A Hybrid Model of a 4-Cylinder Engine for Idle Speed Control[1]

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Abstract

The present document describes the first version of the *Idle Speed Control* case study, developed by PARADES within the "Computation and Control" European Project during year 2002. The specification for the idle control problem for automotive engines is to maintain the crankshaft speed within a given range in the presence of load changes. A cycle-detailed hybrid model of the engine that captures well the interactions between the discrete phenomena of torque generation and spark ignition, and the continuous evolution of the power-train and air dynamics, is proposed.
After introducing the idle-speed control problem as a safety one, a simulation of the engine's open-loop behaviour under the action of a disturbance torque is discussed.

1 Introduction

The synthesis of a control strategy for an internal combustion engine in the idle regime is one of the most challenging problems in engine control. The goal is to maintain the engine speed as close as possible to a reference constant engine speed despite load torque disturbances (i.e. the air conditioning system, the steering wheel servo-mechanism) and engagements and disengagements of the trasmission occurring when the driver operates on the clutch. In order to achieve the best fuel economy, the reference engine speed is chosen at the minimum value that yields acceptable combustion and emission quality, and noise, vibration and harshness (NVH) characteristics.
For indirect injection engines, assuming a stoichiometric air-to-fuel ratio, stability of the engine speed is achieved by controlling the amount of air supplied to the engine and the spark ignition times. In modern cars, the amount of air is controlled by directly acting on the throttle via a DC or a stepper motor(electronic throttle control). This control can provide large control authority but it is relatively slow since it is subject to the dynamics of the cylinders filling and the delay due to the mix compression. On the contrary, spark control is much faster and is actuated by anticipating/retarding the spark ignition with respect to a nominal retard from the maximum spark-to-torque efficiency value. Hence, sudden loads can be much better compensated with spark ignition than with air inflow, while air inflow can be used to control the engine in steady state. However, spark control has a limited authority since combustion inefficiency and catalyst overheating must be avoided.
In order to design a controller able to reduce substantially emission and gas consumption while maintaining idle regime stability, an accurate cycle-to-cycle model of a four–stroke gasoline engine is here proposed. The model has a "natural" hybrid representation because (i) pistons have four modes of operation corresponding to the stroke they are in, and (ii) power–train and air dynamics are continuous–time processes.
In addition, these processes interact tightly: in fact, the timing of the transitions between two phases of the pistons is determined by the continuous motion of the power–train, which, in turn, depends on the torque produced by each piston. The adoption of a hybrid formalism allows to describe the cyclic behavior of the engine, thus capturing the effect of each spark ignition on the generated torque, the interaction between the discrete torque generation and the continuous power-train and air dynamics, and the discrete changes in the power-train. The torque that is generated by each piston and applied to the engine crankshaft can be assumed to be a function of the spark ignition time, and of the air-fuel mixture mass loaded in the cylinder during the intake phase so that it can be modified by appropriately acting on the throttle valve and on the spark ignition times. Hence, the outputs of the hybrid controller are the input voltage to the DC motor moving the throttle valve (continuous-time variable) and the spark ignition times for the four spark plugs (discrete-time variables).

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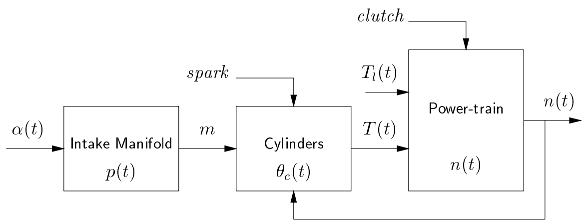

Figure 1: The engine blocks.

2 Hybrid model of the engine

In this section, a hybrid system that describes the behavior of an automotive 4-cylinder engine in idle is illustrated. Figure 1 shows the interactions between the three main subsystems which compose the engine: the intake manifold, the cylinders and the power–train.
The pressure p of the intake manifold depends on the throttle valve angle α and determines the mass of air-fuel mixture m loaded by the cylinders. In each stroke of the 4–stroke engine cycle, the cylinders evolution is described in terms of the angle $\theta \in [0, 180]$, which represents the location of the piston within the given stroke. The torque T generated by the cylinders depends on both the mass m and the spark ignition time. Finally, the power-train dynamics and the crankshaft revolution speed n, controlled by the generated torque T, are subject to the sum of load torques T_l and depends on the position of the clutch. The gear is supposed to be in idle position.

The hybrid model of the engine in idle is characterized by a set of discrete modes (corresponding to the FSM states), a set of continuous variables (subject to a continuous–time dynamics, CTS) and a set of symbolic constants (whose discrete–time evolution is described by a DES). The control inputs are: the throttle angle α and the spark command *spark*(which can be applied from 20^0 before the end of the compression stroke to 15^0 after the beginning of the expansion stroke). The disturbance inputs are: the load torque T_l (assumed to be nonegative and upper bounded by some T_l^M) and the *clutch*. Hence, the controller and the environment act on the system with two kinds of inputs: the continuous inputs, α and T_l, affect the continuous dynamics; the discrete inputs, *spark* and *clutch*, determine the discrete mode transitions, the resetting of continuous variables and the setting of symbolic constants.

The engine FSM has six modes S_-, S, S_+, S_-^L, S^L, and S_+^L, see Figure 2. The three modes S_-, S, and S_+ are used to describe the interleaving between spark ignitions and dead–centers (the pistons are at their uppermost/lowermost position) in the cylinders evolution and are related to the clutch in open position. Modes S_-^L, S^L, and S_+^L are the corresponding modes with clutch closed.

The DES has a 4–dimensional state, whose components (T, m_C, m_E, φ) stand, respectively, for the generated torque, the mass of mixture in the cylinder which is in the compression stroke, the mass of mixture in the cylinder which is in the expansion stroke, and the applied spark advance.

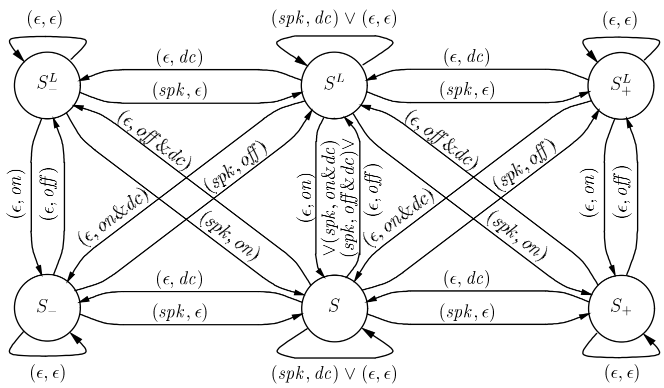

Figure 2: Engine FSM.

The components of the CTS state are: the manifold pressure p, the crankshaft speed n and the piston position θ. Such variables are subject to the following dynamics

$$\dot{p}(t) = a_p p(t) + b_p \alpha(t) \tag{1}$$

$$\dot{n}(t) = a_n n(t) + b_n (T_g(t) - T_l(t)) \tag{2}$$

$$\dot{\theta}(t) = k_c n(t) \tag{3}$$

where $T_g(t) = T(t) - T_p(t)$ represents the effective torque generated by the engine, while $T_p(t)$ stands for the loss of generated torque due both to pumping and friction. If θ is in degrees and n is in revolutions per minute, then $k_c = 6$, $a_n = -Bb_n$ and

$$b_n = \begin{cases} 1/J_0 & \text{if the clutch is open} \\ 1/J_c & \text{if the clutch is closed} \end{cases} \tag{4}$$

with J_0 (J_c) and B denoting the inertial momentum and the viscous friction coefficient of the segment of the power-train from the crankshaft to the clutch (of the primary drive-line, respectively). In (2) the DES variable T plays the role of a parameter.
Formally, the overall engine hybrid model is a tuple $H = (\{Q, X, \Xi\}, \{\Sigma_c, U\}, \{M_c^{disc}, M_c^{cts}\}, \{\Sigma_e, D\}, \{M_e^{disc}, M_e^{cts}\}, \{f, \delta\})$, where:

- the **state space** is composed of: the finite set of FSM modes $Q = \{S_-, S, S_+, S_-^L, S^L, S_+^L\}$, the space of CTS variables $X = \{(p, n, \theta) \mid (p, n, \theta) \in \mathbb{R}^3\}$, and the space of DES variables $\Xi = \{(T, m_C, m_E, \varphi) \mid (T, m_C, m_E, \varphi) \in \mathbb{R}^4\}$. An element (q, x, ξ) in the space $Q \times X \times \Xi$ is called a *configuration*;
- the **control inputs** $\alpha(t)$ and *spark* can be described by means of: the domain of continuous input values $U = [0, \alpha^M]$, the finite domain of *discrete control events* $\Sigma_c = \{spk\}$ with $\Sigma_c^\epsilon = \Sigma_c \cup \{\epsilon\}$ being the set of *discrete control moves* and the special ϵ move

being the *silent* move; the *discrete controller feasible move function*[2] $M_c^{disc} : Q \times X \times \Xi \to 2^{\Sigma_c^\epsilon} \setminus \{\}$ as in Table 1; the *continuous controller feasible move function* $M_c^{cts} : Q \times X \times \Xi \to 2^U \setminus \{\}$ described as follows: $M_c^{cts}(q, x, \xi) = \{\alpha \mid \alpha \in [0, \alpha^M]\}, \ \forall (q, x, \xi)$;

- the **disturbance inputs** $T_l(t)$ and *clutch* can be described by means of: the domain of *continuous disturbance values* $D = [0, T_l^M]$, the finite set of *discrete disturbance events* $\Sigma_e = \{on,\ off,\ dc,\ on\&dc,\ off\&dc\}$ (where the events *on* and *off* represent opening and closing the clutch, the event *dc* represents reaching a dead–center when $\theta = 180$, the events *on&dc* and *off&dc* represent the simultaneous occurrence of a clutch operation and a dead–center), and $\Sigma_e^\epsilon = \Sigma_e \cup \{\epsilon\}$ is the set of *discrete disturbance moves*; the *discrete disturbance move function*[3] $M_e^{disc} : Q \times X \times \Xi \to 2^{\Sigma_e^\epsilon} \setminus \{\}$ given in Table 1; the *continuous disturbance feasible move function*: $M_e^{cts} : Q \times X \times \Xi \to 2^D \setminus \{\}$ described as follows: $M_e^{cts}(q, x, \xi) = \{T_l \mid T_l \in [0, T_{lM}]\}, \ \forall (q, x, \xi)$;

- the **transitions** are described by $f : Q \times X \times \Xi \times U \times D \to \mathbb{R}^n$ that models the CTS dynamics, which depends on the mode, and the *transition function* $\delta = (\delta|_Q, \delta|_X, \delta|_\Xi) : Q \times X \times \Xi \times \Sigma_c^\epsilon \times \Sigma_e^\epsilon \to 2^{Q \times X \times \Xi} \setminus \{\}$ modeling the discrete dynamics. For each $q \in Q$, $f(q, x, \xi, \alpha, T_l)$ is given by (1), (2) and (3), where in (2) the parameters a_n and b_n change by (4) according to whether $q \in \{S, S_+, S_-\}$ or $q \in \{S^L, S_+^L, S_-^L\}$. The transition function δ is described in Figure 2 (FSM next–state function $\delta|_Q$) and in Table 2 (CTS state reset function $\delta|_X$ and DES dynamics $\delta|_\Xi$)).

3 The idle-speed control problem

The goal of an idle speed controller is to keep the crankshaft revolution speed within a specified range around a reference value. The idle speed control problem can be stated as a safety one, within the Maximal Safe Set's theoretical setting. Given a subset G_0 of the state space, composed by all the acceptable configurations according to a prescribed criterion, the Maximal Safe Set is defined as the largest subset S_0 of G_0 for which there always exists a control law that, starting from a state in S_0, is able to keep the system's trajectory within G_0 indefinitely and in spite of any possible action of the disturbance.
In our case, the set G_0 is represented by a specified range $[n_0 - \Delta, n_0 + \Delta]$ around the desired crankshaft revolution speed. A design procedure for a closed-loop controller based on the Maximal Safe Set technique is exposed in detail in [2] and [4].

4 A Simulink/Stateflow model

In order to provide a Simulink representation for the system depicted in Figure 1, we choosed to decompose the Engine FSM (see Figure 2) as the product of a three-location one, capturing the engine phases, and a two-location one, capturing the state of the clutch. That's because the two FSM thus obtained must be placed in different blocks: while the former has its natural placement in the 'Cylinders' block, the latter is a part of the 'Power-train' block.
We built two different Simulink models: the first, named "States3Linear.mdl", has only the three-location FSM, the clutch being supposed always in the 'close' position; the second model, named "States6Linear.mdl", is complete.
The whole set of parameters, taken from a commercial car, is given in Table 3, while the set-point values of the state and input variables are given in Table 4.
The evolution of the crankshaft speed and generated torque under the action of the disturbance load torque represented in Figure 3 are reported respectively in Figure 4 and Figure 5.

References

[1] A. Balluchi, L. Benvenuti, M.D. Di Benedetto, C. Pinello, and A.L. Sangiovanni–Vincentelli, "Automotive engine control and hybrid systems: Challenges and opportunities," *to appear in Proceedings of the IEEE*, 2000.

[2] A. Balluchi, L. Benvenuti, H. Wong-Toi, T. Villa, and A. L. Sangiovanni-Vincentelli, "Controller synthesis for hybrid systems with lower bounds on event separation," in *Proc. 38th IEEE Conference on Decision and Control*, Phoenix, Arizona, USA, December 1999, pp. 3984–3989.

[3] A. Balluchi, L. Benvenuti, M. D. Di Benedetto, G. M. Miconi, U. Pozzi, T. Villa, H. Wong-Toi, and A. L. Sangiovanni-Vincentelli, "Maximal safe set computation for idle speed control of an automotive engine," in *HSCC'00, Hybrid Systems: Computation and Control*.

[4] J. Lygeros, C. Tomlin and S. Sastry, "Controllers for Reachability Specifications for Hybrid Systems," in *Automatica*, March 1999, vol.35, num. 3.

[2] Notice that $M_c^{disc}(S_-, \theta = 15) = \{spk\}$ is a discrete control move required when the spark was not given yet and must be given now, since it is the last valid ignition time instant.

[3] Notice that $M_e^{disc}(S_+, \theta = 180) = \{dc, off\&dc\}$, $M_e^{disc}(S_+, \theta = 180) = \{dc, on\&dc\}$, $M_e^{disc}(S, \theta = 180) = \{dc, off\&dc\}$ and $M_e^{disc}(S^L, \theta = 180) = \{dc, off\&dc\}$ model discrete moves forced by the continuous state.

q	θ	M_c^{disc}	q	θ	M_e^{disc}
S	$[0,160)$	$\{\epsilon\}$	S	$[0,180)$	$\{\epsilon, off\}$
	$[160,180]$	$\{\epsilon, spk\}$		180	$\{dc, off\&dc\}$
S^L	$[0,160)$	$\{\epsilon\}$	S^L	$[0,180)$	$\{\epsilon, on\}$
	$[160,180]$	$\{\epsilon, spk\}$		180	$\{dc, on\&dc\}$
S_+	$[0,180]$	$\{\epsilon\}$	S_-	$[0,15]$	$\{\epsilon, off\}$
S_+^L	$[0,180]$	$\{\epsilon\}$	S_-^L	$[0,15]$	$\{\epsilon, on\}$
S_-	$[0,15)$	$\{\epsilon, spk\}$	S_+	$[0,180)$	$\{\epsilon, off\}$
	15	$\{spk\}$		180	$\{dc, off\&dc\}$
S_-^L	$[0,15)$	$\{\epsilon, spk\}$	S_+^L	$[0,180)$	$\{\epsilon, on\}$
	15	$\{spk\}$		180	$\{dc, on\&dc\}$

Table 1: Discrete move functions M_c^{disc} and M_e^{disc}.

q	$(spk, clutch)$	$\delta\vert_X$	$\delta\vert_\Xi$
S	(ϵ, off)		
	(ϵ, dc) $(\epsilon, off\&dc)$	$\theta := 0$	$m_E := m_C,\ m_C := G_{mp}p + M_0,\ T := 0$
	(spk, ϵ) (spk, off)		$\varphi := 180 - \theta$
	(spk, dc) $(spk, off\&dc)$	$\theta := 0$	$m_C := G_{mp}p + M_0,\ T := (Gm_C + T_0)\eta(0)$
S^L	(ϵ, on)		
	(ϵ, dc) $(\epsilon, on\&dc)$	$\theta := 0$	$m_E := m_C,\ m_C := G_{mp}p + M_0,\ T := 0$
	(spk, ϵ) (spk, on)		$\varphi := 180 - \theta$
	(spk, dc) $(spk, on\&dc)$	$\theta := 0$	$m_C := G_{mp}p + M_0,\ T := (Gm_C + T_0)\eta(0)$
S_-	(ϵ, off)		
	(spk, ϵ) (spk, off)		$T := (Gm_E + T_0)\eta(-\theta)$
S_-^L	(ϵ, on)		
	(spk, ϵ) (spk, on)		$T := (Gm_E + T_0)\eta(-\theta)$
S_+	(ϵ, off)		
	(ϵ, dc) $(\epsilon, off\&dc)$	$\theta := 0$	$m_C := G_{mp}p + M_0,\ T := (Gm_C + T_0)\eta(\varphi)$
S_+^L	(ϵ, on)		
	(ϵ, dc) $(\epsilon, on\&dc)$	$\theta := 0$	$m_C := G_{mp}p + M_0,\ T := (Gm_C + T_0)\eta(\varphi)$

Table 2: Transition functions $\delta|_X$ and $\delta|_\Xi$.

Crankshaft momentum of inertia (clutch open)	J_0	$0.9\ Kg\ m^2$
Crankshaft momentum of inertia (clutch closed)	J_c	$1.04\ Kg\ m^2$
Crankshaft viscosity coefficient	B	$0.02\ Kg\ m^2\ s$
Pumping and friction torque	T_p	$20\ Nm$
Cylinders' torque gain	G	$390000\ Nm\ Kg^{-1}$
Cylinders' torque offset	T_0	$-14.2\ Nm$
Spark efficiency function	$\eta(\phi)$	$\eta = 0.73 + \sqrt{0.083 + 0.005\phi}$
Manifold dynamic parameter	a_p	$-2.33\ s^{-1}$
Manifold dynamic parameter	b_p	$15369\ Pa\ s^{-1}\ deg^{-1}$
Compression mass/pressure gain	G_{mp}	$4.5 \cdot 10^{-9}\ Kg\ Pa^{-1}$
Compression mass offset	M_0	$-2.08 \cdot 10^{-5}\ Kg$

Table 3: Parameters of Simulink schemes.

Manifold pressure	p	$2.47 \cdot 10^4\ Pa$
Torque	T	$21.34\ Nm$
Compression mass	m_c	$9 \cdot 10^{-5}\ Kg$
Expansion mass	m_e	$0\ Kg$
Spark advance	ϕ	20^0
Crankshaft speed	n_0	$800\ RPM$
Throttle angle	α	3.8^0
Spark advance	φ	20^0

Table 4: Set-point values of state and input variables.

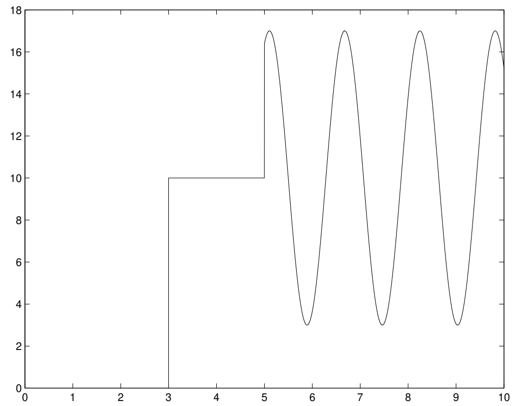

Figure 3: Load torque $T_l(t)$.

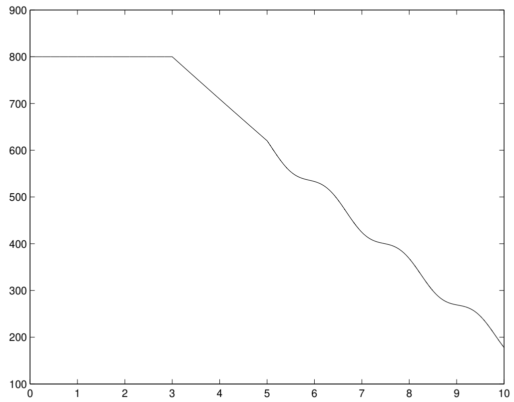

Figure 4: Crankshaft revolution speed $n(t)$.

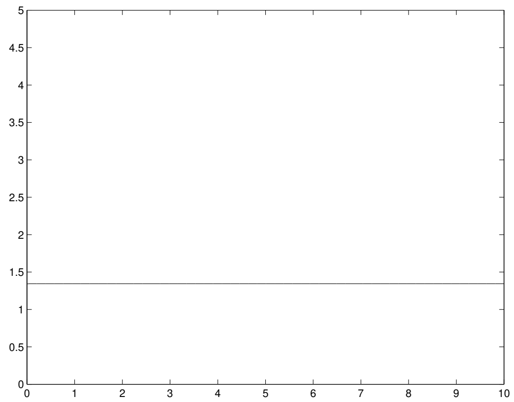

Figure 5: Generated torque $T_g(t)$.