Control of a Car Engine

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Outline

- Introduction. Motivation of investigation.
- Problem formulation.
- Piecewise-affine hybrid system of car engine.
- Dynamic system properties.
- Control synthesis.
- Concluding remarks.

Motivation

- EU.IST.CC Project. Automotive applications. Hybrid system.
- Research with Parades, Rome, Italy.
- Magneti-Marelli, company of auto parts. Part of FIAT.
- Technology allows for high-performance control of engine.
- Close control of car engine, for effective steering, environmental conditions, and for economic reasons.
- Model of car engine is hybrid system.
- Control problem.
- Work in progress.

Engineering model

Remarks

- Hybrid system character due to periodic behavior of crankshaft rotation. Time of revolution not constant.
- Model more detailed than models in literature.

Submodels

- **Gas pressure model:** From throttle to gas pressure.
- **Cylinder model:** From gas pressure to torque.
- **Power train model:** From torque to crankshaft speed and angle.

Engineering model of car engine

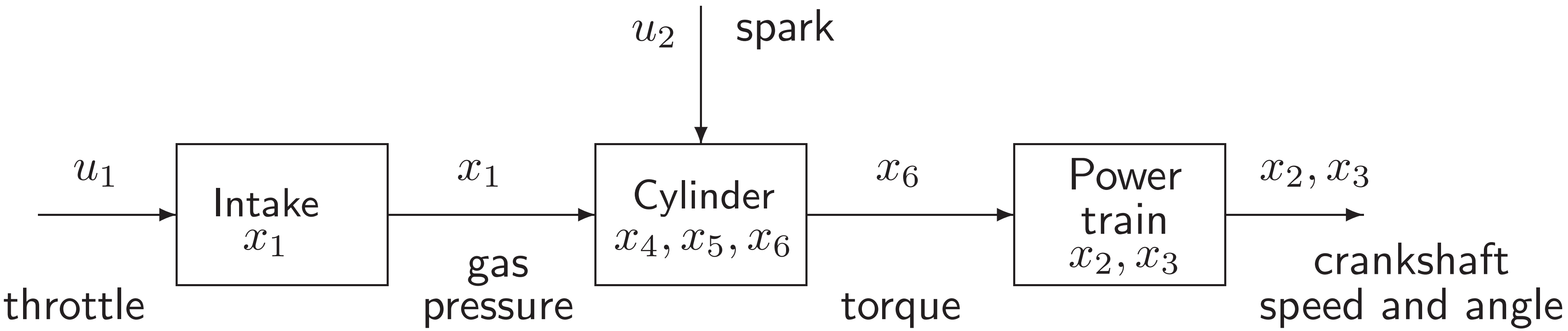

Figure. Engineering model of car engine.

Def. **Piecewise-affine hybrid system** (PAHS, CT, Time-invariant)

$$
\begin{aligned}
Q & \quad \text{finite state set, } U \subseteq \mathbb{R}^m,\ Y \subseteq \mathbb{R}^p,\ \text{polyhedral sets,} \\
X(q) & \subseteq \mathbb{R}^{n(q)},\ \forall q \in Q,\ \text{closed polyhedral sets,} \\
\dot{x}_q(t) & = A(q)x_q(t) + B(q)u(t) + a(q),\ x_q(t_o) = x_q^+, \\
y(t) & = C(q)x_q(t) + D(q)u(t) + c(q), \\
e & \in E_{cd},\ \text{if } x(t_1) \in G_q(e) \subset \partial X(q), \\
& \quad \text{event generated by continuous dynamics; then transition,} \\
q^+ & = f(q^-, e, x_{q^-}^-),\ q_0, \\
x_{q^+}^+ & = A_r(q^-, e, q^+)x_{q^-}^- + b_r(q^-, e, q^+).
\end{aligned}
$$

Remark PAHS defined above is **switched system** rather than general hybrid system.

Model of car engine - States

$$
\begin{aligned}
Q &= \{1,2,3\}, \quad 1 = S^-,\ 2 = S,\ 3 = S^+, \\
X &= \prod_{i=1}^{6} X_i \subset \mathbb{R}^6_+, \\
x_1 &= \text{gas pressure}, \\
x_2 &= \text{crankshaft speed}, \\
x_3 &= \text{crankshaft angle}, \\
x_4 &= \text{mass of gas mixture in cylinder during compression stroke}, \\
x_5 &= \text{mass of gas mixture in cylinder during expansion stroke}, \\
x_6 &= \text{torque}, \\
u_1 &= \text{throttle angle}, \quad U(1) = [u_{1,min}, u_{1,max}] \subset (0,\infty), \\
u_2 &= \text{spark angle advance or delay}, \quad U(2) = [u_{2,min}, u_{2,max}] \subset (0,\infty).
\end{aligned}
$$

Continuous dynamics

Identical in all three discrete states,

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu_1(t), \quad x(t_0) = x_0, \\
&= \begin{pmatrix} A_p x_1(t) + B_p u_1(t) \\ A_n x_2(t) + B_n x_6(t) \\ K_c x_2(t) \\ 0 \\ 0 \\ 0 \end{pmatrix}, \\
z(t) &= e_2^T x(t), \ \textbf{controlled output, } \text{crankshaft speed,} \\
& \quad A_p, A_n \in (-\infty, 0), \ B_p, B_n \in (0, \infty); \\
\eta(u_2) &= -0.73 + (0.083 + 0.005u_2)^{1/2}, \ \textbf{spark efficiency function}.
\end{aligned}
$$

Dynamics consists of one first order positive system,
three hold systems, followed by two first order positive systems.

Discrete dynamics

At discrete input $u_2 = 0$. Discrete state $q = 2$ only.

$$
\begin{aligned}
q &= 2, \\
X(2, u_2) &= \{x \in X | x_3 \in [0, \pi]\}, \\
2 &= f(2, e(2, 2), x^-), \\
X_0(2, u_2) &= \{x \in X(2, u_2) | x_3 = 0\}, \\
G(2, e(2, 2)) &= \{x \in X(2, u_2) | x_3 = \pi\}, \\
x^+ &= \begin{pmatrix} x_1^- \\ x_2^- \\ 0 \\ G_{mp} x_1^- + M_0 \\ x_5^- \\ G\eta(0) x_4^- + T_0 \eta(0) \end{pmatrix}.
\end{aligned}
$$

Discrete dynamics

At discrete input $u_2 < 0$. Discrete state $q = 1, 2$.

$$\begin{aligned}
q &= 1, \\
X(1, u_2) &= \{x \in X | x_3 \in [0, -u_2]\}, \\
2 &= f(1, e(2,1), x^-), \\
X_0(2, u_2) &= \{x \in X(1, u_2) | x_3 = 0\}, \\
G(2, e(2,2)) &= \{x \in X(1, u_2) | x_3 = -u_2 > 0\}, \\
x^+ &= \begin{pmatrix} x_1^- \\ x_2^- \\ x_3^- \\ x_4^- \\ x_5^- \\ G\eta(u_2)x_5^- + T_0\eta(u_2) \end{pmatrix}.
\end{aligned}$$

Discrete dynamics

At discrete input $u_2 < 0$.

$$\begin{aligned}
q &= 2, \\
X(2, u_2) &= \{x \in X | x_3 \in [-u_2, \pi]\}, \\
1 &= f(2, e(1,2), x^-), \\
X_0(2, u_2) &= \{x \in X(2, u_2) | x_3 = -u_2\}, \\
G(2, e(1,2)) &= \{x \in X(2, u_2) | x_3 = \pi\}, \\
x^+ &= \begin{pmatrix} x_1^- \\ x_2^- \\ 0 \\ G_{mp} x_1^- + M_0 \\ x_4^- \\ 0 \end{pmatrix}.
\end{aligned}$$

Discrete dynamics

At discrete input $u_2 > 0$. Discrete state $q = 2, 3$.

$$\begin{aligned}
q &= 2, \\
X(2, u_2) &= \{x \in X | x_3 \in [0, \pi - u_2]\}, \\
3 &= f(2, e(3,2), x^-), \\
X_0(2, u_2) &= \{x \in X(2, u_2) | x_3 = 0\}, \\
G(2, e(3,2)) &= \{x \in X(2, u_2) | x_3 = \pi - u_2\}, \\
x^+ &= x^-.
\end{aligned}$$

Discrete dynamics

At discrete input $u_2 > 0$.

$$
\begin{aligned}
q &= 3, \\
X(3, u_2) &= \{x \in X | x_3 \in [\pi - u_2, \pi]\}, \\
2 &= f(3, e(2,3), x^-), \\
X_0(2, u_2) &= \{x \in X(3, u_2) | x_3 = \pi - u_2\}, \\
G(3, e(2,3)) &= \{x \in X(3, u_2) | x_3 = \pi\}, \\
x^+ &= \begin{pmatrix} x_1^- \\ x_2^- \\ 0 \\ G_{mp} x_1^- + M_0 \\ x_5^- \\ G\eta(u_2) x_4^- + T_0 \eta(u_2) \end{pmatrix}.
\end{aligned}
$$

Dynamics over cycles

Time axis replaced by axis for crankshaft angle, x_3.

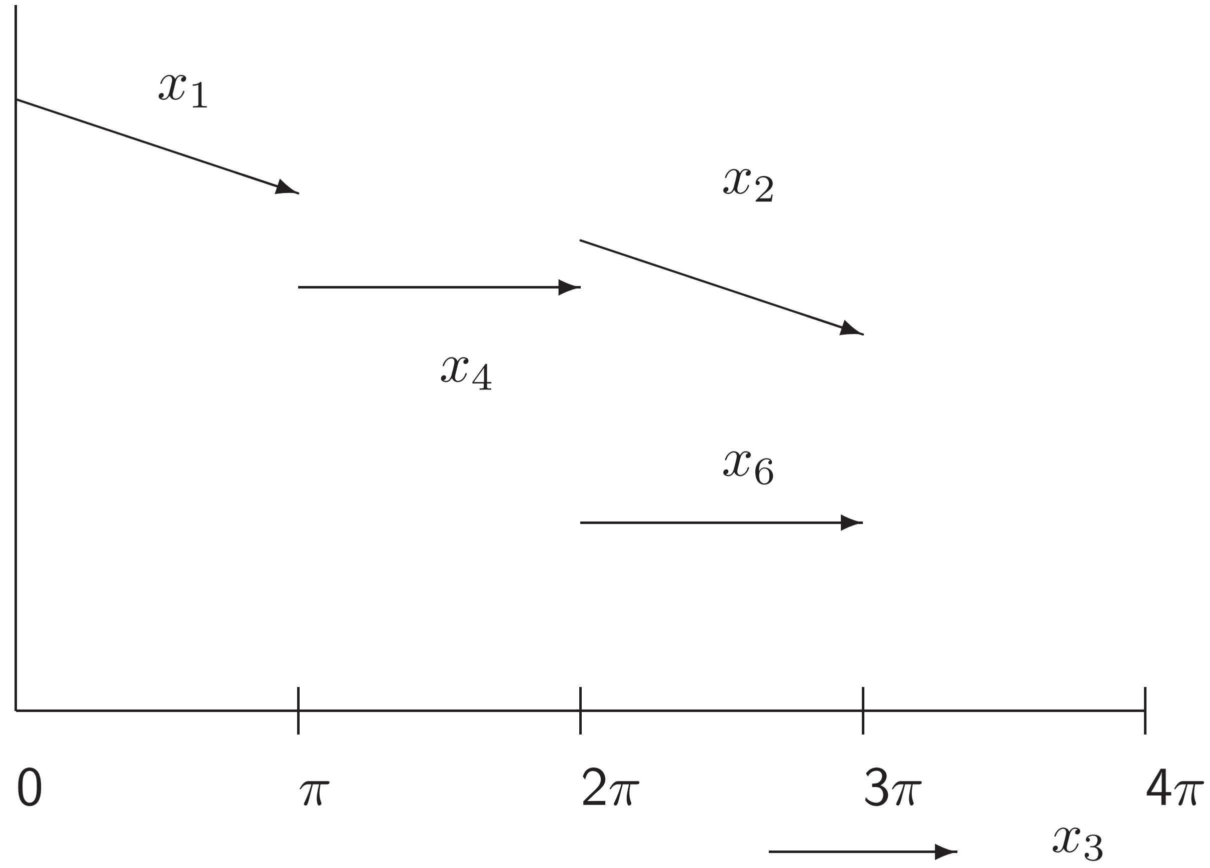

Figure. Diagram of dynamics of continuous-space system.

Dynamic system properties - Stationary state

Problem

Consider a crankshaft speed $z_s \in (0, \infty)$.

For all $u_2 \in U(2)$, spark angle advance,

determine the throttle input $u_{1,s} \in U(1) = \mathbb{R}_+$

and a stationary state $x_s \in X$

such that when the throttle input equals $u_{1,s}$

then the state is stationary and equals x_s, in particular $x_{2,s} = z_s$,

with the crankshaft running at constant speed $x_{2,s} = z_s$.

Existence stationary state

Theorem Consider crankshaft speed $z_s \in (0, \infty)$ sufficiently large.

(a) Case $u_2 < 0$. Existence of periodic solution for the state trajectory. Details to be worked out.

(b) Case $u_{2,s} = 0$. Stationary state and throttle input are:

$$
\begin{aligned}
x_{2,s} &= z_s, \quad x_{6,s} = -(A_n/B_n)x_{2,s}, \\
x_{4,s} &= -[x_{6,s} - T_0\eta(0)]/G\eta(0), \; x_{5,s} \text{ not relevant for this case,} \\
x_{1,s} &= [x_{4,s} - M_0]/G_{mp}, \; u_{1,s} = -(A_p/B_p)x_{1,s}.
\end{aligned}
$$

(c) Case $u_{2,s} > 0$. Stationary state and throttle input are:

$$
\begin{aligned}
x_{2,s} &= z_s, \quad x_{6,s} = -(A_n/B_n)x_{2,s}, \\
x_{4,s} &= -[x_{6,s} - T_0\eta(u_{2,s})]G\eta(u_{2,s}), \\
x_{1,s} &= [x_{4,s} - M_0]/G_{mp}, \; u_{1,s} = -(A_p/B_p)x_{1,s}.
\end{aligned}
$$

Theorem Forward invariant sets.

Consider the PAHS for the car engine. Consider the case $u_2 = 0$.

There exists

a subset of the state set $X^{inv} \subset X$ and

a subset of the input set $U^{inv}(1) \subset U(1)$

such that with any input $u_1 \in U^{inv}(1)$,

the subset X^{inv} is **forward invariant:**

if $x_0 \in X^{inv}$ then for all $t \in T$, $x(t) \in X^{inv}$.

Formulas for X^{inv} are stated in paper.

Problem Control to maintain crankshaft speed at set point

Motivation. Unforeseen disturbances affect the crankshaft speed: road surface irregularities, windgusts, etc.

Control objective: Maintain crankshaft speed at set point.

Approaches

- Control law which steers crankshaft speed back to set point in a few steps. Done. Theorem with formulas in paper.
- Fast yet simple control law. Asymptotically stability. Not yet worked out.

Problem Control to transfer to a new set point

Situation: Lower crankshaft speed to new set point,
lower than current one, $0 < z_s < z_{s,old}$.

Control objectives

1. Set point z_s reached.
2. No undershoot, under a specified value $z_{s,low} \in (0, \infty)$.
3. Minimal or relatively small transfer time.

Control to transfer to a new set point - Approaches

Case 1. $u_2 = 0$.

Def. **Control law 1**.

$$g(x) = u_{1,s} \in U(1), \quad z_s \mapsto u_{1,s}.$$

Proposition Control law 1 meets control objectives 1 and 2, but control objective 3 not so well due to slowness.

Proof Monotone decline of crankshaft speed to new set point.

Control to transfer to a new set point - Approaches

Case 1. $u_2 = 0$.

Def. **Control law 2**.

$$\begin{aligned} g(x) &= u_{1,min} I_{(x_{1,a} < x_1(t))} + u_{1,max} I_{(x_{1,b} < x_1(t) \leq x_{1,a})} + \\ &\quad + u_{1,s} I_{(x_1(t) \leq x_{1,b})}. \end{aligned}$$

Parameters of control law to be determined.

Performance of closed-loop system for control objectives to be determined.

Optimality with respect to minimal transfer time?

Control to transfer to a new set point - Approaches

Case 1. $u_2 = 0$.

Def. Control law 3. Control to be based on control-to-facet.

Partition state set into multivariable rectangles.

For each rectangle determine a control law transferring the state to a specified facet.

Control objectives 1 and 2 are then met.

To be carried out in detail.

Control to transfer to a new set point - Approaches

Approach Optimization over spark advance input $u_2 \in U(2)$.

To be done.

Concluding remarks

Results

- Model as hybrid system. Dynamic system properties.
- Control law for maintaining set point.
- Control law for transfer to a new set point, special case.

Plan for research

- Control law for transfer to a new set point. Optimal control. Optimize over spark angle.
- Control law for maintaining set point. Asymptotic stability. Optimal control.