Analysis and Lyapunov-based hybrid control design for stabilization of the ABB Benchmark

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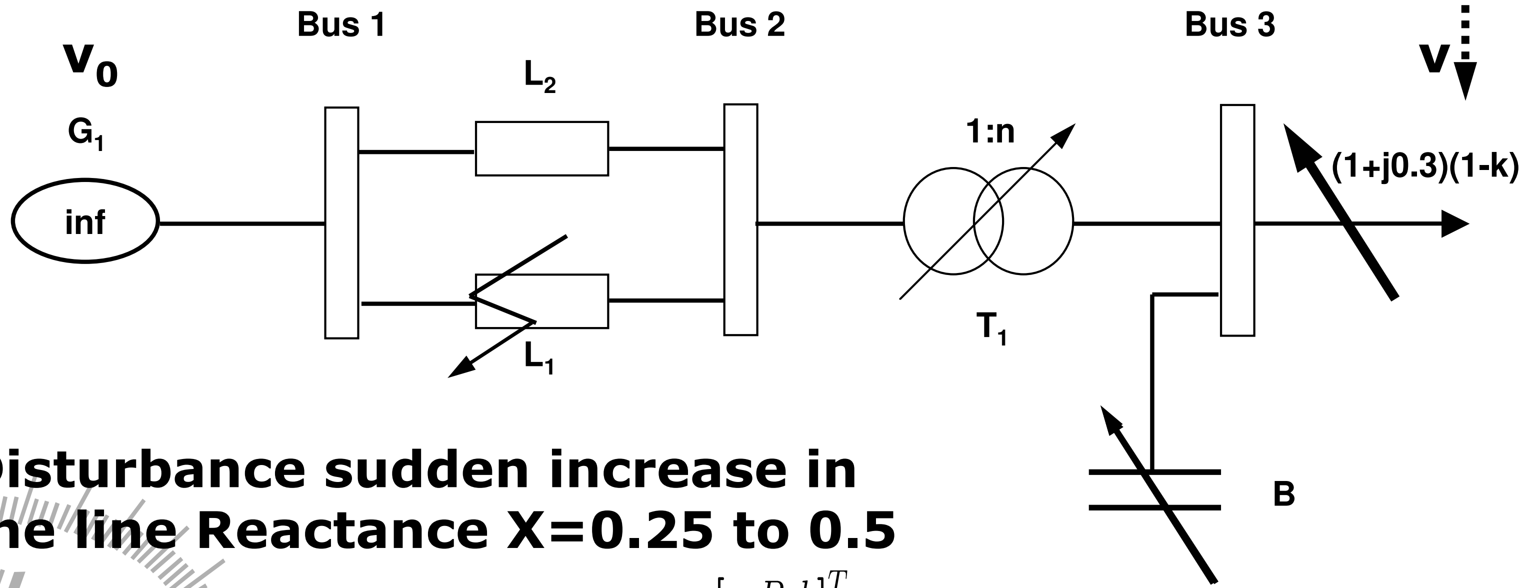
Outline

- Simplified benchmark presentation
- Model properties
- Lyapunov-based hybrid control design
- Simulation results
- Conclusions and future work

System Presentation



Disturbance sudden increase in the line Reactance X=0.25 to 0.5

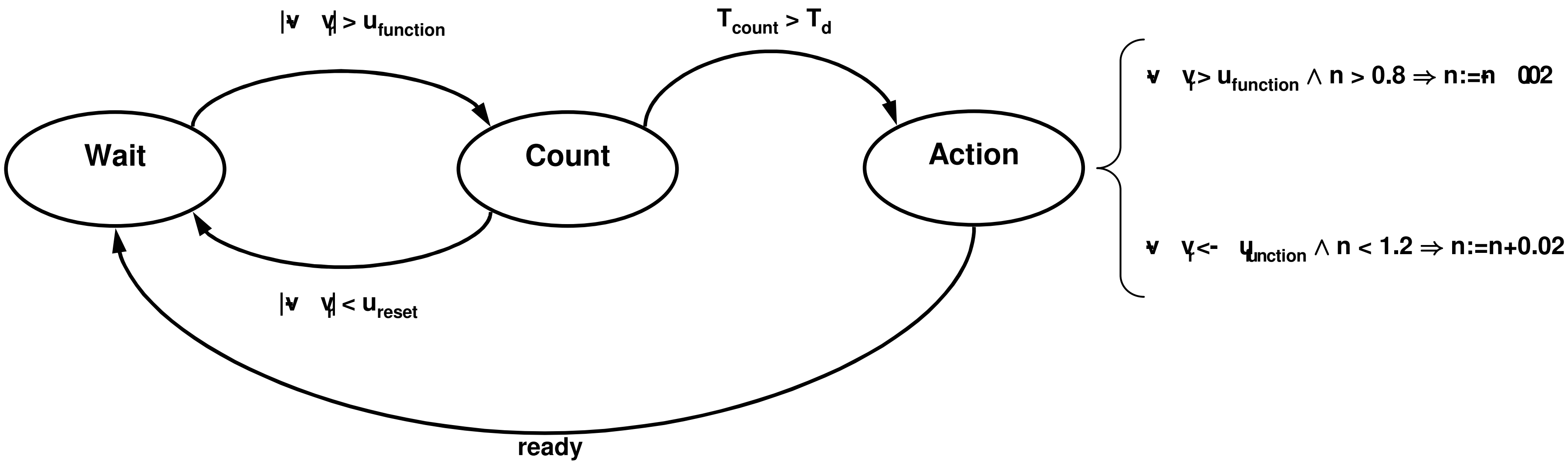
$$u = [n\ B\ k]^T$$

$n \in [n_m\ n_M]$ $\quad n_m = 0.8, \quad n_M = 1.2$ With steps of 0.02

$B \in \{0, 0.1, 0.2, 0.3\}$

$k \in \{0, 0.05, 0.1, 0.15\}$

OLTC dynamics



Which can be approximated by

$$\frac{dn}{dt} = \Gamma sgn(\nu, n, \Delta)$$

Model properties

The dynamic load model

The system is described by the following

$$\dot{x}_p = -\frac{x_p}{T_p} + P_0 \left(v^{\alpha_s} - v^{\alpha_t}\right)$$
$$\dot{x}_q = -\frac{x_q}{T_q} + Q_0 \left(v^{\beta_s} - v^{\beta_t}\right)$$

$$P = (1-k)\left(\frac{x_p}{T_p} + P_0 v^{\alpha_t}\right)$$
$$Q = (1-k)\left(\frac{x_q}{T_q} + Q_0 v^{\beta_t}\right)$$

P_0 : steady state value of the active power demand
Q_0 :steady state of the reactive power demand
$\alpha_s=\beta_s=0$, $\alpha_t=\beta_t=2$,$T_p=T_q$= 60 sec
k $\in$ {0,0.05,0.1,0.15} is the load shedding control input

The DAE nature of the problem

The load flow equations add the following constraints to the dynamics

$$(1-k)\left(\frac{x_p}{T_p}+P_0v^2\right) = -\frac{v_0 v}{nX}\sin(\delta)$$
$$(1-k)\left(\frac{x_q}{T_q}+Q_0v^2\right) = \frac{v_0 v}{nX}\cos(\delta) - \frac{v^2}{n^2X} + Bv^2$$

So that the model can be written in the DAE framework

$$\dot{x} = f(x,v)$$
$$0 = g(x,u,v,\theta)$$

An invariance property

It can be shown that

$$F(x_p, x_q) = P_0 x_q - Q_0 x_p = 0$$

is an invariant manifold

So that under $x(0) \in F$

The system reduces to

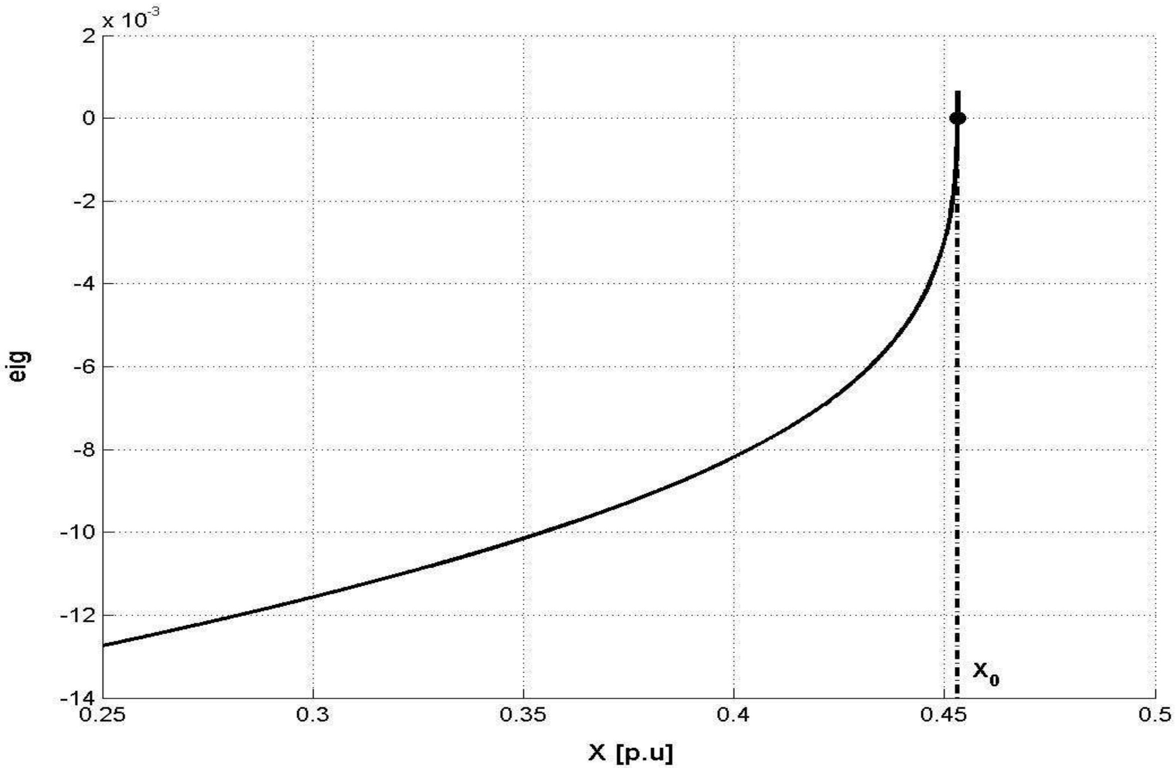
$$\begin{aligned} \dot{x} &= -\frac{x}{T_p} + P\left[1 - y(x,u)\right] \\ 0 &= y^2 + 2\alpha_1(x,u)y + \alpha_0(x,u) \end{aligned}$$

where $y = v^2$

Equilibrium points

Linearization of the system around an equilibria x^*

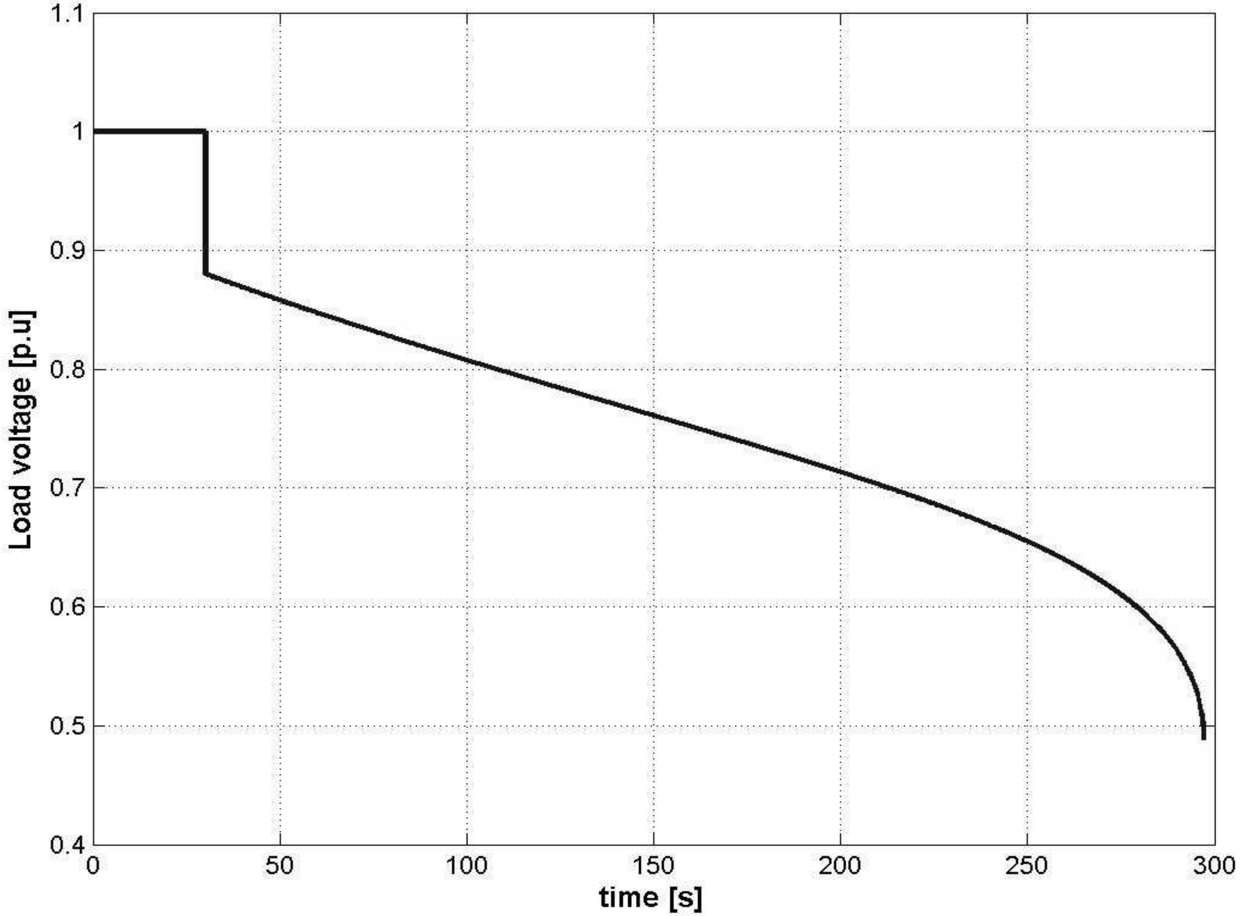
$$\dot{\tilde{x}} = c\tilde{x}$$



c as a function of the line reactance X
for n_n=1, B_n= 0, k_n=0

Voltage collapse dynamics

Time domain simulation



Voltage profile for X=0.5

Voltage collapse dynamics

State space interpretation

From equation

$$\frac{dg}{dt} = \frac{\partial g}{\partial y}\ \dot{y} + \frac{\partial g}{\partial x}\ \dot{x} = 0$$

One can writes

$$\dot{y} = -\left(\frac{\partial g}{\partial y}\right)^{-1} \frac{\partial g}{\partial x}\ \dot{x}$$

and define

$$S = \{(x, y) \in L : \Delta(x, y) = det\frac{\partial g}{\partial y} = 0\}$$

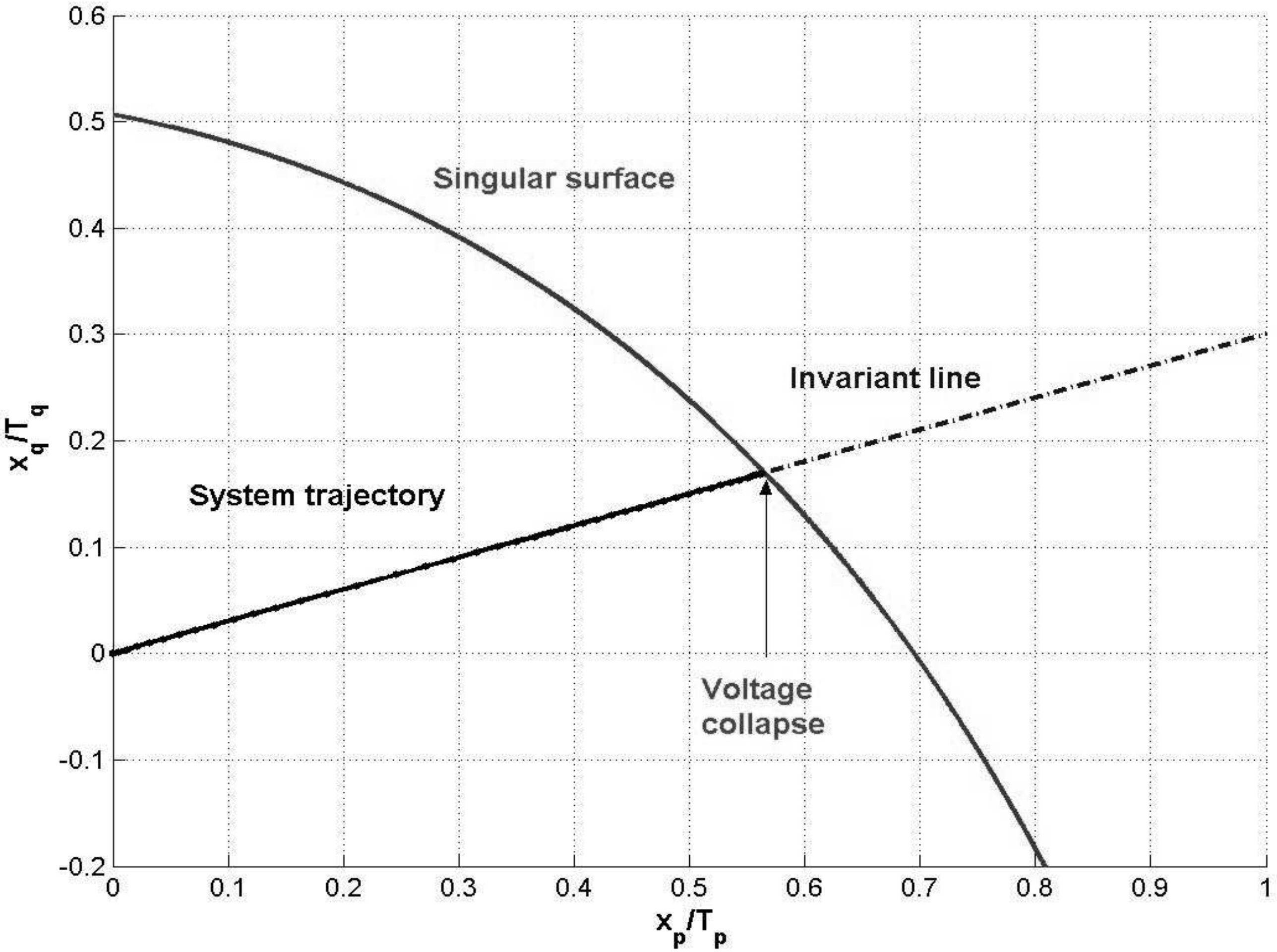
as the singular surface

Which gives explicitly

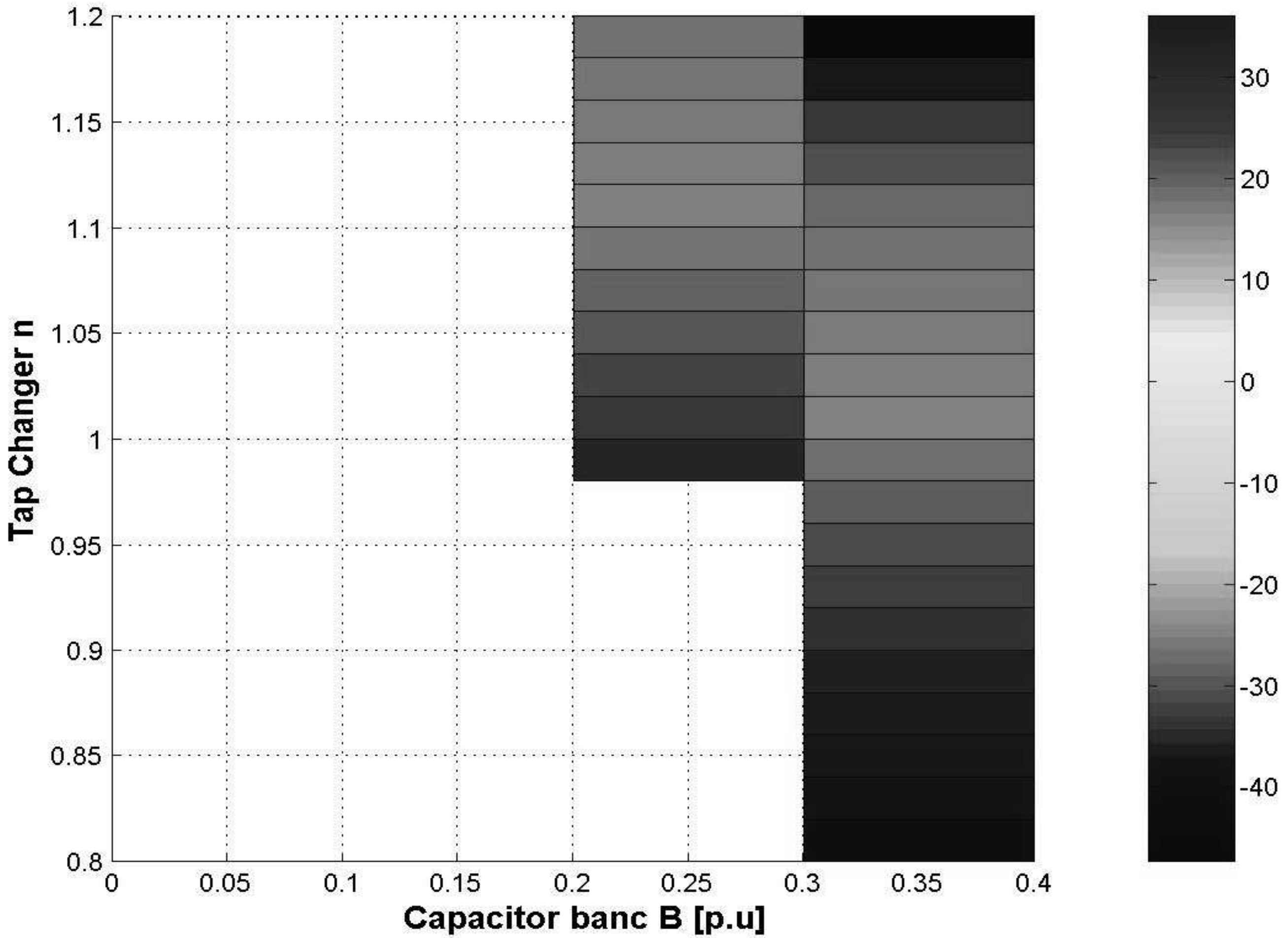
$$\dot{y} = \frac{1}{2(y+\alpha_1(x,u))}\left[2\frac{\partial \alpha_1}{\partial x}(x,u)y + \frac{\partial \alpha_0}{\partial x}(x,u)\right]\dot{x}$$

Voltage collapse dynamics

State space interpretation

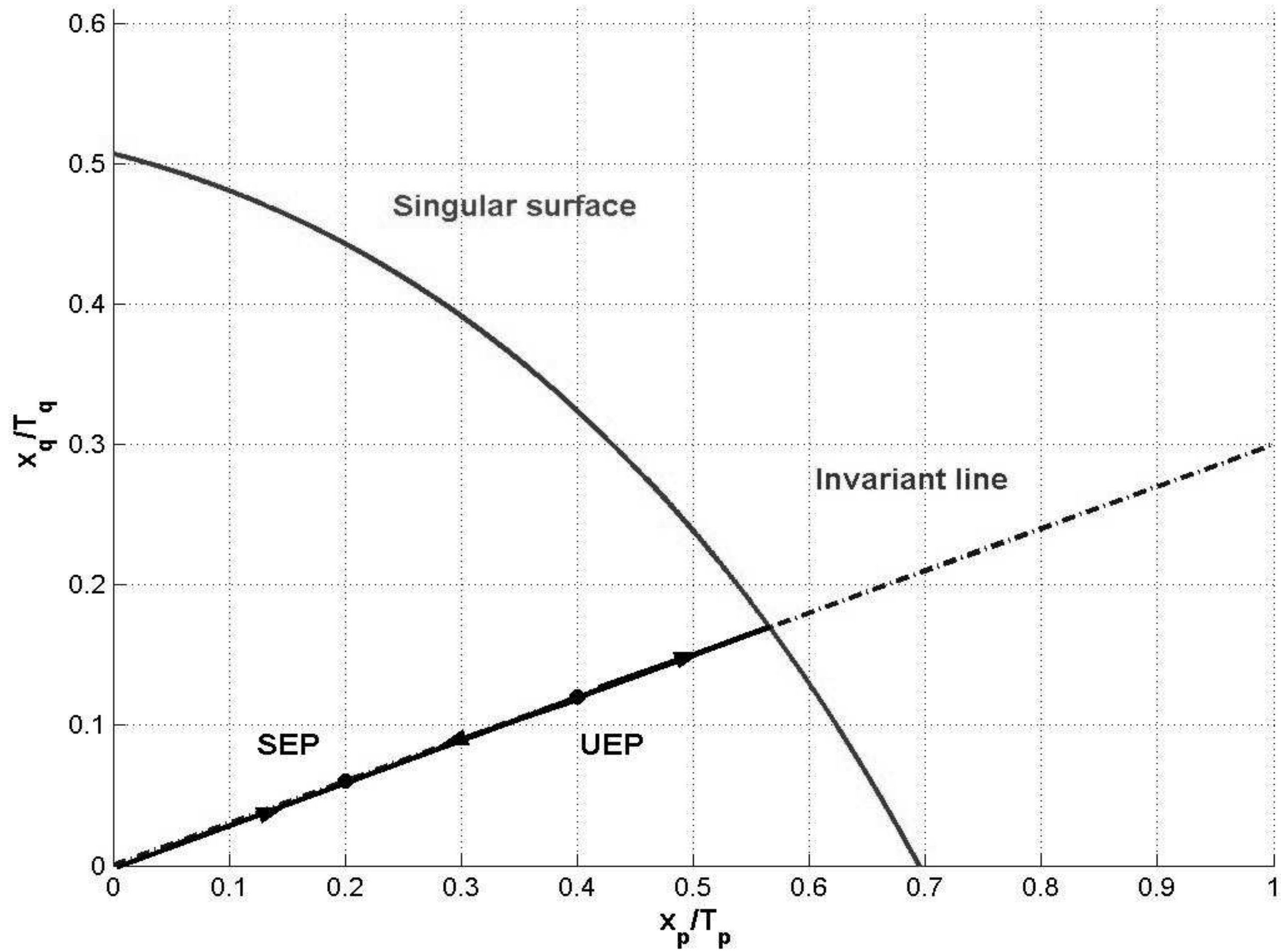


Stationary Points



Stationary points for X=0.5, k=0

A possible strategy



Lyapunov-based hybrid control

- The general idea is based on the control Lyapunov functions

    ✓ Take a Lyapunov function candidate

    ✓ Synthesize a feedback law such that the Lyapunov candidate is decreasent

Algorithm formulation

For convenience the model is written

$$\begin{aligned}\dot{x} &= -ax + b\left[1 - y(x,u)\right] \\ 0 &= y^2 + 2\alpha_1(x,u)y + \alpha_0(x,u)\end{aligned}$$

With $a=1/T_p$ and $b=P_0$

$$u^T = [n, B, k]$$

Algorithm formulation

For convenience the control constraints are repeated

- the tap ratio n $\in$ [**0.8**, **1.2**] with steps of **0.02 p.u**.
- the capacitor banc B$\in$ [**0**, **0.3**] with steps of **0.1** p.u of reactive power compensation.
- the load shedding k $\in$ [**0**, **0.15**] with steps of **0.05** p.u.

Algorithm formulation

For synthesis purposes the following model

$$\frac{dn}{dt} = \Gamma sgn(\nu, n, \Delta)$$

Where $sgn(\cdot,\cdot,\cdot)$ is defined as

$$sgn(\nu, n, \Delta) = \begin{cases} 1 \;\; \text{if} \;\; (\nu > \Delta) \wedge (n < n_M) \\ -1 \;\; \text{if} \;\; (\nu < -\Delta) \wedge (n > n_m) \\ 0 \;\; \text{if} \;\; (|\nu| \leq \Delta) \vee (n = n_m) \vee (n = n_M) \end{cases}$$

Practically $v_r = \nu + v$ is used to drive the OLTC

Algorithm formulation

The other control inputs **B** and **k** are casted in a strictly ordered set

k/B	0.0	0.1	0.2	0.3
0.00	$j = 1$	$j = 2$	$j = 3$	$j = 4$
0.05	$j = 5$	$j = 6$	$j = 7$	$j = 8$
0.10	$j = 9$	$j = 10$	$j = 11$	$j = 12$
0.15	$j = 13$	$j = 14$	$j = 15$	$j = 16$

A particular selection of a pair (B, k) will be denoted u_j and is henceforth labelled by an integer j.

Algorithm formulation

Suppose when the system is in fault

$$\exists \quad (x^*, u^*) \qquad \text{s.t}$$

$$\begin{aligned} 0 &= -ax^* + b\left[1 - y^*\right] \\ 0 &= y^{*2} + 2\alpha_1^* y^* + \alpha_0^* \end{aligned}$$

The set of possible (x^* , u^*) is in general not a singleton

► The **output**, the voltage in this case is **above** or **between certain limits**;

► The **minimum** value of the **capacitor banc** and **load shedding** is used to achieve **a certain closed loop characteristics**.

Algorithm formulation

The dynamic equations can be written in the error coordinates

$$\tilde{x} = x - x^*, \qquad \tilde{u} = u - u^*$$

as

$$\begin{aligned} \dot{\tilde{x}} &= -a\tilde{x} + b[y^* - y^+(x,u)] \\ \dot{\tilde{n}} &= \Gamma\, sgn(\nu, n, \Delta) \\ 0 &= y^2 + 2\alpha_1(x,u)y + \alpha_0(x,u) \end{aligned}$$

where

$$y^+(x,u) = \max y_{1,2}(x,u)$$

Control design

Consider the following Lyapunov function candidate

$$V(z) = \frac{z^T z}{2}$$

$$z^T = [\dot{\tilde{x}} \ \tilde{x} \ \tilde{n}]$$

So that

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial z}\dot{z} \\ &= \dot{\tilde{x}}\ddot{\tilde{x}} + \tilde{x}\dot{\tilde{x}} + \tilde{n}\dot{\tilde{n}} \end{aligned}$$

Using

$$\dot{\tilde{x}} = -a\tilde{x} + b[y^* - y^+(x,u)]$$

We have

$$\dot{V} = -\left(a + b\frac{\partial \tilde{y}}{\partial x}\right)\dot{\tilde{x}}^2 - \left(b\frac{\partial \tilde{y}}{\partial n} - \tilde{n}\right)\dot{\tilde{n}} + \tilde{x}\dot{\tilde{x}} - b\dot{\tilde{x}}\left(\frac{\partial \tilde{y}}{\partial u_j}\right)^T \dot{u}_j$$

Control design

Taking into account the update law

$$\dot{\tilde{n}} = \Gamma \, sgn(\nu, n, \Delta)$$

The signal ν is taken as

$$\nu = \left(b \frac{\partial \tilde{y}}{\partial n} - \tilde{n} \right)$$

So that

$$\dot{V} = -\left(a + b \frac{\partial \tilde{y}}{\partial x} \right) \dot{\tilde{x}}^2 - \varphi \left(b \frac{\partial \tilde{y}}{\partial n} - \tilde{n}, \Delta \right) + \tilde{x} \dot{\tilde{x}} - b \dot{\tilde{x}} \left(\frac{\partial \tilde{y}}{\partial u_j} \right)^T \dot{u}_j$$

where φ is a function returning the absolute value of its first argument dead zoned with the parameter Δ.

Control design

Defining

$$f_{u_j}^T = \dot{\tilde{x}} b \left(\frac{\partial \tilde{y}}{\partial u_j}\right)^T, \ \delta_{u_j} = \dot{u}_j$$

And

$$W = \left(a + b\frac{\partial \tilde{y}}{\partial x}\right)\dot{\tilde{x}}^2 + \varphi\left(b\frac{\partial \tilde{y}}{\partial n}\dot{\tilde{x}} + \tilde{n}\right) - \tilde{x}\dot{\tilde{x}}$$

We have

$$\dot{V} = -W(x, n, u_j) - f_{u_j}^T \delta_{u_j}$$

Lyapunov Based Logic Switching Strategy LBLSS

- Make use of the concept of supervisory control at the higher level
  - The Lyapunov function is used as a monitoring signal
  - Hysteresis switching
- Solve a constrained optimization problem at the lower level
  - Decayedness of the Lyapunov function
  - Respect a safety margin
  - Penalty on switching

LBLSS

1. **Let $t:=0$; $J = \{1,2,\cdots,N\}$; $R \in \Re_+$; $[\varepsilon_1\ \varepsilon_2\ \varepsilon_3]^T \in \Re^3{}_+$; $\varepsilon_2 > \varepsilon_1$; $d_0 \in \Re_+$;**
2. **$t:=t+1$;**
3. **$C_k := \sigma(V,\varepsilon_1,\varepsilon_2)$;**
4. **If C_k**

   a) **Among the admissible values for j_k find j_k^* from**

$$j_k^* = \arg\min_{jk \in \mathcal{J}} \{j_k^2 + R\,(j_k - j_{k-1}^*)^2\}$$

under

   i. $W = W_{jk^*} \geq \varepsilon_3$

   ii. $d \triangleq x_{s2}(j_k^*) - x \geq d_0$

5. **else**

   a) **Among the admissible values for j_k find j_k^* from**

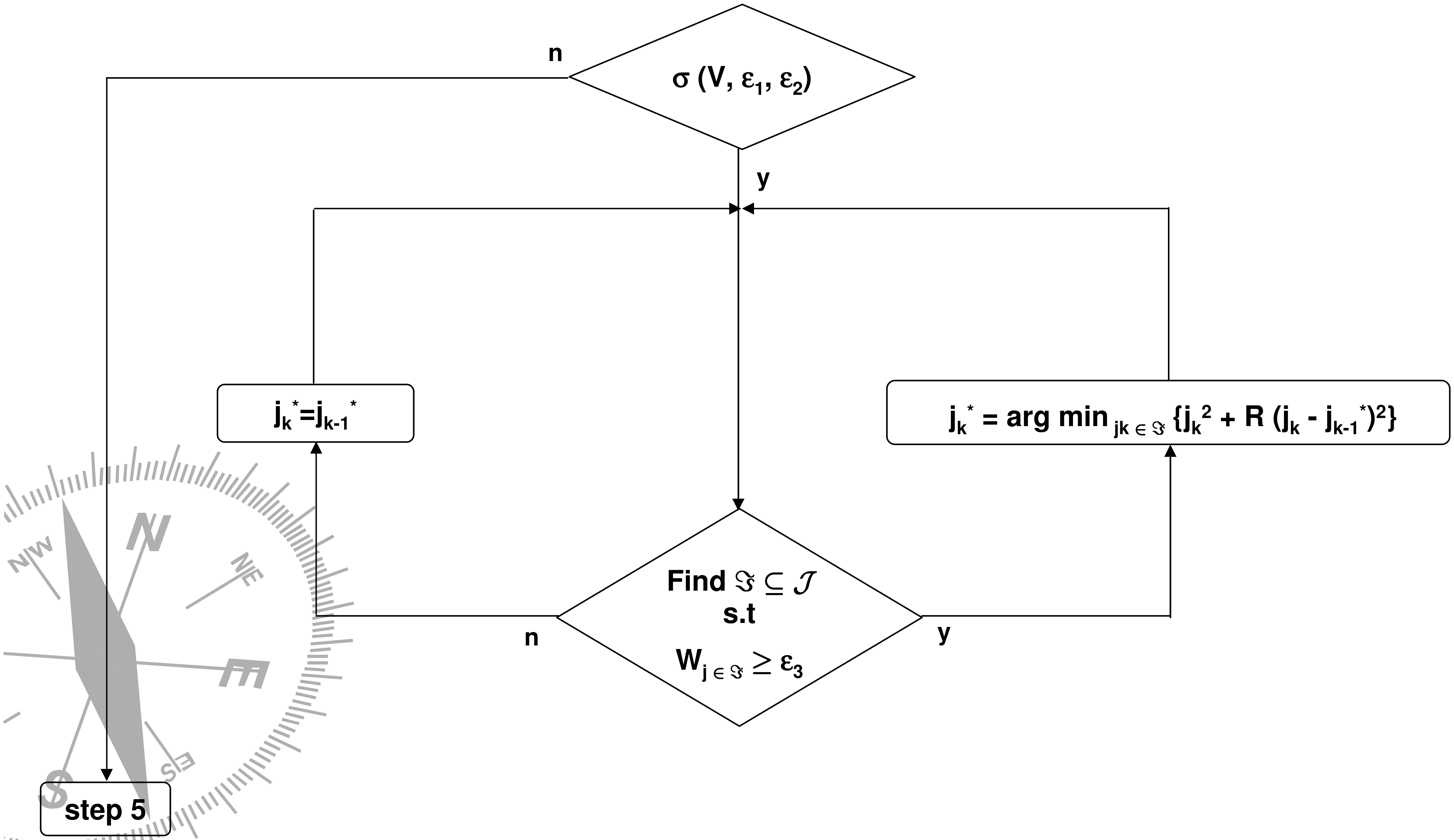
$$j_k^* = \min\{j_k : j_k \in \mathcal{J}\}$$

for which

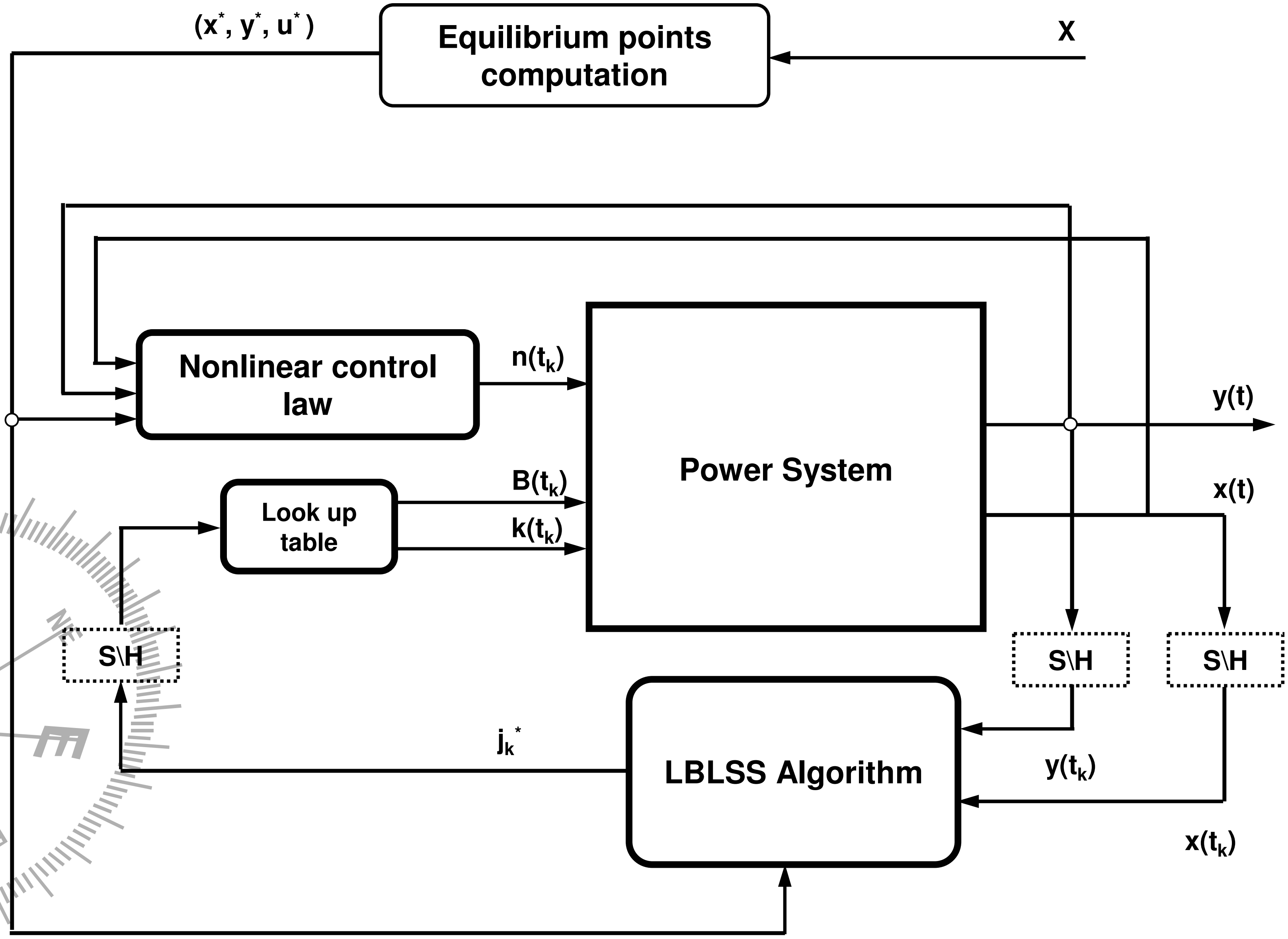
$$0 = f(x^*, y^*, j_k^*)$$
$$0 = y^{*2} + \alpha_{1,jk^*}\, y^* + \alpha_{0,jk^*}$$

6. **Goto 2;**

Computer diagram

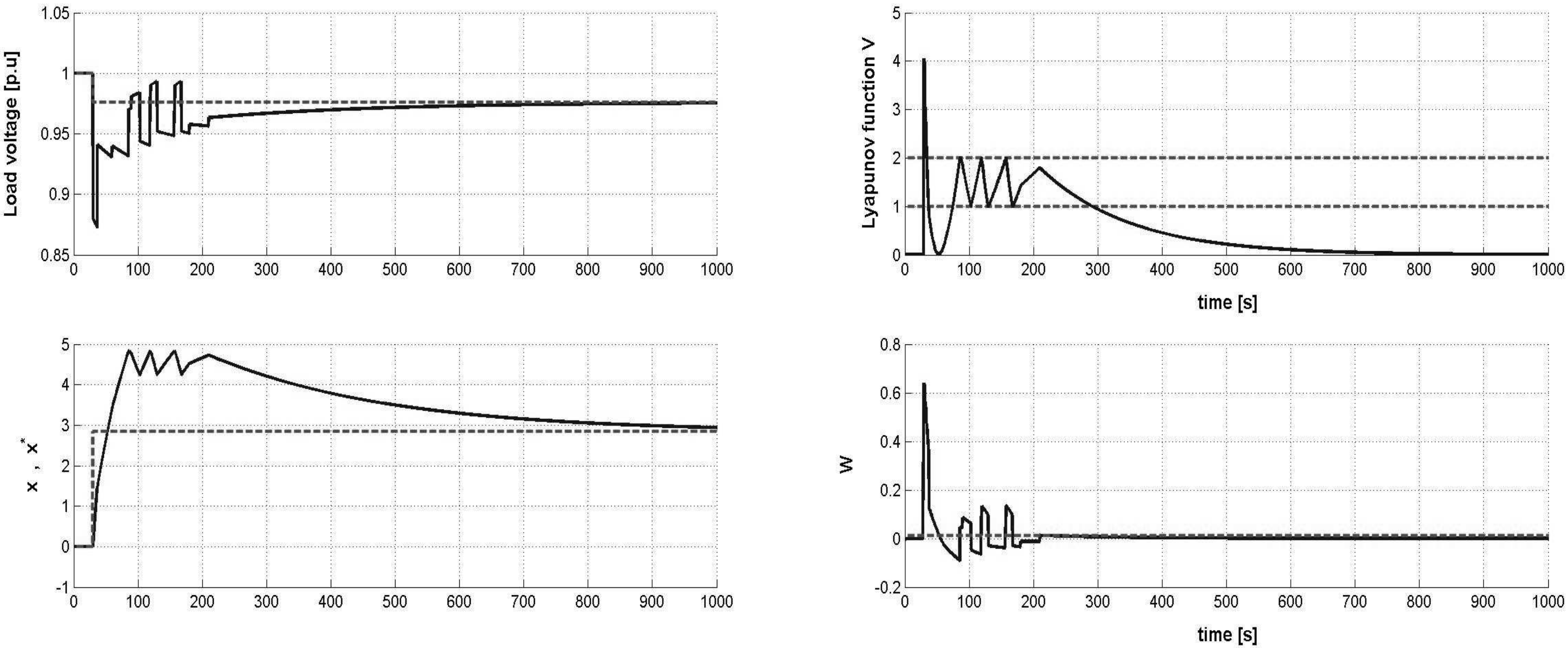


Closed loop system

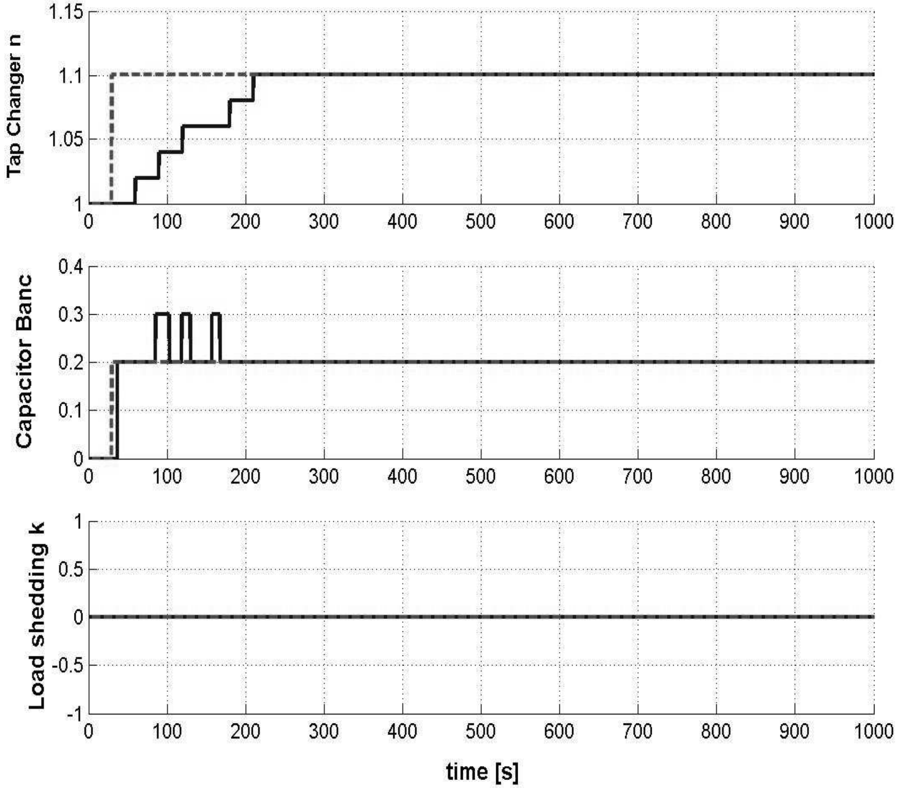


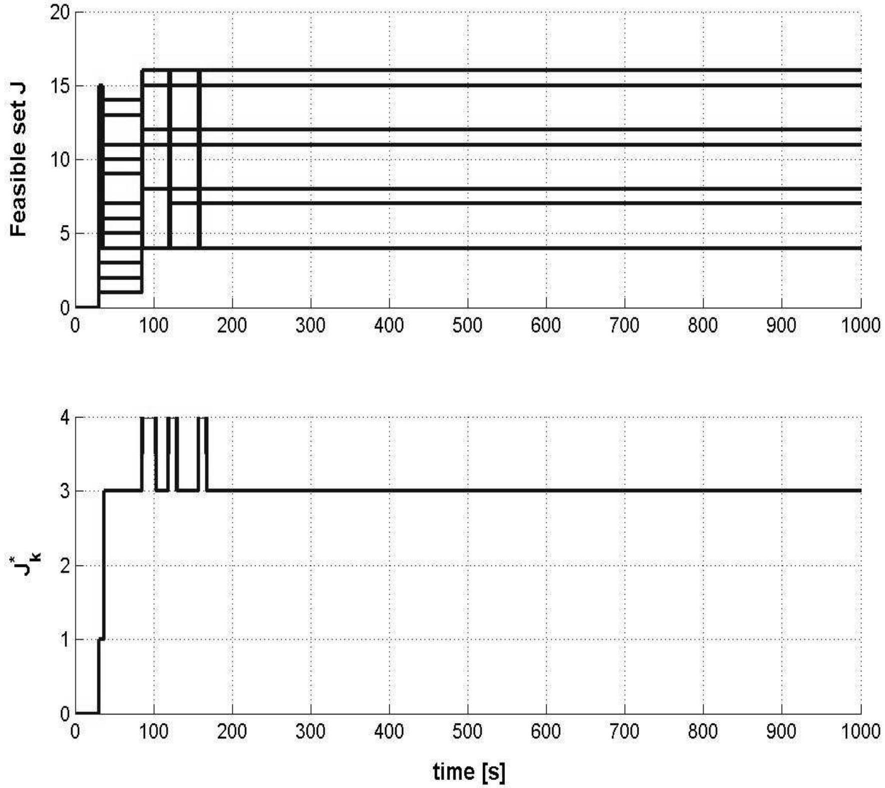
Simulation results

Nominal Case

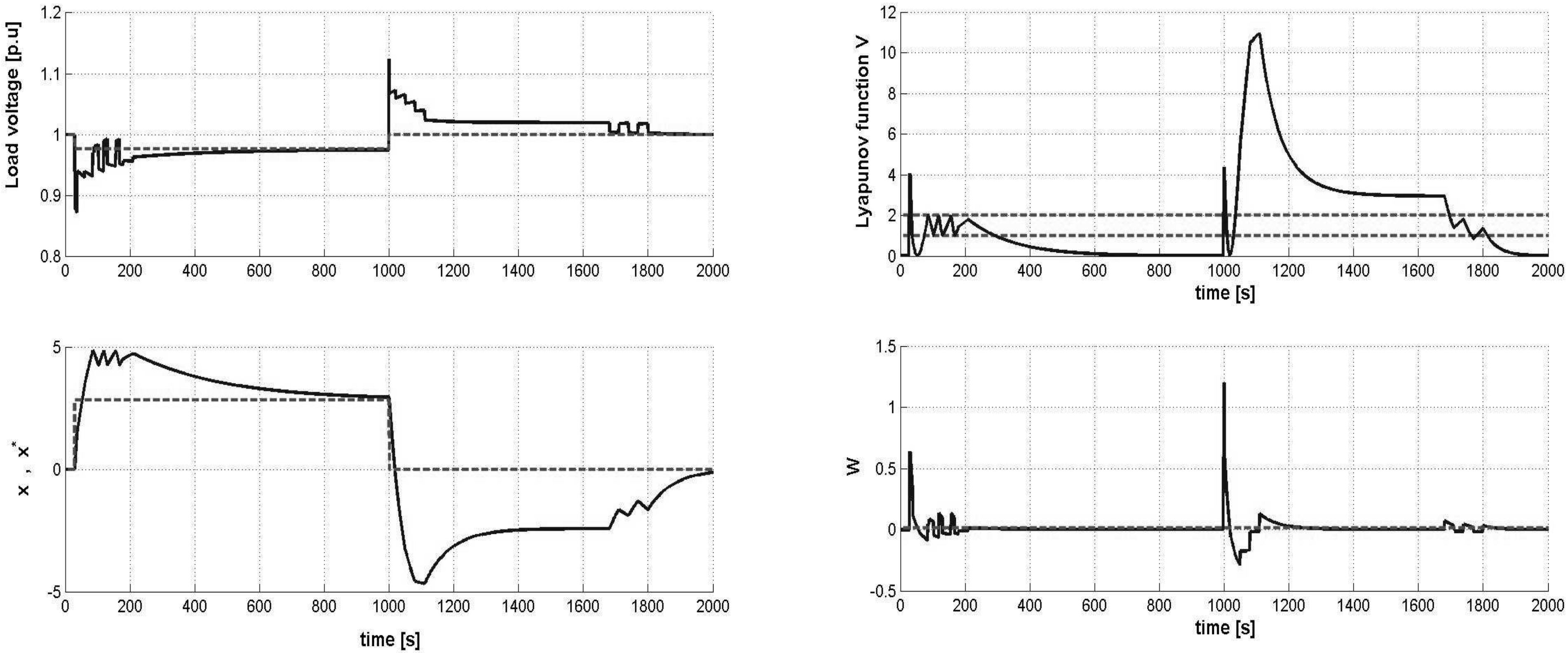


Nominal Case

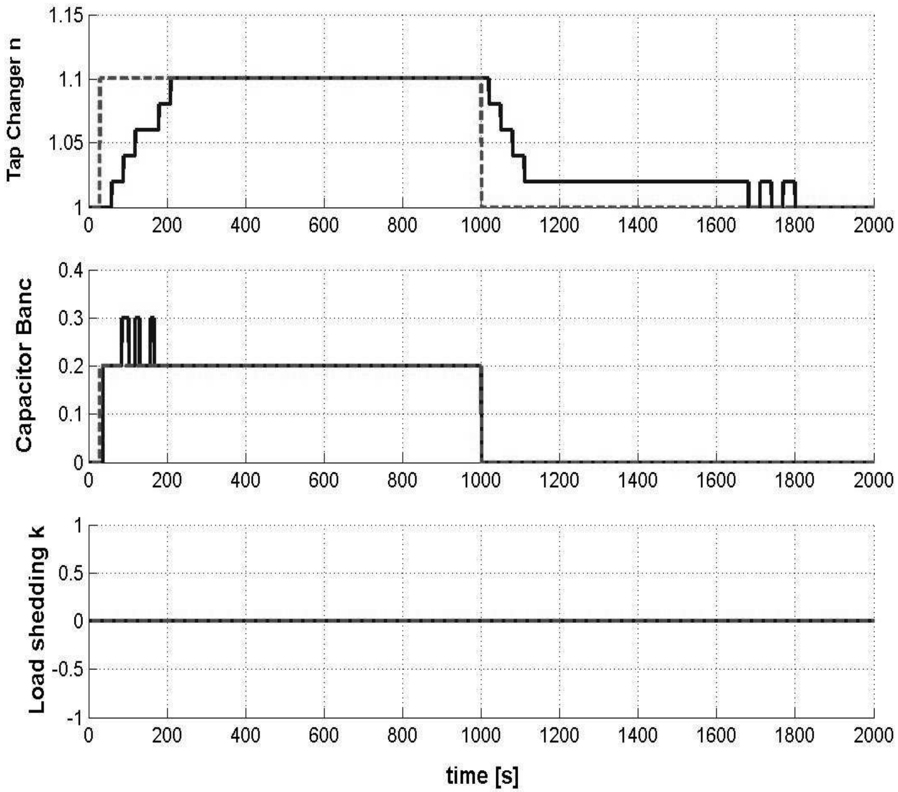


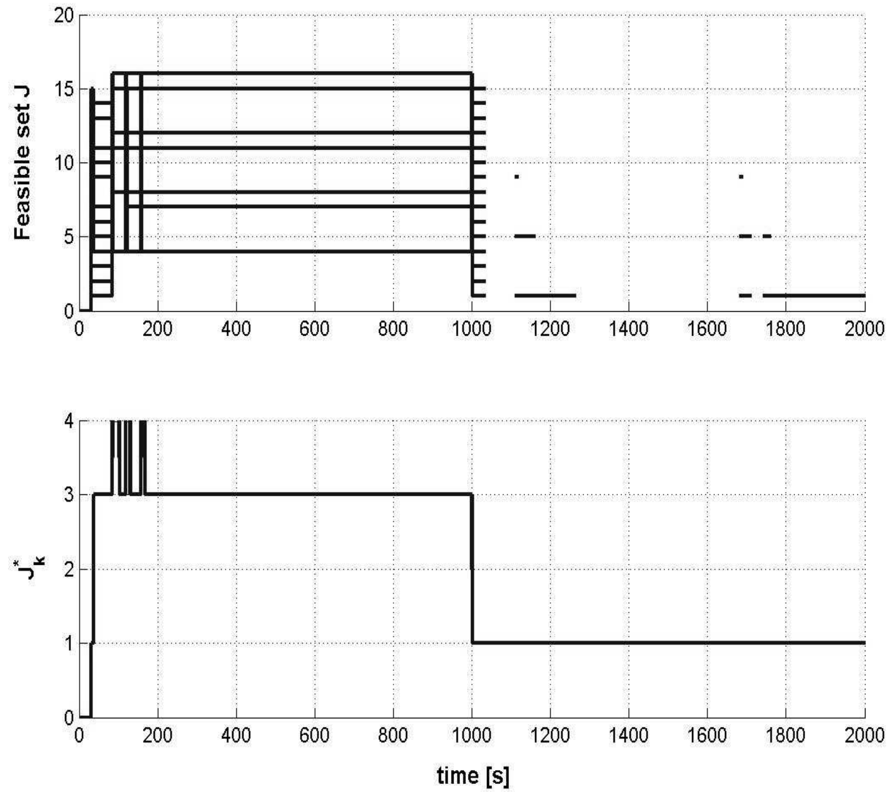


Fault clearance

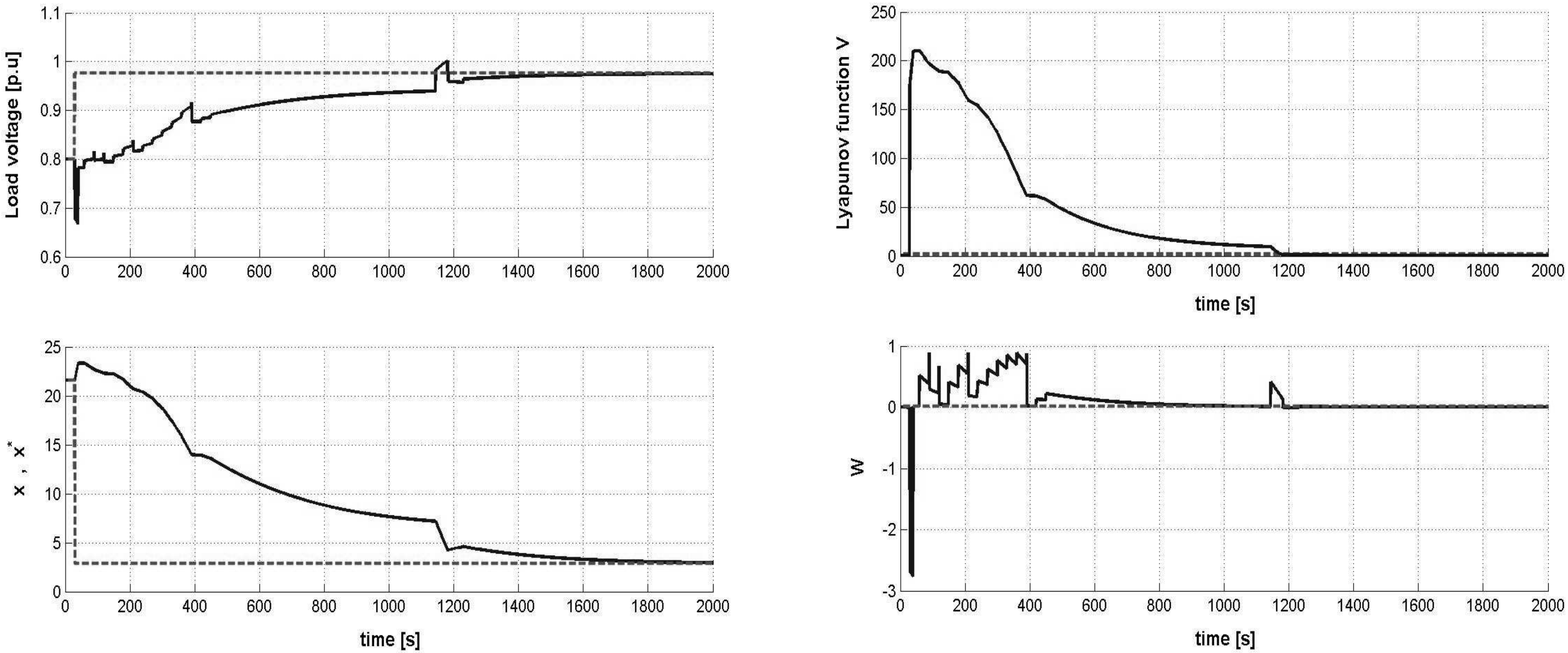


Fault clearance

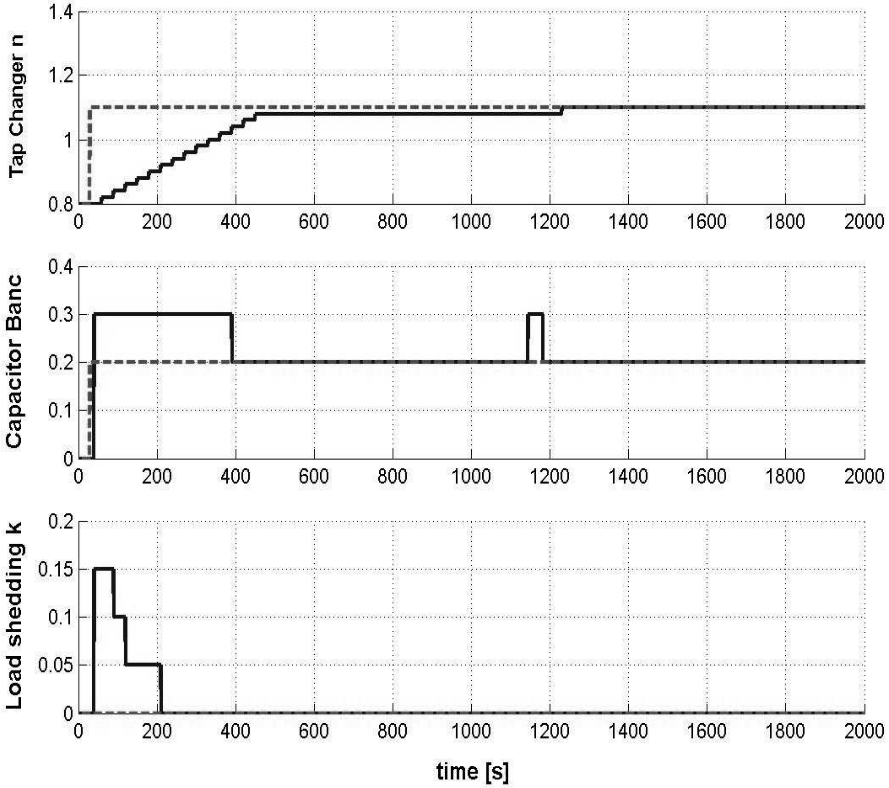




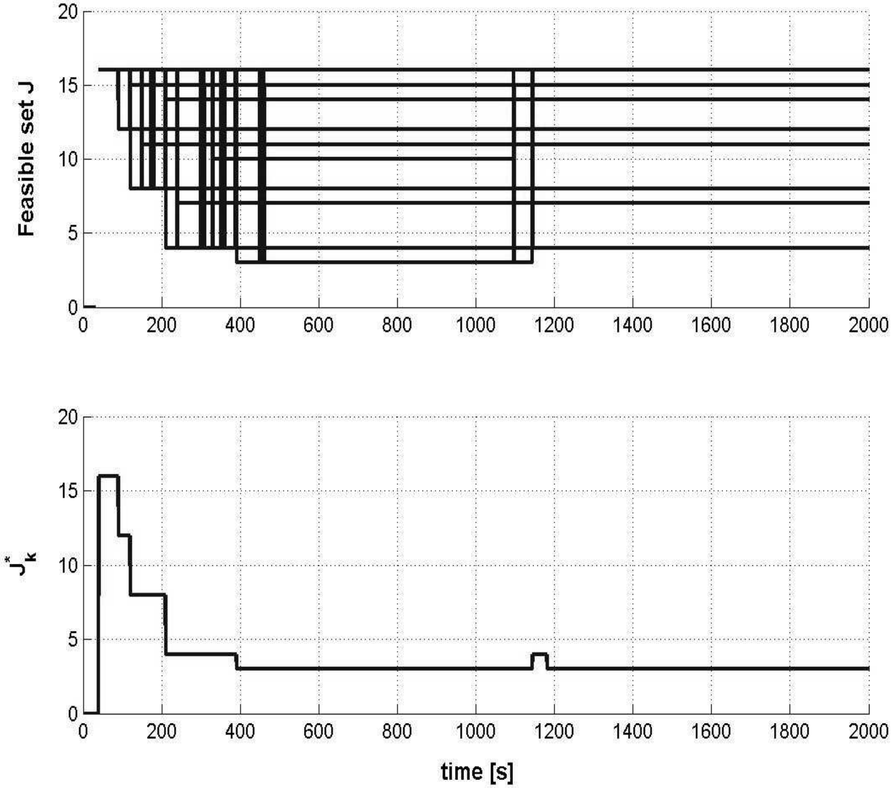
Worst case


Load voltage [p.u]
1.1
1
0.9
0.8
0.7
0.6
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Lyapunov function V
250
200
150
100
50
0
time [s]
x , x*
25
20
15
10
5
0
time [s]
w
1
0
-1
-2
-3
time [s]

Worst case



Tap Changer n
Capacitor Banc
Load shedding k
time [s]



Feasible set J
J^*_k
time [s]

Conclusions and future work

- Stabilization proved possible by simulating different cases
- The approach proposed is tractable from the computational view point

- Need to assert stability
- Need to assert that switching is finite
  - More systematic way to tune the parameters
  - Find more suitable monitoring signals