"Grab The Ball"

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May 31, 2005

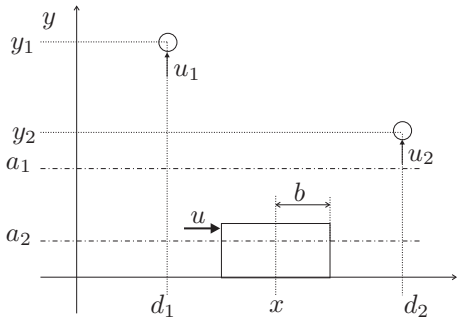

Figure 1: Hybrid model

Consider the system depicted in Figure 1, where a container of mass M is free to move along the x-axis with viscous friction β and pushed by a force u, $M\ddot{x} = -\beta\dot{x} + u$, and two balls are falling down with constant velocity. The goal is to move the container in order to grab the balls. Let y_1, y_2 be the height of ball #1 and #2, respectively, d_1, d_2 their x-coordinate, and x the position of the container. A ball can be grabbed only if $|x - d_i| \leq b$ and $a_1 \leq y_i \leq a_2$, $i = 1, 2$. The vertical motion of the balls may be slowed down by activating air flows u_i, $i = 1, 2$, $\dot{y}_i = -c_i + \gamma u_i$, where c_1, c_2, γ are constants. The following constraint are imposed on the system: $-u_{\max} \leq u \leq u_{\max}$, $0 \leq u_i \leq \text{jet}_{\max}$, $i = 1, 2$.

1. Describe the model as a discrete-hybrid automaton in HYSDEL, by sampling the dynamics using forward Euler approximation $\frac{dx}{dt} \approx \frac{x(t+1)-x(t)}{T_s}$, with sampling time T_s, and using the values reported in Table 1.

2. Set up an MPC controller with horizon $N = 5$ steps in order to grab both balls and park the container at the origin, under the condition that ball #2 must be catched before ball #1. Simulate the closed-loop system starting from the initial condition $x(0) = 0$, $\dot{x}(0) = 0$, $y_1(0) = 5$, $y_2(0) = 5$.

3. Remove the condition that ball #2 must be catched before ball #1 and design an MPC controller with prediction horizon $N \leq 3$. Simulate the closed-loop system.

T_s	0.3
β	0.2
M	1
a_1	2
a_2	1
d_1	1
d_2	3
b	0.5
c_1	6
c_2	8
γ	0.5
$u_{\max}$	50
$\text{jet}_{\max}$	10

Table 1: Model parameters