Networked Control Systems:

an emulation approach to controller design

Dragan Nesic

The University of Melbourne

Electrical and Electronic Engineering

Acknowledgements:

My collaborators: A.R. Teel, M. Tabbara, D. Dacic, D. Liberzon, M. Heemels, N. van de Wouw, D. Carnevale, R. Postoyan, T. Kameneva, P. Tabuada, A. Anta

Outline:

- Motivation & background
- Emulation for NCS (NCS model)
- NCS protocols
- Main result with remarks
- An example
- Extensions
- Summary

Background

- Control loop closed via LAN - increasingly used
- Packet based communication (no quantization)
- Fly-by-wire aircraft; drive-by-wire cars; and so on

Pros:
- Easier maintenance & installation
- Lower cost, weight and volume

Cons:
- Design harder: hybrid, delays, dropouts
- Performance may deteriorate

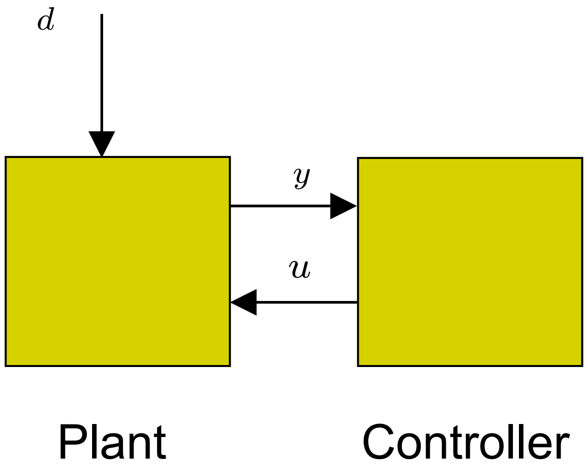
Motivation

CDS Panel Report, 2002:

"Control distributed across multiple computational units, interconnected through packet based communications, will require new formalisms for ensuring stability, performance and robustness."

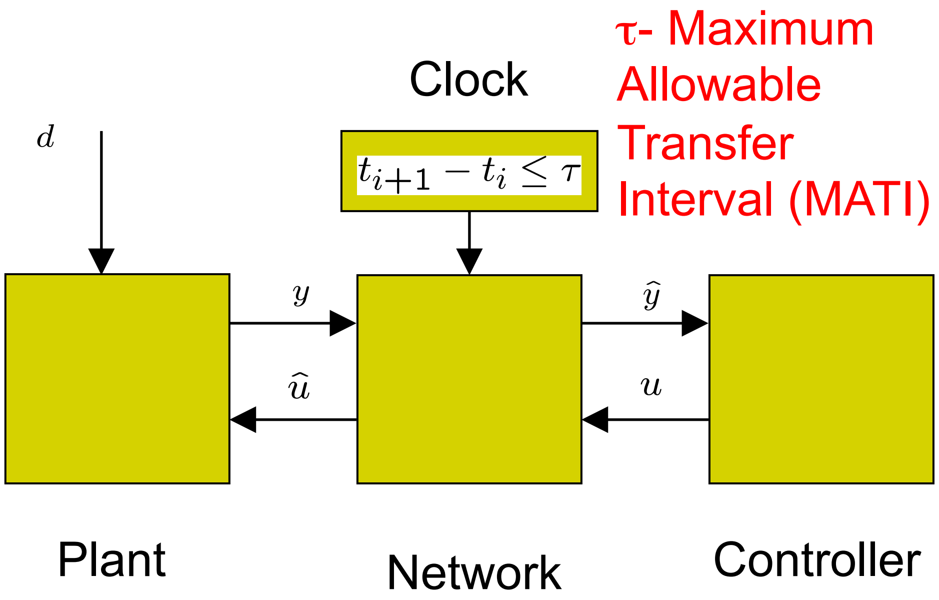
Our goal: present an approach for achieving stability, performance & robustness (Lp stability or ISS).

Classical vs Networked



Classical Control System

Point-to-point dedicated connections



Networked Control System

Serial bus/channel connection

We concentrate on:

- Varying sampling periods - item (i)
- Network effects (scheduling) – item (iv)

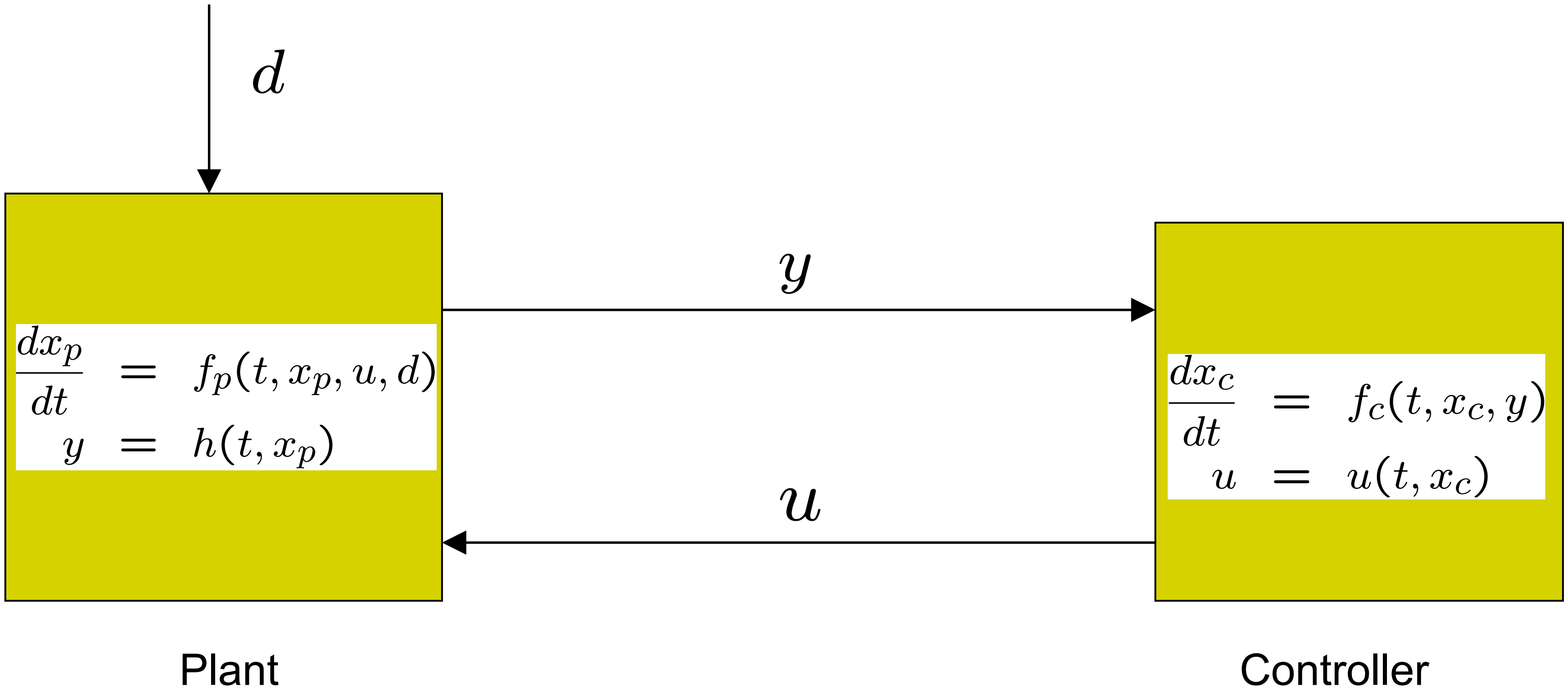
- Delays, dropouts, quantization ignored but can be dealt with within the same framework.

An emulation approach

Background

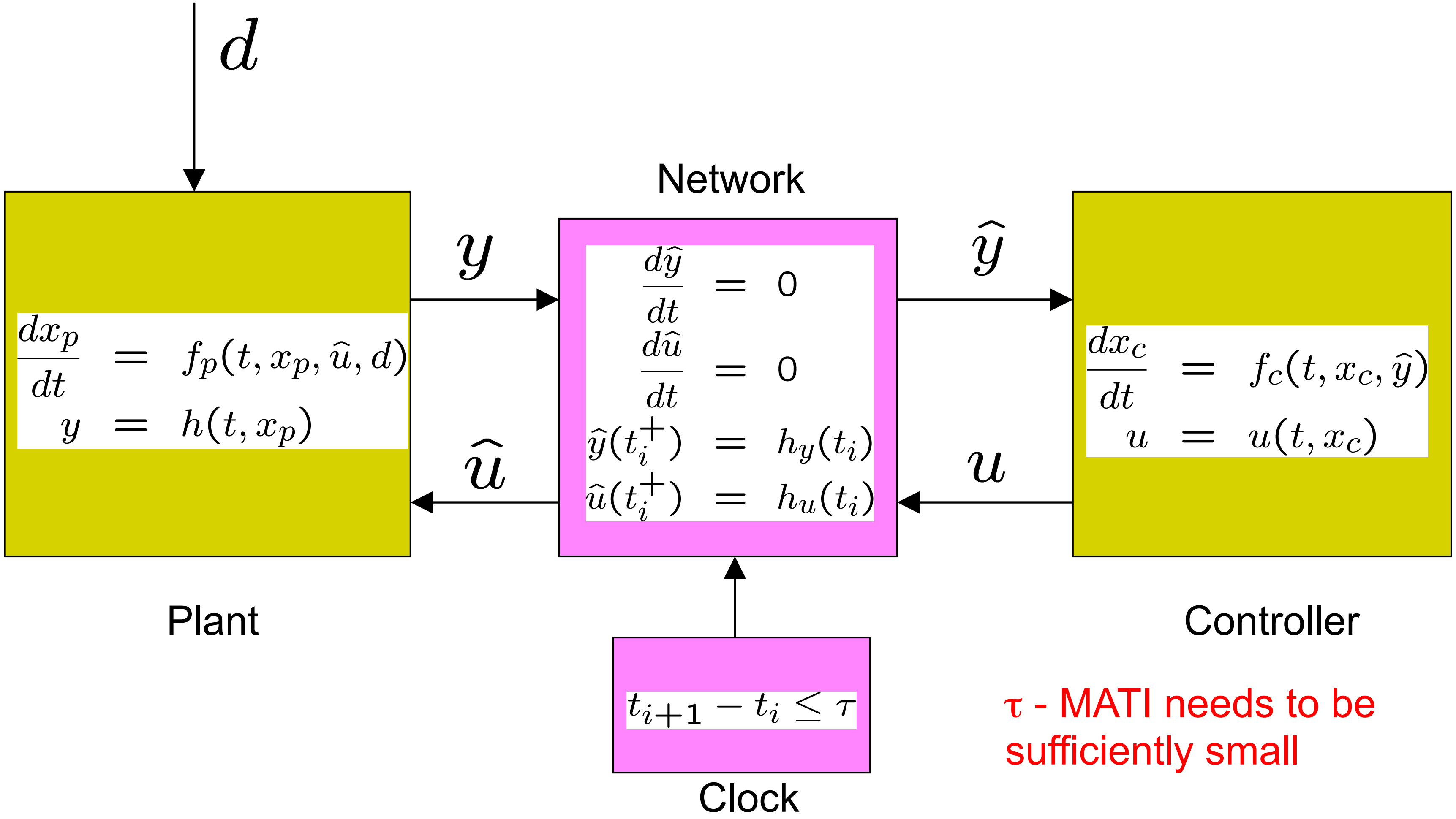
- Proposed by Walsh et al.,
  (IEEE TAC 2001 & IEEE TCST 2002).
- Further developed by Nesic & Teel,
  (IEEE TAC 2004 & Automatica 2004).
- A generalization of sampled-data systems
- Numerous extensions summarized later.

Step 1: Design the controller ignoring the network.



Step 2a: Implement the same controller over network

Step 2b: Find sufficiently small MATI so that the closed loop system is stable in an appropriate sense.



τ - MATI needs to be sufficiently small

Impulsive model of NCS
(Nesic & Teel, IEEE TAC 2004)

$$\Sigma_x : \quad \frac{dx}{dt} = f(t, x, e, d) \qquad t \in [t_i, t_{i+1}]$$

$$\Sigma_e : \quad \frac{de}{dt} = g(t, x, e, d) \qquad t \in [t_i, t_{i+1}]$$

$$\Sigma_p : \quad e(t_i^+) = h(i, e(t_i))$$

$$\Sigma_c : \quad 0 < \epsilon \leq t_{i+1} - t_i \leq \tau$$

$$x := \begin{pmatrix} x_p \\ x_c \end{pmatrix} \qquad e := \begin{pmatrix} \widehat{y} - y \\ \widehat{u} - u \end{pmatrix}$$

The jump equation Σ_p describes a “protocol”

Goal:

- Provide checkable conditions on Σ_x, Σ_e, Σ_p and Σ_c that guarantee stability.
- The conditions on Σ_p should cover a range of protocols.
- Small MATI is essential in implementing the emulated controller – we provide estimates that guarantee stability.

NCS Protocols

Main assumption

- Network protocol arbitrates access to the network.
- "Node" is a group of inputs/outputs that are always transmitted together

$$e = \begin{pmatrix} e_1 \\ \vdots \\ e_\ell \end{pmatrix}$$

ℓ - the number of "nodes"

ASSUMPTION: When a node k gets access to the network at time t_j, then we have

$$e_k(t_j^+) = 0$$

Protocol models

- A large class of protocols has the form:

$$h(i,e) = (I - \Psi(s))e; \qquad s = s(i,e)$$

$$\Psi(s) := diag\{\delta_{1s} I_{n_1}, \ldots, \delta_{\ell s} I_{n_\ell}\}$$

I_{n_j} are the identity matrices

δ_{is} are Kronecker symbols

$s : N \times R^{n_e} \to \{1, \ldots, \ell\}$ is the scheduling function

Example 1: Round robin (RR)

- Suppose we have 2 nodes.

$$h(i,e) = \begin{cases} \begin{pmatrix} 0 \\ e_2 \end{pmatrix}; & i = 0, 2, 4, \ldots \\ \begin{pmatrix} e_1 \\ 0 \end{pmatrix}; & i = 1, 3, 5, \ldots \end{cases}$$

- In this case $s = s(i) = \begin{cases} 1 & \text{if } i \text{ is even} \\ 2 & \text{if } i \text{ is odd} \end{cases}$

Example 2: Try-Once-Discard

(TOD) Walsh et al 2001

- Suppose we have 2 nodes.

$$h(i,e) = \begin{cases} \begin{pmatrix} 0 \\ e_2 \end{pmatrix}; & |e_1| \geq |e_2| \\ \begin{pmatrix} e_1 \\ 0 \end{pmatrix}; & |e_1| < |e_2| \end{cases}$$

- In this case $s = s(e) = \mathrm{min}[\mathrm{arg}_i \, \mathrm{max} \, |e_i|]$

UGES Protocols
(Nesic & Teel 2004)

- We introduce an auxiliary system:

$$e^+ = h(i, e)$$

Protocol is W-UGES if there exist W(i,e) and positive numbers a_1, a_2 and ρ 2 [0,1) such that for all (i,e) we have:

$$a_1|e| \;\leq\; W(i,e) \;\leq\; a_2|e|$$
$$W(i+1, h(i,e)) \;\leq\; \rho \cdot W(i,e)$$

Examples of UGES protocols

- RR protocol is UGES.
- TOD protocol is UGES.
- Many other protocols are UGES.

- We construct Lyapunov functions for the above protocols, e. g. for TOD we have:

$$W(i,e) = |e|;\ \ a_1 = a_2 = 1;\ \ \rho = \sqrt{\frac{\ell - 1}{\ell}}$$

Main result

Σ_x assumption

Σ_x is L_p stable $p \in [1,\infty]$ from (d,e) to x with gain γ.

- In other words, there exist K, $\gamma \geq 0$ such that:

$$||x[t_0,t]||_p \leq K|x(t_0)| + \gamma||e[t_0,t]||_p + \gamma||d[t_0,t]||_p$$

$\forall\ x(t_0), d(\cdot)$, $t \geq t_0$. γ is the "gain".

Σ_p assumption

Σ_p is a W-UGES protocol.

- That is, the following holds for $a_1, a_2 > 0$ and $\rho \in [0,1)$

$$a_1|e| \;\leq\; W(i,e) \;\leq\; a_2|e|$$
$$W(i+1, h(i,e)) \;\leq\; \rho \cdot W(i,e)$$

Σ_e assumption

W grows exponentially along Σ_e dynamics.

- That is, there exist L, c ≥ 0 such that:

$$\left\langle \frac{\partial W}{\partial e}, g(t,x,e,d) \right\rangle \leq LW(i,e) + c|x| + c|d|$$

Σ_c assumption

MATI is sufficiently small.

- In other words, the following holds:

$$\tau < \tau^* := \frac{1}{L} \ln \left(\frac{L + \frac{c\gamma}{a_1}}{L\rho + \frac{c\gamma}{a_1}} \right)$$

- γ from Σ_x
- a_1, ρ from Σ_p
- L, c from Σ_e

Main result:

Suppose that:

1. Σ_x assumption holds;
2. Σ_p assumption holds;
3. Σ_e assumption holds;
4. Σ_c assumption holds.

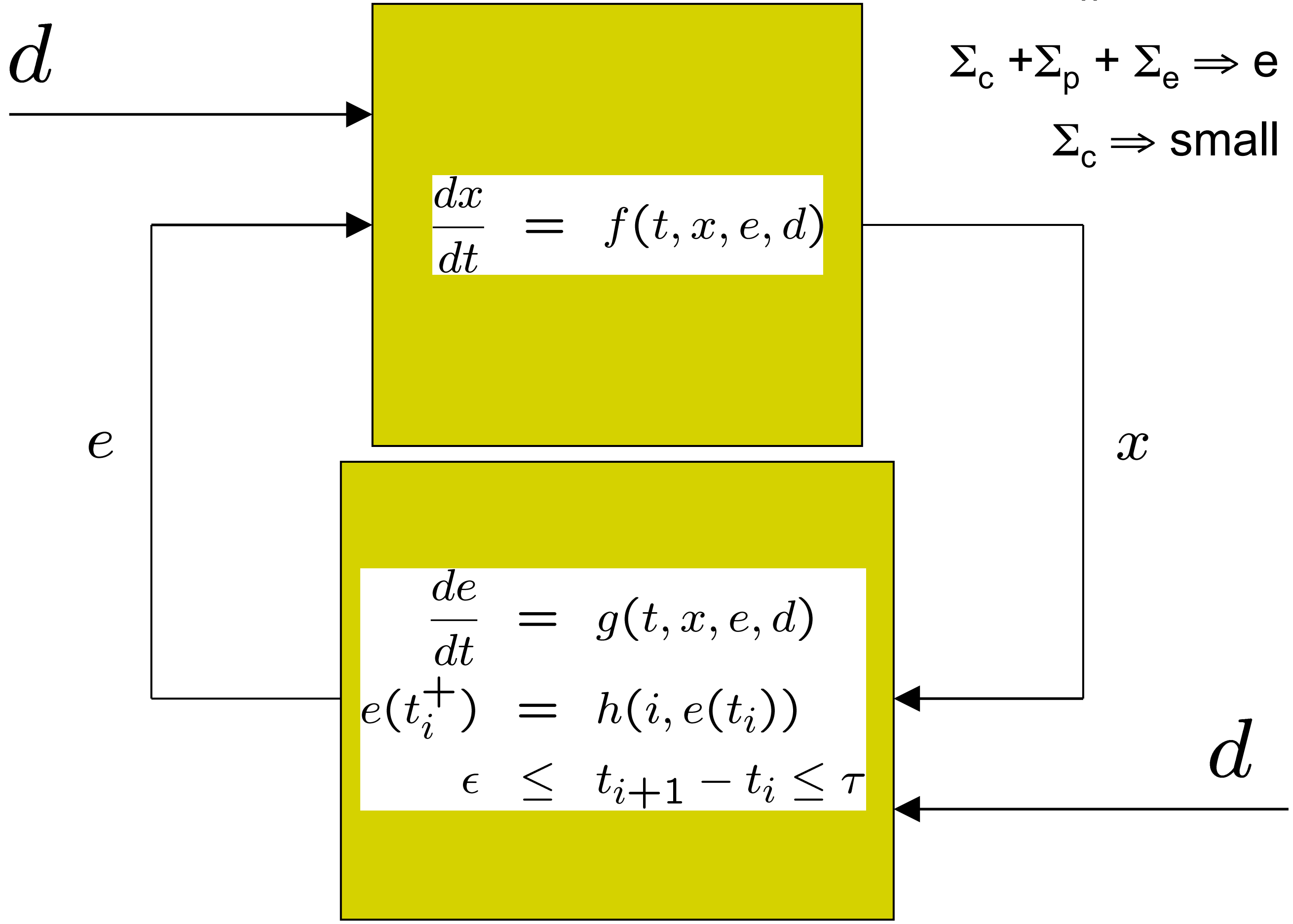
Then, the NCS is L_p stable from d to (x,e).

Sketch of proof:

$\Sigma_x \Rightarrow$ x system L_p stable

$\Sigma_c + \Sigma_p + \Sigma_e \Rightarrow$ e system L_p stable

$\Sigma_c \Rightarrow$ small gain condition holds



Remarks

- MATI bound (clock!) depends on:
  - γ determines robustness of *x* system
  - *L, c* determine the inter-sample growth of *W*
  - ρ, a_1 determine the properties of protocol

- Can conclude exponential stability when $d=0$
- Can state ISS based results
- Can treat dropouts and regional results

Remarks

- A controller design framework achieving L_p stability proved for the first time for NCS
- Similar to emulation in sampled data design
- Attractive for its “modularity” (i.e. simplicity)
- The analysis involves computing MATI
- Our MATI bounds not conservative but can be further improved
- When MATI is reduced, non-networked performance is recovered

Example

Unstable batch reactor

(Green & Limebeer 1995)

- 4^{th} order linear plant, 2^{nd} order controller (MIMO)
- 2 outputs sent via the network

MATI with TOD protocol:

Walsh *et al*	Nesic & Teel	Simulations
0.00001 sec	0.01 sec	0.06 sec

MATI with RR Protocol:

Walsh *et al*	Nesic & Teel	Analytical
0.00001 sec	0.0082 sec	0.0657 sec

Extensions

Extensions:

- ISS for NCS [Nesic & Teel, Automatica '04]
- Wireless NCS [Tabbara, Nesic, Teel, TAC '07]
- BMIs for stability [Dacic, Nesic, Automatica '07]
- BMIs for observers [Dacic, Nesic, Automatica '08]
- Lyapunov based proof [Carnevalle, Teel, Nesic TAC '07]
- Stochastic NCS [Hespahna, Teel '06]; [Tabbara, Nesic '08]
- Unified with quantized control [Nesic, Liberzon '07]
- Delays [Heemels, Teel, De Wouw, Nesic '08]; [Chaillet, Bicci '08]
- Observers [Postoyan & Nesic]
- Special cases [Hemels et al]
- Event driven sampling [Postoyan, Tabuada, Anta & Nesic]

[Nesic & Teel, Automatica '04]

- UGAS protocols: there exist W(i,e), $\alpha_1, \alpha_2 \in K_\infty$ and $\rho \in [0,1)$ such that for all (i,e)

$$\alpha_1(|e|) \leq W(i,e) \leq \alpha_2(|e|)$$
$$W(i+1, h(i,e)) \leq \rho W(i,e)$$

If Σ_p is UGAS, Σ_x is ISS and condition similar to Σ_e holds, then the networked closed-loop is semi-globally practically ISS in MATI.

[Tabbara, Nesic, Teel, TAC '07]

- Wireless protocols: the switching function can not depend on e:

$$
\begin{aligned}
e^{+} &= (I - \Psi(s))e \qquad\qquad s = s(i, \hat{e}) \\
\hat{e}^{+} &= \Lambda(i, \hat{e}, \Psi(s)e)
\end{aligned}
$$

If Σ_p is ***persistently exciting*** and conditions similar to Σ_x, Σ_e, Σ_c hold, then the NCS is L_p stable/ISS.

[Dacic, Nesic, Automatica ’07]

- Assume:
  - Plant is linear;
  - Controller is a linear dynamical system (to be designed);
  - Sampling period fixed;

If a certain BMI is feasible, then we design a TOD like protocol and controller so that the closed-loop is quadratically stable.

[Carnevalle,Teel,Nesic TAC'07]

- Suppose:
  - We know L_2 Lyapunov function for Σ_x
  - We know Lyapunov function for Σ_p
  - Condition similar to Σ_e holds

Then, for sufficiently small (better!) MATI we construct a Lyapunov function for NCS.

[Hespahna & Teel '07] [Tabbara & Nesic TAC '07]

- Various stochastic versions of the presented results (e.g. control over Ethernet):

  - Plant and protocol are stochastic [Hespanha & Teel]

  - Plant is deterministic but protocol is stochastic [Tabarra & Nesic]

[Nesic & Liberzon ’07]

- A unifying framework for systems involving quantization and time scheduling.
- Cross-fertilization:

- UGES/UGAS quantization protocols.
- Small gain proof.
- Combined quantization and time scheduling protocols, e.g. TOD + zooming protocols.
- MATI bounds explicit.

Summary

- L_p stability proved (stability, performance & robustness)
- Our results can be used as a framework for controller and/or protocol design in NCS
- Novel proof technique yields much better bounds on MATI
- Goal: develop systematic designs for NCS
- Various extensions & improvements available and being developed

Thank you!