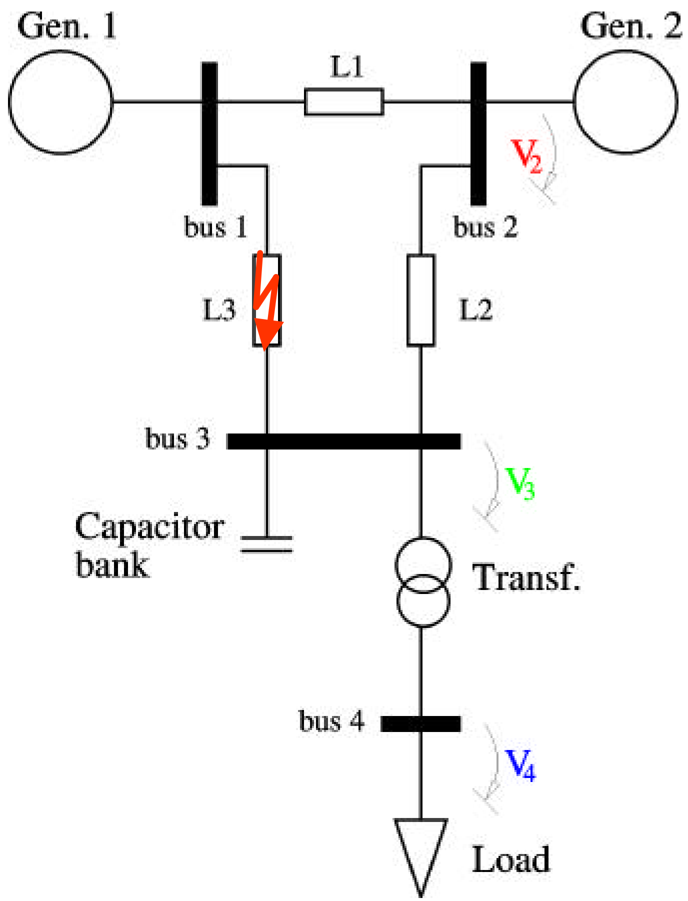
Generator 2:
- internal controller (AVR)

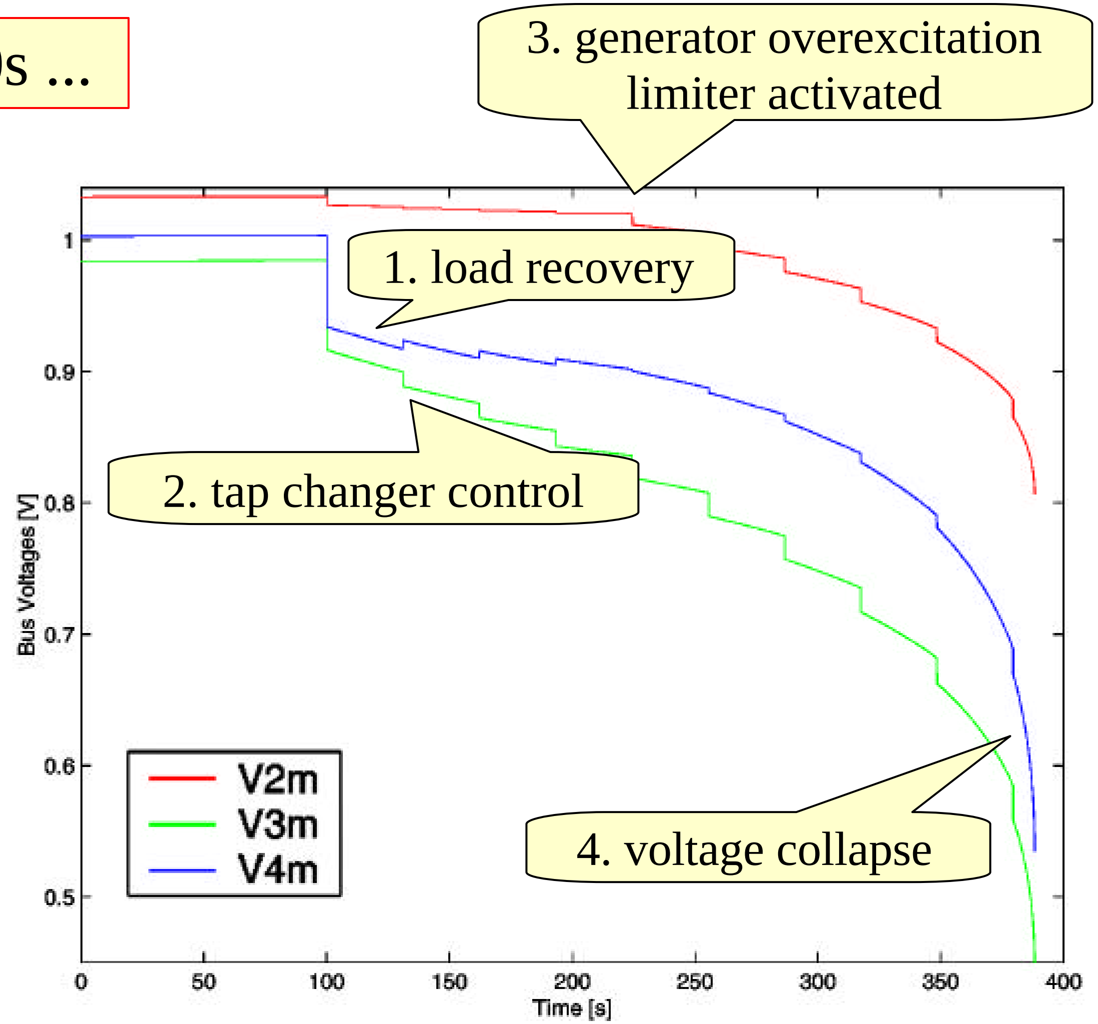
Transformer:
- internal controller (FSM)

Load:
- aggregate dynamic load

Collapse Scenario

Line tripping (L3) at t=100s ...





... leads to voltage collapse.

Why do we need Control?

- Power system designed for N-1 stability
- Power system collapse extremely expensive
- Time-constants rather small
- Power system operated closer to stability limit because of
  - deregulation of energy market
  - environmental concerns
  - increase of electric power demand

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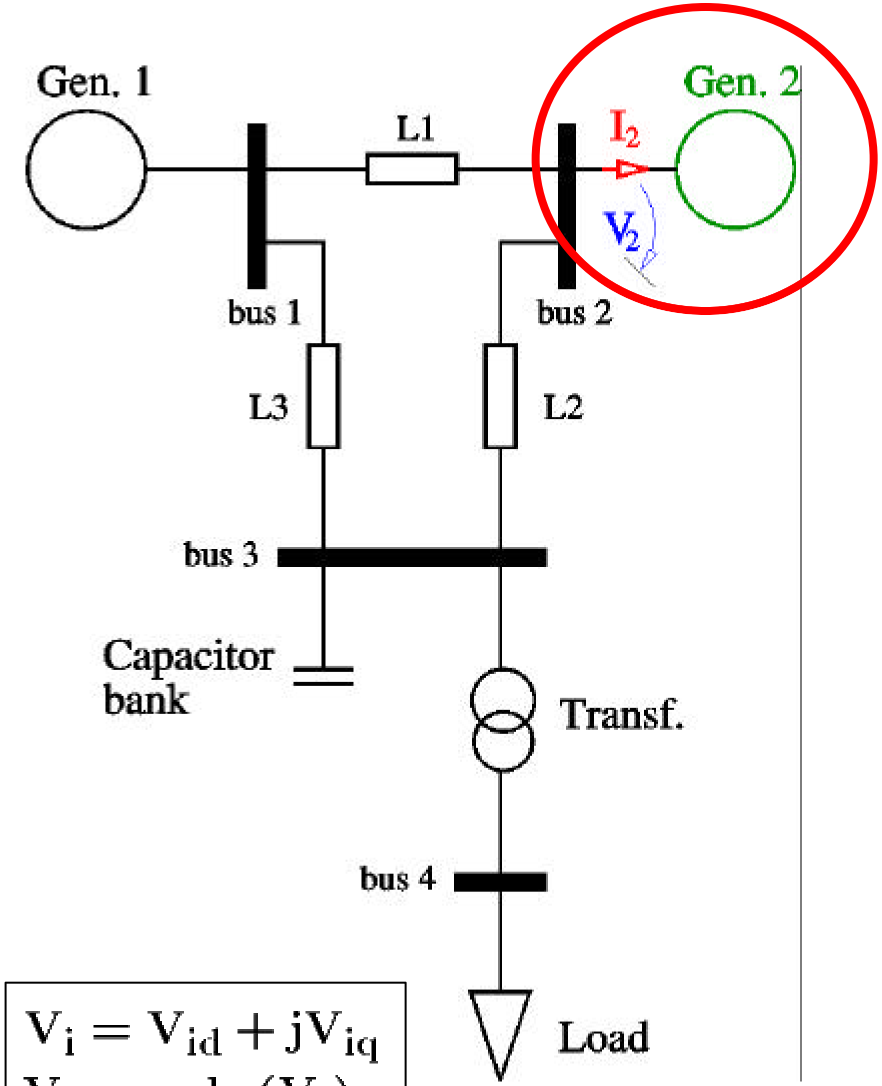
Generator 2: Overview



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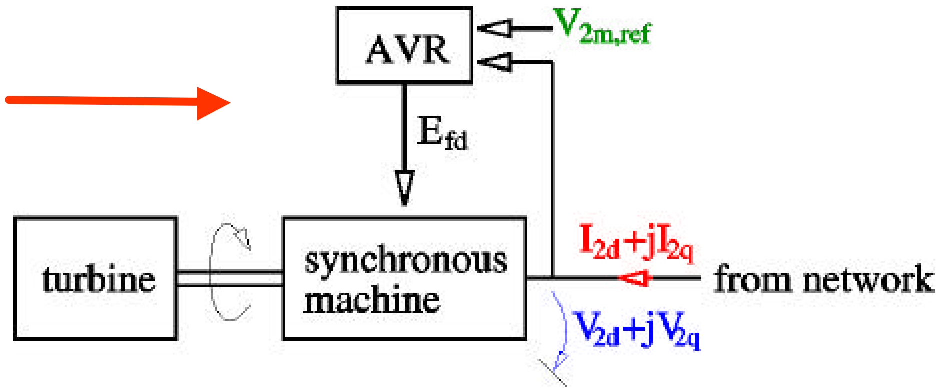
$$V_{im} = \mathrm{abs}(V_i)$$



purpose:

- generates limited power

assume:

- quasi-steady state behaviour

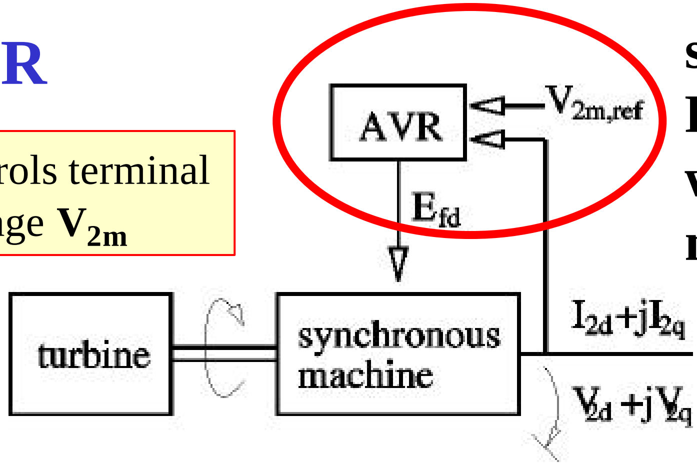
2 (static) components:

- automatic voltage regulator (AVR)
- synchronous machine

Generator 2: Model

AVR

controls terminal voltage V_{2m}

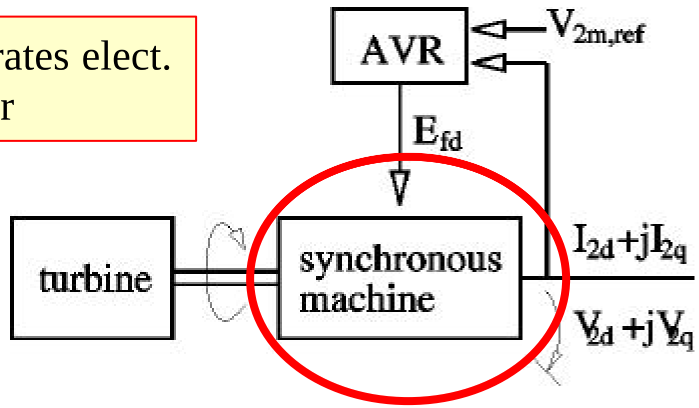


saturating P-controller with input nonlinearity

$$
\begin{aligned}
V_{2m} &= \sqrt{V_{2d}^2 + V_{2q}^2} \\
dE_{fd} &= 50(V_{2m,ref} - V_{2m}) \\
E_{fd} &= min(E_{f,max},\ dE_{fd} + E_{f0})
\end{aligned}
$$

Synchronous machine

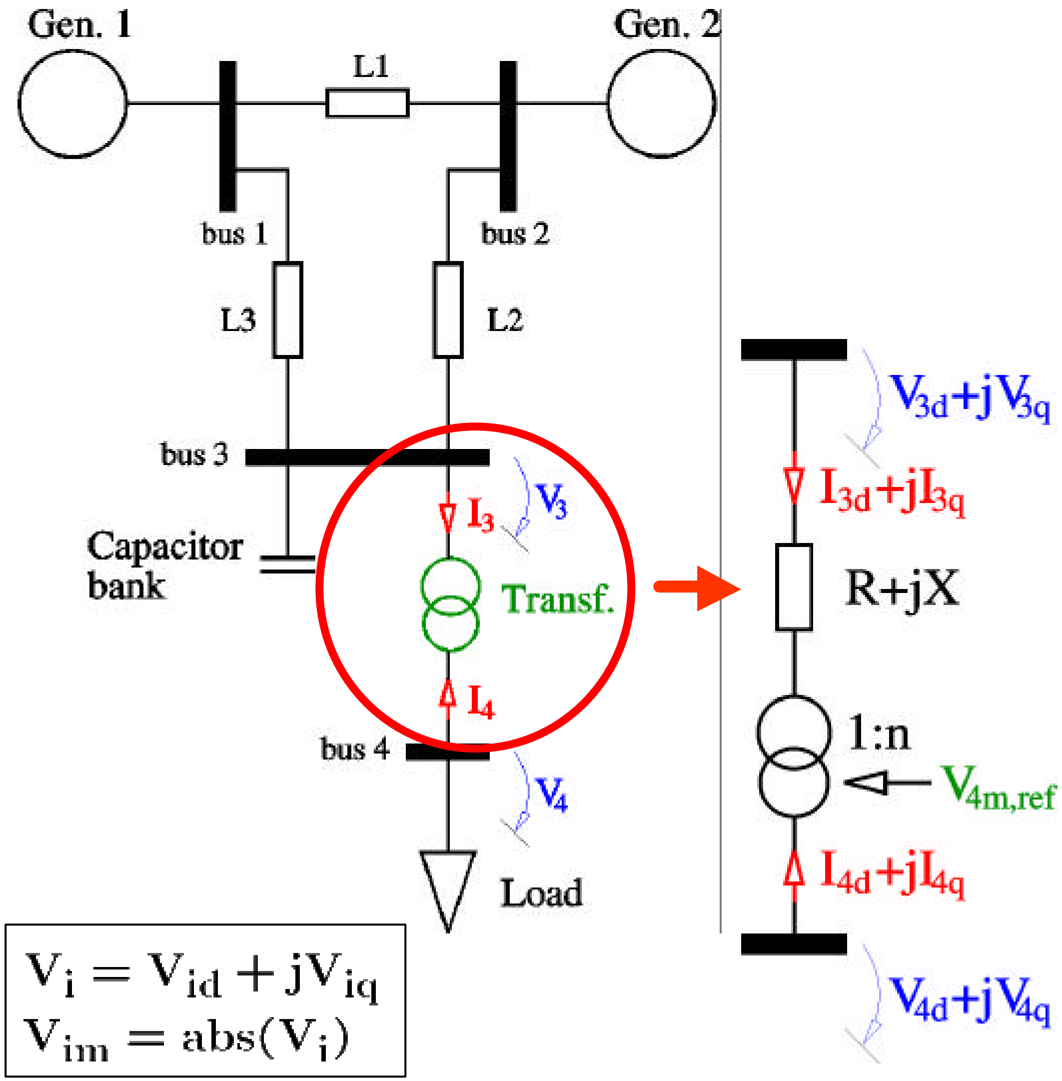
generates elect. power



static I/O-behaviour

$$
\begin{aligned}
P_{m0} &= E_{fd} i_q + (x_d - x_q) i_d i_q \\
v_d &= -x_q i_q \\
v_q &= E_{fd} + x_d i_d \\
-I_{2d} &= -sin(\delta) i_d + cos(\delta) i_q \\
-I_{2q} &= cos(\delta) i_d + sin(\delta) i_q \\
V_{2d} &= -sin(\delta) v_d + cos(\delta) v_q \\
V_{2q} &= cos(\delta) v_d + sin(\delta) v_q
\end{aligned}
$$

Transformer: Overview



$$V_i = V_{id} + jV_{iq}$$
$$V_{im} = \mathrm{abs}(V_i)$$

purpose:

- steps down voltage
- controls load-voltage V_{4m} by adjusting tap ratio 1:n

2 components:

- transformer
- controller of tap ratio

manipulated variable:

- voltage reference $V_{4m,ref}$

Transformer: Controller

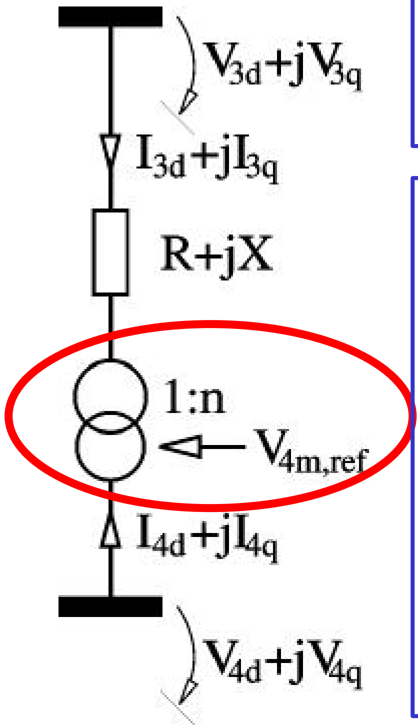
input nonlinearity: $V_{4m} = \sqrt{V_{4d}^2 + V_{4q}^2}$

logic:

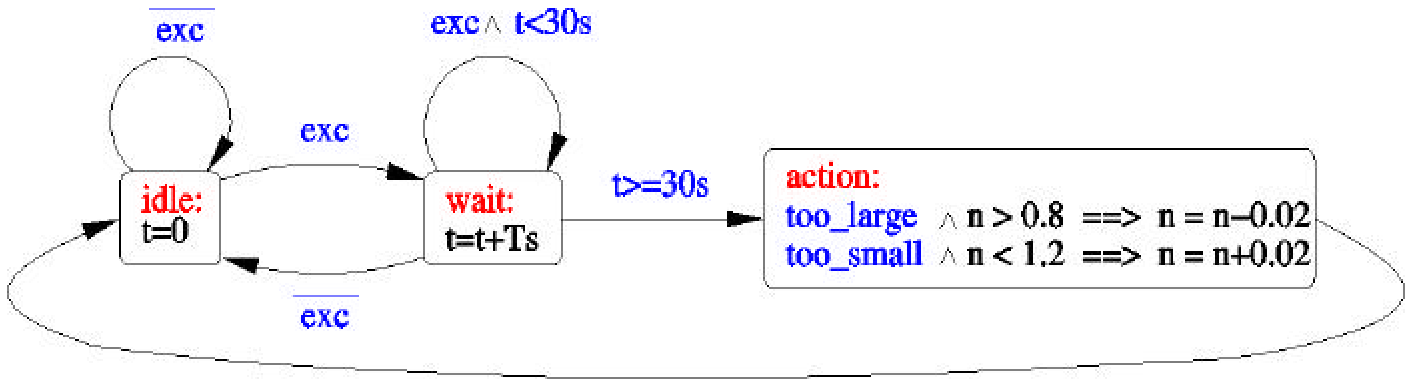
$$too\_large = \begin{cases} 1, & \text{if } V_{4m} > V_{4m,ref} + V_{4m,err} \\ 0, & \text{else} \end{cases}$$

$$too\_small = \begin{cases} 1, & \text{if } V_{4m} < V_{4m,ref} - V_{4m,err} \\ 0, & \text{else} \end{cases}$$

$$exc = too\_large \vee too\_small$$



finite state machine:



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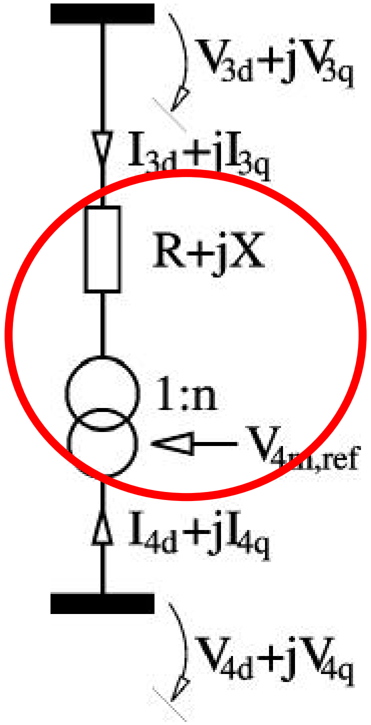
Transformer: Equations

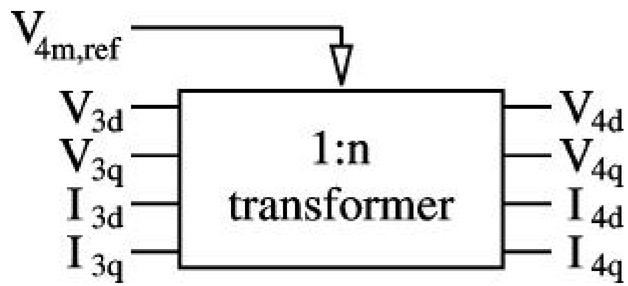


transformer equations:

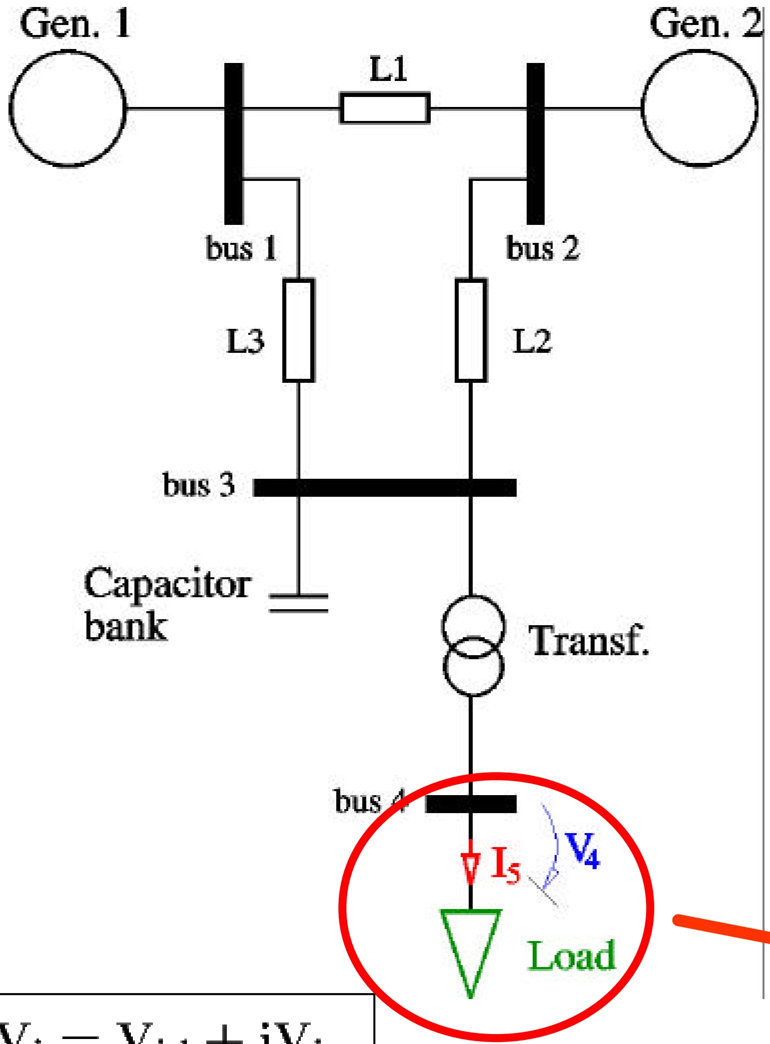
- nonlinear equations
- relate $\mathbf{V_3}$, $\mathbf{V_4}$, $\mathbf{I_3}$, $\mathbf{I_4}$ depending on **n**

$$
\begin{aligned}
V_{3d} &= \frac{1}{n}V_{4d} - RnI_{4d} + XnI_{4q} \\
V_{3q} &= \frac{1}{n}V_{4q} - XnI_{4d} - RnI_{4q} \\
I_{3d} &= -nI_{4d} \\
I_{3q} &= -nI_{4q}
\end{aligned}
$$

block diagram:



Load: Overview



$V_i = V_{id} + jV_{iq}$

$V_{im} = \text{abs}(V_i)$

purpose:

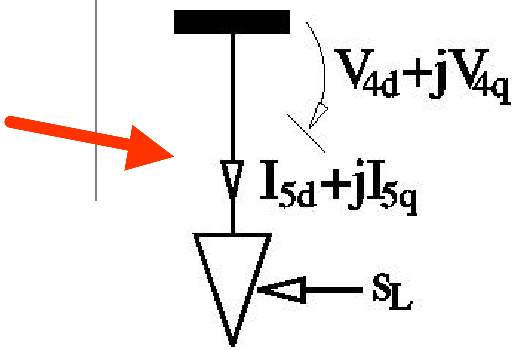
- power consumption

assume: *aggregate dynamic* load

- aggregate: distribution network with various loads (motors, heating, lighting)
- dynamic: self-restoring following a disturbance

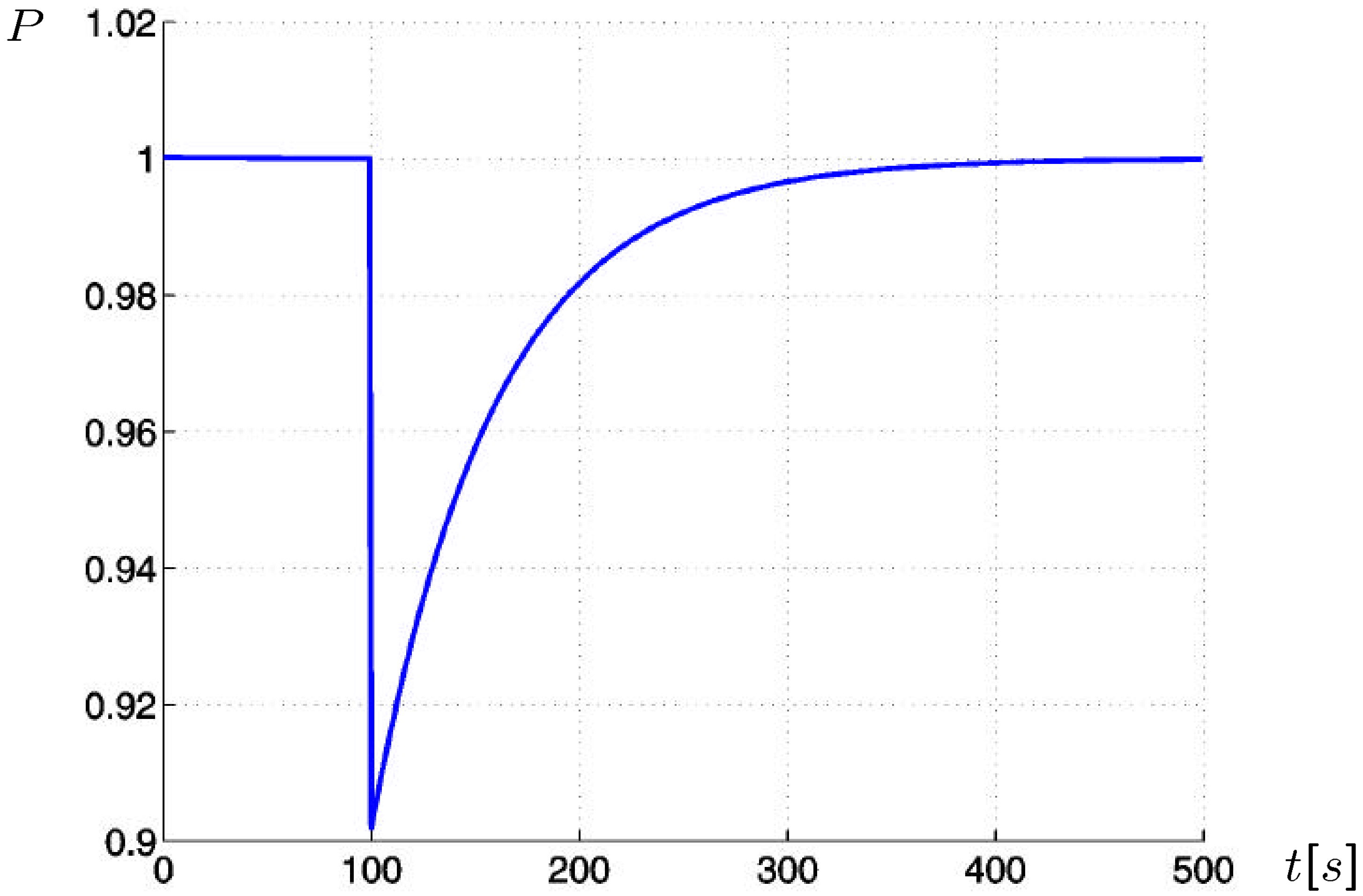
manipulated variable:

- load shedding s_L



Load: Self Restoration

Following a disturbance in the supply voltage, the active and reactive powers drawn by the load are restored by internal controllers (like thermostats).



Load: Model

active power:

$$\dot{x}_p = -\frac{x_p}{T_p} + P_{L0}(1 - V_{4m}^2)$$

$$P_L = (1 - 0.05 \cdot s_L)(\frac{x_p}{T_p} + P_{L0} \cdot V_{4m}^2)$$

reactive power:

$$\dot{x}_q = -\frac{x_q}{T_q} + Q_{L0}(1 - V_{4m}^2)$$

$$Q_L = (1 - 0.05 \cdot s_L)(\frac{x_q}{T_q} + Q_{L0} \cdot V_{4m}^2)$$

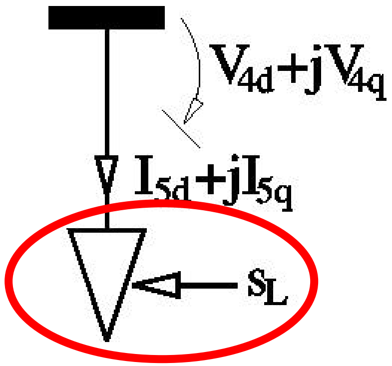
where $V_{4m}^2 = V_{4d}^2 + V_{4q}^2$

and s_L = load shedding (discrete manipulated var.)

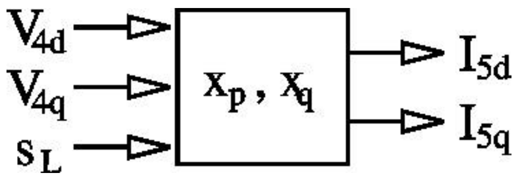
output equations:

$$I_{5d} = \frac{P_L V_{4d} + Q_L V_{4q}}{V_{4d}^2 + V_{4q}^2}$$

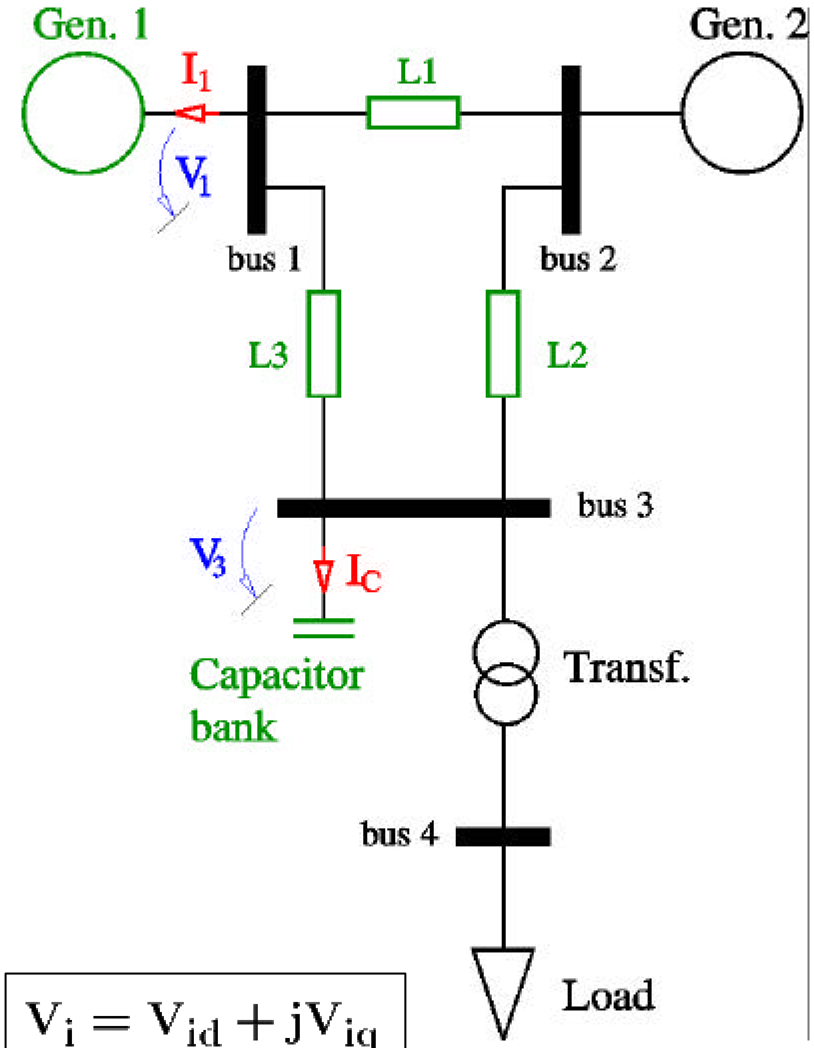
$$I_{5q} = \frac{P_L V_{4q} - Q_L V_{4d}}{V_{4d}^2 + V_{4q}^2}$$



block diagram:



Linear Submodels



$$V_i = V_{id} + jV_{iq}$$
$$V_{im} = \text{abs}(V_i)$$

generator 1:

- infinite bus
- supplies constant voltage $\mathbf{V_1} = 1.03$

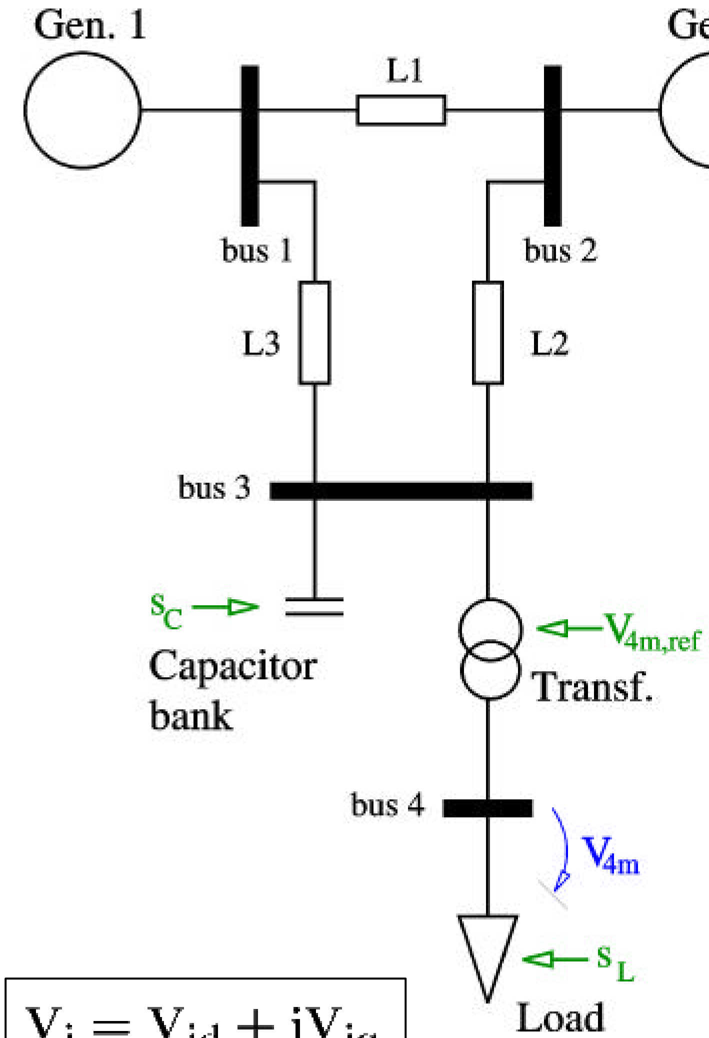
capacitor bank:

- stabilizes power system
- discrete manipulated variable

network:

- algebraic equations at the buses according to Kirchhoff's laws

Hybrid Model



$$V_i = V_{id} + jV_{iq}$$
$$V_{im} = \mathrm{abs}(V_i)$$

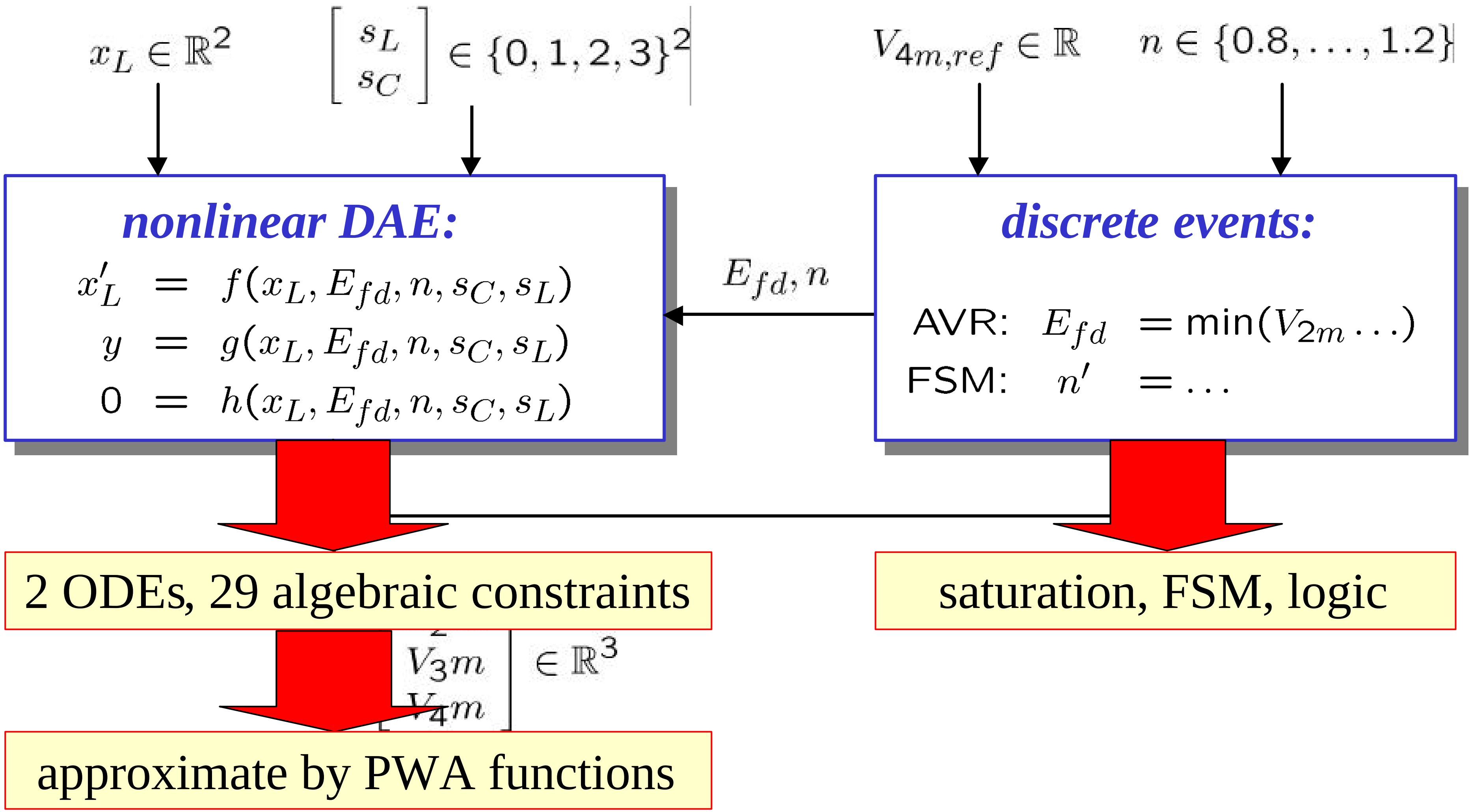
Hybrid system:

- saturation
- finite state machine with logic
- nonlinearities → pwa functions
- discrete manipulated variables

Dimensions:

- 2 ordinary diff. equations
- 29 algebraic equations:
  - 11 linear, 18 nonlinear
- 3 states:
  - 2 continuous, 1 discrete
- 3 manipulated variables:
  - 1 continuous, 2 discrete

Hybrid Model (ctd.)



MLD Formulation

$$\begin{aligned} x(t+1) &= Ax(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \\ y(t) &= Cx(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t) \\ E_2 \delta(t) + E_3 z(t) &\leq E_4 x(t) + E_1 u(t) + E_5 \end{aligned}$$

where: $\delta(t) \in \{0,1\}^{n_d}$ auxiliary binary variables

$z(t) \in R^{n_z}$ auxiliary continuous variables

if problem is ***well-posed***:

for a given $x(t)$ and $u(t)$ the inequality

$$E_2 \delta(t) + E_3 z(t) \leq E_4 x(t) + E_1 u(t) + E_5$$

defines uniquely $\delta(t)$ and $z(t)$.

… leads to 49 δ , 86 z variables and 409 constraints

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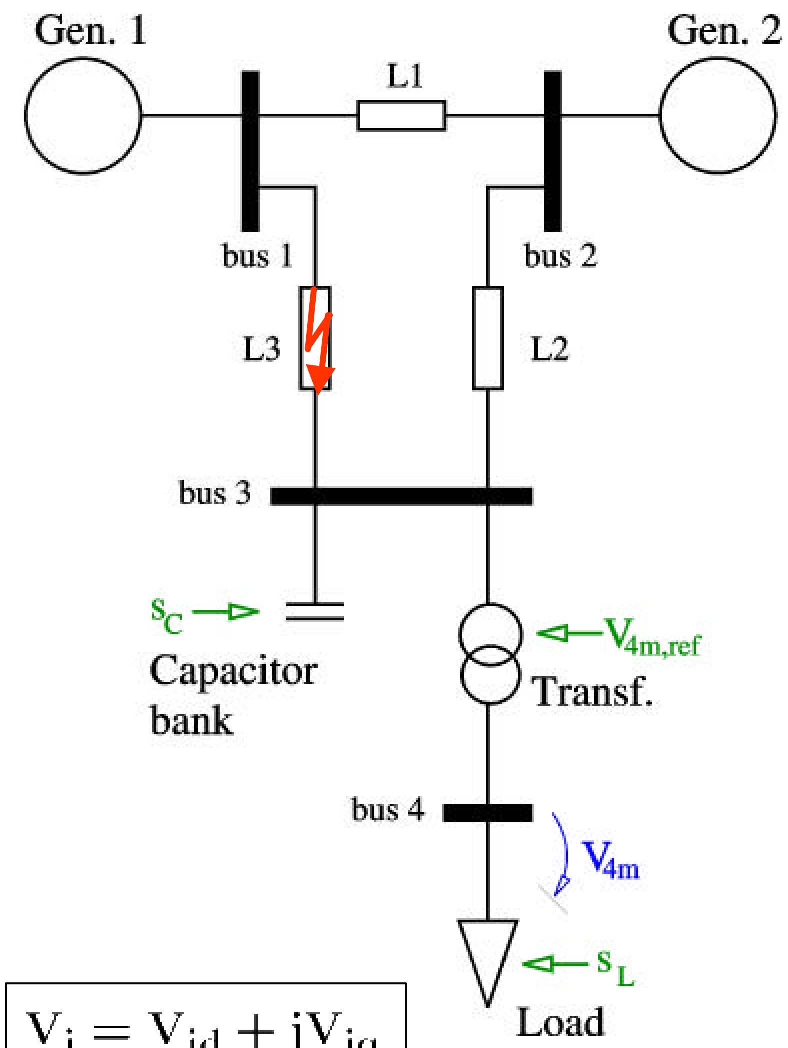
Control Problem



$$V_i = V_{id} + jV_{iq}$$
$$V_{im} = abs(V_i)$$

- **Control objectives:**
  - stabilize V_{4m}
  - min. load shedding s_L
  - keep bus voltages within certain limits V_{2m}, V_{3m}, V_{4m}
- **Manipulated variables:**
  - ultc voltage reference: $V_{4m,ref}$
  - capacitor switching: s_C
  - load shedding: s_L
- **Fault:**
  - line outage

Model Predictive Control

$$J = \sum_{k=0}^{N-1} \left(\|V_{4m}(t+k|t) - 1\| + \|\Delta u(t+k|t)\|_{R,\infty} \right) + \sum_{k=1}^{N-1} S(t+k|t)$$

subject to

- **MLD model:**

$$\begin{aligned} x(t+1) &= Ax(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \\ y(t) &= Cx(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t) \\ E_2 \delta(t) + E_3 z(t) &\leq E_4 x(t) + E_1 u(t) + E_5 \end{aligned}$$

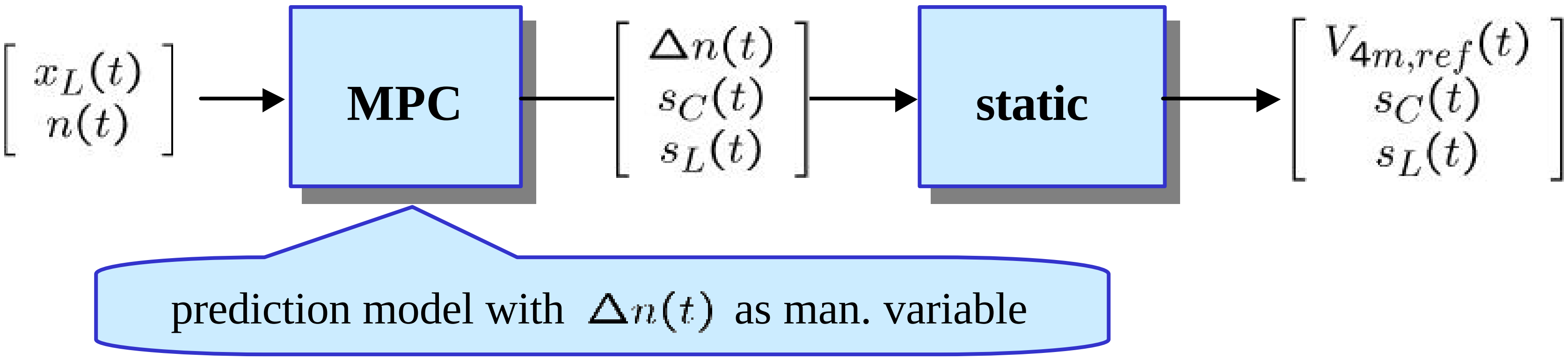
- **Soft constraints on bus voltages:**

$$\begin{aligned} V_{2m} &\in [0.95..1.05] \\ V_{3m}, V_{4m} &\in [0.9..1.1] \end{aligned}$$

Controller Structure

Controller of tap changer highly sensitive to $V_{4m,ref}$

→ Decompose MPC in cascaded controller



- MPC sets tapping strategy $\Delta n(t)$
- Static controller chooses $V_{4m,ref}$ accordingly
- Mechanical wear results from tap changes

Tuning of Cost Function

Penalty on u:

$$J = \sum_{k=0}^{N-1} \left(\|V_{4m}(t+k|t)-1\| + \|\Delta u(t+k|t)\|_{R,\infty} \right) + \sum_{k=1}^{N-1} S(t+k|t)$$

$$= \begin{bmatrix} \Delta n \\ \Delta s_C \\ \Delta s_L \end{bmatrix}$$

choose R such that:

➢ *nominal control* (no constraint violated):

allow ultc voltage reference and capacitor bank

➢ *emergency control* (constraints violated):

allow all controls including load-shedding

Tuning of Cost Function (ctd.)

Penalty on violation of soft constraints:

$$J = \sum_{k=0}^{N-1} \left(\|V_{4m}(t+k|t) - 1\| + \|\Delta u(t+k|t)\|_{R,\infty} \right) + \sum_{k=1}^{N-1} S(t+k|t)$$

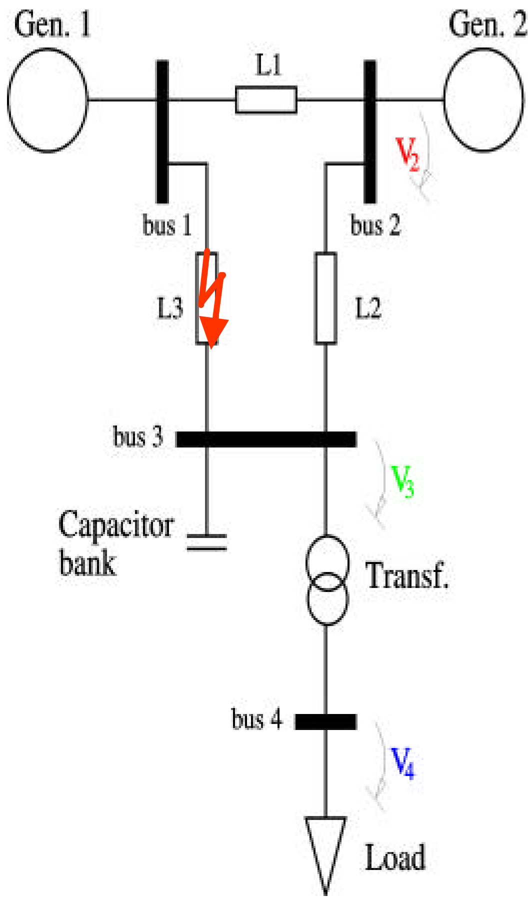
$$= p \sum_{i=2}^{4} \left\{ b_i(t+k|t) + sl_i(t+k|t) \right\}$$

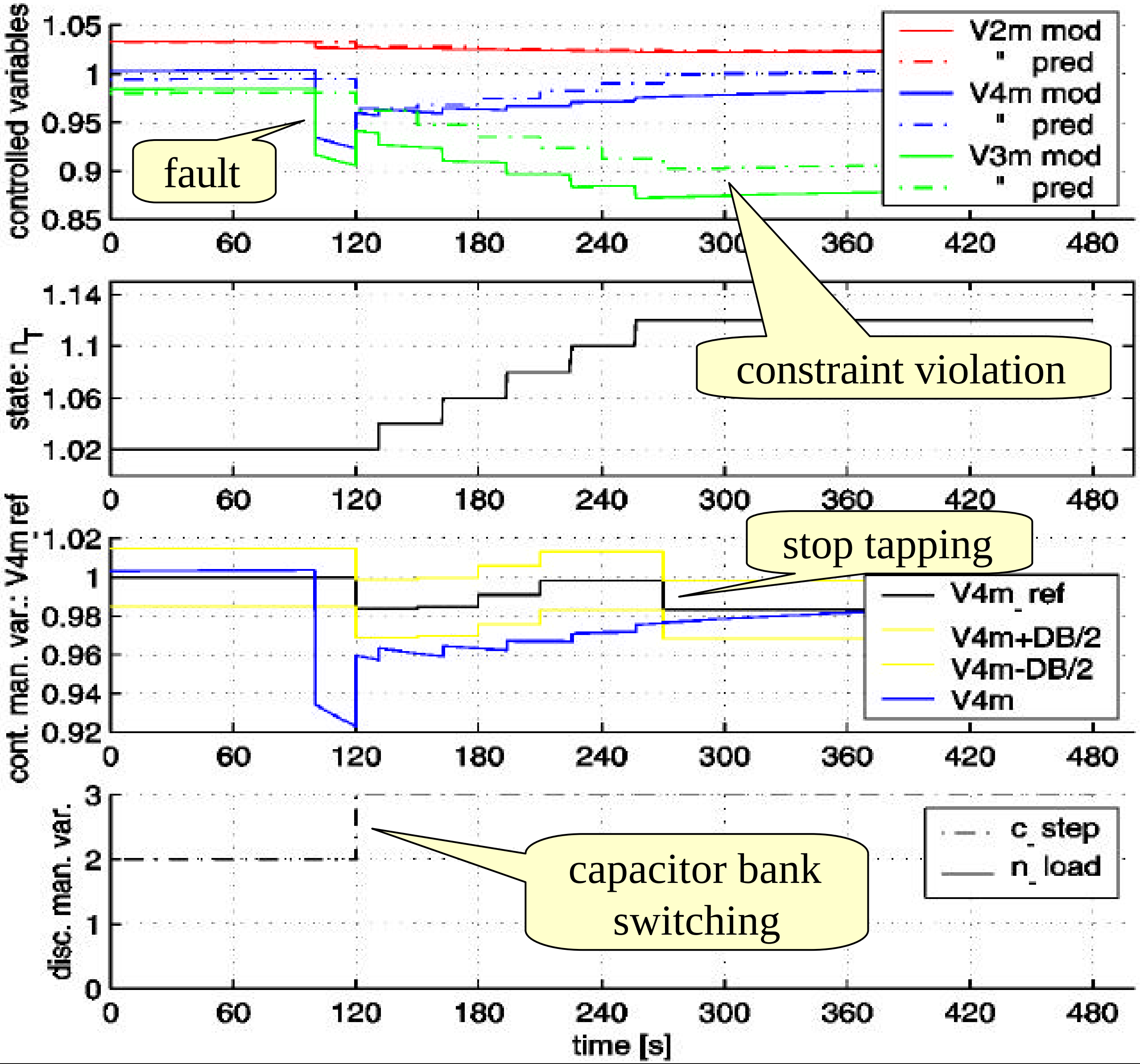
p: $>> \|\Delta u_{max}\|_{R,\infty}$

b_i: violation of i-th constraint $\in \{0, 1\}$

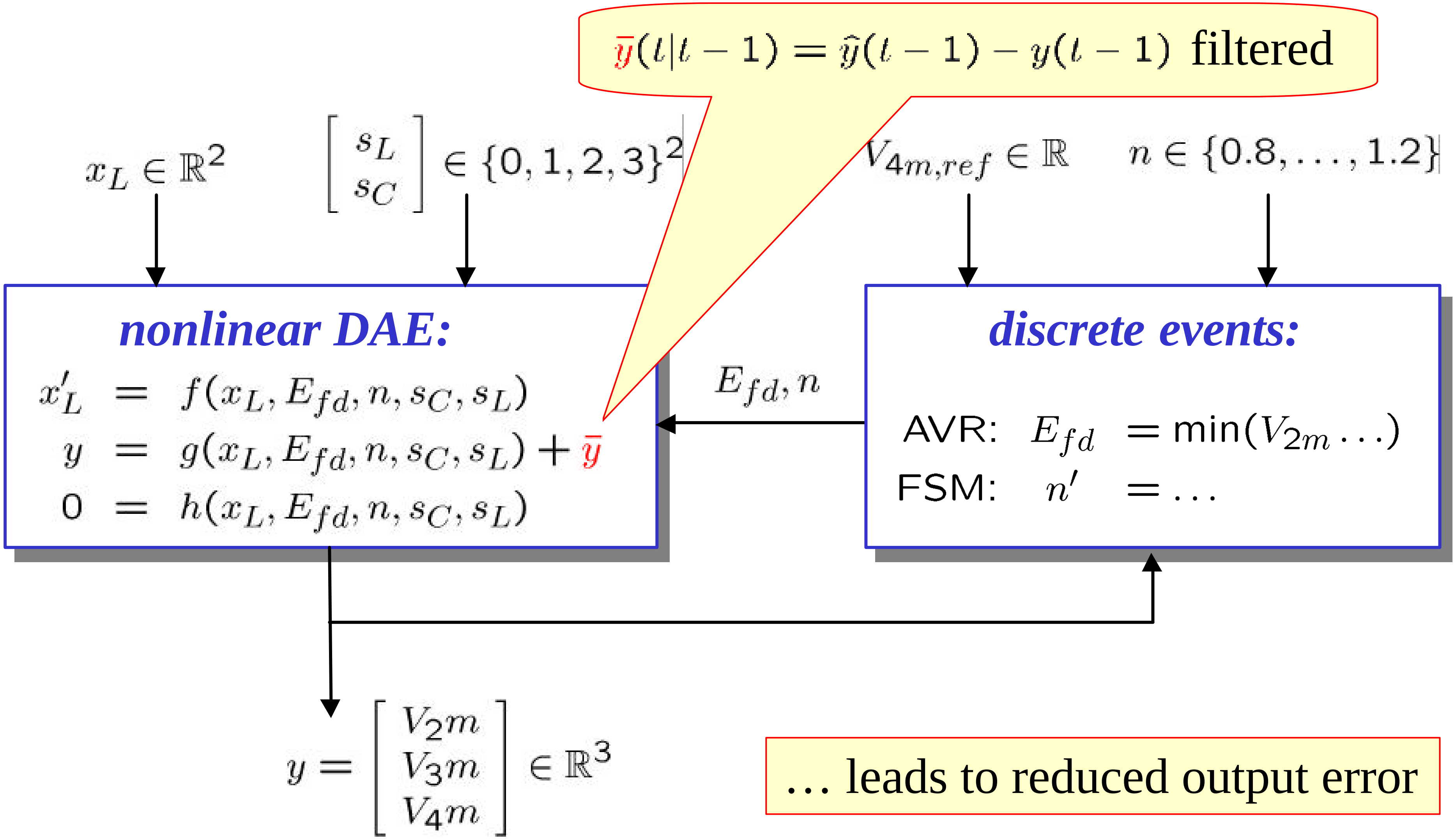
sl_i: slack $\in R^+$

Preliminary Results





Compensation for Output Error



… leads to reduced output error

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Nonlinear Hybrid MPC

Questions:

- Is the MPC cost function tuned properly?
- How large is the max. tolerable approximation error?

→ Simulate nonlinear hybrid MPC with exact model

Implementation:

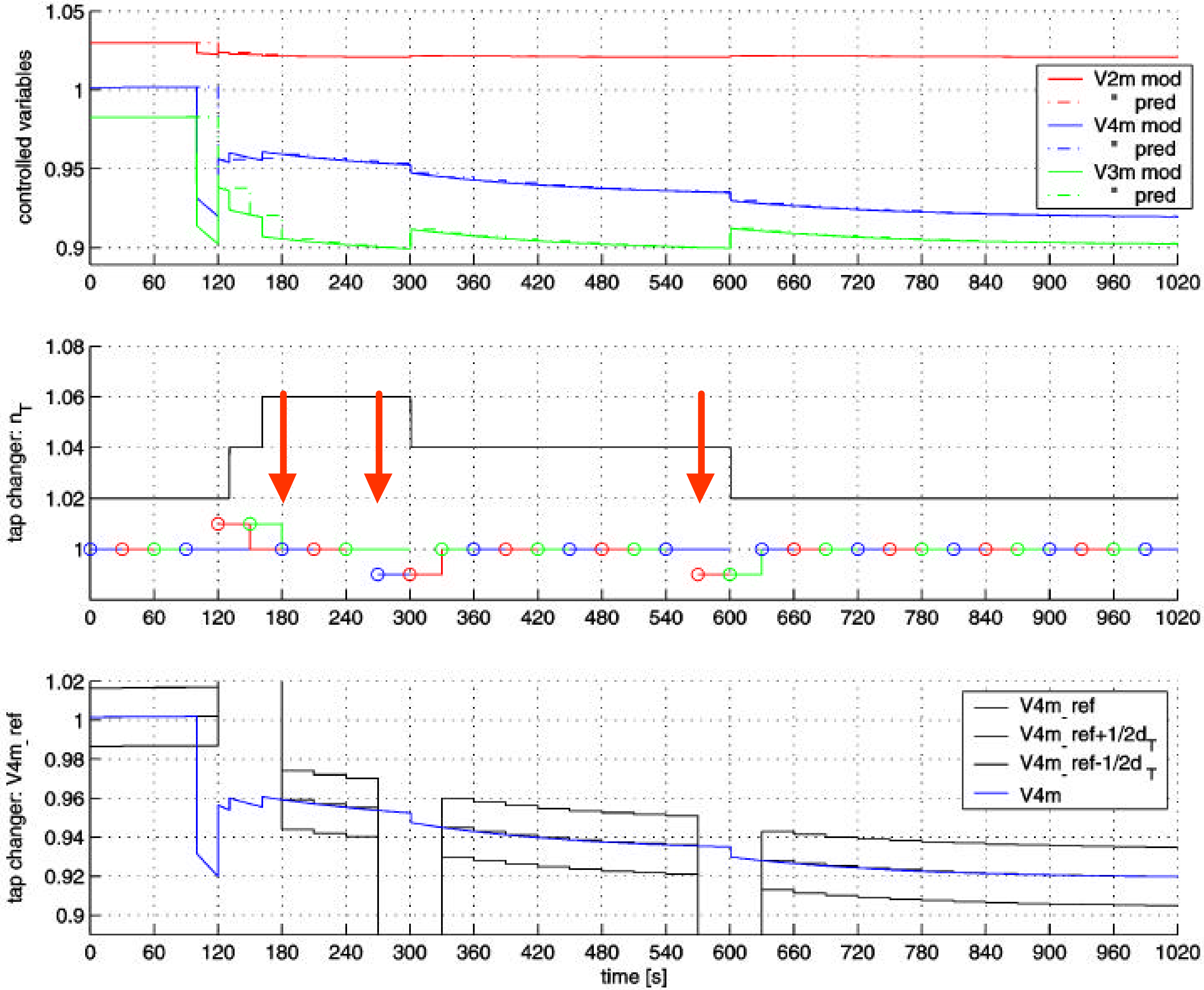
- branch on discrete inputs over prediction horizon
- use bound techniques
- Simulink (Modelica): used to simulate the model response and to evaluate the cost function

Nominal Case

t=120s:
cap. switching

t=180, 570s:
constraint viol.
predicted

no load shed.

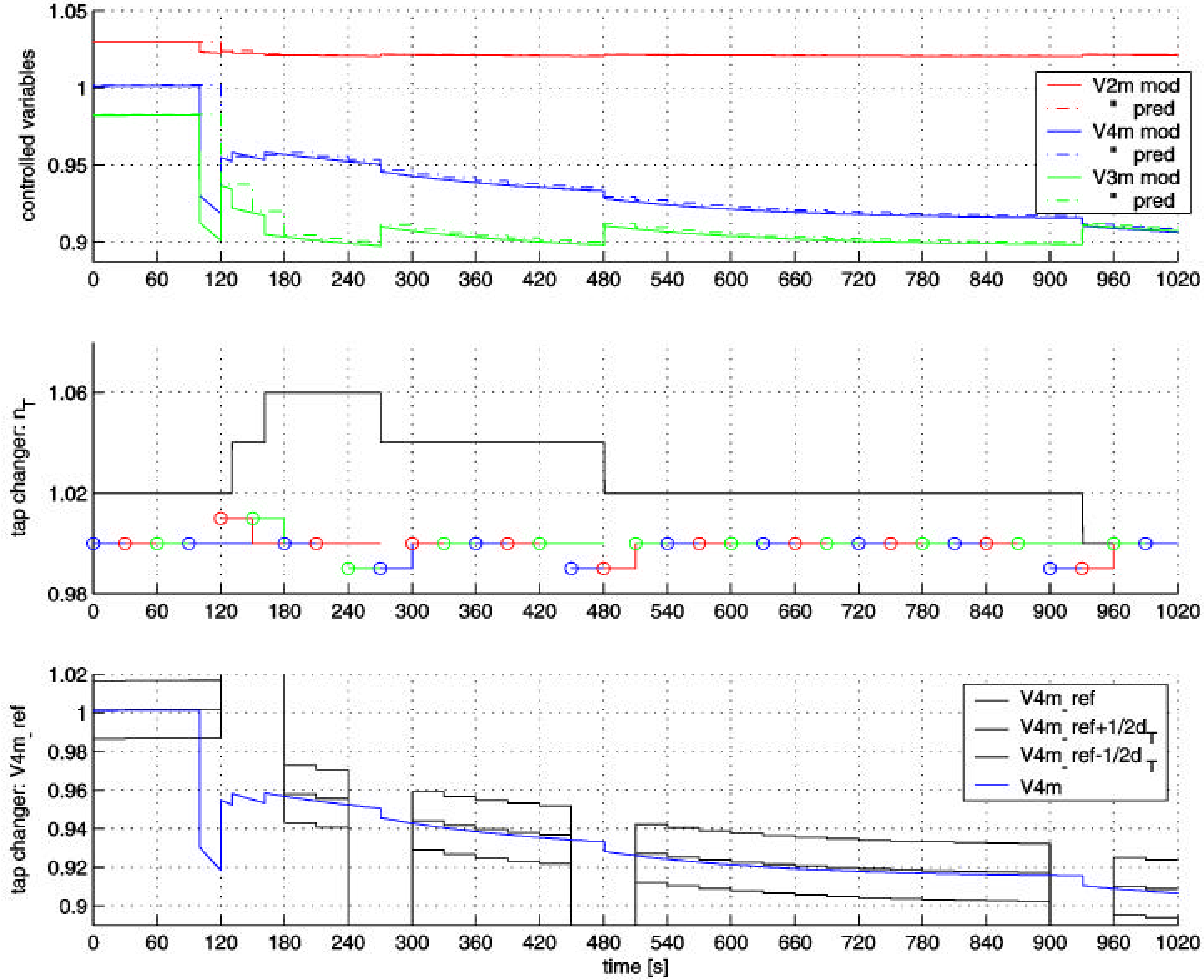


Parameter Uncertainty: P0 +0.5%

t=120s:
cap. switching

no load shed.

small offset in
bus voltages



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Conclusions

- MPC cost function:
  - use cascaded control scheme,
  - penalize tap changes,
  - allow for short constraint violations and
  - prediction horizon N=2 sufficient.

- Max. tolerable approximation error:
  - small parameter uncertainties lead to output offset,
  - if P0 > +1%: nominal control moves can't stabilize system,
  - system parameters and structure are known very accurately (PMU)

    except P0 (variations of up to 0.3% per minute)

  → high accuracy for PWA approximation mandatory

Outlook

- ➢ Reformulate MLD model using subdivided model
- ➢ Compensate for output error
- ➢ Reduce computational time
  - search heuristics
  - exploit model structure

- ➢ Large-scale power system
- ➢ Reachability analysis
- ➢ Controllability and observability

Reformulation of MLD Model

$x_L \in \mathbb{R}^2$

$\begin{bmatrix} s_L \\ s_C \end{bmatrix} \in \{0,1,2,3\}^2$

$V_{4m,ref} \in \mathbb{R}$

$n \in \{0.8, \ldots, 1.2\}$

nonlinear DAE:

$$\begin{aligned} x'_L &= f(x_L, E_{fd}, n, s_C, s_L) \\ y &= g(x_L, E_{fd}, n, s_C, s_L) \\ 0 &= h(x_L, E_{fd}, n, s_C, s_L) \end{aligned}$$

E_{fd}, n

discrete events:

AVR: $E_{fd} = \min(V_{2m} \ldots)$

FSM: $n' = \ldots$

approximate by PWA functions
with 4 real inputs
for all 16 combinations of
discrete inputs

formulate as MLD model

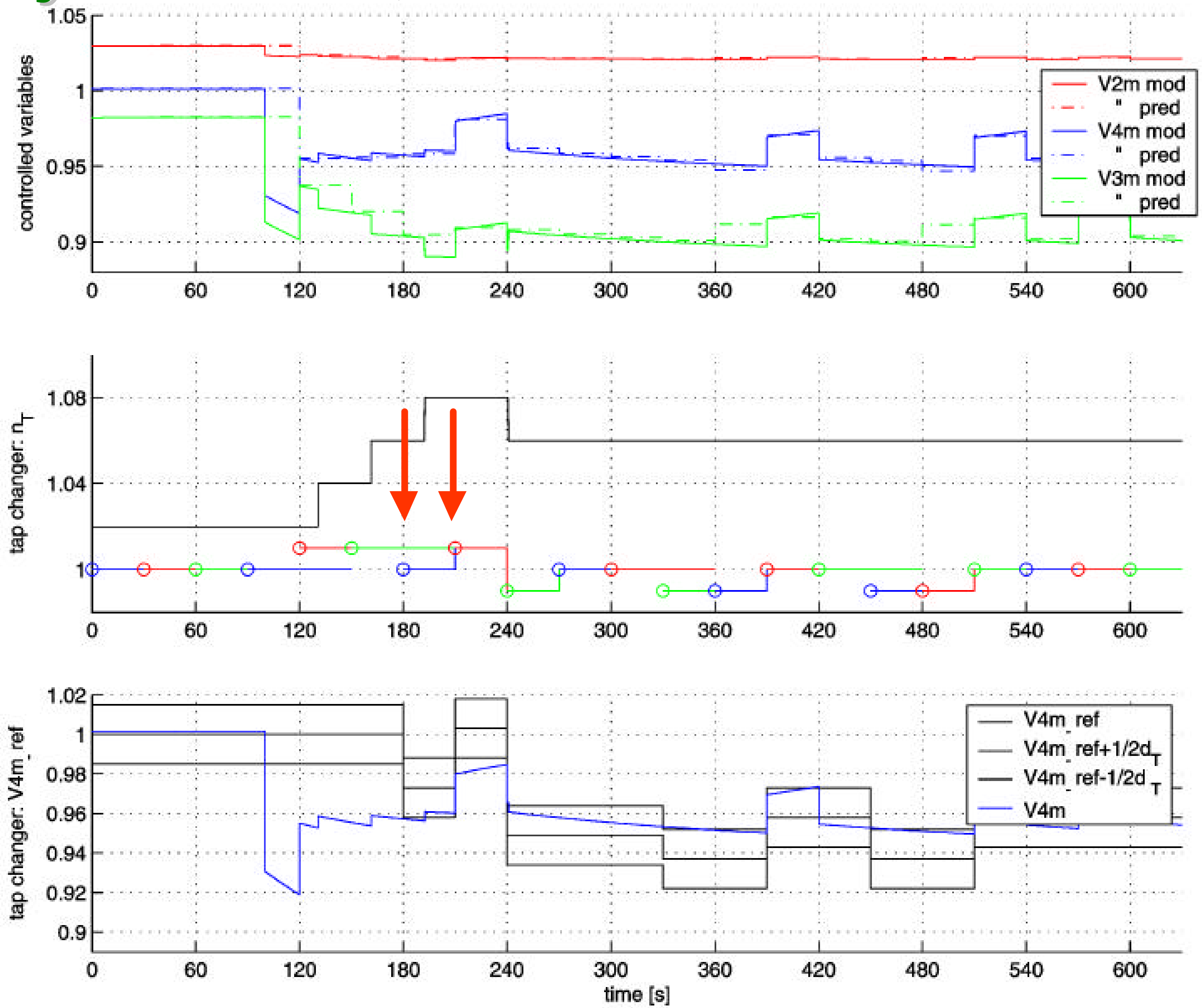
The end

Penalty on V4m,ref and P0 +0.5%

t=120s:
cap. switching

t=180s:
constraint viol. predicted, but change in V4m,ref not effective

t=210s:
load shed.



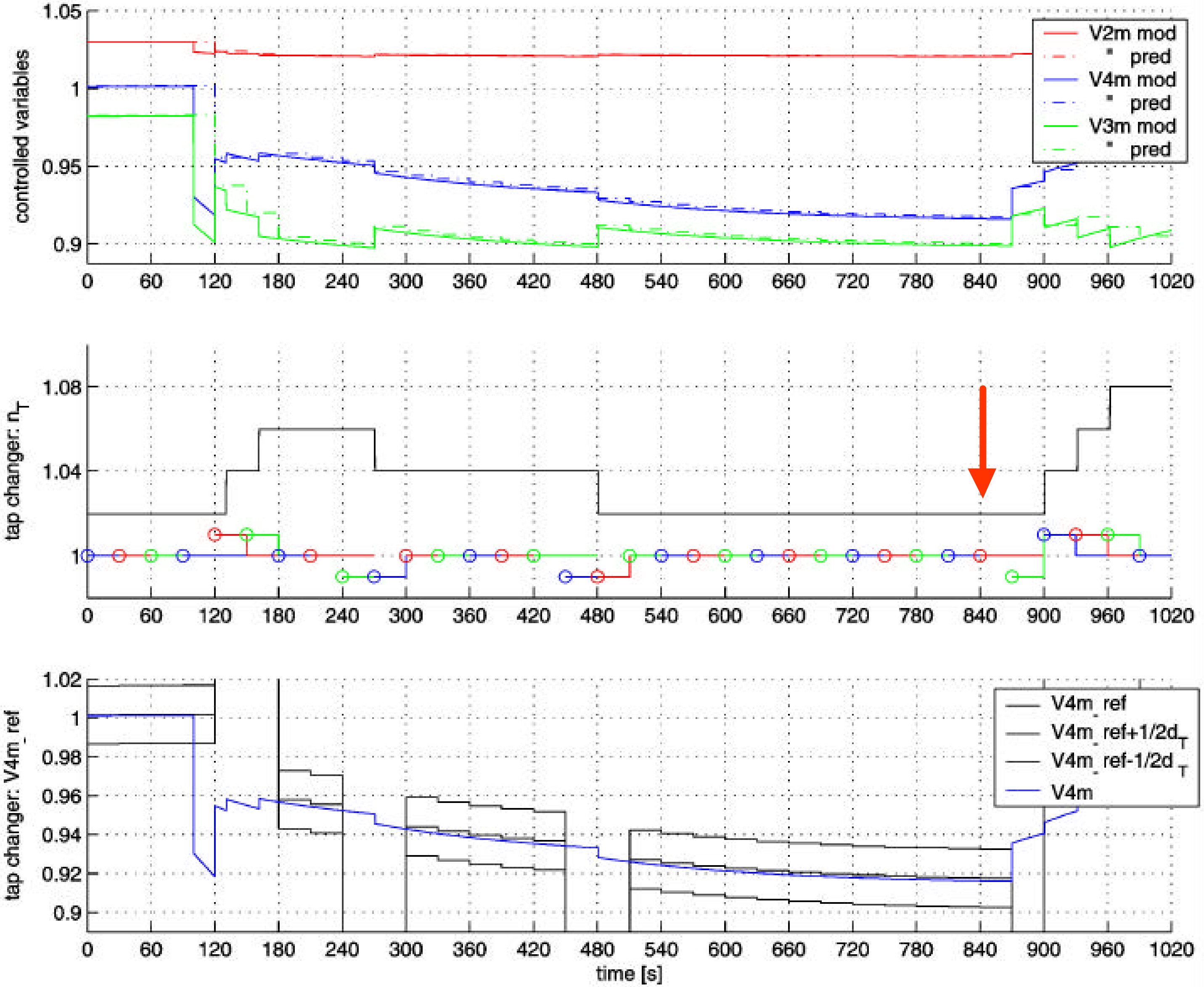
Penalize all Constraint Viol. and P0 +0.5%

constraint viol. for k=0 penalized, too

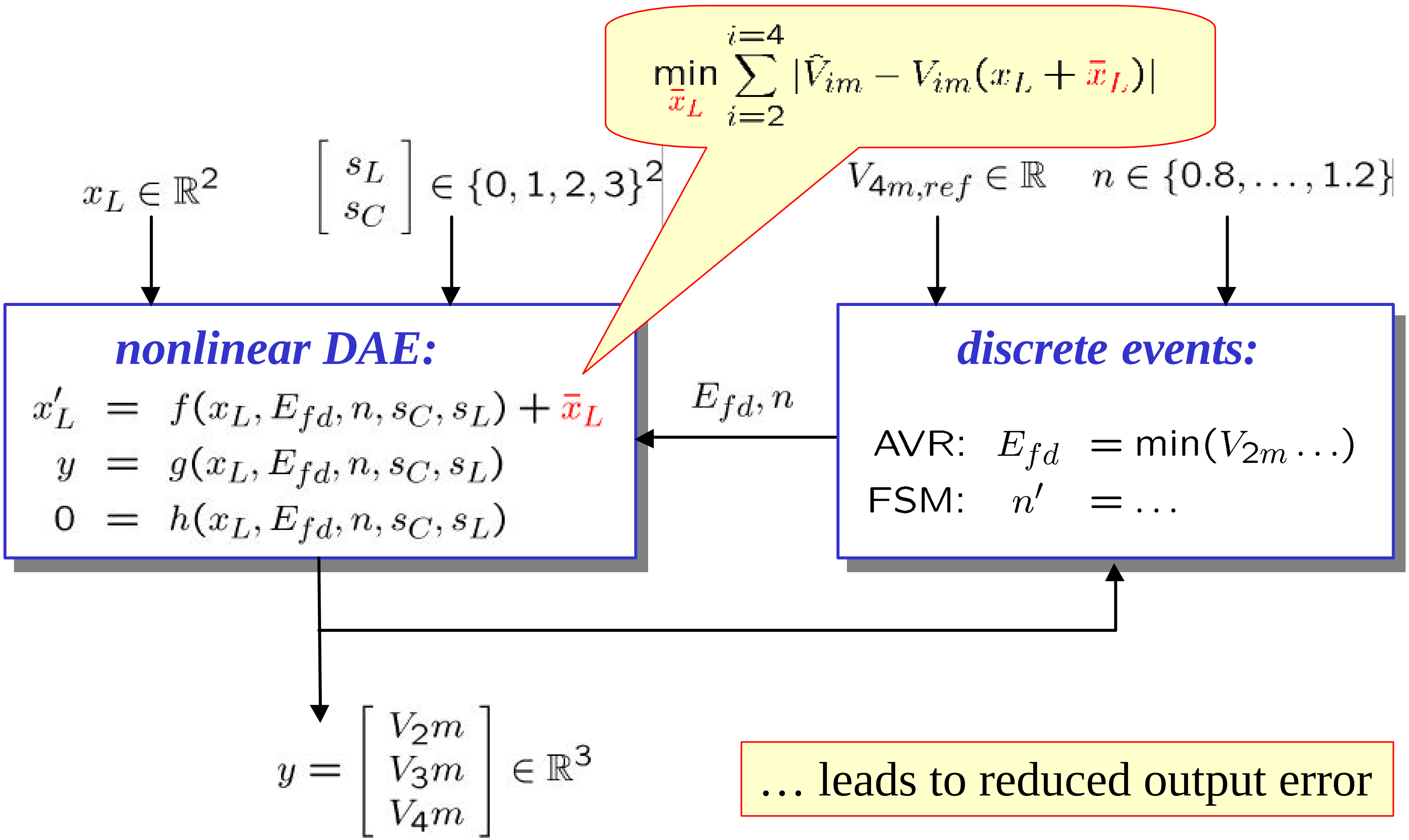
t=120s: cap. switching

t=840s: constraint viol. NOT predicted

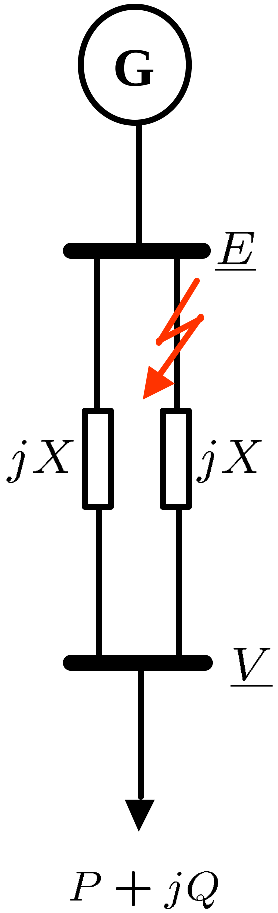
t=870s: load shedding



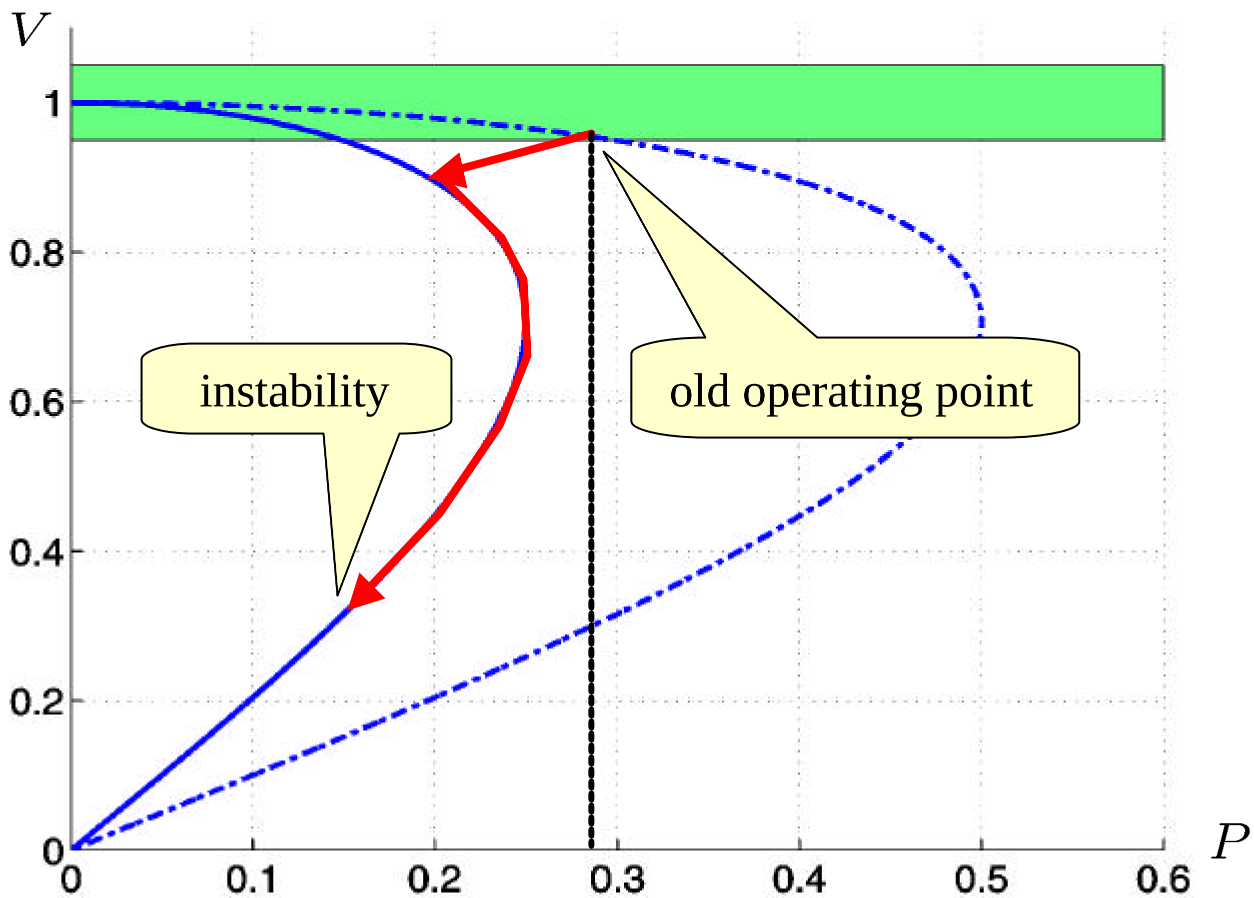
Static State-Estimation



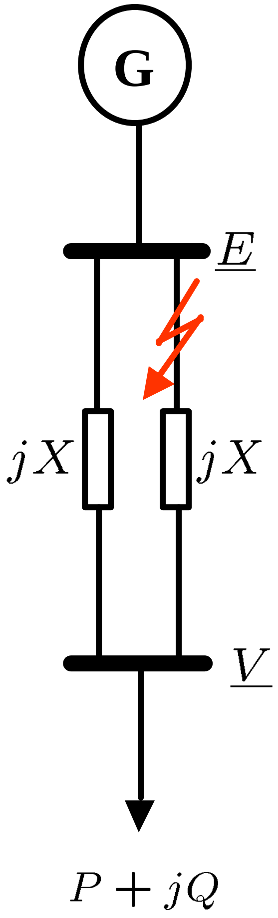
Instability: line outage



$$\tan\phi = 0 : V^2 = \frac{1}{2} \pm \sqrt{\frac{1}{4} - P^2 X^2}$$



Countermeasures: load shedding



$$\tan\phi = 0 : V^2 = \frac{1}{2} \pm \sqrt{\frac{1}{4} - P^2 X^2}$$