Making Stochastic Process Algebras Count: Modelling Collective Dynamics

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and Centre for Systems Biology at Edinburgh
University of Edinburgh

24th May 2011

Joint work with Stephen Gilmore and Mirco Tribastone

Outline

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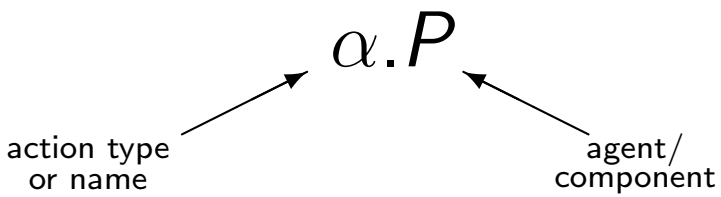
Process Algebra

- Models consist of agents which engage in actions.



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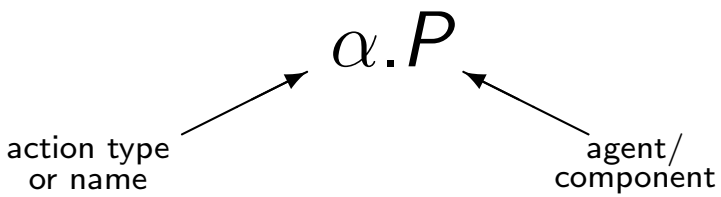

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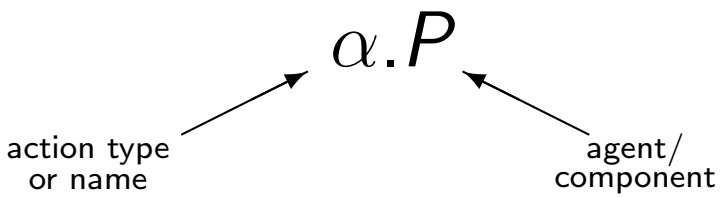

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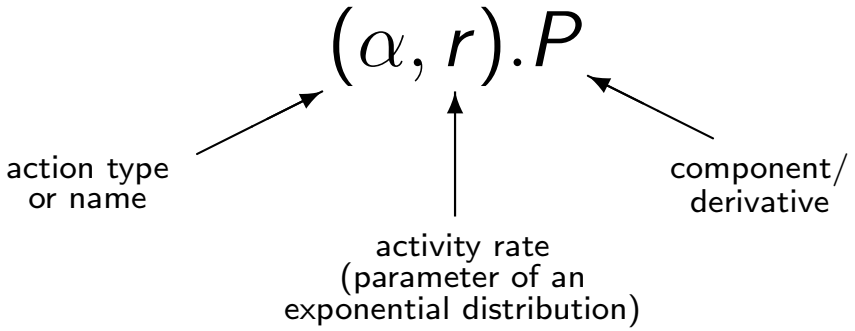
Process algebra model $\xrightarrow{\text{SOS rules}}$ Labelled transition system

Stochastic process algebras

Process algebras where models are decorated with quantitative information used to generate a stochastic process are stochastic process algebras (SPA).

Stochastic Process Algebra

- Models are constructed from components which engage in activities.



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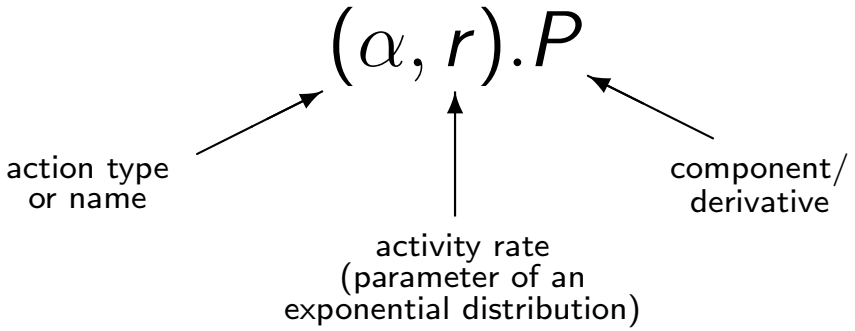

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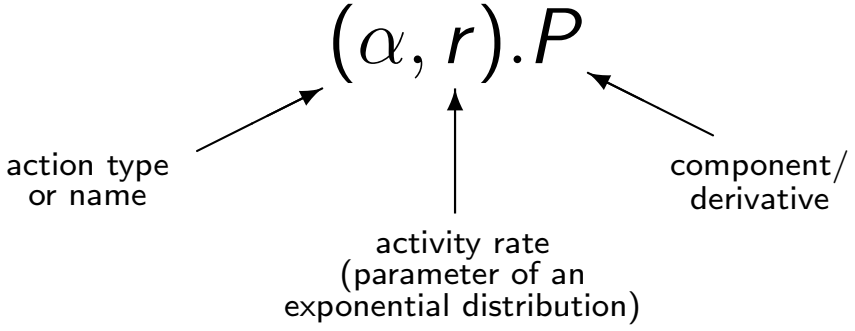

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Integrated analysis

Qualitative verification can now be complemented by quantitative verification.

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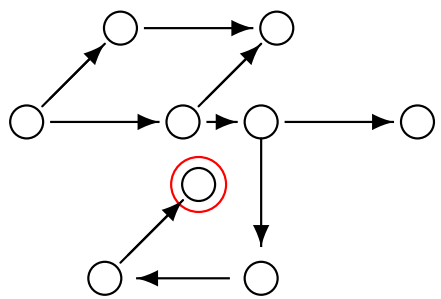
Reachability analysis

How long will it take for the system to arrive in a particular state?



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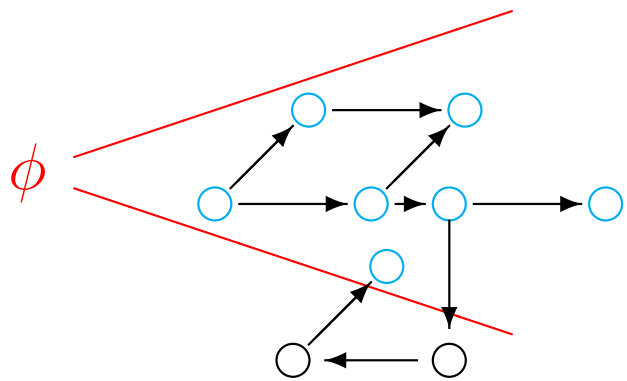
Model checking

Does a given property ϕ hold within the system with a given probability?

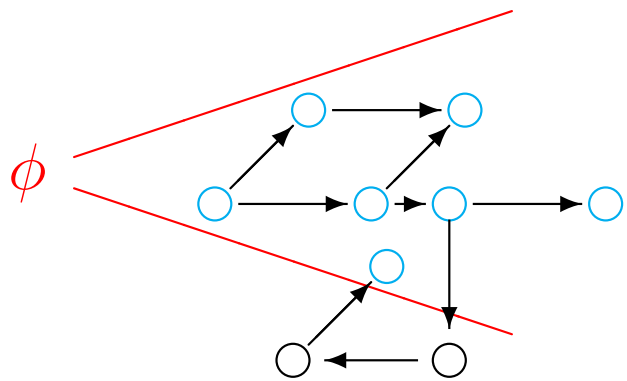


Integrated analysis

Qualitative verification can now be complemented by quantitative verification.

Model checking

For a given starting state
how long is it until
a given property ϕ holds?



Performance Evaluation Process Algebra (PEPA)

$(\alpha, f).P$	Prefix
$P_1 + P_2$	Choice
$P_1 \underset{L}{\bowtie} P_2$	Co-operation
P/L	Hiding
X	Variable

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When working with large numbers of entities, we write $P[n]$ to denote an array of n copies of P executing in parallel.

$$P[5] \equiv (P \parallel P \parallel P \parallel P \parallel P)$$

A simple example: processors and resources

$$
\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\
Res_1 &\stackrel{def}{=} (reset, r_4).Res_0
\end{aligned}
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$$Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

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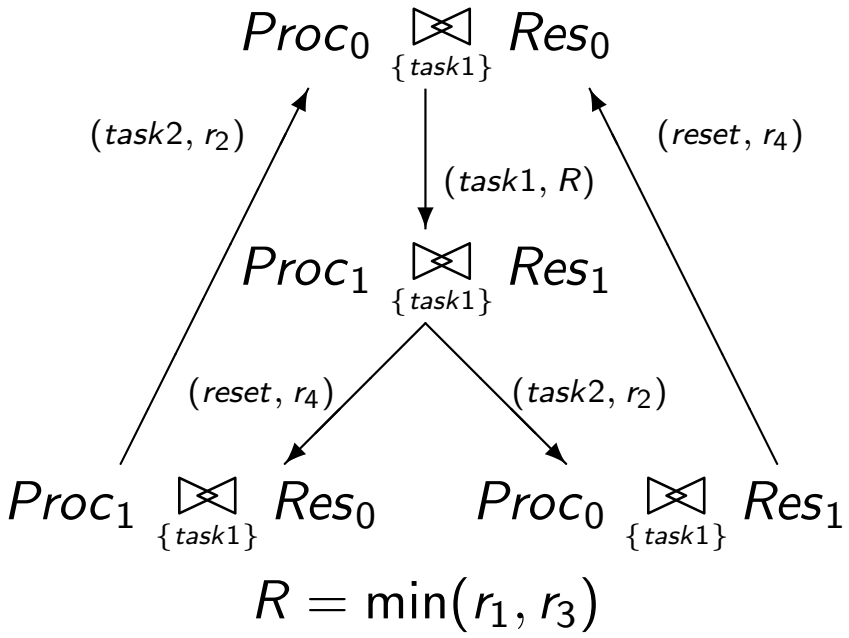

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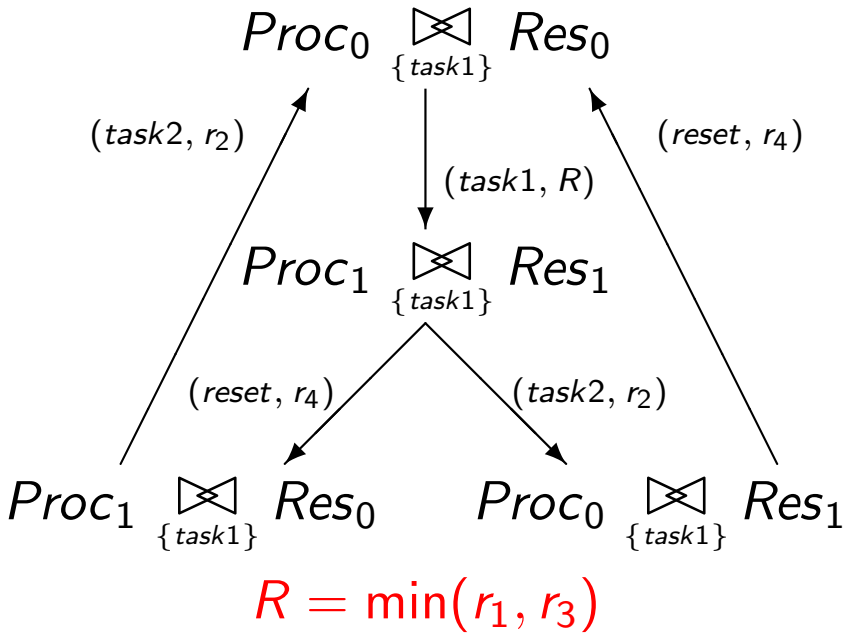

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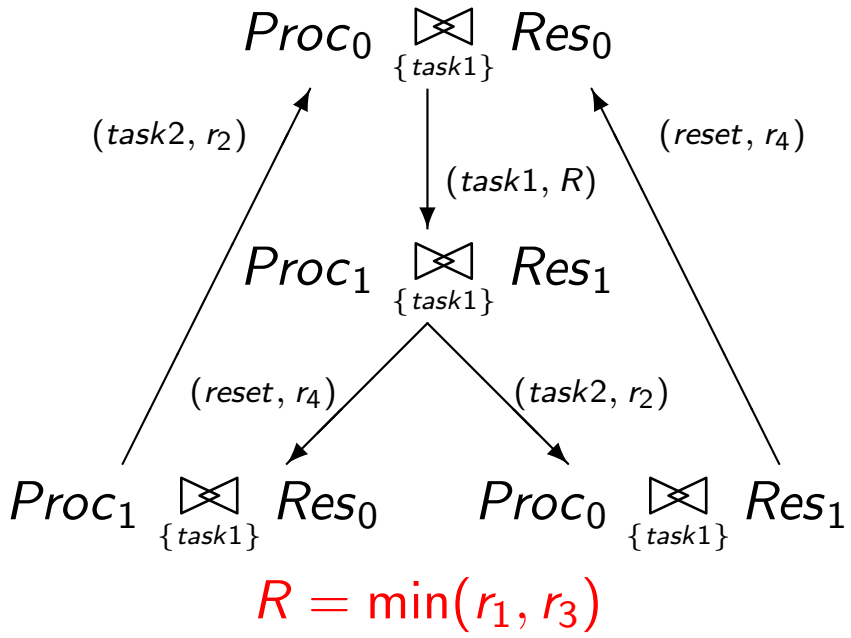
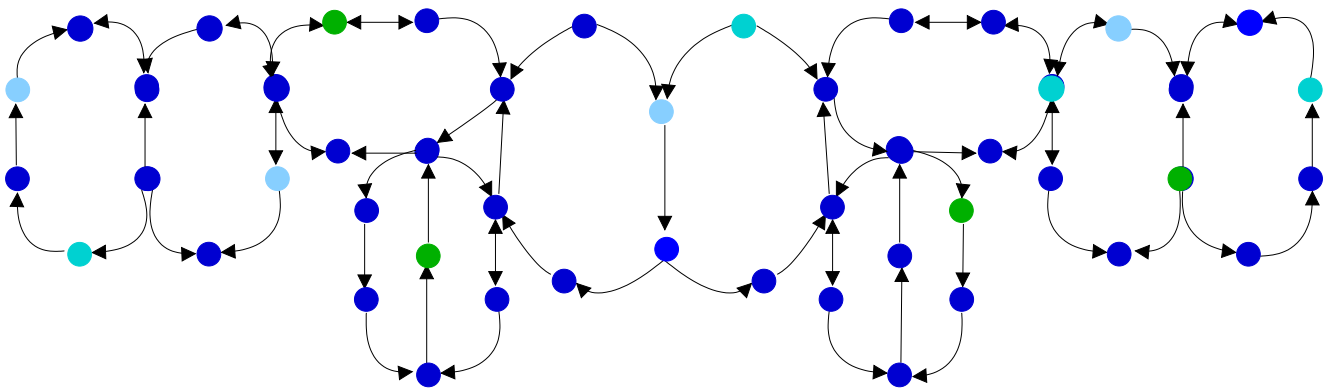


$$\mathbf{Q} = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix}$$

Collective Dynamics

The behaviour of many systems can be interpreted as the result of the collective behaviour of a large number of interacting entities.



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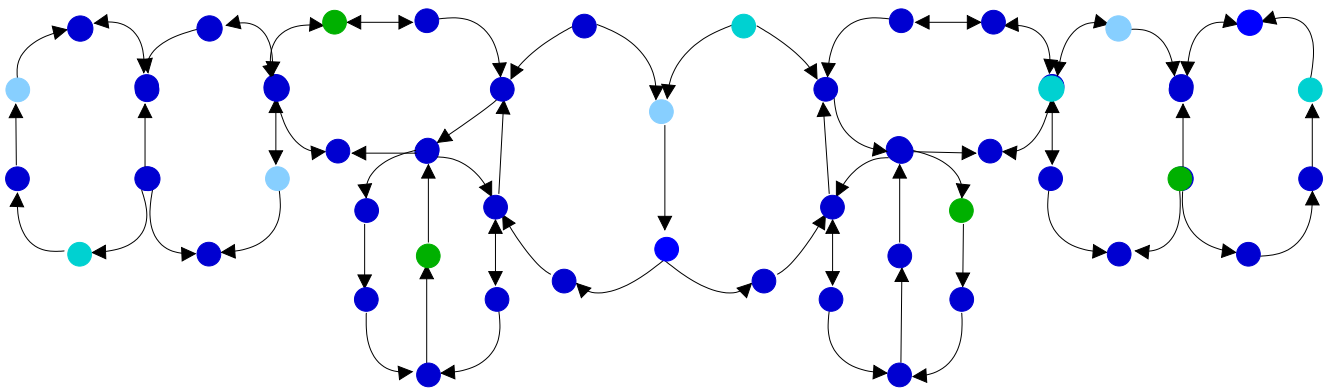

For such systems we are often as interested in the population level behaviour as we are in the behaviour of the individual entities.

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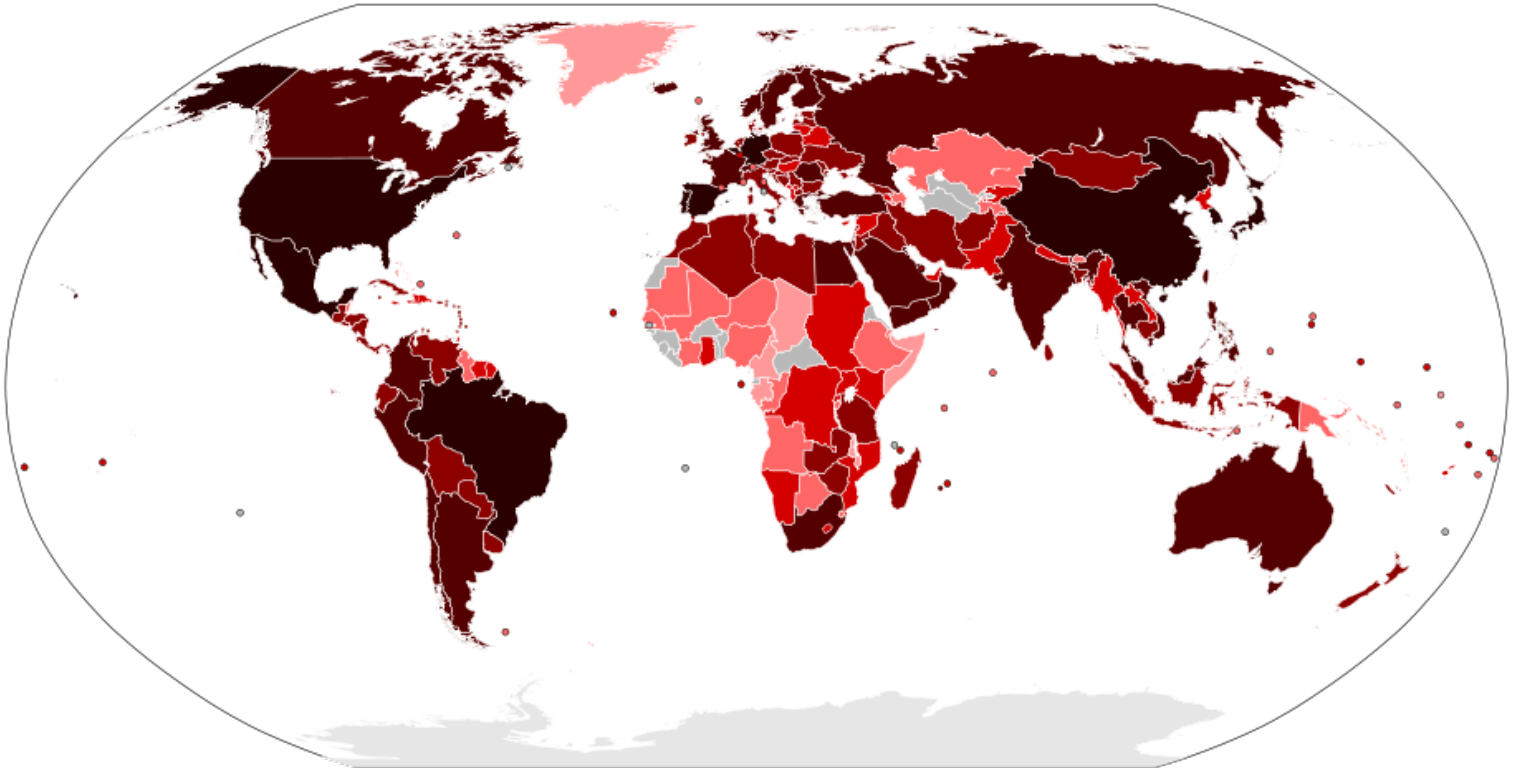

Spread of H1N1 virus in 2009

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Love Parade, Germany 2006

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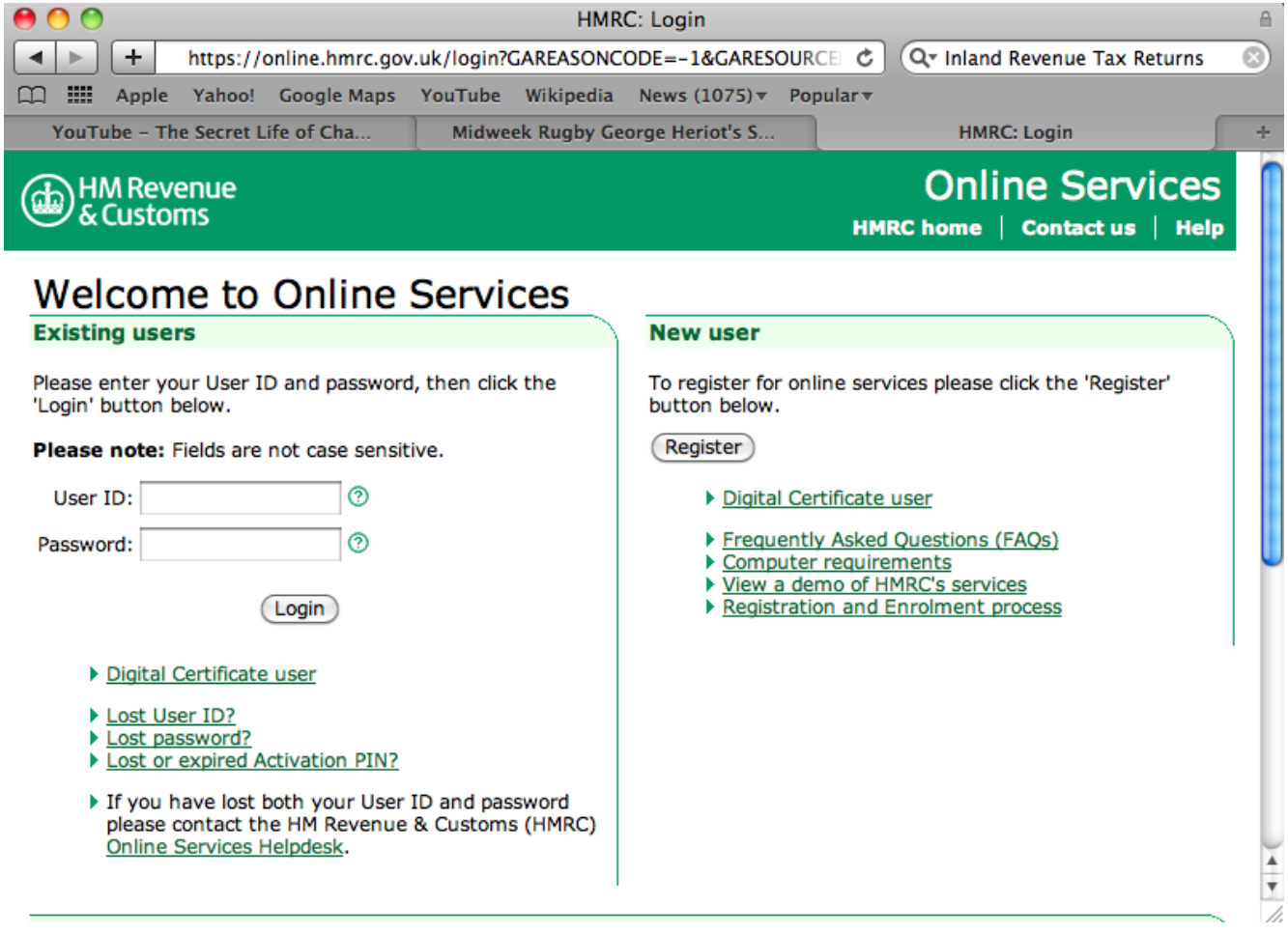

Self assessment tax returns 31st January each year

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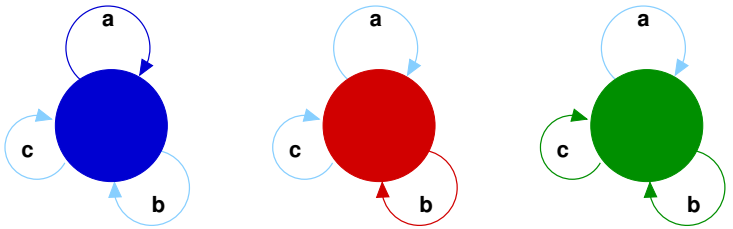
Unfortunately, whilst theoretically possible, the analysis of such systems through the standard process algebra approaches — explicitly building the state space — is not generally feasible.

Solving discrete state models

Under the SOS semantics a SPA model is mapped to a CTMC with global states determined by the local states of all the participating components.



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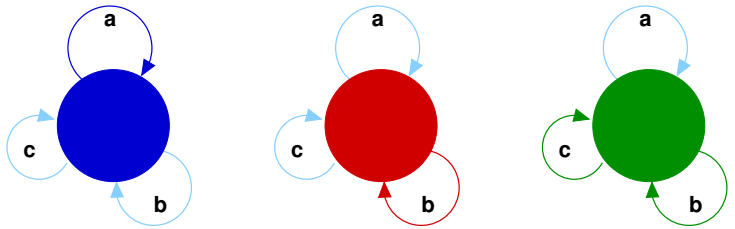

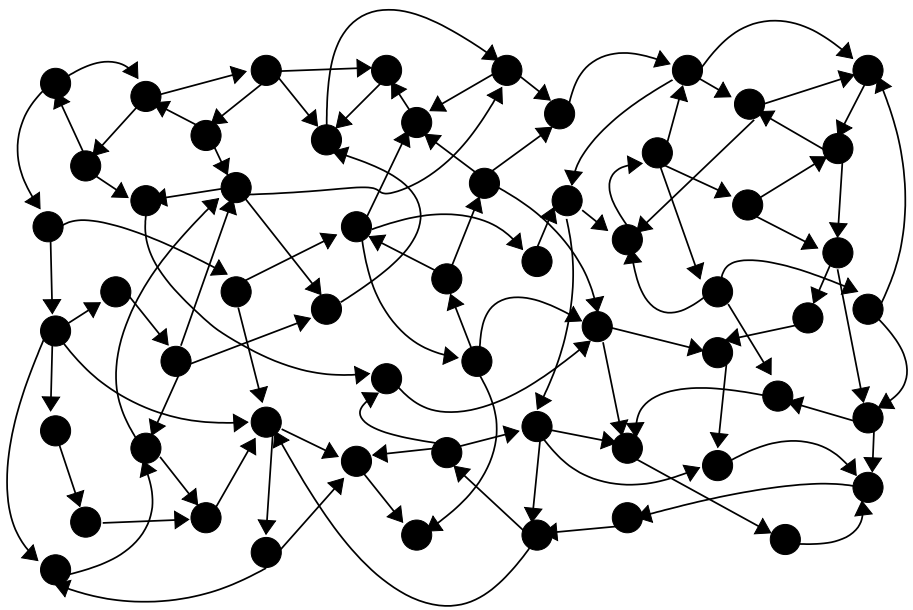

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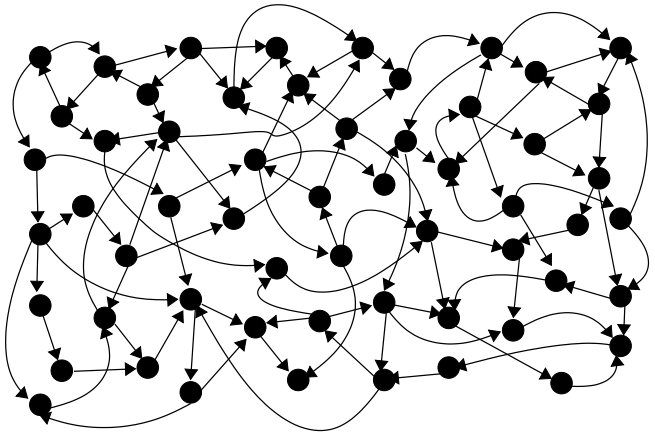

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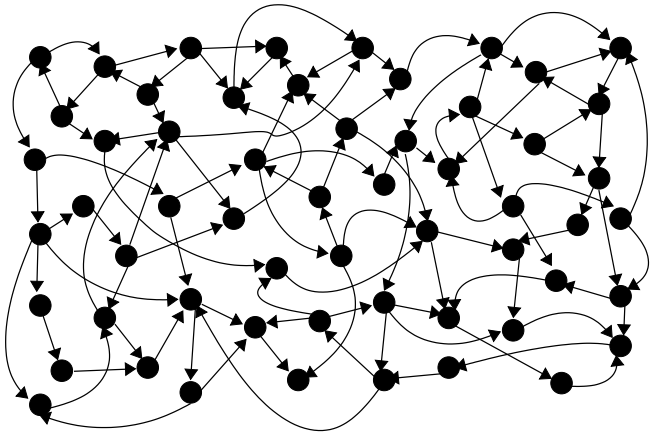

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$$Q = \begin{pmatrix} q_{1,1} & q_{1,2} & \cdots & q_{1,N} \\ q_{2,1} & q_{2,2} & \cdots & q_{2,N} \\ \vdots & \vdots & & \vdots \\ q_{N,1} & q_{N,2} & \cdots & q_{N,N} \end{pmatrix}$$

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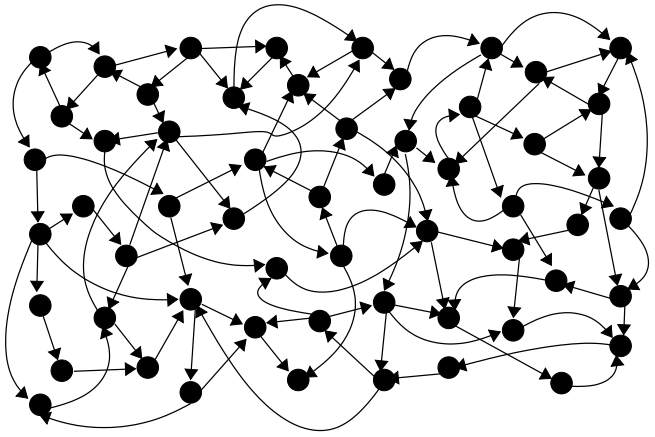

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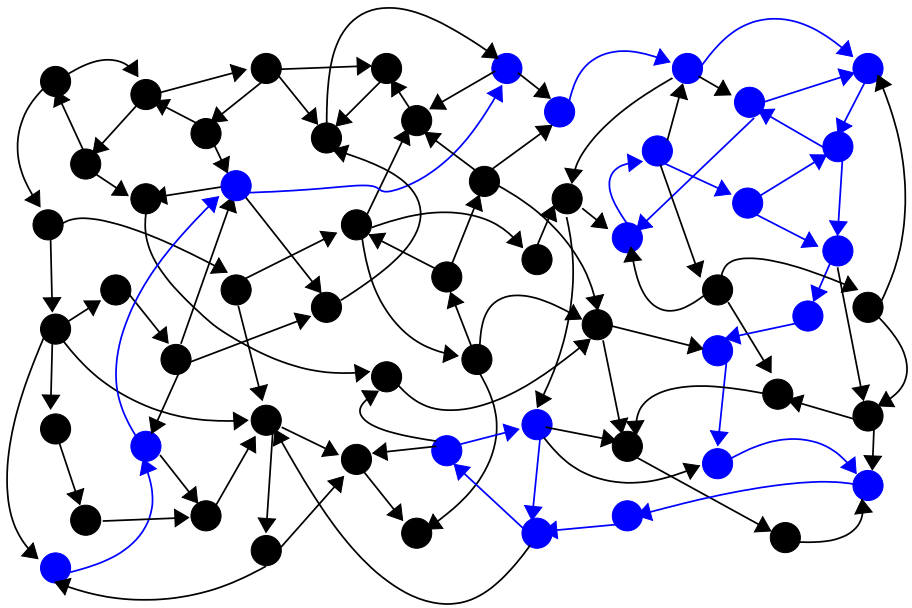
$$\pi(t) = (\pi_1(t), \pi_2(t), \ldots, \pi_N(t))$$

Solving discrete state models

Alternatively they may be studied using stochastic simulation. Each run generates a single trajectory through the state space. Many runs are needed in order to obtain average behaviours.



State space explosion

As the size of the state space becomes large it becomes infeasible to carry out numerical solution and extremely time-consuming to conduct stochastic simulation.

The CODA project

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Another approach is to make a shift to population statistics.

Population statistics: emergent behaviour

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This allows us to model much larger systems than previously possible but in making the shift we are no longer able to collect any information about individuals in the system.

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To characterise the behaviour of a population we count the number of individuals within the population that are exhibiting certain behaviours rather than tracking individuals directly.

Furthermore we make a continuous approximation of how the counts vary over time.

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Large scale software systems
Issues of scalability are important for user satisfaction and resource efficiency but such issues are difficult to investigate using discrete state models.

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Biochemical signalling pathways
Understanding these pathways has the potential to improve the quality of life through enhanced drug treatment and better drug design.

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Epidemiological systems
Improved modelling of these systems could lead to improved disease prevention and treatment in nature and better security in computer systems.

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Crowd dynamics
Technology enhancement is creating new possibilities for directing crowd movements in buildings and urban spaces, for example for emergency egress, which are not yet well-understood.

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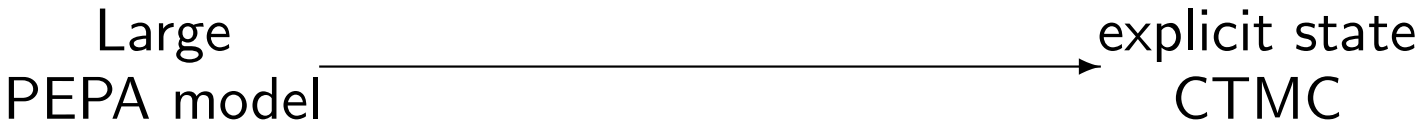
Alternative Representations



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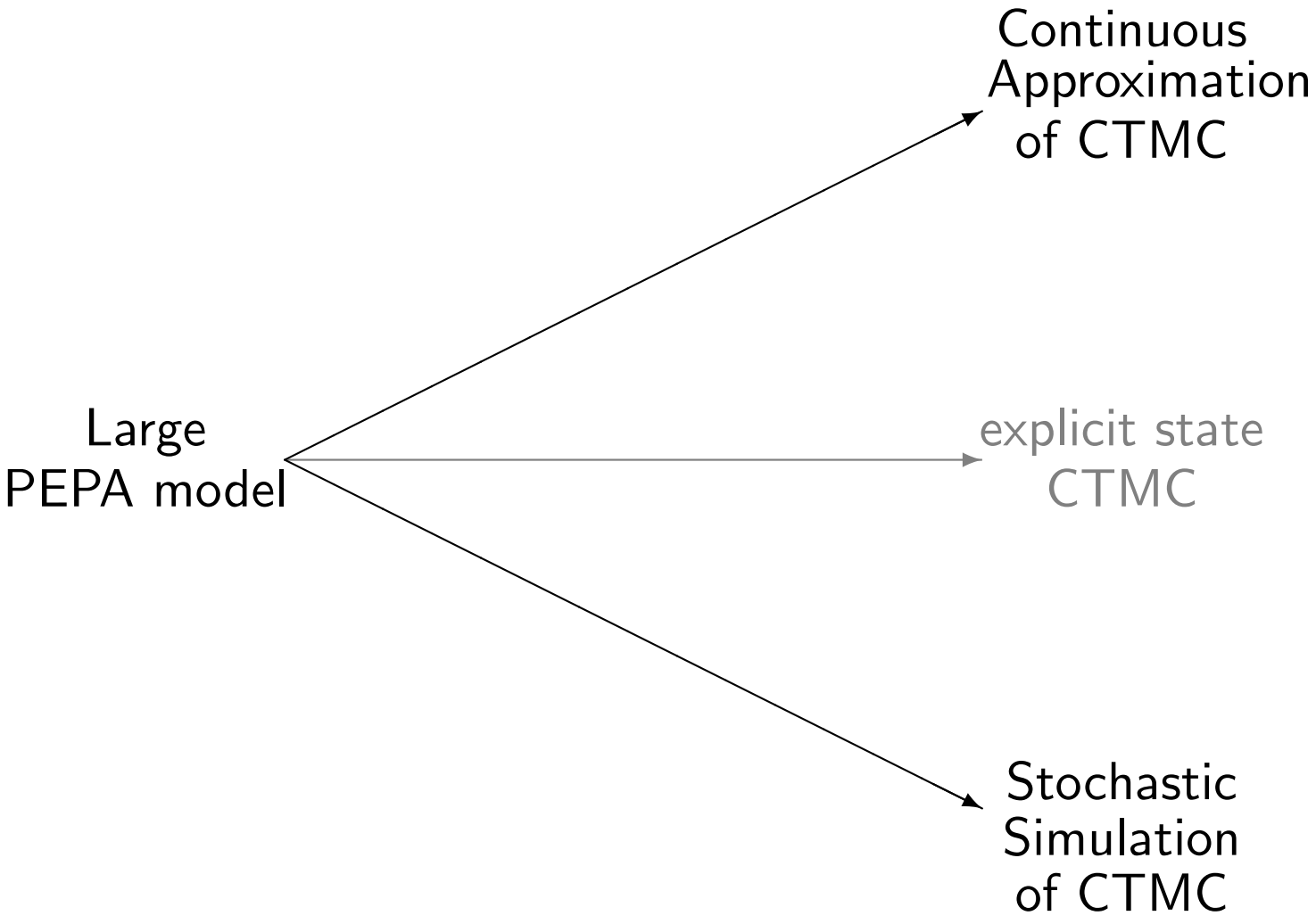

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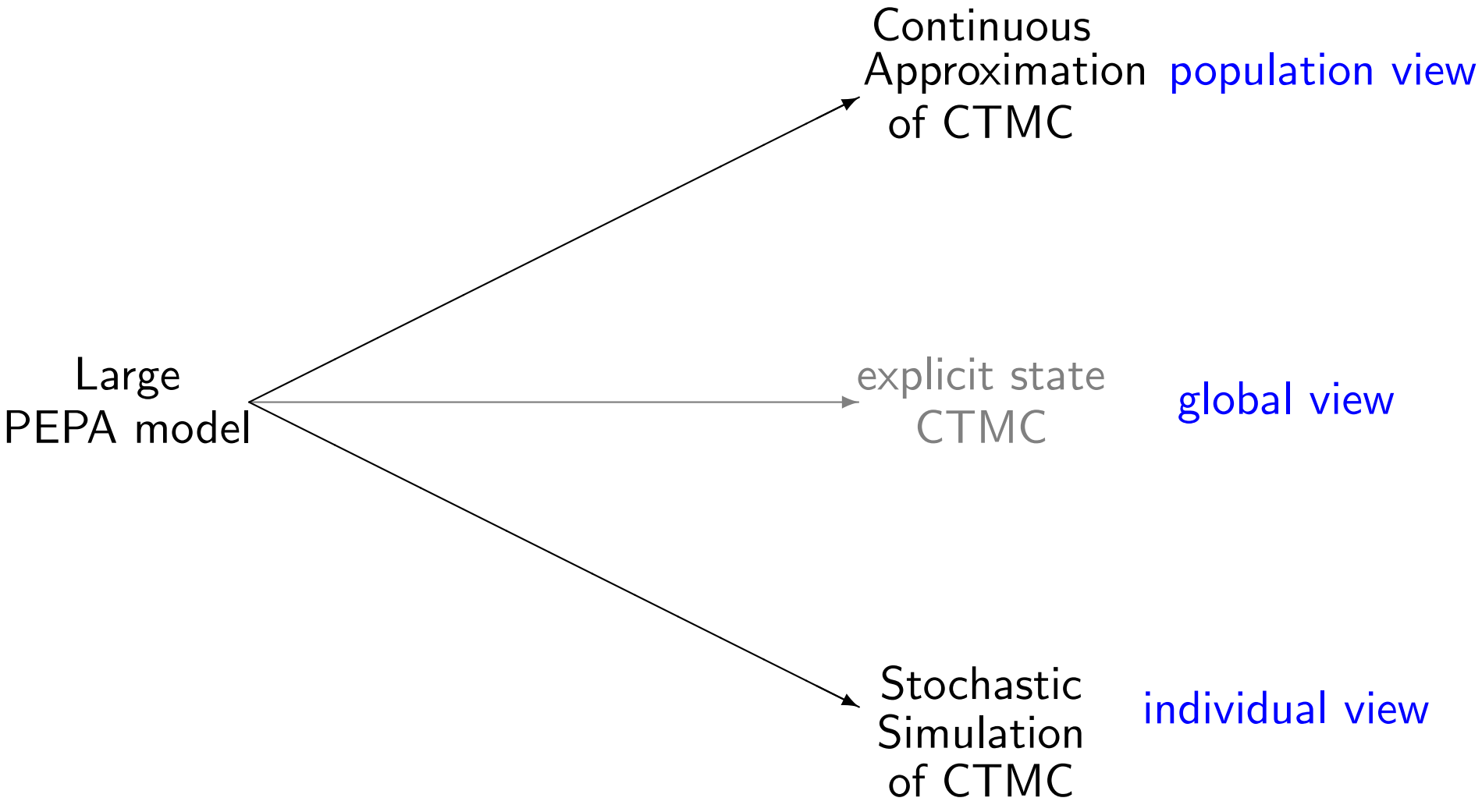

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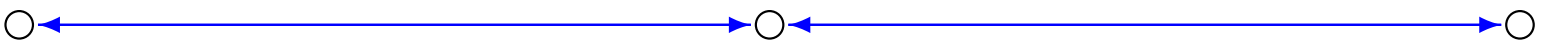

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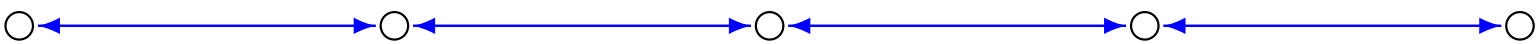

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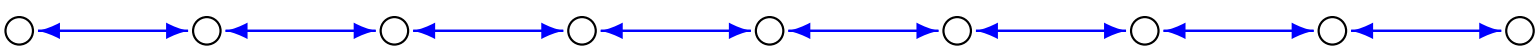

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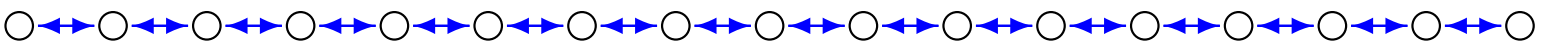

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Use continuous state variables to approximate the discrete state space.

Use ordinary differential equations to represent the evolution of those variables over time.

New mathematical structures: differential equations

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4. Instead the trajectory of the ODEs estimates the expected behaviour of the CTMC.

Appropriate for models in which there are large numbers of components of the same type, i.e. models of populations and situations of collective dynamics.

Models suitable for counting abstraction

- In the PEPA language multiple instances of components are represented explicitly — we write $P[n]$ to denote an array of n copies of P executing in parallel.

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- The impact of an action of a counting variable is
  - decrease by 1 if the component participates in the action
  - increase by 1 if the component is the result of the action
  - zero if the component is not involved in the action.

Models suitable for counting abstraction

- In Bio-PEPA components are parameterised with a counting variable and the definition of an action records the impact that an action has on the counting variable.

Simple example revisited

$$
\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\
Res_1 &\stackrel{def}{=} (reset, r_4).Res_0
\end{aligned}
$$

$$
Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R]
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$$

$$
Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R]
$$

CTMC interpretation

Processors (N_P)	Resources (N_R)	States ($2^{N_P+N_R}$)
1	1	4
2	1	8
2	2	16
3	2	32
3	3	64
4	3	128
4	4	256
5	4	512
5	5	1024
6	5	2048
6	6	4096
7	6	8192
7	7	16384
8	7	32768
8	8	65536
9	8	131072
9	9	262144
10	9	524288
10	10	1048576

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\end{aligned}
$$

$$Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R]$$

- $task1$ decreases $Proc_0$ and Res_0
- $task1$ increases $Proc_1$ and Res_1
- $task2$ decreases $Proc_1$
- $task2$ increases $Proc_0$
- $reset$ decreases Res_1
- $reset$ increases Res_0

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$$
Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R]
$$

$$
\begin{aligned}
\frac{dx_1}{dt} &= -\min(r_1\, x_1, r_3\, x_3) + r_2\, x_2 \\
x_1 &= \text{no. of } Proc_1
\end{aligned}
$$

- $task1$ decreases $Proc_0$
- $task1$ is performed by $Proc_0$ and Res_0
- $task2$ increases $Proc_0$
- $task2$ is performed by $Proc_1$

Simple example revisited

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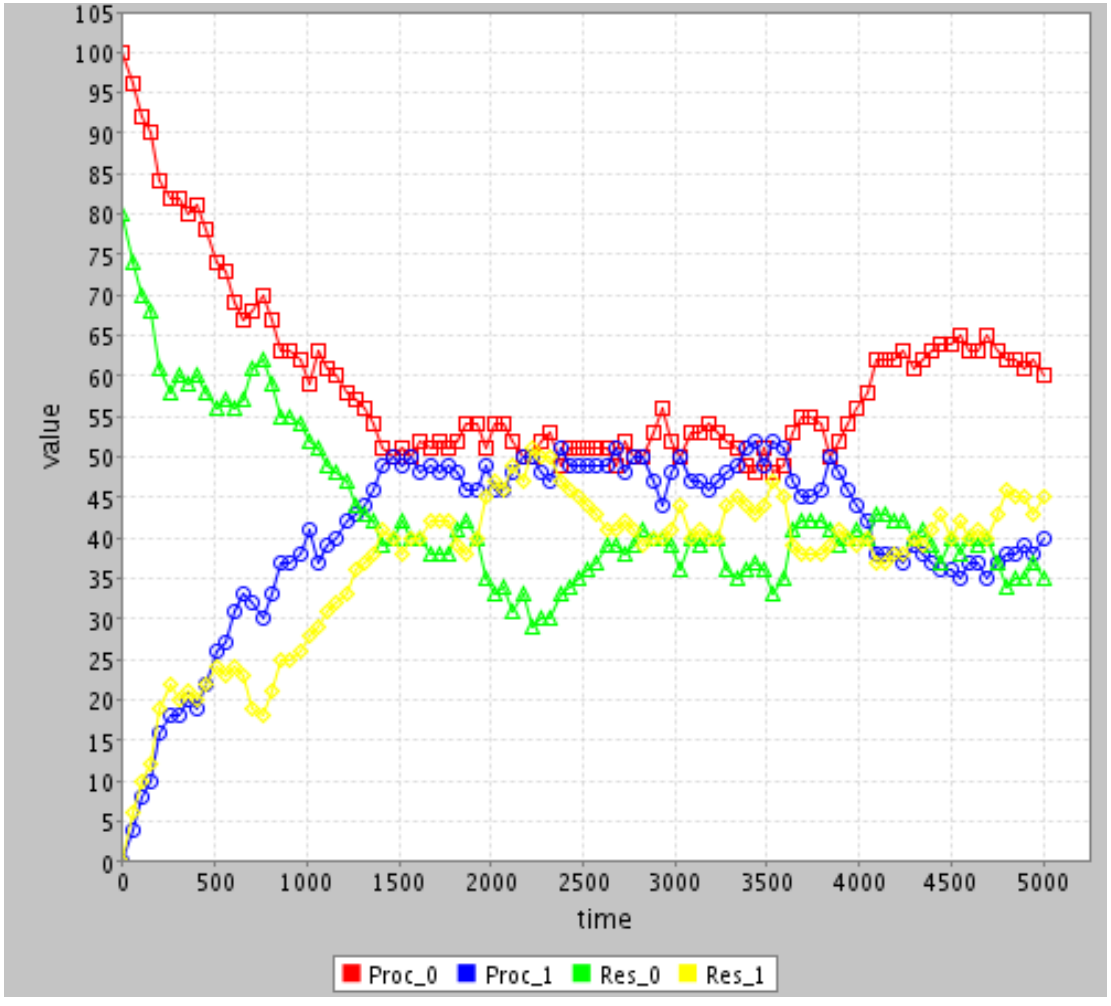
Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R]
$$

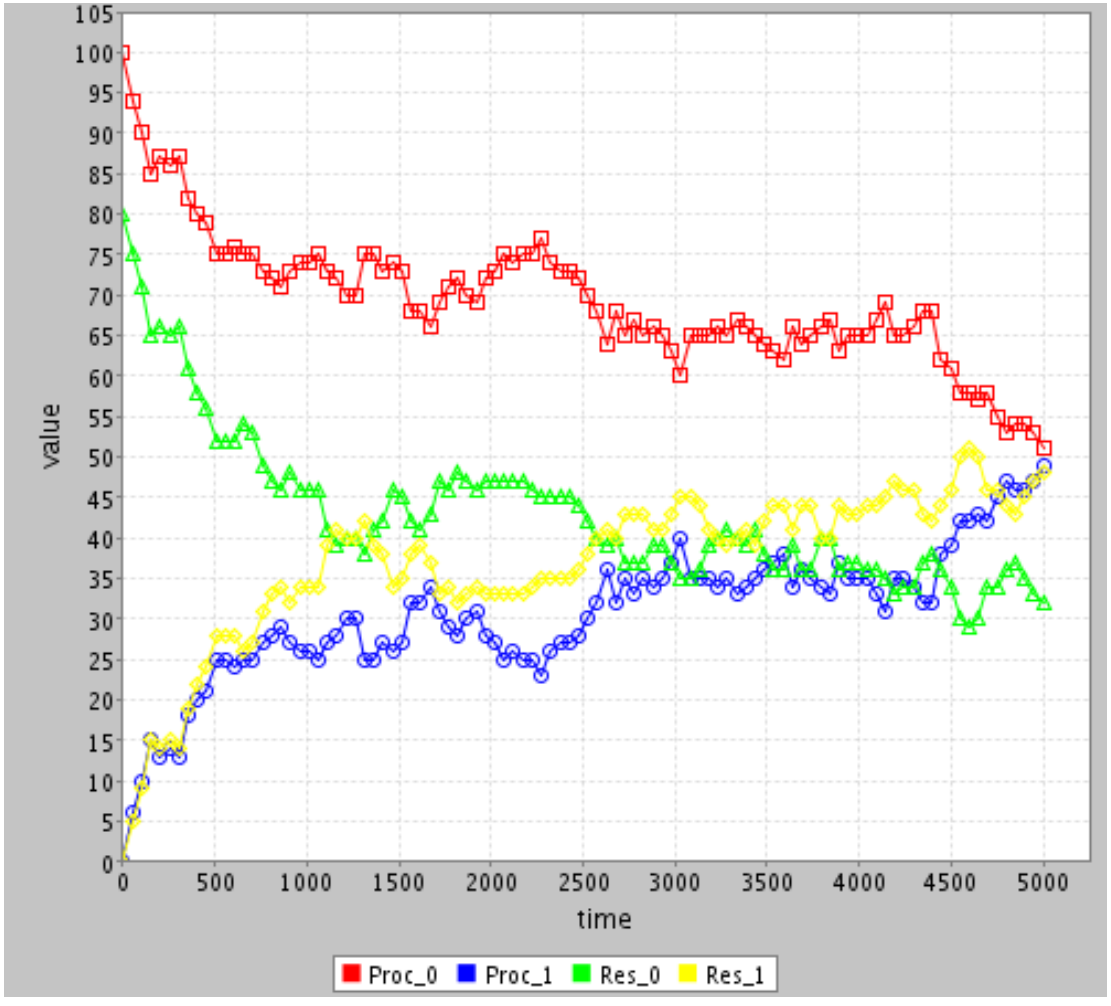
ODE interpretation

$$
\begin{aligned}
\frac{dx_1}{dt} &= -\min(r_1\, x_1, r_3\, x_3) + r_2\, x_2 \\
& \quad x_1 = \text{no. of } Proc_1 \\
\frac{dx_2}{dt} &= \min(r_1\, x_1, r_3\, x_3) - r_2\, x_2 \\
& \quad x_2 = \text{no. of } Proc_2 \\
\frac{dx_3}{dt} &= -\min(r_1\, x_1, r_3\, x_3) + r_4\, x_4 \\
& \quad x_3 = \text{no. of } Res_0 \\
\frac{dx_4}{dt} &= \min(r_1\, x_1, r_3\, x_3) - r_4\, x_4 \\
& \quad x_4 = \text{no. of } Res_1
\end{aligned}
$$

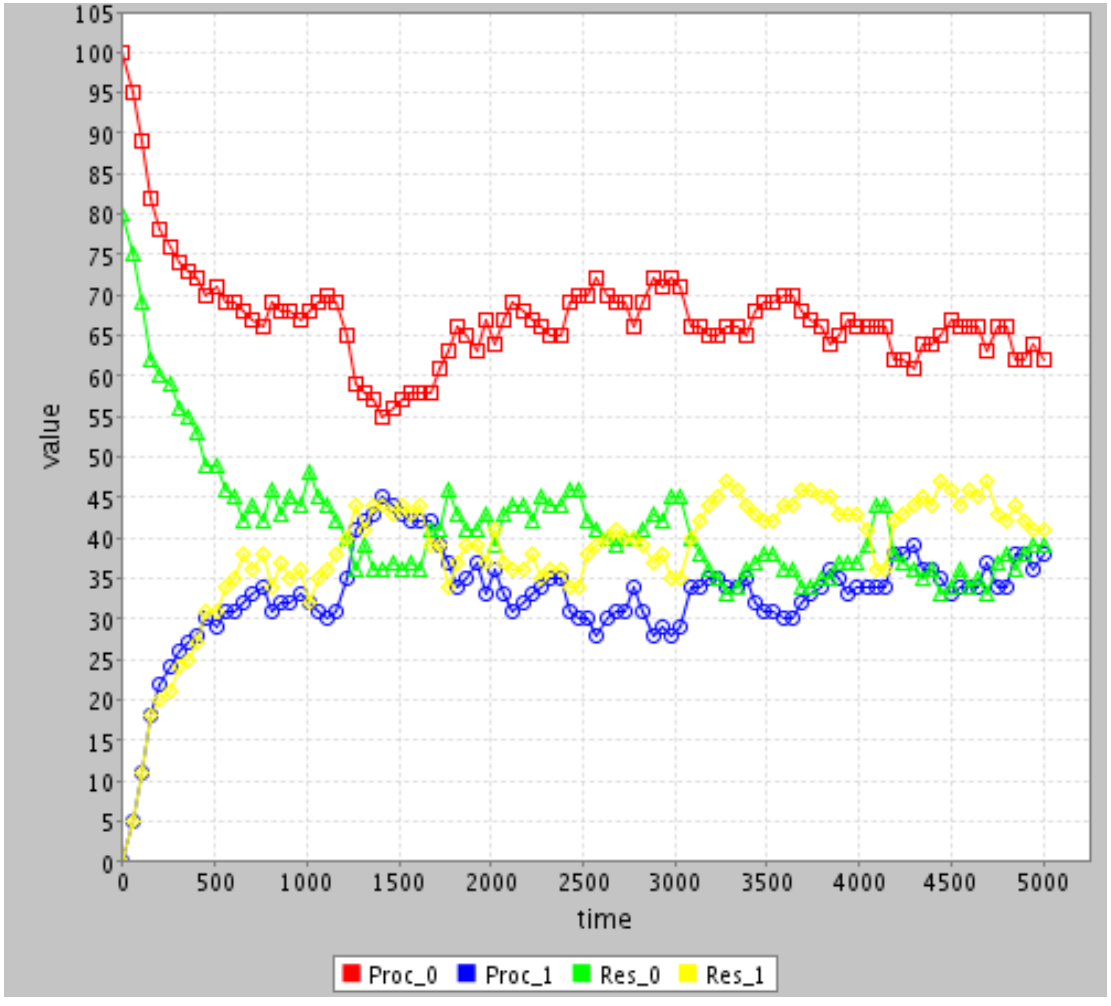
100 processors and 80 resources (simulation run A)



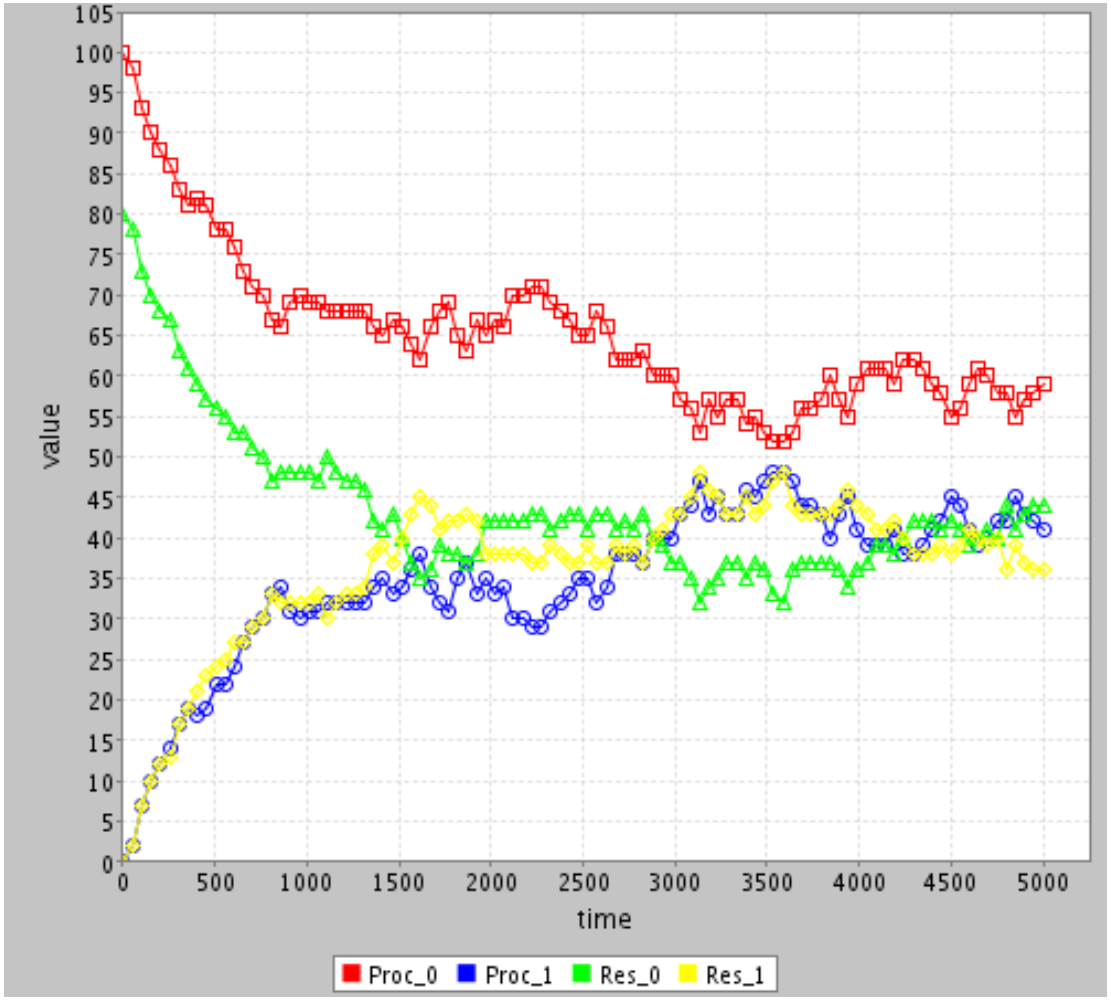
100 processors and 80 resources (simulation run B)



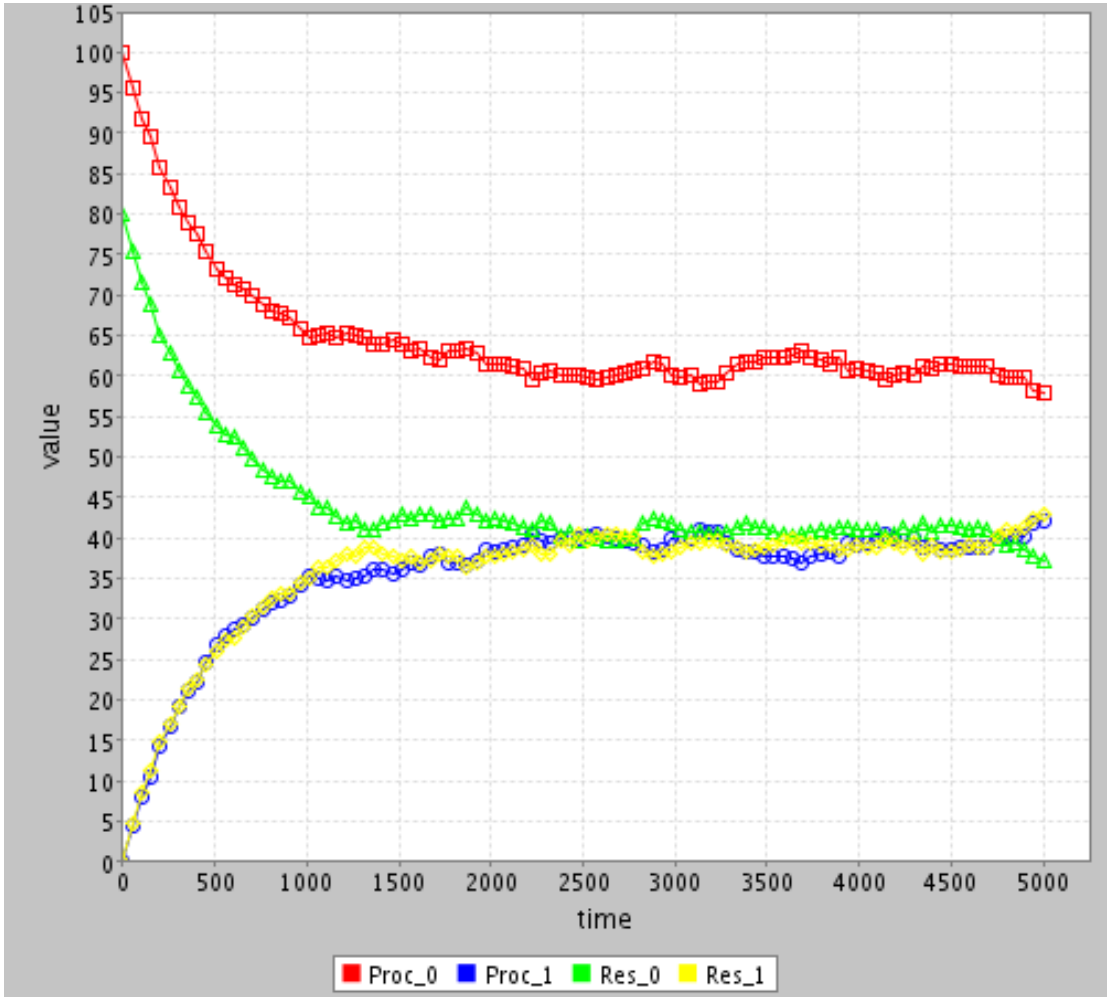
100 processors and 80 resources (simulation run C)



100 processors and 80 resources (simulation run D)



100 processors and 80 resources (average of 10 runs)



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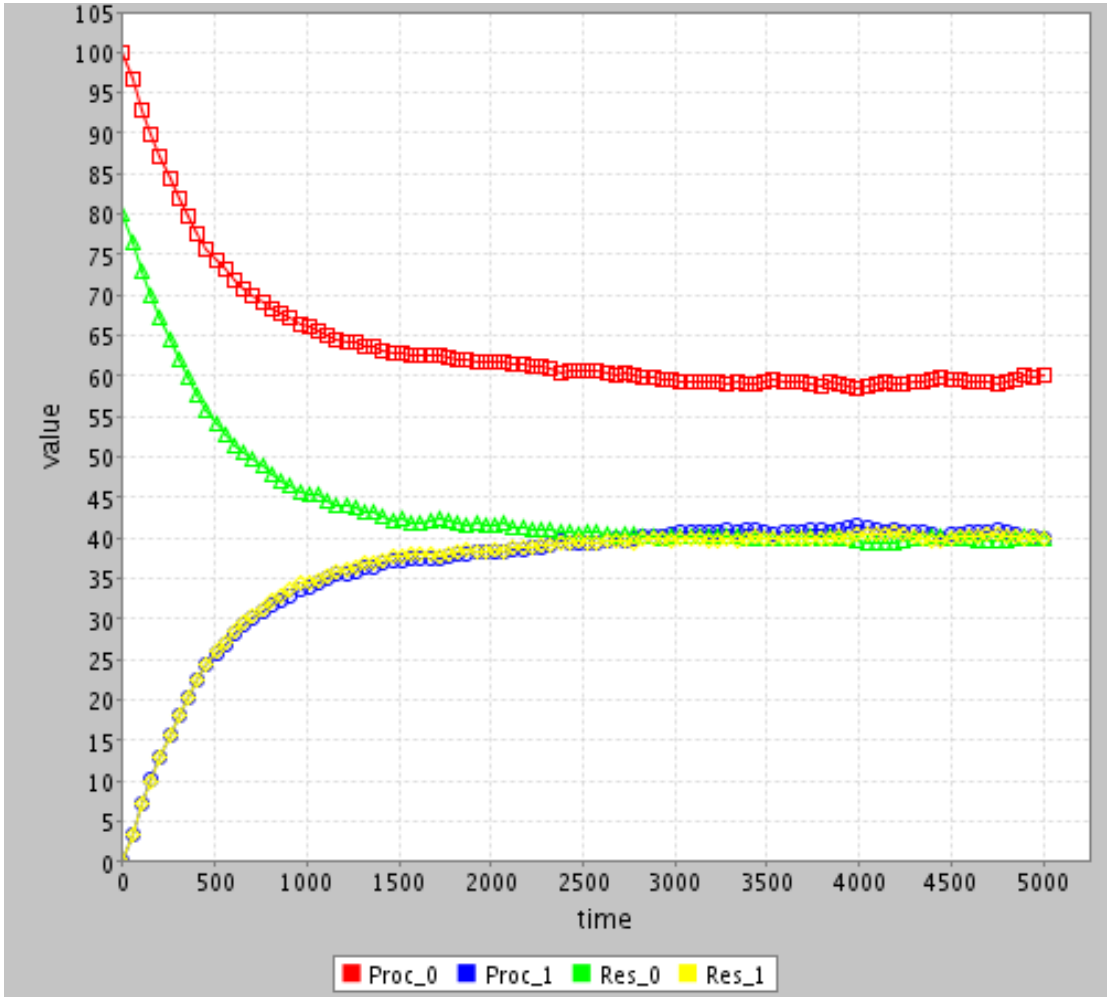
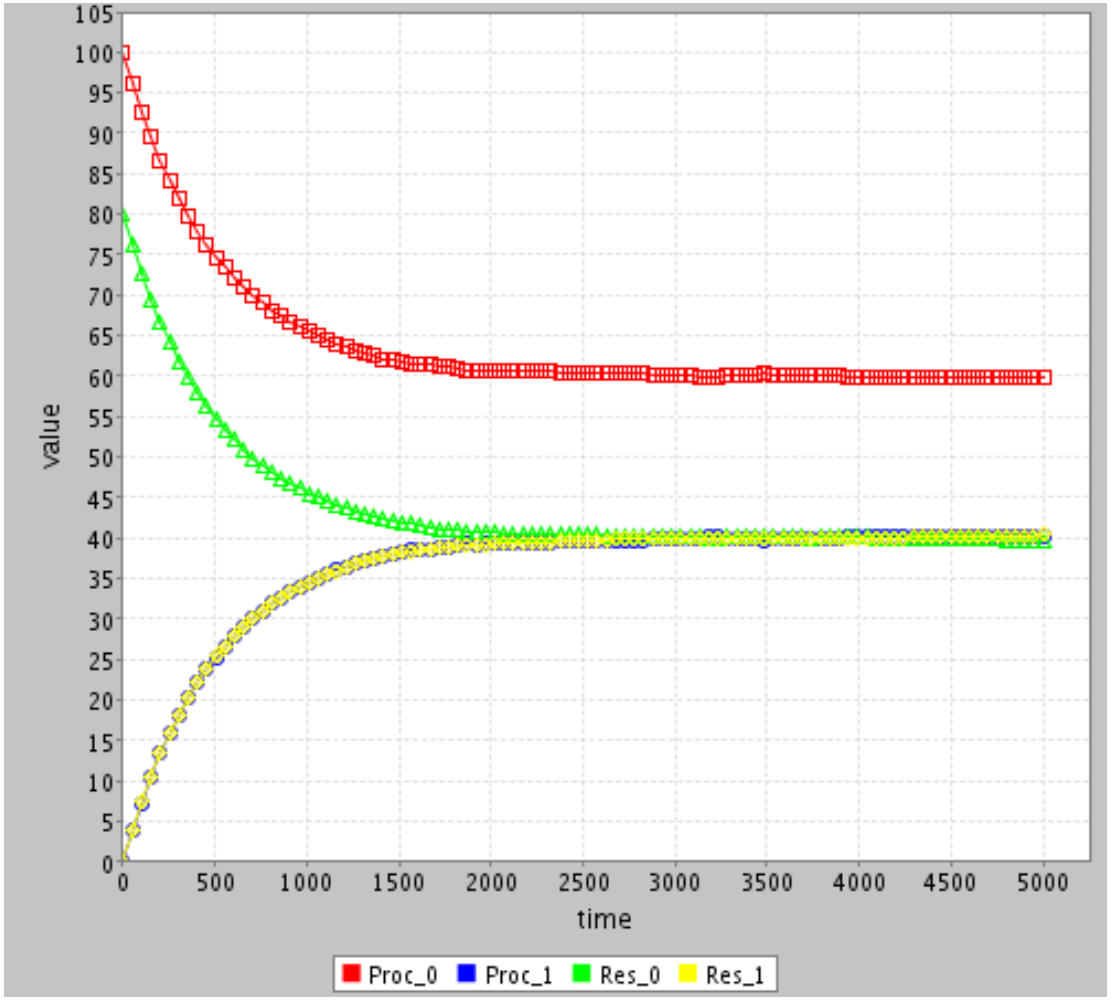
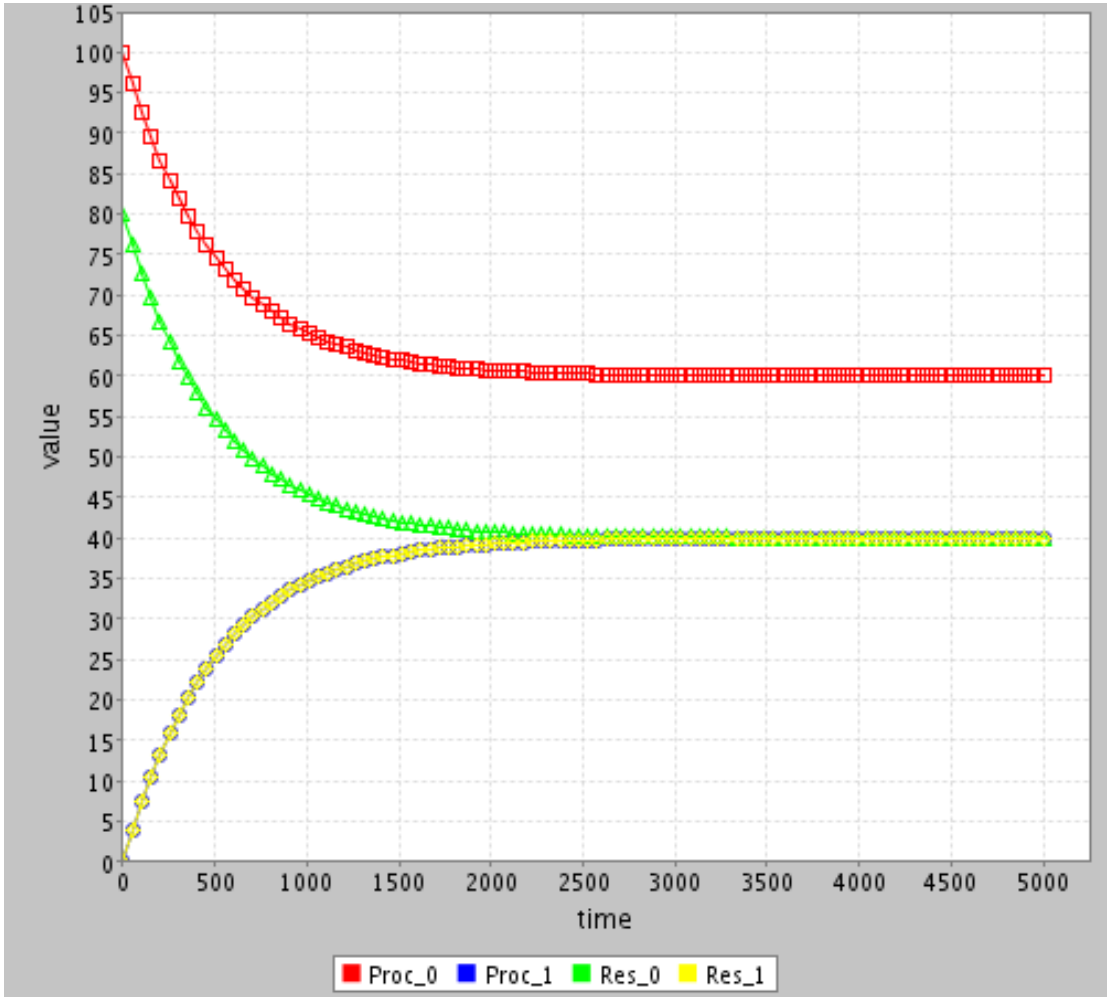


100 processors and 80 resources (average of 1000 runs)



100 processors and 80 resources (ODE solution)



Outline

Deriving a Fluid Approximation of a PEPA model

The aim is to represent the CTMC implicitly (avoiding state space explosion), and to generate the set of ODEs which are the fluid limit of that CTMC.

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1. Make the counting abstraction (*Context Reduction*)
2. Collect the transitions of the reduced context (*Jump Multiset*)
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Once this is done we can extract the vector field $F_{\mathcal{M}}(x)$ from the jump multiset.

Context Reduction

$$
\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
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System &\stackrel{def}{=} Proc_0[N_P] \underset{\{transfer\}}{\bowtie} Res_0[N_R]
\end{aligned}
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$$\Downarrow$$

$$\mathcal{R}(System) = \{Proc_0, Proc_1\} \underset{\{task1\}}{\bowtie} \{Res_0, Res_1\}$$

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Population Vector

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$$

Location Dependency

$$System \stackrel{def}{=} Proc_0[N'_C] \underset{\{task1\}}{\bowtie} Res_0[N_S] \parallel Proc_0[N''_C]$$

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Population Vector

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)$$

Fluid Structured Operational Semantics by Example

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$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$$

$$\frac{Proc_0 \xrightarrow{task1, r_1} Proc_1}{Proc_0 \xrightarrow{task1, r_1 \xi_1}_* Proc_1}$$

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$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$$

$$
\frac{Proc_0 \xrightarrow{task1, r_1} Proc_1 \qquad Res_0 \xrightarrow{task1, r_3} Res_1}{Proc_0 \xrightarrow{task1, r_1 \xi_1}_* Proc_1 \qquad Res_0 \xrightarrow{task1, r_3 \xi_3}_* Res_1}
$$

$$
Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1
$$

Apparent Rate Calculation

$$\frac{\dfrac{Proc_0 \xrightarrow{task1, r_1} Proc_1}{Proc_0 \xrightarrow{task1, r_1 \xi_1}_* Proc_1} \qquad \dfrac{Res_0 \xrightarrow{task1, r_3} Res_1}{Res_0 \xrightarrow{task1, r_3 \xi_3}_* Res_1}}{Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1}$$

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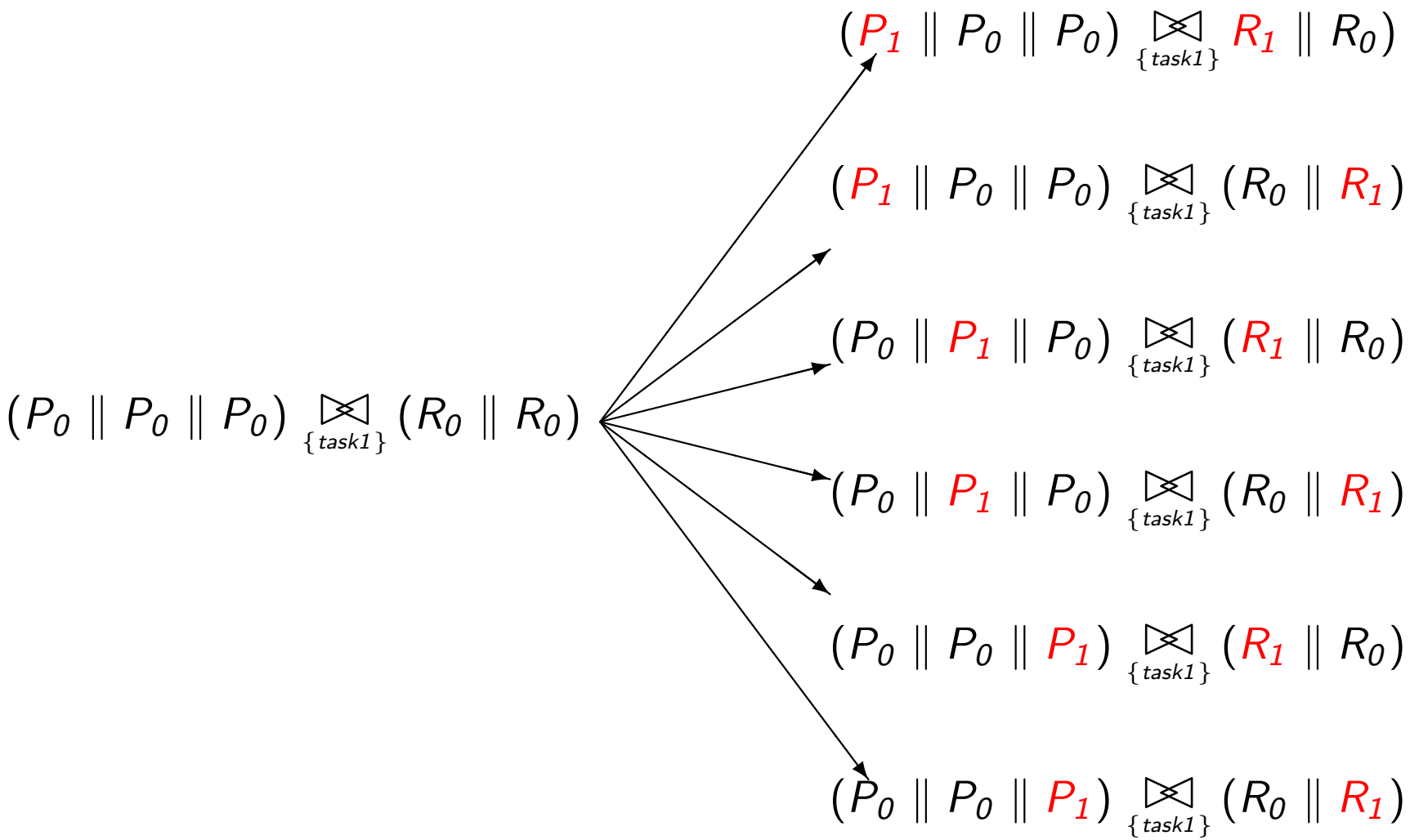
$$r(\xi) = \min\left(r_1\xi_1, r_3\xi_4\right)$$

$f(\xi, l, \alpha)$ as the Generator Matrix of the *Lumped* CTMC



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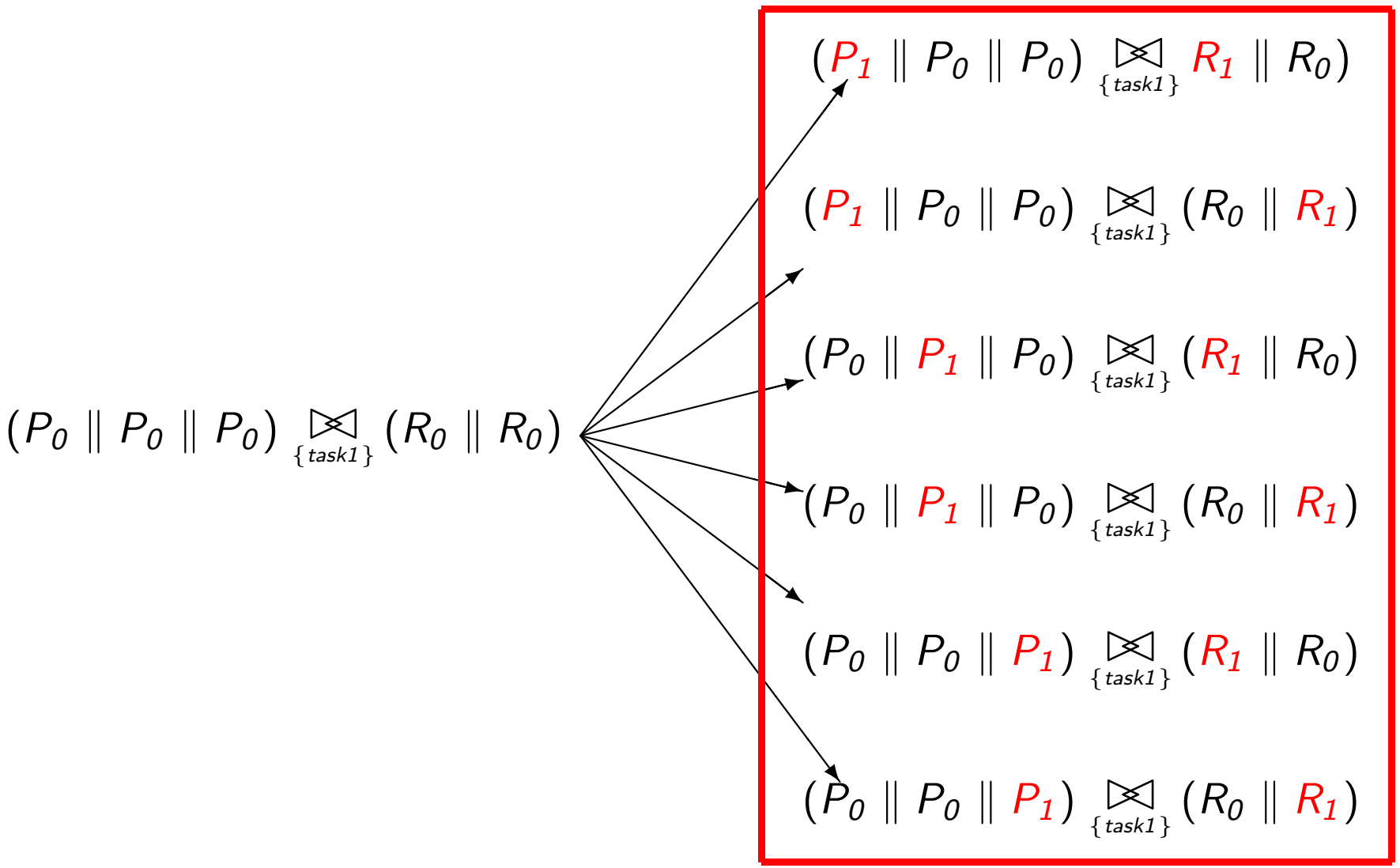

$f(\xi, l, \alpha)$ as the Generator Matrix of the *Lumped* CTMC

$$(3,0,2,0) \xrightarrow{\min(3r_1, 2r_3)} (2,1,1,1)$$

$$(P_0 \parallel P_0 \parallel P_0) \underset{\{task1\}}{\bowtie} (R_0 \parallel R_0)$$

$$(P_1 \parallel P_0 \parallel P_0) \underset{\{task1\}}{\bowtie} (R_1 \parallel R_0)$$

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Jump Multiset

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_{*} Proc_1 \underset{\{task1\}}{\bowtie} Res_1$$

$$r(\xi) = \min\left(r_1\xi_1, r_3\xi_3\right)$$

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$$Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1$$

$$r(\xi) = \min\left(r_1\xi_1, r_3\xi_3\right)$$

$$Proc_1 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task2, \xi_2 r_2}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

Jump Multiset

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1$$

$$r(\xi) = \min\left(r_1\xi_1, r_3\xi_3\right)$$

$$Proc_1 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task2, \xi_2 r_2}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

Equivalent Transitions

Some transitions may give the same information:

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$
$$Proc_1 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_0$$

i.e., Res_1 may perform an action independently from the rest of the system.

This is captured by the procedure used for the construction of the generator function $f(\xi, l, \alpha)$

Construction of $f(\xi, l, \alpha)$

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

Construction of $f(\xi, l, \alpha)$

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

- Take $l = (0, 0, 0, 0)$

Construction of $f(\xi, l, \alpha)$

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

- Take $l = (0, 0, 0, 0)$
- Add -1 to all elements of l corresponding to the indices of the components in the lhs of the transition

$$l = (-1, 0, 0, -1)$$

Construction of $f(\xi, l, \alpha)$

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

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$$l = (-1, 0, 0, -1)$$

- Add $+1$ to all elements of l corresponding to the indices of the components in the rhs of the transition

$$l = (-1 + 1, 0, +1, -1) = (0, 0, +1, -1)$$

Construction of $f(\xi, l, \alpha)$

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$$l = (-1 + 1, 0, +1, -1) = (0, 0, +1, -1)$$

$$f\big(\xi, (0, 0, +1, -1), reset\big) = \xi_4 r_4$$

Construction of $f(\xi, l, \alpha)$

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$$Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1$$

$$f(\xi, (-1, +1, -1, +1), task1) \quad = \quad r(\xi)$$

Construction of $f(\xi, l, \alpha)$

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$$Proc_1 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task2, \xi_2 r_2'}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

$$\begin{aligned} f(\xi, (-1, +1, -1, +1), task1) &= r(\xi) \\ f(\xi, (+1, -1, 0, 0), task2) &= \xi_2 r_2 \end{aligned}$$

Construction of $f(\xi, l, \alpha)$

$$
\begin{aligned}
Proc_0 \underset{\{task1\}}{\bowtie} Res_0 &\xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1 \\
Proc_1 \underset{\{task1\}}{\bowtie} Res_0 &\xrightarrow{task2, \xi_2 r_2'}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \\
Proc_0 \underset{\{task1\}}{\bowtie} Res_1 &\xrightarrow{reset, \xi_4 r_4}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0
\end{aligned}
$$

$$
\begin{aligned}
f(\xi, (-1, +1, -1, +1), task1) &= r(\xi) \\
f(\xi, (+1, -1, 0, 0), task2) &= \xi_2 r_2 \\
f(\xi, (0, 0, +1, -1), reset) &= \xi_4 r_4
\end{aligned}
$$

Capturing behaviour in the Generator Function

$$
\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\
Res_1 &\stackrel{def}{=} (reset, r_4).Res_0 \\
System &\stackrel{def}{=} Proc_0[N_P] \underset{\{transfer\}}{\bowtie} Res_0[N_R]
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Numerical Vector Form

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4) \in \mathbb{N}^4, \quad \xi_1 + \xi_2 = N_P \text{ and } \xi_3 + \xi_4 = N_R$$

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Generator Function

$$
f(\xi, l, \alpha): \quad
\begin{aligned}
f(\xi, (-1, 1, -1, 1), task1) &= \min(r_1\xi_1, r_3\xi_3) \\
f(\xi, (1, -1, 0, 0), task2) &= r_2\xi_2 \\
f(\xi, (0, 0, 1, -1), reset) &= r_4\xi_4
\end{aligned}
$$

Extraction of the ODE from f

Generator Function

$$\begin{aligned} f(\xi, (-1,1,-1,1), \mathit{task1}) &= \min(r_1\xi_1, r_3\xi_3) \\ f(\xi, (1,-1,0,0), \mathit{task2}) &= r_2\xi_2 \\ f(\xi, (0,0,1,-1), \mathit{reset}) &= r_4\xi_4 \end{aligned}$$

Differential Equation

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} = F_{\mathcal{M}}(x) &= \sum_{l \in \mathbb{Z}^d} l \sum_{\alpha \in \mathcal{A}} f(x, l, \alpha) \\ &= (-1,1,-1,1)\min(r_1x_1, r_3x_3) + (1,-1,0,0)r_2x_2 \\ &\quad + (0,0,1,-1)r_4x_4 \end{aligned}$$

Extraction of the ODE from f

Generator Function

$$\begin{aligned} f(\xi, (-1, 1, -1, 1), \mathit{task1}) &= \min(r_1\xi_1, r_3\xi_3) \\ f(\xi, (1, -1, 0, 0), \mathit{task2}) &= r_2\xi_2 \\ f(\xi, (0, 0, 1, -1), \mathit{reset}) &= r_4\xi_4 \end{aligned}$$

Differential Equation

$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}t} &= -\min(r_1x_1, r_3x_3) + r_2x_2 \\ \frac{\mathrm{d}x_2}{\mathrm{d}t} &= \min(r_1x_1, r_3x_3) - r_2x_2 \\ \frac{\mathrm{d}x_3}{\mathrm{d}t} &= -\min(r_1x_1, r_3x_3) + r_4x_4 \\ \frac{\mathrm{d}x_4}{\mathrm{d}t} &= \min(r_1x_1, r_3x_3) - r_4x_4 \end{aligned}$$

Consistency results

- The vector field $\mathcal{F}(x)$ is Lipschitz continuous i.e. all the rate functions governing transitions in the process algebra satisfy local continuity conditions.

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- We can prove this using Kurtz's theorem:
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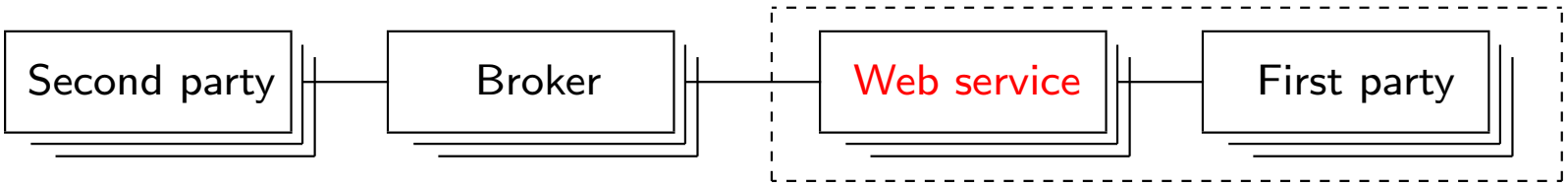
- Moreover Lipschitz continuity of the vector field guarantees existence and uniqueness of the solution to the initial value problem.

Outline

Example: Secure Web Service use



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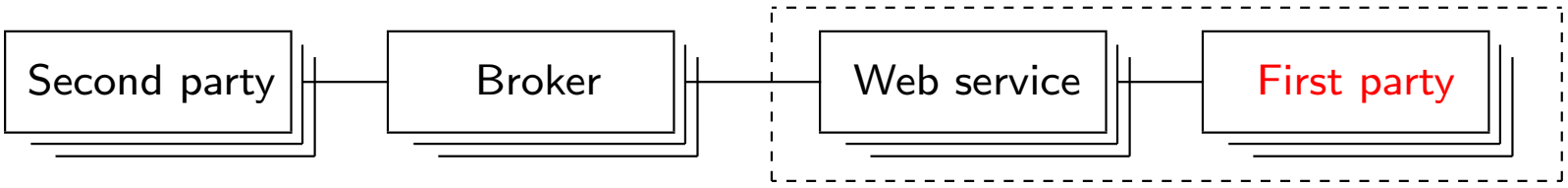

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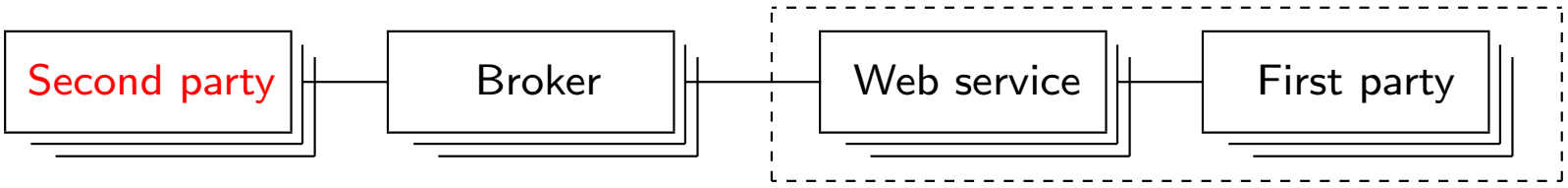

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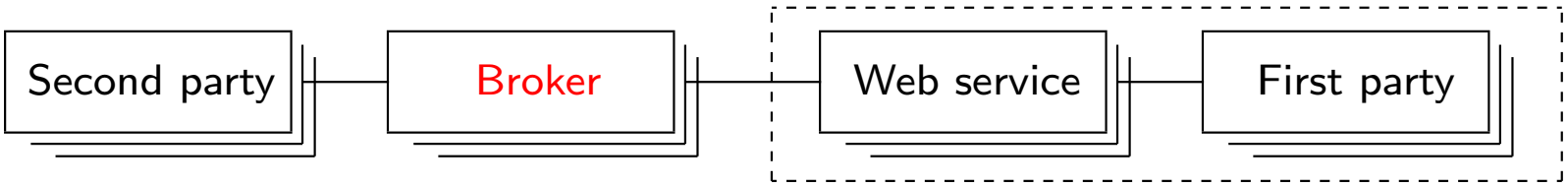

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  - first party application clients which access the web service across a secure intranet, and
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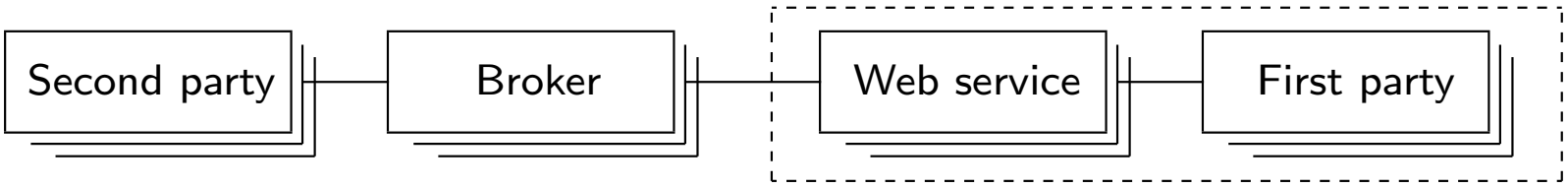
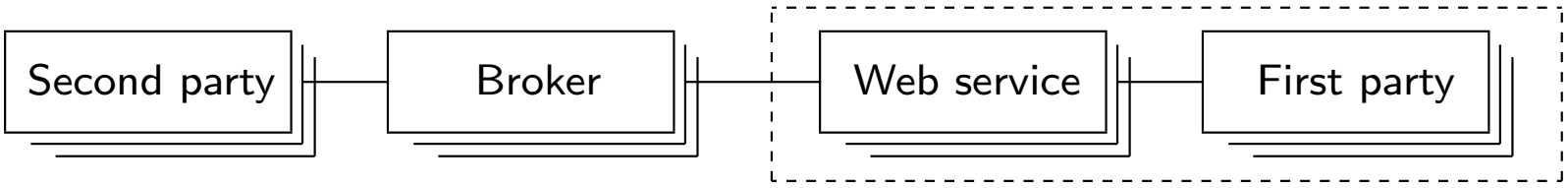
- Second party clients route their service requests via trusted brokers.

Scalability and replication



- To ensure scalability the Web service is replicated across multiple hosts.

Scalability and replication



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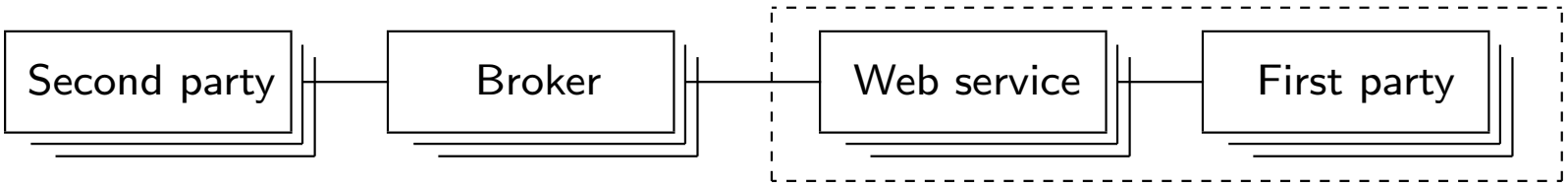

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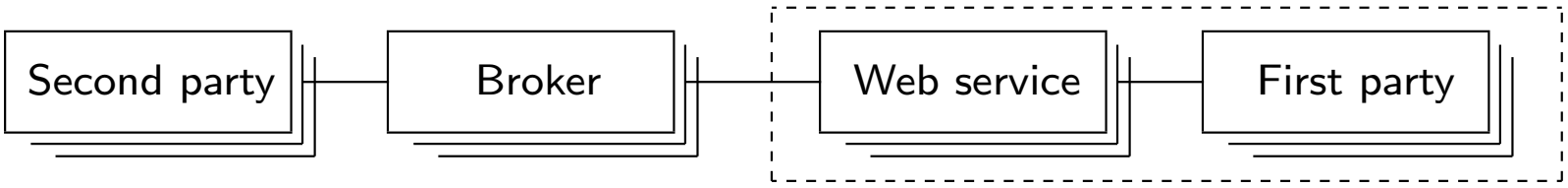

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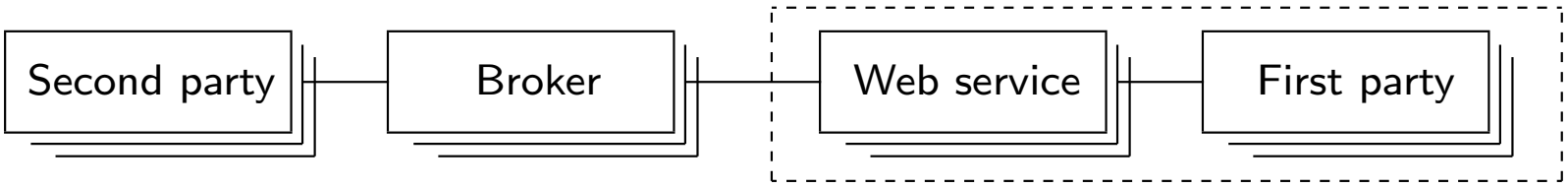
- There are numerous second party clients outside the firewall.

Security and use of encryption



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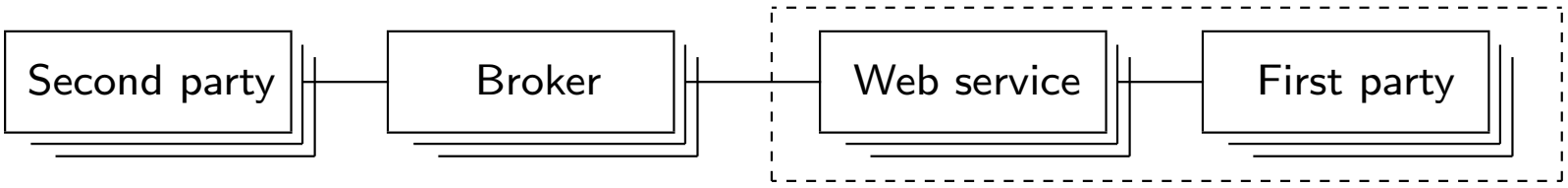

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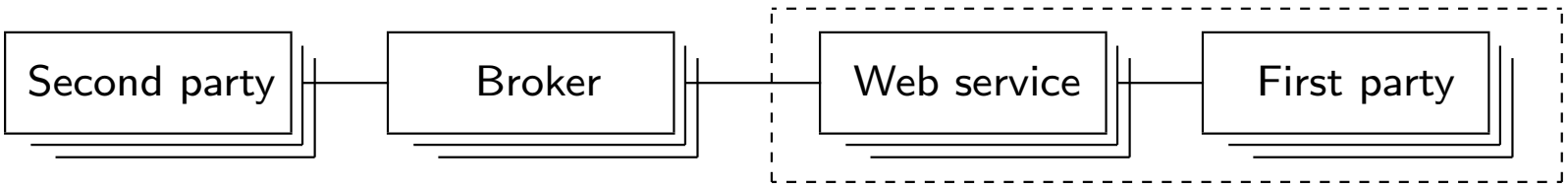

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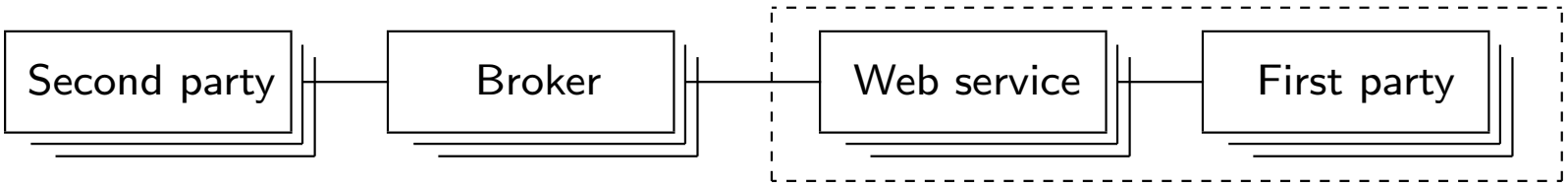

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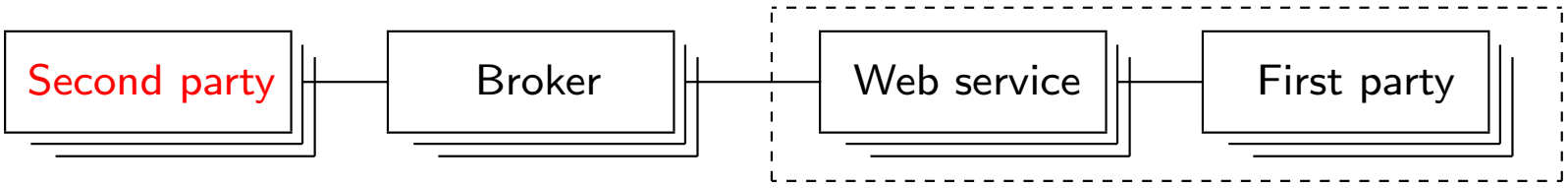
  - When processing a request from a second party client brokers decrypt the request before re-encrypting it for the Web service.
  - When the response to a request is returned to the broker it decrypts the response before re-encrypting it for the client.

PEPA model: Second party clients



- A second party client composes service requests, encrypts these and sends them to its broker.

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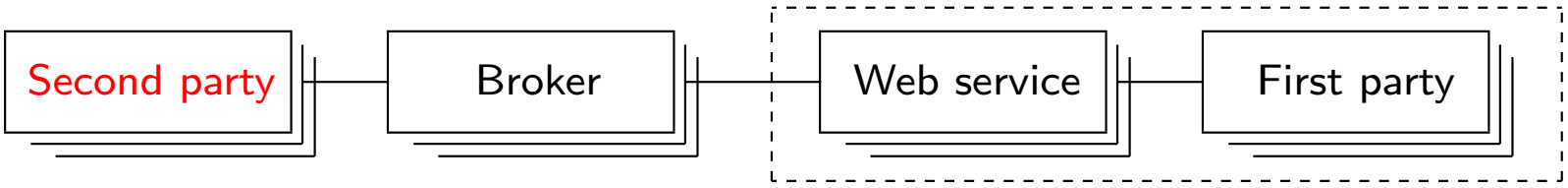

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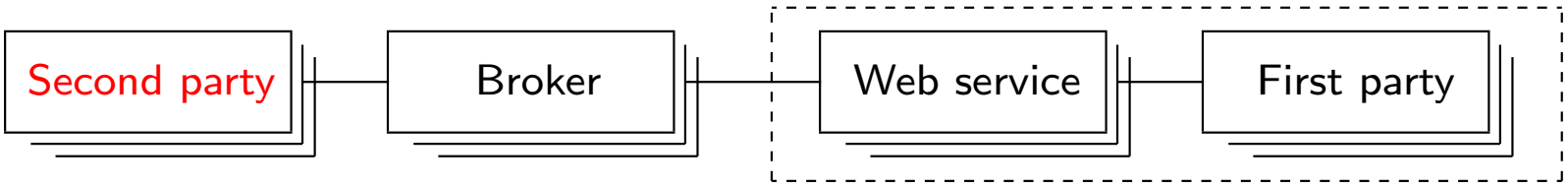

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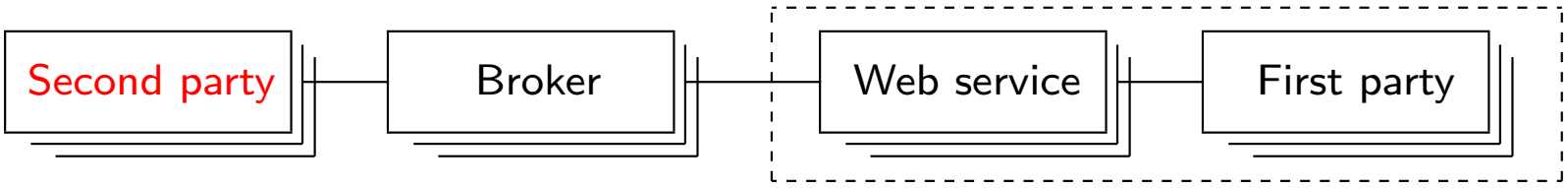

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- The rate at which responses are produced is determined by the interaction of the broker and the service endpoint.

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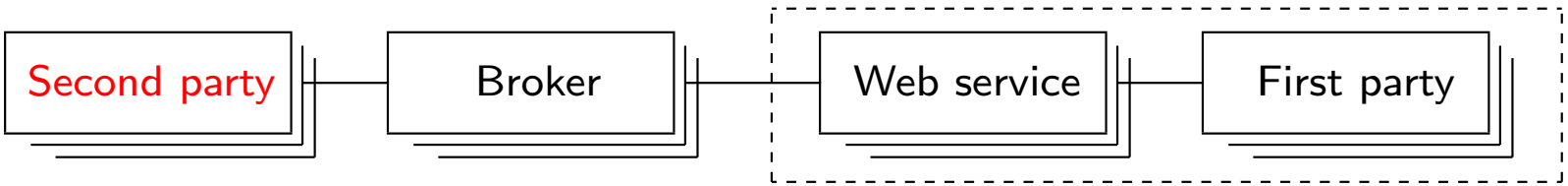


$$SPC_{idle} \stackrel{def}{=} (compose_{sp}, r_{sp\_cmp}).SPC_{enc}$$

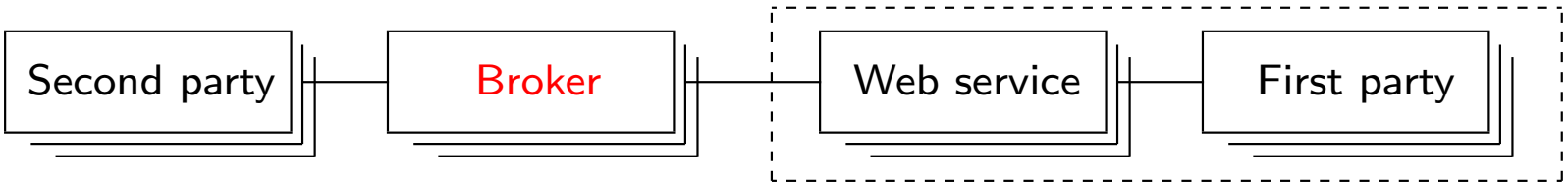
$$SPC_{enc} \stackrel{def}{=} (encrypt_b, r_{sp\_encb}).SPC_{sending}$$

$$SPC_{sending} \stackrel{def}{=} (request_b, r_{sp\_req}).SPC_{waiting}$$

$$SPC_{waiting} \stackrel{def}{=} (response_b, \top).SPC_{dec}$$

$$SPC_{dec} \stackrel{def}{=} (decrypt_b, r_{sp\_decb}).SPC_{idle}$$

PEPA model: Brokers



- The broker is inactive until it receives a request.

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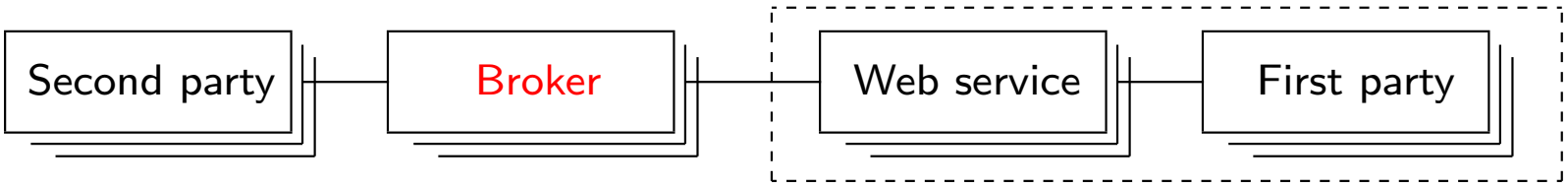

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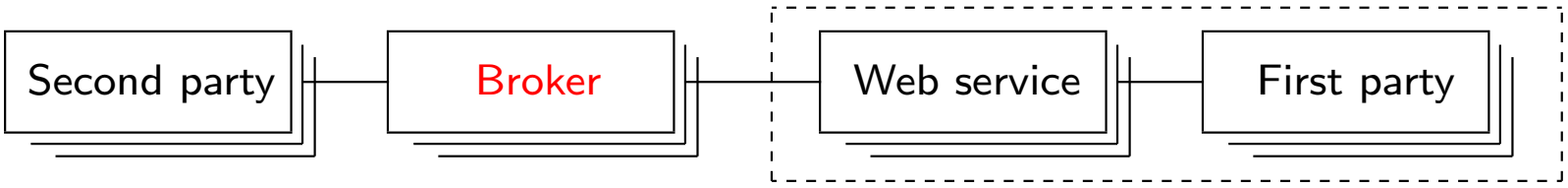

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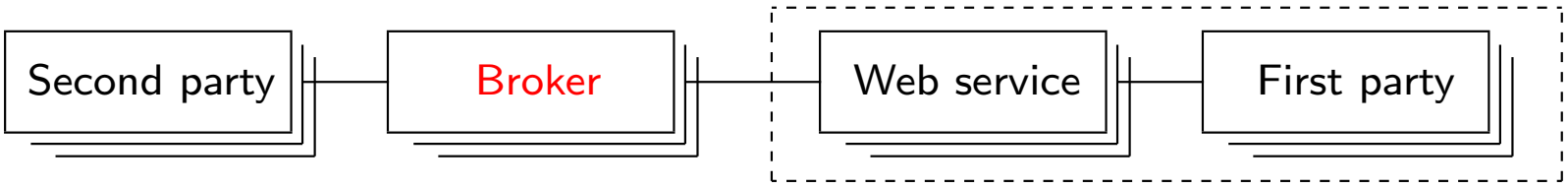

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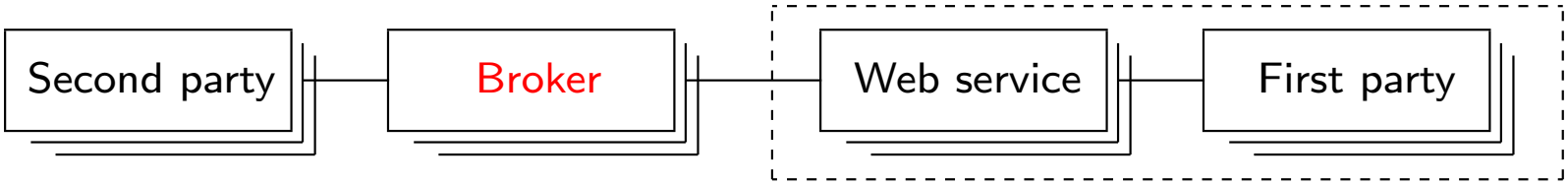
- It then decrypts the request before re-encrypting it for the Web service to ensure end-to-end security.
- It forwards the request to the Web service and then waits for a response.
- The corresponding decryption and re-encrytion are performed before returning the response to the client.

PEPA model: Brokers



$$
\begin{aligned}
Broker_{idle} &\stackrel{def}{=} (request_b, \top).Broker_{dec\_input} \\
Broker_{dec\_input} &\stackrel{def}{=} (decrypt_{sp}, r_{b\_dec\_sp}).Broker_{enc\_input} \\
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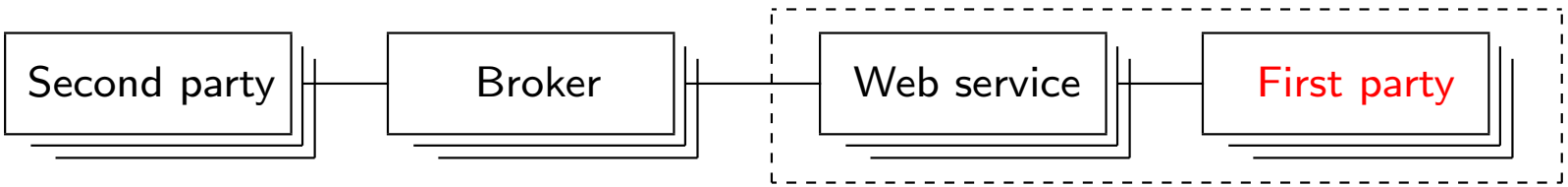
\end{aligned}
$$

PEPA model: First party clients



- The lifetime of a first party client mirrors that of a second party client except that encryption need not be used when all of the communication is conducted across a secure intranet.

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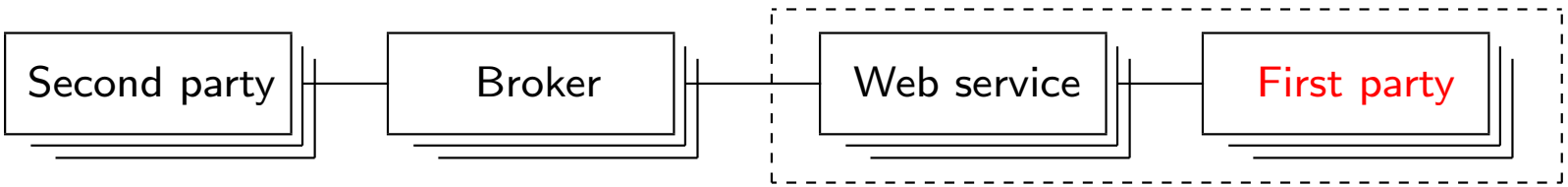

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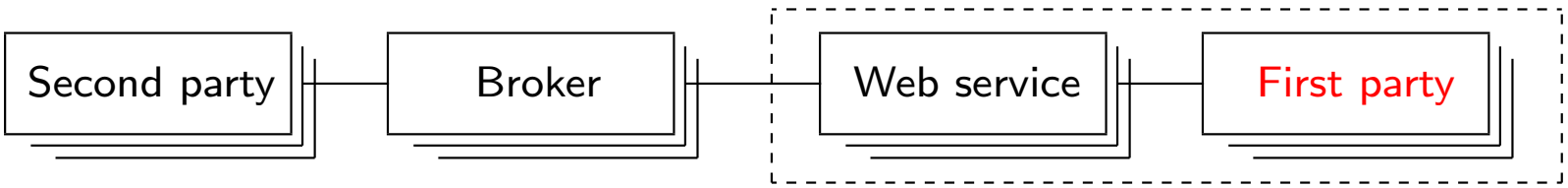
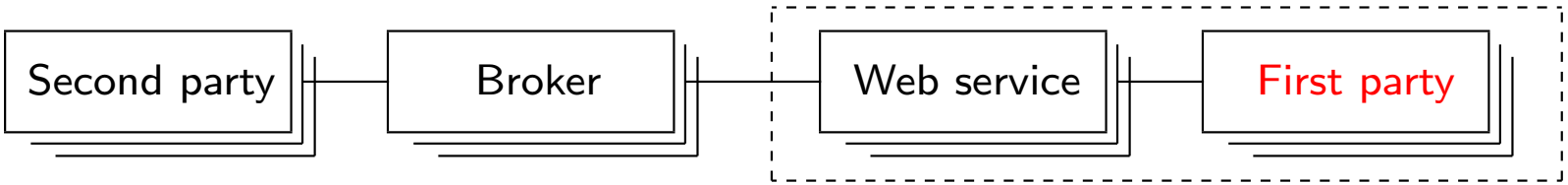


- The lifetime of a first party client mirrors that of a second party client except that encryption need not be used when all of the communication is conducted across a secure intranet.
- Also the service may be invoked by a remote method invocation to the host machine instead of via HTTP.
- Thus the first party client experiences the Web service as a blocking remote method invocation.

PEPA model: First party clients

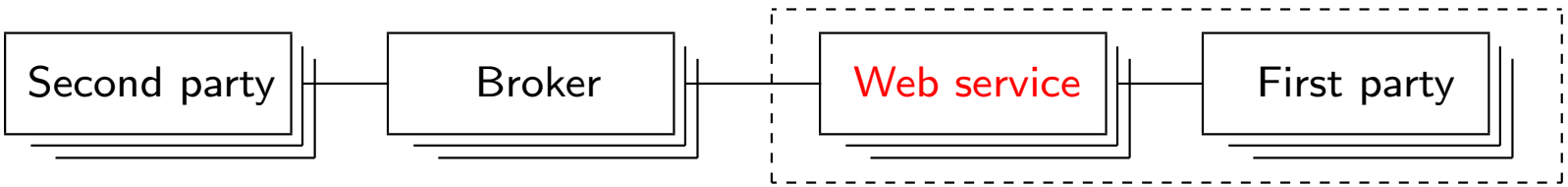


$$FPC_{idle} \stackrel{def}{=} (compose_{fp}, r_{fp\_cmp}).FPC_{calling}$$

$$FPC_{calling} \stackrel{def}{=} (invoke_{ws}, r_{fp\_inv}).FPC_{blocked}$$

$$FPC_{blocked} \stackrel{def}{=} (result_{ws}, \top).FPC_{idle}$$

PEPA model: Web service



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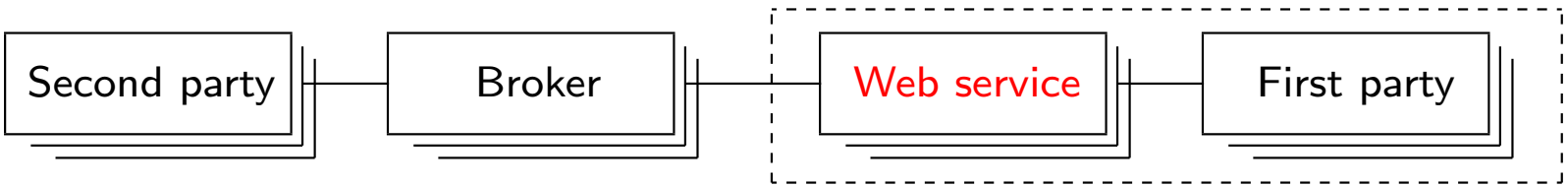
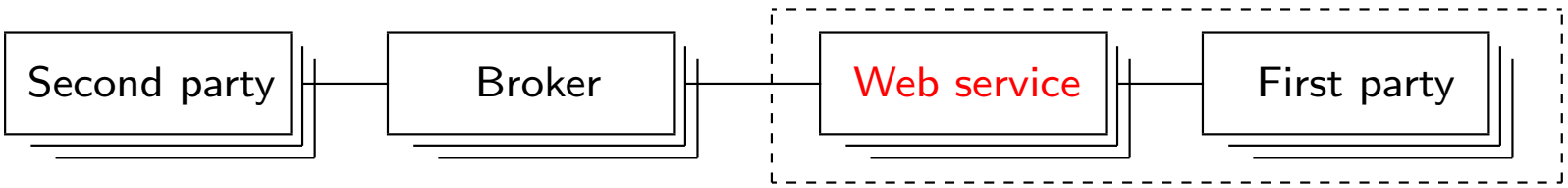


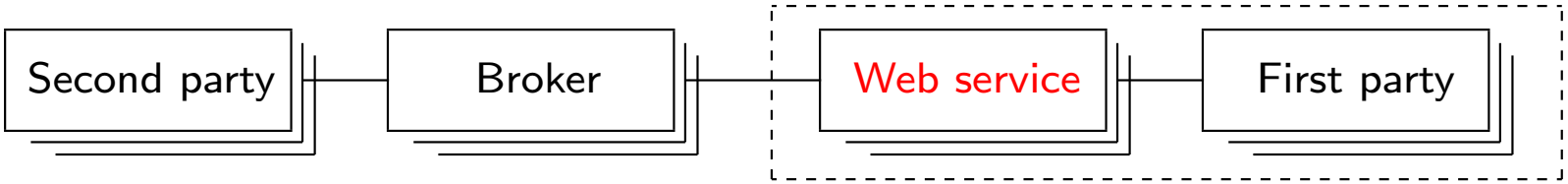
- There are two ways in which the service is executed, leading to a choice in the process algebra model taking the service process into one or other of its two modes of execution.
- In either case, the duration of the execution of the service itself is unchanged.

PEPA model: Web service



- There are two ways in which the service is executed, leading to a choice in the process algebra model taking the service process into one or other of its two modes of execution.
- In either case, the duration of the execution of the service itself is unchanged.
- The difference is only in whether encryption is needed and whether the result is delivered via HTTP or not.

PEPA model: Web service



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\begin{aligned}
WS_{idle} &\stackrel{def}{=} (request_{ws}, \top).WS_{decoding} \\
&\quad + (invoke_{ws}, \top).WS_{method} \\
WS_{decoding} &\stackrel{def}{=} (decryptReq_{ws}, r_{ws\_dec\_b}).WS_{execution} \\
WS_{execution} &\stackrel{def}{=} (execute_{ws}, r_{ws\_exec}).WS_{securing} \\
WS_{securing} &\stackrel{def}{=} (encryptResp_{ws}, r_{ws\_enc\_b}).WS_{responding} \\
WS_{responding} &\stackrel{def}{=} (response_{ws}, r_{ws\_resp\_b}).WS_{idle} \\
WS_{method} &\stackrel{def}{=} (execute_{ws}, r_{ws\_exec}).WS_{returning} \\
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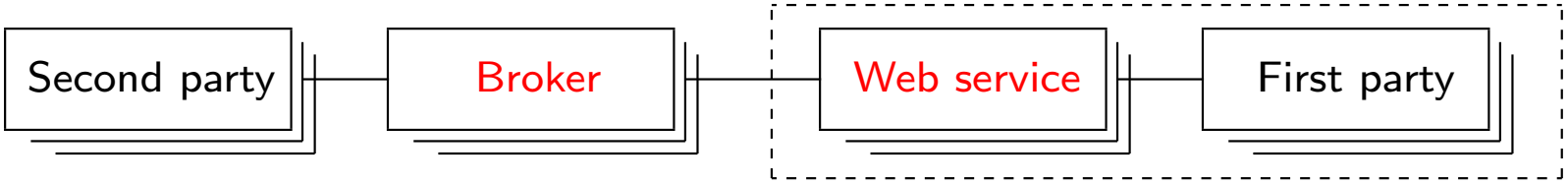

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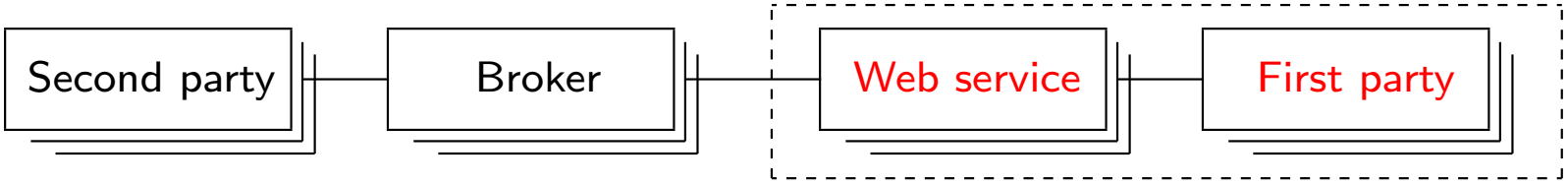

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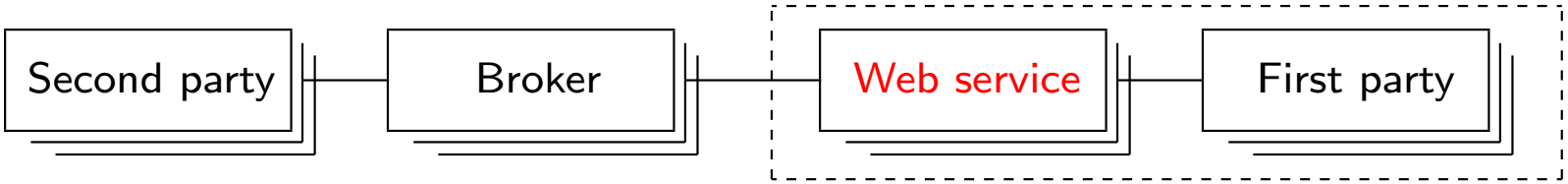

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PEPA model: System composition

In the initial state of the system model we represent each of the four component types being initially in their idle state.

$$System \stackrel{def}{=} (SPC_{idle} \underset{\mathcal{K}}{\bowtie} Broker_{idle}) \underset{\mathcal{L}}{\bowtie} (WS_{idle} \underset{\mathcal{M}}{\bowtie} FPC_{idle})$$

$$\begin{array}{lrcl} \text{where} & \mathcal{K} & = & \{ request_b, response_b \} \\ & \mathcal{L} & = & \{ request_{ws}, response_{ws} \} \\ & \mathcal{M} & = & \{ invoke_{ws}, result_{ws} \} \end{array}$$

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This model represents the smallest possible instance of the system, where there is one instance of each component type. We evaluate the system as the number of clients, brokers, and copies of the service increase.

Cost of analysis

- We compare fluid approximation, ODE-based, evaluation against other techniques which could be used to analyse the model.

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- We compare fluid approximation, ODE-based, evaluation against other techniques which could be used to analyse the model.
- We compare against steady-state and transient analysis based on an explicit state representation,as implemented by the PRISM probabilistic model-checker (which provides PEPA as one of its input languages). We also compare against Monte Carlo Markov Chain simulation.

Comparison of analysis types

- We report only a single run of the simulation analysis. In practice, due to the stochastic nature of the analysis, this would need to be re-run multiple times to produce results comparable to the ODE-based analysis.

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- We report only a single run of the simulation analysis. In practice, due to the stochastic nature of the analysis, this would need to be re-run multiple times to produce results comparable to the ODE-based analysis.
- Moreover, note that the number of ODEs is constant regardless of the number of components in the system, whilst the state space grows dramatically.

Running times from analyses (in seconds)

Second party clients	Brokers	Web service instances	First party clients	Number of states in the full state-space	Number of states in the aggregated state-space	Sparse matrix steady-state	Matrix/MTBDD steady-state	Transient solution for time $t = 100$	MCMC simulation one run to $t = 100$	ODE solution
1	1	1	1	48	48	1.04	1.10	1.01	2.47	2.81

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1000	100	500	1000	–	–	–	–	–	3.72	2.77

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Time series analysis via ODEs

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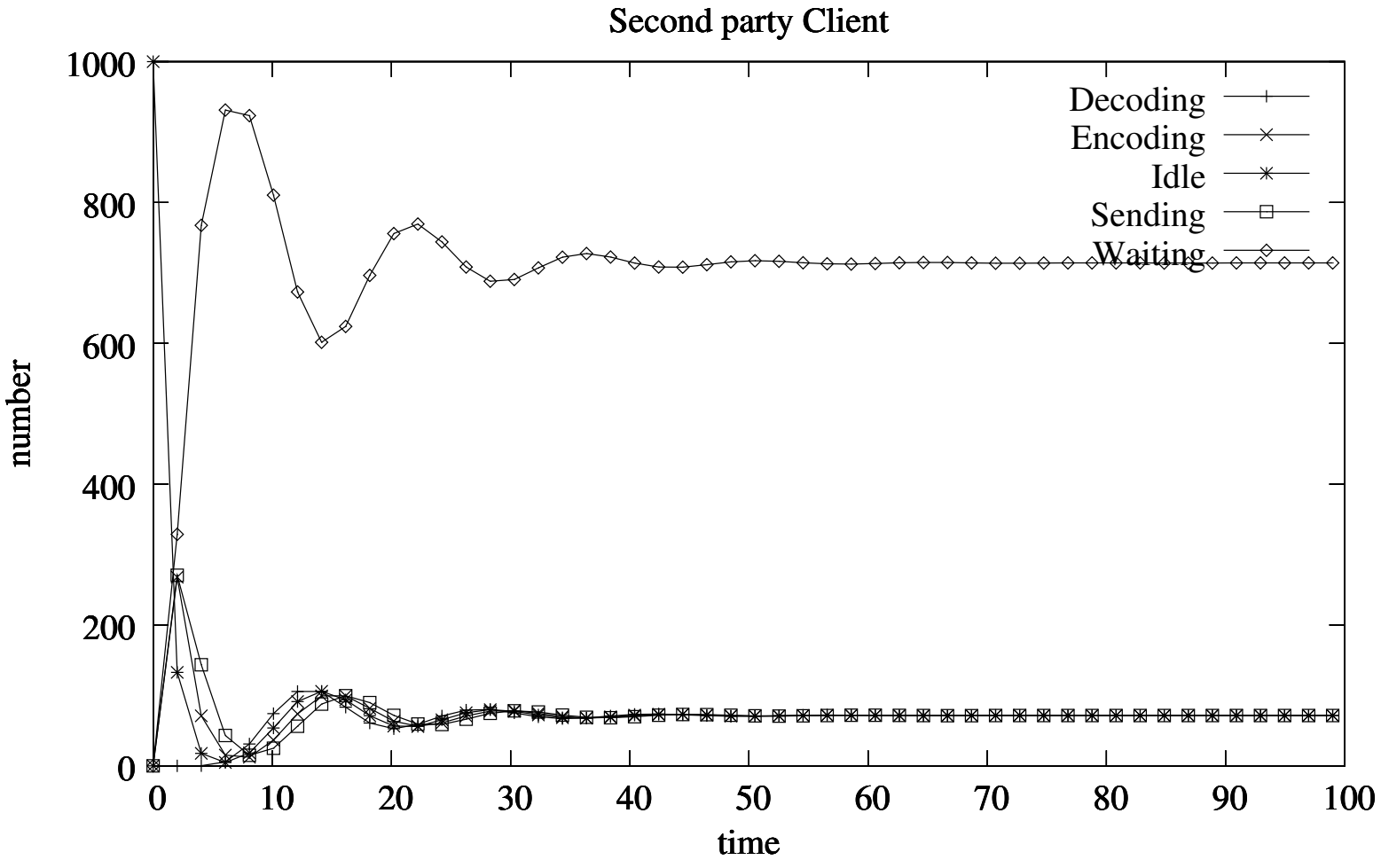
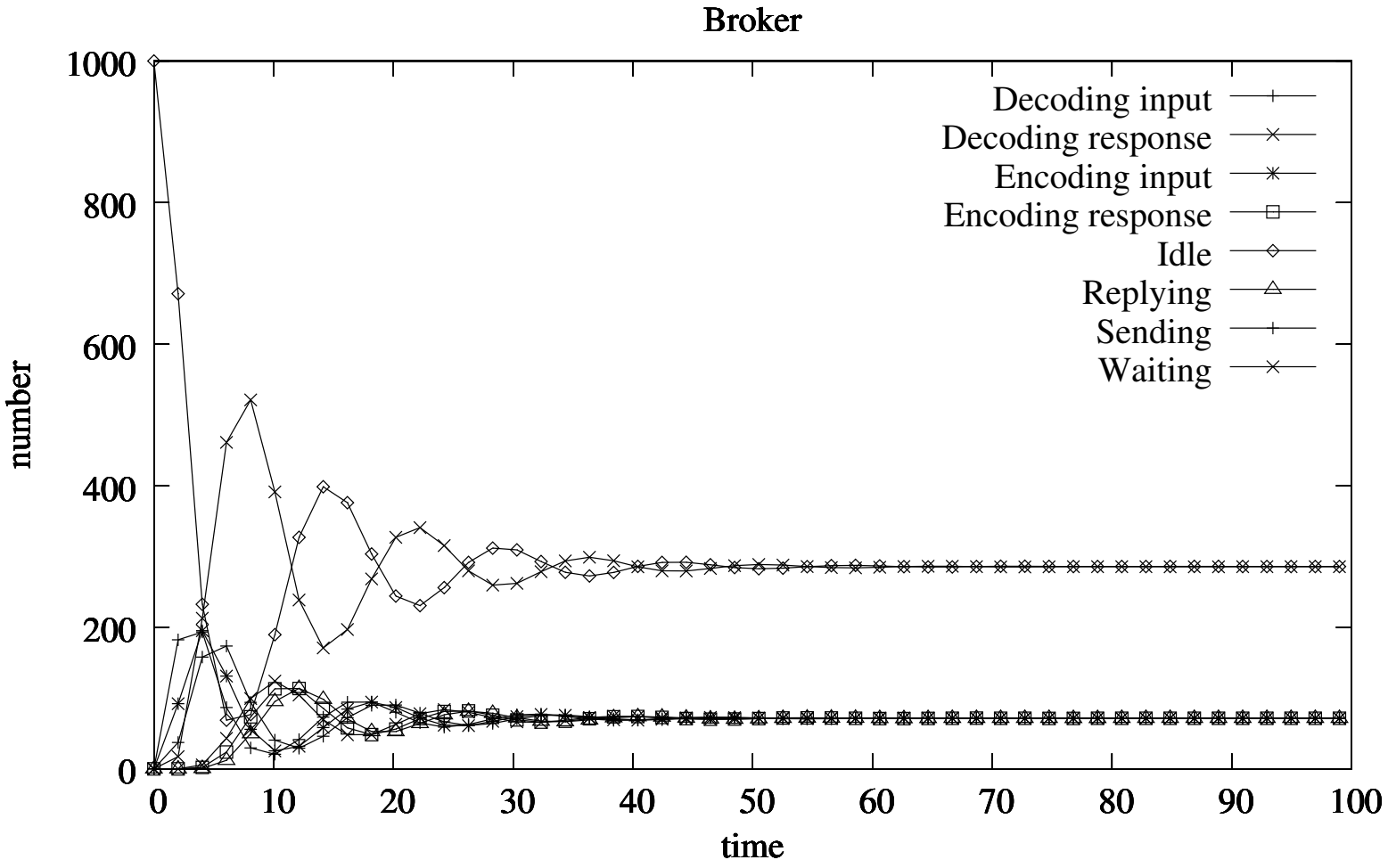
- The results as presented from our ODE integrator are time-series plots of the number of each type of component behaviour as a function of time.
- We can observe an initial flurry of activity until the system stabilises into its steady-state equilibrium at time (around) $t = 50$.

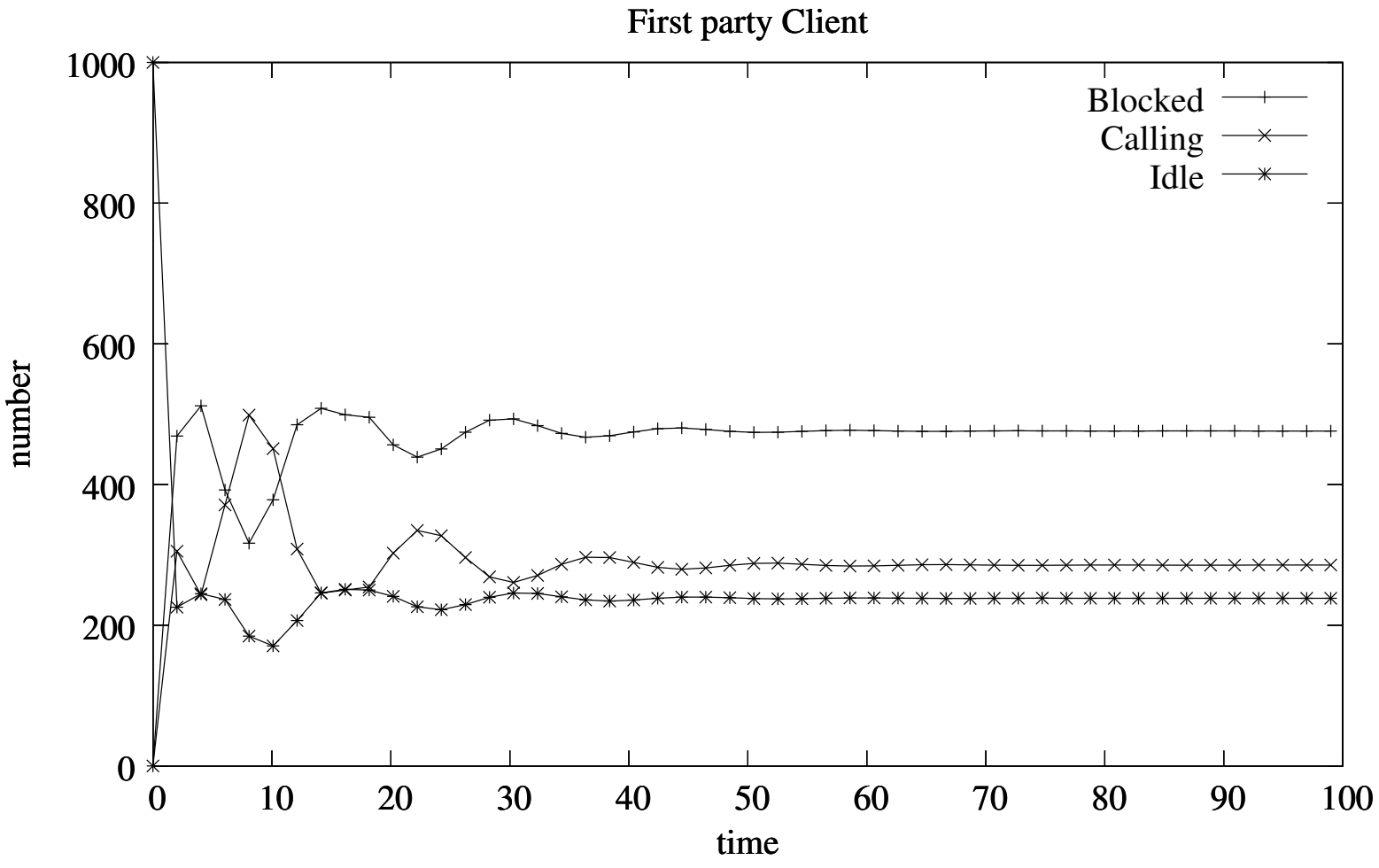
Second party clients



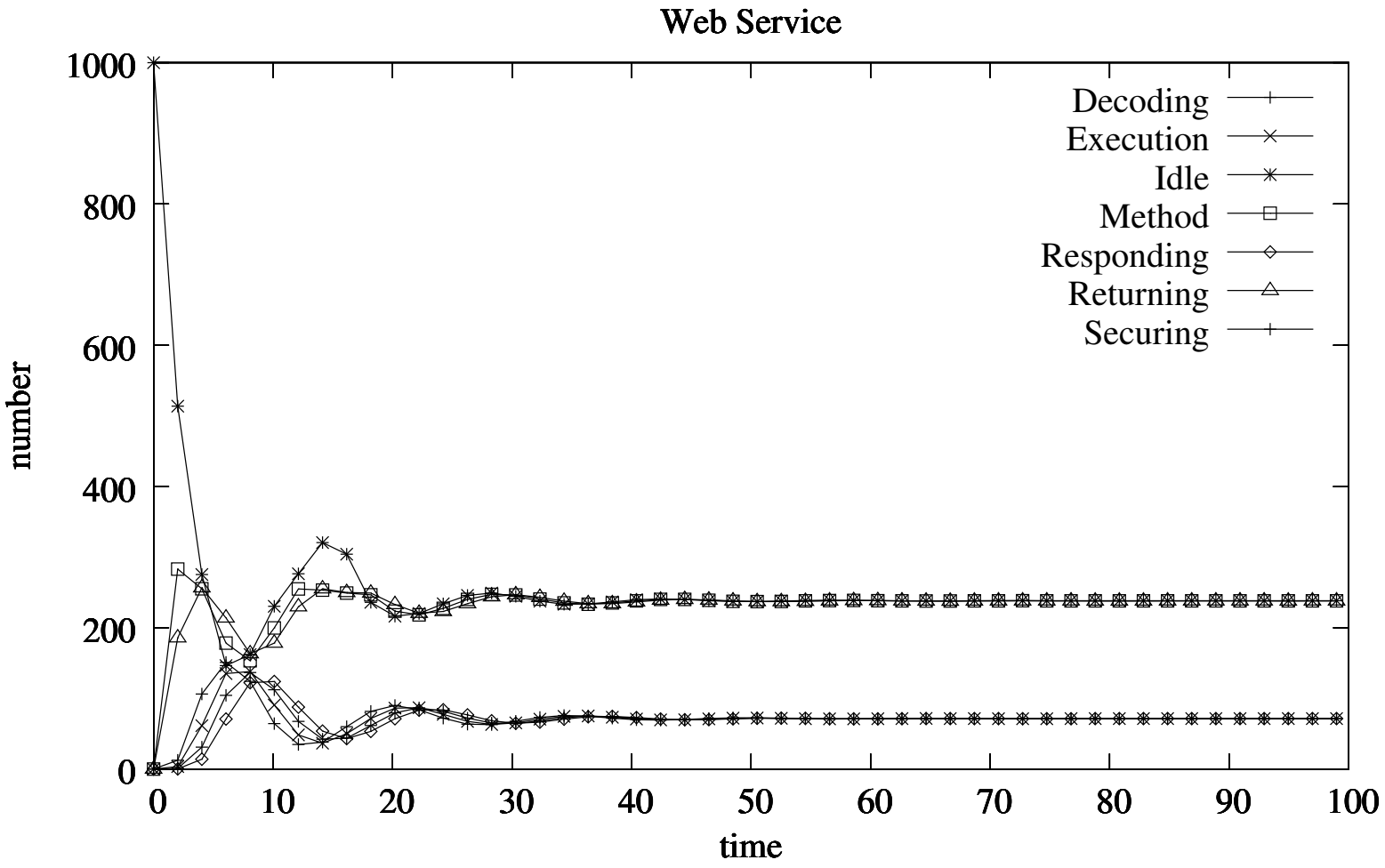
Brokers


Broker
number
time
0
200
400
600
800
1000
0
10
20
30
40
50
60
70
80
90
100
Decoding input
Decoding response
Encoding input
Encoding response
Idle
Replying
Sending
Waiting

First party clients



Web service


Web Service
number
time
0
200
400
600
800
1000
0
10
20
30
40
50
60
70
80
90
100
Decoding
Execution
Idle
Method
Responding
Returning
Securing

Outline

Conclusions

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Conclusions

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- Process algebras, such as PEPA and Bio-PEPA, are well-suited to modelling the behaviour of such systems in terms of the individuals and their interactions.
- Continuous approximation allows a rigorous mathematical analysis of the average behaviour of such systems.
- This alternative view of systems has opened up many and exciting new research directions.

On-going work

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- On-going work is investigating the use of probes to query the model by adding components to the model whose sole purpose is to gather statistics.
- Using this technique we can now derive measures such as cumulative response time from the fluid approximation of the model.

Thanks!

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Acknowledgements: collaborators
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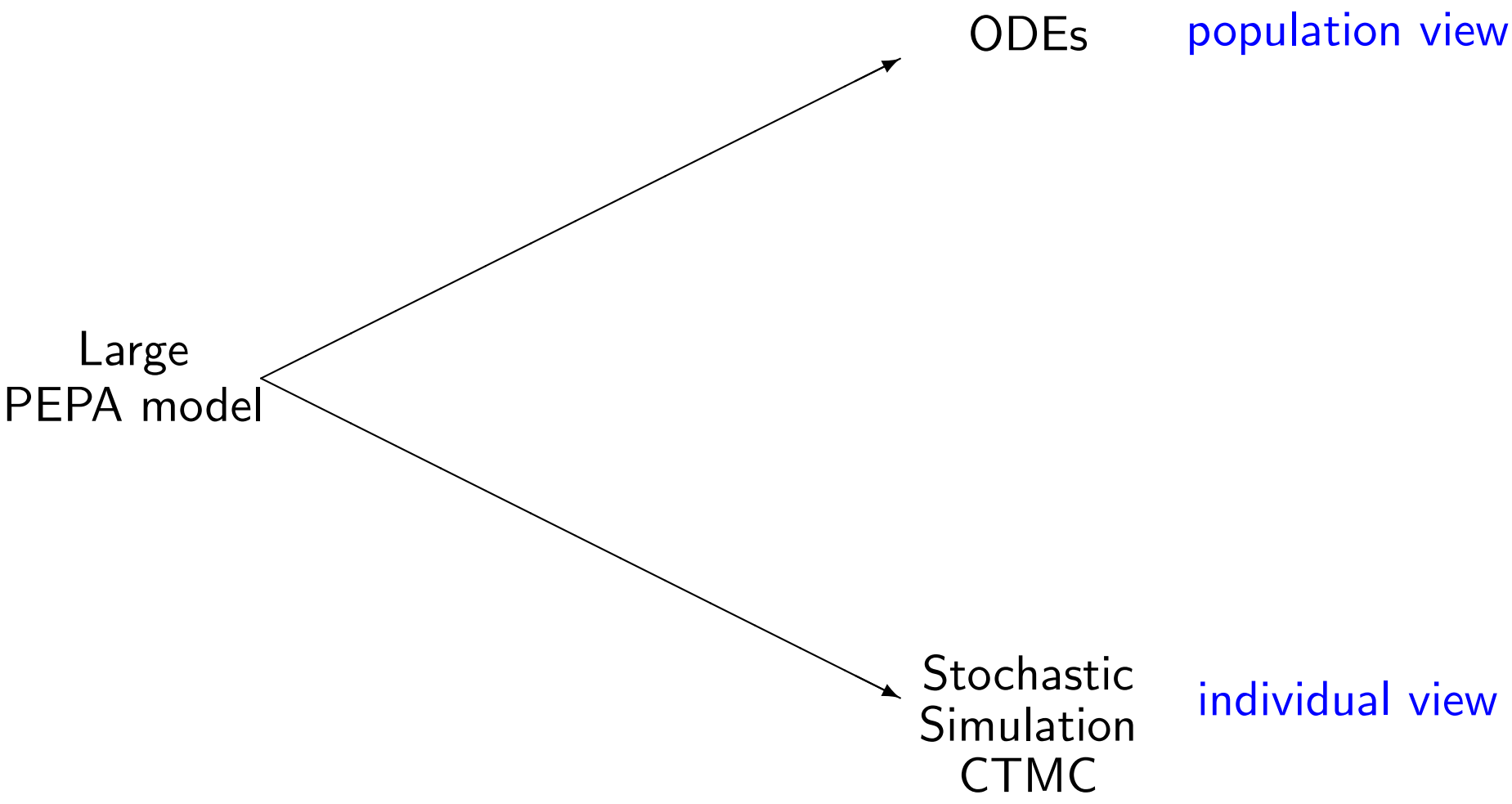
More information:

http://www.dcs.ed.ac.uk/pepa

Alternative Representations



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